COMPUTER AIDED FILTER DESIGN

by

Liu Hsueh-wen

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Thesis Advisor: Chin-Hwa Lee

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Digital filter design has become instrumental in many fields of electronics. The objectives of this work is to study the benefit of using computer design tools to find equivalent digital filter realizations of analog filters. An analog video bandpass filter for TV Intermediate Frequency signal extraction was selected as an example.

First we introduce the Finite Impulse Response Filter (FIR) and the Remez program that is used for optimal FIR filter design. Other computer design tools such as ILS are used to design a number of existing bandpass filters. Emphasis was placed on the use of zero-one decomposition of filters to eliminate the number of multipliers and thus reduce hardware requirements.
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by

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ABSTRACT

Digital filter design has become instrumental in many fields of electronics. The objectives of this work is to study the benefit of using computer design tools to find equivalent digital filter realizations of analog filters. An analog video bandpass filter for TV Intermediate Frequency signal extraction was selected as an example.

First we introduce the Finite Impulse Response Filter (FIR) and the Remez program that is used for optimal FIR filter design. Other computer design tools such as ILS are used to design a number of existing bandpass filters. Emphasis was placed on the use of zero-one decomposition of filters to eliminate the number of multipliers and thus reduce hardware requirements.
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I. INTRODUCTION

Digital filter design is important in many electronic fields. With advances in digital circuits, digital filter design can be applied to many new areas. What kind of tools are the best and what kind of methodology can be used to design filters effectively are the issues addressed in this thesis. First, the Finite Impulse Response Filter (FIR) and the Remez program for optimal FIR filter design are introduced. Then, existing computer aided design tools are used to design several bandpass filter examples. A multi-band analog video bandpass filter for TV Intermediate Frequency (IF) signal extraction was selected as an example. The objective of this work is to study the benefit of using computer design tools to find an equivalent digital filter that is similar to the analog one. In terms of hardware, it is important to examine the implementation issues of the digital filter as well, with interest concentrated on zero-one decomposition of a filter to eliminate using multipliers.

A. BACKGROUND OF FIR FILTER AND CHARACTERISTICS

The design of nonrecursive FIR digital filters is based on the following relationships [Ref. 1]. For a causal nonrecursive system the FIR filter can be described by the difference equation,
\[ y(n) = \sum_{k=0}^{k=L} b_k x(n-k) \quad (1.1) \]

It is found that if the input is \( x(n) = e^{jn\theta} \) then the steady-state system output is,

\[ y(n) = ejn\theta H(e^{j\theta}) \quad (1.2) \]

where, \( H(e^{j\theta}) \) is the system frequency response. Expanding the right-side of equation (1.1) for the same input \( x(n) = e^{jn\theta} \) yields the following steady-state output

\[ e^{jn\theta} H(e^{j\theta}) = b_0 e^{jn\theta} + b_1 e^{jn\theta} e^{-j\theta} + \ldots + b_L e^{jn\theta} e^{-jL\theta} \quad (1.3) \]

Eliminating \( e^{jn\theta} \) yields,

\[ H(e^{j\theta}) = \sum_{n=0}^{n=L} h(n) e^{-jn\theta} \quad (1.4) \]

where \( h(n) = b_n \) is the impulse response.

For a casual filter, \( h(n) = 0 \) for \( n < 0 \), with a finite integer of duration \( L \). Equation (1.4) establishes a direct relationship between the impulse response of a FIR filter and the system transfer function.

The goal of any design is to determine the filter coefficients (or weights) \( b_0, b_1, \ldots, b_n \) such that a
desired frequency response characteristic $H(e^{j\omega})$ can be achieved. But, why is an FIR digital filter chosen instead of other forms of digital filter? The advantages of the FIR filter can be described in the following. [Ref. 2]

1. FIR filters with exact linear phase can be easily designed. Linear phase filters are important for applications where frequency dispersion due to a nonlinear phase shift is harmful, e.g., video and speech signal processing.

2. FIR filters, realized nonrecursively by direct convolution, are always stable.

The possible disadvantages of FIR filters are:

1. A large value of $N$, the impulse response duration, is required to approximate sharp cutoff filters. Hence, an expensive implementation is required to realize such filters.

2. The delay of linear phase FIR filters may not be an integer number of samples. This noninteger delay can lead to problems in signal processing applications. Consequently, a filter with an odd number length to yield integer delay is usually preferred.

B. IMPLEMENTATIONS OF FIR FILTER

There are several methods for hardware realization of an FIR filter. The goal is to speed up the execution and make possible "real time" simulations in desired digital systems with lowest costs. In this section various digital hardware realizations of digital filters including direct and cascade FIR realizations are discussed.

1. Direct Form FIR Hardware

One realization for a direct form FIR filter is shown in Figure 1. Figure 2 shows a simple structure for realizing the filter using a single computational element.
It consists of a multiplier and an adder. A shift register is used to hold the filter states, and a ROM is used for the coefficients. By means of a multiplier and an accumulator, a single output sample can be computed by successive addition as the shift register content circulates.

![Diagram of Direct form FIR filter](attachment:direct_form_fir_filter.png)

**Figure 1** Direct form FIR filter. [Ref. 2]

![Diagram of Structure for implementation of direct form FIR filter](attachment:structure_direct_form_fir_filter.png)

**Figure 2** Structure for implementation of direct form FIR filter. [Ref. 2]
2. **Cascade Form FIR Hardware**

Figure 3 shows an example of a three section filter, each section being fourth order. Figure 1 shows the direct implementation of \((1 + a_1 z^{-1} + a_2 z^{-2} + \ldots + a_{12} z^{-12})\).

Figure 3 is the cascade implementation of the factored form of the above the equation, i.e.,

\[
(1 + b_1 z^{-1} \ldots + b_4 z^{-4})(1 + c_1 z^{-1} \ldots + c_4 z^{-4})
\]

\[
(1 + d_1 z^{-1} \ldots + d_4 z^{-4})
\]

The implementation in Figure 3 needs multipliers and adders. Sometimes, it is difficult to achieve real time response in this implementation due to the speed limitation of the multiplier.

How to eliminate the need for multiplication in a bandpass filter design? It is possible to accomplish this objective using zero-one decomposition methodology [Ref. 3].

![Figure 3 Cascade FIR filter.](image)

Figure 4 shows an implementation of a bandpass filter,

\[
1 - 2z^{-1} - z^{-2} + 5z^{-3} - 2z^{-4} - 5z^{-5} + 4z^{-6} + 4z^{-7} - 5z^{-8} - 2z^{-9} + 5z^{-10} - z^{-11} - 2z^{-12} + z^{-13} \quad (1.5)
\]
This bandpass filter has frequency response similar to the desired transfer function for color signal used in a TV receiver system. The same transfer function can be written as:

\[(1 - z^{-2})(1 + z^{-4})(1 + z^{-3})(1 - z^{-1})(1 - z^{-1})(1 - z^{-2})\]

Equation (1.6) shows a special way to decompose the transfer function where all coefficients are +1, -1 or 0. If the desired filter transfer function can be decomposed into this kind of form, both processing time and hardware implementation cost can be saved. For example, according to the expression in (1.5), implementing it in direct form required 12 multipliers and 13 adders. Implementing the decomposed form in (1.6) required only 6 adders. What procedure can be used to obtain this decomposed expression? Is this the best way to simulate an original filter? These are the issues discussed in the Chapter IV. In Chapter II, optimization principle and the characteristics of the Remez program are summarized. In Chapter III cascade and parallel implementation of the design example mentioned in Chapter II will be discussed. Finally all studies are summarized in Chapter IV.
Figure 4 A bandpass filter for video signal uses zero-one decomposition that eliminates the multiplication inherent in the filter implementation. [Ref. 3]
II. DESCRIPTION OF ALGORITHMS

In this chapter the characteristics of the computer tools for designing FIR digital filters are discussed. The uses of this computer tool and the advantages of using this tool for digital filter design are the important issues addressed here. Using the video Intermediate Frequency (IF) analog filter as an example we investigate the necessary steps to implement an equivalent digital filter.

A. COMPUTER-AIDED DESIGN USING THE REMEZ EXCHANGE ALGORITHM

Optimization techniques are used widely in operation research, economics, and other related fields. However, only recently have such techniques gained acceptance as tools for the design and evaluation of electronic circuits.

1. Optimal Filter Principle

Since we are only concerned with FIR filter design, the Remez exchange algorithm will be chosen as the computer aided design tool. The following discussion consists of excerpts from the book of Theory and Application of Digital Signal Processing [Ref. 2] and the thesis by J. V. England. [Ref. 1]. The basis of this computer aided design (CAD) technique is optimization. A desired filter frequency response is approximated by a particular filter whose coefficients are to be determined. The accuracy of the
approximation is evaluated according to some criterion, usually an error function, that indicates how large a disparity exists between the desired filter frequency response and the approximating filter frequency response. Variable parameters of the approximating function are then "adjusted" to optimize the filter design in terms of this criterion. In other words, the solution is optimal (in the sense that the peak approximation error over the entire interval of approximation is minimized) and unique. And the mathematical basis for this algorithm is the weighted Chebyshev approximation.

A summary of the approximation and error functions for this algorithm follows. It has been shown in (2.1) [Ref. 2] that the frequency response for the four cases of linear phase filter, i.e., even or odd symmetry with an even or odd number of terms, can be written in the form:

$$H(e^{j\theta}) = e^{-j\theta(n-1)/2} e^{j(\pi/2) \hat{H}(e^{j\theta})}$$

(2.1)

where \( \hat{H}(e^{j\theta}) \) is a real-valued function used to approximate the desired filter's magnitude specifications and the remaining terms approximate the desired phase.

A measure of how well the designed filter frequency response approximates the desired filter frequency response is required. The weighted Chebyshev approximation uses an error function defined as follows:

$$E(\theta) = W(\theta) [\hat{H}_D(e^{j\theta}) - \hat{H}(e^{j\theta})]$$

(2.2)
where,

\[ H_D(e^{j\theta}) = \text{the desired frequency response} \]
\[ \hat{H}(e^{j\theta}) = \text{the designed frequency response} \]
\[ W(\theta) = \text{weighting function} \]
\[ E(\theta) = \text{error function} \]

Thus, the Chebyshev approximation performed by the Remez exchange algorithm can be stated as follows: Find the set of filter coefficients that minimizes the maximum absolute value of the error, \( E(\theta) \), over the frequency range of interest.

\[ E_{\text{opt.}} = \min \left( \max |E(\theta)| \right) \quad (2.3) \]

At this point, discussion of the weighting function, \( W(\theta) \), is in order. The purpose of this weighting function is to ensure a small tolerance for error in critical frequency ranges.

If \( W(\theta) \) is large, this means a large deviation from the desired frequency response, \( H_D(e^{j\theta}) \), can not be tolerated. Looking at equation (2.2), we see that if \( W(\theta) \) is large, the difference between the desired and designed frequency response, \( |H_D(e^{j\theta}) - \hat{H}(e^{j\theta})| \), has to be small to keep the weighted error small. Conversely, if \( W(\theta) \) is small, the difference \( |H_D(e^{j\theta}) - \hat{H}(e^{j\theta})| \) can be large and still meet the error criterion. Small values for \( W(\theta) \) would be used in frequency bands where close approximation to the desired frequency response is not critical.
2. **Characteristics of the Remez Program**

The Remez exchange program requires the following parameter specifications for a bandpass filter design. Frequencies are normalized with respect to the sampling frequency. For a bandpass filter the design procedure consists of specifying $N$ the filter length, the left and rightside cutoff frequency ($F_{LP}$, $F_{RP}$), the left and rightside stopoff frequency ($F_{LS}$, $F_{RS}$), and the ripple ratio, $K = \frac{\delta_1}{\delta_2}$, which determines the desired weighting function $W(e^{j\theta})$ as:

$$W(e^{j\theta}) = \begin{cases} 1/K = \frac{\delta_2/\delta_1}{1/K} & 0 \leq \theta \leq 2\pi F_{LS} \\ 1 & 2\pi F_{LP} \leq \theta \leq 2\pi F_{RP} \\ 1/K = \frac{\delta_2/\delta_1}{1/K} & 2\pi F_{RS} \leq \theta \leq \pi \end{cases}$$

Here $\delta_1$ is the passband ripple, and $\delta_2$ is the stopband ripple, and $\Delta F$ is the transition band size. Figure 5 shows the frequency response of a bandpass filter and the frequency specifications. Since this optimal filter design procedure is based on the Chebyshev approximation which has ripples in the passband and stopband, the stopoff and cutoff frequencies are dependent on the specified magnitude of the desired ripple. It is not possible to specify the 3-db frequency position for the designed filter directly in the Remez program.
Figure 5 Frequency response of an optimal (minimax error) bandpass filter [Ref. 2].
To investigate the relationship between the filter length, \( N \), and the transition band size, \( \Delta F \), the Remez program was used for a BPF with the following specifications:

\[
\begin{align*}
F_{LS} &= 0.1 \\
F_{LP} &= 0.2 \\
F_{RP} &= 0.3 \\
F_{RP} &= 0.4 \\
K &= 1
\end{align*}
\]

Figures 6, 7, and 8, illustrate the frequency response obtained for \( N=31, 61, 81 \), respectively. The other specifications are kept the same. Looking at the left transition region we note, the longer length filter has a sharper transition band.

![Figure 6](image-url)

Figure 6  The relation between filter length and transition band when \( N = 31 \).
Figure 7 The relation between filter length and transition band when $N = 61$. 

Figure 8 The relation between filter length and transition band when $N = 81$. 

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From the above study, it is known that if the filter length \( N \) is sufficiently large, the desired transition slope can be achieved. The exact 3 db cutoff frequency of the designed passband is dependent on the filter length \( N \) and the stopband and passband specification. As we can see, it seems that there are relationships among the transition band \( \Delta F \), filter length \( N \) and band rejection \( B. R. \) in FIR filter design using the Remez program. These will be discussed in Chapter III.

B. STUDY EXAMPLE: SPECIFICATIONS FOR A VIDEO INTERMEDIATE FREQUENCY (I. F.) FILTER

The main effort of this section will be to take an analog filter and redesign the filter using the Remez
exchange technique. The first step is to convert the analog requirements to digital requirements using the formula
\[ \theta = \alpha T, \]
where \( T = 1 / \text{sampling frequency} \) and \( \alpha = \text{analog frequency}. \) There are advantages of using digital filters over analog filters. Namely, they are less likely to be affected by noise and are therefore more reliable than analog filters. Additionally, digital filters have many possible implementations, and the signal data can be stored without loss of accuracy.

1. Analog Specifications

The analog BPF commonly used for video signals has been selected as an example. The objective is to investigate whether it is possible to implement a traditional analog BPF with digital circuits. In practice, two stages of I. F. amplifiers and the associated filters are used for processing of composite television signals. Figure 10 illustrates the transfer function curve of a full 4.25 MHz bandwidth amplifier. As can be seen it meets the following specifications. [Ref. 5]:

![Figure 10](image-url)

**Figure 10** Typical I. F. amplitude response curve for full 4.25-MHz bandwidth.
a. Over-all Gain

The overall gain of the video I.F. amplifier is determined based on the noise and signal voltage from the mixer stage. The total gain in the picture I. F. amplifier must be 12,500 times in voltage or 82 db. If we use a digital filter instead of the original analog amplifier and filters, this means that the band rejection ratio should be 1:12,500, thus making the rejection ratio less than 20 \log (1/10,000) = -80 db.

b. Gain-bandwidth Product

The gain-bandwidth product of a typical I.F. stage is 53. This means that a gain of about 13 can be obtained at a nominal bandwidth of 4 MHz, or a gain of 17.5 at 3 MHz, or a gain of 17.5 at 3 MHz bandwidth. It thus appears that four stages are necessary to achieve I. F. gains above 10,000 with an over-all bandwidth of 4 MHz. However, recent improvements in I.F. amplifier circuit arrangements have increased the gain-bandwidth product to about 75, making possible a gain of 10,000 in three stages at a full bandwidth of 4 MHz [Ref. 7]. The present tendency is to design the I.F. amplifier for the maximum attainable bandwidth, consistent with the selectivity requirements, and thus assure maximum resolution in the image. In the design case considered here a passive digital filter is used to achieve the transfer function requirement. The gain-bandwidth product is not of concern.
c. I. F. Traps

The third specification relates to the selectivity of the I. F. amplifier, and particularly to discrimination against three undesired carriers: the associated sound carrier at 41.25 MHz whose total attenuation is -65 db, the lower adjacent channel sound I.F. carrier at 47.25 MHz whose total attenuation is -20 db, and the upper adjacent channel picture I.F. carrier at 45.75 MHz whose total attenuation is from -10 to -15 db. In other words, the analog I-F amplifier specifications are:

1. Stop band 1: 0 (MHz) ------ 41.3 (MHz)
   Pass band 2: 41.73 (MHz) ------ 45 (MHz)
   Stop band 3: 46.5 (MHz) ------ 100 (MHz)

2. Band rejection ratio: 1/12,500 = 0 db/-80 db

3. Phase delay: It is required to be linear in the passband on both sides of the cutoff frequency from -0.87 to 0.87 radians. Use of an FIR filter will guarantee linear phase and constant delay for the frequency component.

2. Translated Digital Specification

To translate the analog specifications, the following are determined:

1. Nyquist sampling frequency: \( F = 100 \text{ (MHz)} \)

   This is an arbitrary choice dictated by forthcoming technology.

2. Band 1: \( \theta_1 = \frac{(2\pi \times 0)}{(100 \times 10^6)} = 0 \)
   \( \theta_2 = \frac{(2\pi \times 41.3 \times 10^6)}{(100 \times 10^6)} = 0.826\pi \)

   Band 2: \( \theta_3 = \frac{(2\pi \times 41.73 \times 10^6)}{(100 \times 10^6)} = 0.834\pi \)
   \( \theta_4 = \frac{(2\pi \times 45 \times 10^6)}{(100 \times 10^6)} = 0.9\pi \)
Band 3: \[ \theta_5 = \frac{(2\pi \times 46.5 \times 10^6)}{(100 \times 10^6)} = 0.93\pi \]
\[ \theta_6 = \frac{(2\pi \times 50 \times 10^6)}{(100 \times 10^6)} = \pi \]

For convenience, we normalized them to \(\pi\) radians.

Band 1: \[ \theta_1 = 0 \quad \Rightarrow \quad \theta_1 = 0 \]
\[ 0.826\pi/\pi = \theta_2/0.5 \quad \Rightarrow \quad \theta_2 = 0.413 \]

Band 2: \[ 0.834\pi/\pi = \theta_3/0.5 \quad \Rightarrow \quad \theta_3 = 0.417 \]
\[ 0.9\pi/\pi = \theta_4/0.5 \quad \Rightarrow \quad \theta_4 = 0.45 \]

Band 3: \[ 0.93\pi/\pi = \theta_5/0.5 \quad \Rightarrow \quad \theta_5 = 0.465 \]
\[ \pi/\pi = \theta_6/0.5 \quad \Rightarrow \quad \theta_6 = 0.5 \]

3. **Band Rejection Ratio and Phase Delay**

The band rejection is chosen based on the design. For a four stage implementation, each individual filter stage rejection ratio should be at least -20 db. If the three stage method is chosen, then each individual filter rejection should be at least -26.64 db, \((10,000 = 10^4 = 21.5^3, 20 \log 21.5 = 26.64\text{dB})\). If two stages are chosen, any one rejection should be at least 40 db.

As mentioned before, the phase delay requirement is automatically satisfied using FIR filters. Present Analog to Digital (A/D) converter technology can provide a maximum sampling frequency on the order of 50 MHz. The 100 MHz sampling frequency desired is not available at this time, however, it may soon be possible in the future. Although it is impossible to use this design in digital video processing now because of the high sampling rate requirement (100 MHz),
the objectives of using this design illustration is to develop the FIR digital filter design methodology.
III. BANDPASS FILTER DESIGN

The main purpose of this chapter is to use the analog specifications of an I.F. filter as mentioned before in Chapter II and design a digital bandpass filter. To achieve this goal, several different methods are tried, i.e., single stage and multiple stage methods are considered for this design.

A. SINGLE STAGE FILTER DESIGN

1. Experimental design

Prior to using the Remez program for filter design purposes, it is necessary to understand what band rejection characteristics are exhibited by the filter designed using this program. In other words, what effect does varying the program parameters have on the resultant filter characteristics? To achieve this familiarization, the following experimental filters are used to show the characteristics of the design using the Remez program.

Filter 1 is a given bandpass filter with transition band=0.1. The normalized sampling frequency is 0.5. The specifications of this filter are as follows:

- Stop band 1: 0.0 to 0.18
- Pass band: 0.28 to 0.32
- Stop band 2: 0.42 to 0.5
Filter 2 is another given bandpass filter with transition band=0.01. The specifications are:

<table>
<thead>
<tr>
<th>Stop band 1:</th>
<th>0.0 to 0.27</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pass band:</td>
<td>0.28 to 0.32</td>
</tr>
<tr>
<td>Stop band 2:</td>
<td>0.33 to 0.5</td>
</tr>
</tbody>
</table>

Filter 3 is a bandpass filter with transition band=0.05. The specifications are as follows:

<table>
<thead>
<tr>
<th>Stop band 1:</th>
<th>0.0 to 0.23</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pass band:</td>
<td>0.28 to 0.32</td>
</tr>
<tr>
<td>Stop band 2:</td>
<td>0.37 to 0.5</td>
</tr>
</tbody>
</table>

These three filters are then designed using the Remez program with different filter lengths, N, and weights. Table 3.1 contains a summary of the results obtained from these experiments. Different N's are chosen because it is interesting to reveal the effect of varying the filter length. Two different sets of weights, \( W_1 \) and \( W_2 \), are chosen to control the passband and the stopband ripple. The transition band between stopband and passband (\( \Delta F = \text{cutoff frequency} - \text{stopoff frequency} \)) is also varied. The purpose here is to examine the interdependency of the three parameters: \( \Delta F \), N and Band Rejection (B. R.). The relations are just like a triangle shown in Figure 11, but what are the exact dependencies among these three parameters?

From Table 3.1, the limits of the band rejection with respect to the filter length can be obtained and are summarized in Table 3.2.
Figure 11 Filter length $N$, transition band $\Delta F$, and achieved band rejection (B.R.).

TABLE 3.1 CHARACTERISTICS OF FILTERS IN EXPERIMENT (SAMPLING FREQUENCY=100MHz)

<table>
<thead>
<tr>
<th>$N$ (Filter length)</th>
<th>$W$ (Weighting)</th>
<th>$\Delta F$ (Transition band)</th>
<th>Band Rejection</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$W_1$ $W_2$ (pass band)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>31</td>
<td>1 1</td>
<td>0.1</td>
<td>-58dB</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.05</td>
<td>-30dB</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.01</td>
<td>-48dB</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.0</td>
<td>-68dB</td>
</tr>
<tr>
<td>61</td>
<td>1 10</td>
<td>0.05</td>
<td>-18dB</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.01</td>
<td>-8dB</td>
</tr>
<tr>
<td></td>
<td>10 1</td>
<td>0.05</td>
<td>-113dB</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.01</td>
<td>-62dB</td>
</tr>
<tr>
<td>91</td>
<td>1 10</td>
<td>0.05</td>
<td>-153dB</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.01</td>
<td>-86dB</td>
</tr>
<tr>
<td></td>
<td>10 1</td>
<td>0.05</td>
<td>-159dB</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.01</td>
<td>-33dB</td>
</tr>
</tbody>
</table>
TABLE 3.2 CHARACTERISTICS OF FILTERS FOR A LIMIT REJECTION OF 20db.

<table>
<thead>
<tr>
<th>W (Weighting)</th>
<th>AF (Transition band)</th>
<th>N (Filter length)</th>
<th>Limit rejection (dB)</th>
</tr>
</thead>
<tbody>
<tr>
<td>W1</td>
<td>W2 (pass band)</td>
<td>0.1</td>
<td>&gt;10</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>0.05</td>
<td>&gt;25</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.01</td>
<td>&gt;91</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.05</td>
<td>&gt;27</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.01</td>
<td>&gt;150</td>
</tr>
<tr>
<td></td>
<td>10</td>
<td>0.01</td>
<td>&gt;15</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.1</td>
<td>&gt;9</td>
</tr>
<tr>
<td>10</td>
<td>1</td>
<td>0.05</td>
<td>&gt;25</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.01</td>
<td>&gt;25</td>
</tr>
</tbody>
</table>

From the investigation of how the program parameters affect filter characteristics, the following comments are in order:

1. As shown in Table 3.1, when the W (Weighting function) and N (filter length) are fixed, the band rejection is proportional to the $\Delta F$ (transition band). (That is when $\Delta F$ becomes small, the rejection becomes small and poor).

2. When the $\Delta F$ (transition band) and W (Weighting function) are fixed, the band rejection is proportional to the N (filter length). If more band rejection is desired, an increase in N, the length of the filter, is required.

3. When W (weighting function) is fixed to satisfy a certain band rejection, the $\Delta F$ is inversely proportional to the N (filter length). From Table 3.1 we see that in order to satisfy the analog specifications with a band rejection of -80 db as discussed in the previous chapter, it is necessary to choose a filter length, N, ranging from 35 to 41 where the weighting function $w_1 = w_2 = 1$. 

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2. Actual Bandpass Filter Design

The digital specifications of the video I.F filter in Chapter II will be used, however, there is a finite limit on the filter length that we can use due to the characteristics of the Remez program and hardware constraints. Bearing in mind these limitations, a filter with the frequency response illustrated in Figure 12 is obtained. This filter has the following specifications:

- Filter length: 111
- Band specifications:
  - Stop band 1: 0. to 0.413
  - Pass band: 0.417 to 0.45
  - Stop band 2: 0.465 to 0.5
- Weighting factors: 1, 10, 1

From Figure 12 the response has a dip of about -1 db in the passband and a band rejection of only -3 db. The reason why the desired specifications cannot be achieved is that it requires a larger filter length. According to Ref. 4, the required filter length is:

\[ F_2 - F_1 \geq \frac{4\pi}{N} \]

where \( F_1 \) and \( F_2 \) are stopoff and cutoff frequency. Since, \( 0.417-0.413 < 0.465-0.45 \), we select the smaller \( F_2 - F_1 \),

\[ F_2 - F_1 = 0.417-0.413 \text{ and } 0.004 \geq \frac{4\pi}{N} \]

\[ N \geq 1000 \]
The filter length must be greater than 1000 which is impossible to implement using just one digital filter. Therefore it is necessary to pursue another design method.

Figure 12 Single stage filter design of the TV I. F. specifications.
B. MULTIPLE STAGE FILTER DESIGN

1. Cascade Implementation

Using the associative and distributive properties, the equivalent unit sample responses for parallel and cascade implementation are shown in Figure 13. The question is which implementation method can achieve the desired specifications? CAD tools will be used to find our answer. The procedure is as follows. Using the Remez program the cascade response of the two filters is obtained.

1. For example, we select a fourth order bandpass filter with band specifications: 0., 0.1, 0.2, 0.3, 0.4, 0.5 and filter 1's impulse response is obtained.

2. Choosing another fourth order bandpass filter but shifting the band specifications to: 0., 0.09, 0.19, 0.29, 0.39, 0.5, we obtain filter 2's impulse response.

3. Multiplying the two filters together in the frequency domain yields the overall frequency response of Figure 14.

The CAD tools used also allow us to enter the combined filter transfer functions in polynomial form directly, and yield the response shown in Figure 15. After comparing Figure 14 and Figure 15, the procedure to cascade two filters is confirmed to be correct. We now proceed with I-F filter design using the Remez program.

The issue now is to find the shift between the two component filters used in the cascade form implementation of the filter in Figure 12. Assuming that a single 21 point filter is generated as the first filter and that the second
Figure 13 Equivalent systems in parallel and cascade forms. [Ref. 3]

Figure 14 Using FFT to process and get the combined result in cascade form.
filter is the shifted version of the first one, we examine three cases. The three sets of band specifications are:

1. \( \Delta f = 0.1 \)
   
   \[ 0.0, 0.08, 0.18, 0.30, 0.40, 0.50 \]

2. \( \Delta f = 0.05 \)
   
   \[ 0.0, 0.08, 0.13, 0.30, 0.35, 0.50 \]

3. \( \Delta f = 0.01 \)
   
   \[ 0.0, 0.08, 0.09, 0.30, 0.31, 0.50 \]

Figure 15 Using polynomial entry command to get frequency response in cascade form.
With a fixed weighting of 10, 1, and 10 several filters were designed and are summarized in Table 3.3. The frequency response curves of these same filters are illustrated in Figures 16 to 36.

### Table 3.3
SUMMARY OF CASCADE FILTER DESIGN EXPERIMENT

<table>
<thead>
<tr>
<th>Filter length</th>
<th>ΔF transition</th>
<th>Distance (normal freq.)</th>
<th>Band Rejection</th>
<th>Extracted from Fig. No.</th>
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<tr>
<td>0.1</td>
<td>0</td>
<td>(original)</td>
<td>-80dB</td>
<td>16</td>
</tr>
<tr>
<td></td>
<td>0.02</td>
<td></td>
<td>-75dB</td>
<td>18</td>
</tr>
<tr>
<td></td>
<td>0.04</td>
<td></td>
<td>-75dB</td>
<td>20</td>
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<td></td>
<td>0.06</td>
<td></td>
<td>-70dB</td>
<td>22</td>
</tr>
<tr>
<td>0.05</td>
<td>0</td>
<td>(original)</td>
<td>-75dB</td>
<td>23</td>
</tr>
<tr>
<td></td>
<td>0.01</td>
<td></td>
<td>-75dB</td>
<td>25</td>
</tr>
<tr>
<td></td>
<td>0.05</td>
<td></td>
<td>-40dB</td>
<td>27</td>
</tr>
<tr>
<td></td>
<td>0.1</td>
<td></td>
<td>-40dB</td>
<td>29</td>
</tr>
<tr>
<td>0.01</td>
<td>0</td>
<td>(original)</td>
<td>-40dB</td>
<td>30</td>
</tr>
<tr>
<td></td>
<td>0.05</td>
<td></td>
<td>-25dB</td>
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<td>0.1</td>
<td></td>
<td>-25dB</td>
<td>34</td>
</tr>
<tr>
<td></td>
<td>0.15</td>
<td></td>
<td>-25dB</td>
<td>36</td>
</tr>
</tbody>
</table>
Figure 16 is the original bandpass filter with a transition band of 0.1. The specification for the frequency bands are:

- Stop band 1: 0.0 0.08
- Pass band: 0.18 0.30
- Stop band 2: 0.40 0.50

Figure 17 is a shifted version of the original bandpass filter by 0.02 of the normalized frequency. The specifications for the frequency bands are:

- Stop band 1: 0.0 0.1
- Pass band: 0.2 0.32
- Stop band 2: 0.42 0.50

Figure 18 shows the response of a combined cascaded filter using the filters of Figure 16 and Figure 17. The observed band rejection ratio in Figure 18 is about -80 db. The rejection ratios are listed in Table 3.3 along the row with \( N = 21, \Delta F = 0.1, \) and shifted distance = 0.02. The rejection ratios corresponding to the cascade designs illustrated in the remaining figures are listed in the second to the last column of the table.
Figure 16 Original filter, $\Delta F = 0.1$

Figure 17 Shifted filter by 0.02 of the normalized frequency.
Figure 18  Frequency Response for the cascade realization of filters illustrated in Figure 16 and Figure 17.
Figure 16 Original filter, $\Delta F = 0.1$

Figure 19 Shifted filter by 0.04 of the normalized frequency.
Figure 20 Cascade realization of Figure 16 and Figure 19 filters.
Figure 16 Original filter, \( \Delta F = 0.1 \)

Figure 21 Shifted filter by 0.06 of the normalized frequency.
Figure 22 Cascade realization of the original filter of Figure 16 and the filter of Figure 21.
Figure 23 Original filter, $\Delta F = 0.05$

Figure 24 Shifted filter by 0.01 of the normalized frequency.
Figure 25  Cascade realization of the original filter in Figure 23 and the filter of Figure 24.
Figure 23 Original filter, $\Delta F = 0.05$

Figure 26 Shifted filter by 0.05 of the normalized frequency.

40
Figure 27  Cascade realization for the original filter in Figure 23 and the filter of Figure 26.
Figure 23 Original filter, $\Delta F = 0.05$

Figure 28 Shifted filter by 0.1 of the normalized frequency.
Figure 29 Cascade realization of original filter in Figure 23 and the filter of Figure 28.
Figure 30  Original filter, $\Delta F = 0.01$

Figure 31  Shifted figure by 0.05 of the normalized frequency.
Figure 32 Cascade realization for the original filter in Figure 30 and the filter of Figure 31.
Figure 30  Original filter, $\Delta F = 0.01$

Figure 33  Shifted filter by 0.1 of the normalized frequency.
Figure 34 Cascade realization for the original filter in Figure 30 and the filter of Figure 33.
Figure 30 Original filter, $\Delta F = 0.01$

Figure 35 Shifted filter by 0.15 of the normalized frequency.
From the preceding pages, where the cascaded responses are shown in Figures 18, 20, 22, 24, 26, 28, 30, 32, 34, and 36, it appears that to obtain an increase in
bandwidth through the use of cascade filter realization is difficult, however, band rejection characteristics are improved using the cascade implementation. Thus we see that achieving the desired wide and flat passband response requires an alternative design approach.

With analog active amplifier filters it is possible to stagger a filter with the shifted one, and a flat and wide passband response can be obtained as in Figure 37. Figure 38 illustrates two FIR filters overlapped according to the procedure suggested in Figure 37. Figure 39 illustrates their cascade combination, which is not wider than the original passive digital filter. The Remez program for passive digital filters is thus not able to obtain a wider frequency response, as was the case with analog filters.

Figure 37 Stagger-tuned analog design in cascade form. [Ref. 3]
Figure 38 Original filter and the shifted filter design.
2. **Parallel Implementation**

As mentioned at the beginning of this chapter, we can attempt to design our I-F filter using a parallel implementation. The following band specifications and a filter lengths of 21 are used.

- Stop band 1: 0.0 to 0.23671
- Pass band: 0.34071 to 0.3842047
- Stop band 2: 0.4882047 to 0.5

The resultant filter frequency response is illustrated in Figure 40. The filter is then shifted to the left so that a crossover occurs at magnitude 0.5 as shown in Figure 41. Using addition to combine the two filters together, the

---

**Figure 39** Cascade filter combination.
resultant frequency response is depicted in Figure 42. From this figure, it can be seen that the resultant response is wider, as we desired.

Other questions remain, such as, is it possible to obtain the same frequency response using a direct implementation of a single filter transfer function? Which design method is best in terms of hardware implementation? These problems remain to be investigated.

Figure 40 Original filter for parallel implementation.
Figure 41 Shifted filter.

Figure 42 Combination filter in parallel implement.
IV. DECOMPOSITION FOR IMPLEMENTATION

This chapter will investigate the use of the zero-one coefficient decomposition method to obtain any order filter. First, the brute force enumeration method is used to explore the possibility of taking an existing filter transfer function, and finding all possible combinations of decomposed factors for the filter polynomial. The frequency responses are then obtained, and studied to find a relationship between the original transfer function and the decomposed transfer functions. Going through these results to get all the numerous errors lead us to an approach to find the best decomposed filter and save the number of multipliers. From these studies a computer flow chart is proposed where the best 0,1 decomposed filter design can be selected.

A. HARDWARE REQUIREMENT

As seen in Chapter III, using various multi-stage methods to design a filter with a wide flat passband is difficult. The Remez program has a limit in its ability to produce filter transfer function patterns similar to the original analog filter design of Figure 37. It was also shown, using parallel implementation techniques, that good results like that shown in Figure 42 can be produced.
Now, our attention is drawn to implementation of the desired filter using direct design techniques and comparing the results with those obtained using parallel implementation. Figure 42 and Table 3.1 were consulted to determine the direct design specifications: 0.0, 0.125, 0.2575, 0.3541, 0.4779, 0.5 and filter length = 41. The results are illustrated in Figure 43. Comparing these results with the filter of Figure 42, we note the direct design is better. However, a true comparison of the two techniques requires a comparison of the hardware implementation of the two designs.

1. **Direct Design**

\[ H(Z) = \sum_{n=1}^{41} h(n) z^{-n} \]

From the above equation, the direct calculation of the FIR requires the following computations:

- Number of complex multiplications: 41
- Number of complex additions: 40
- Number of total delays: 40

The hardware implementation is illustrated in Figure 44.

2. **For the Two Filters in Parallel Combination**

\[ H(Z) = \sum_{n=1}^{21} h(n) z^{-n} + \sum_{n=1}^{21} h(n) z^{-n} \]

We can get the following:

complex multiplications = 42
complex additions = 40
delays = 20

The hardware implementation is shown in Figure 45. From this comparison, the parallel implementation is shown to be
slightly advantageous in terms of total delay time, which is not a critical factor in most designs. Thus, we can conclude that the direct implementation is superior to both cascade and parallel methods, because it requires less time for design and exhibits optimal frequency response characteristics.

Figure 45 Parallel implementation for a filter of length = 21.

B. ZERO-ONE DECOMPOSITION

As we have seen direct filter design using the Remez program is a convenient tool. [Ref. 2] The question is, is there any method to decompose the desired filter transfer function using only 0, +1, -1 coefficients to save multiplies? [Ref. 3] To investigate this question a brute force enumeration method will be used to approximate the desired filter. (There is no known methodology for decomposition.)
In this chapter a method is proposed to take advantage of the available CAD tools to find a good decomposed filter transfer function that can approximate the desired filter.

A fourth order transfer function was selected as our example:

\[
H(Z) = -0.08705 + 0.012859Z^{-1} + 0.12619Z^{-2} + 0.012859Z^{-3} - 0.087105Z^{-4} \quad (4.1)
\]

The frequency response is plotted in Figure 46 and a direct hardware implementation is shown in Figure 47 where multipliers are needed.

\[
\begin{align*}
\text{Value} & : & 0.0 & -3.333 & -8.667 & -10.00 \\
\text{Frequency x10}^7 \text{ Hz} & : & 0 & 0.625 & 1.25 & 1.875 & 2.5 & 3.125 & 3.75 & 4.375 & 5.0
\end{align*}
\]

Figure 46 Frequency response for the transfer function of equation (4.1).
The total possible number of factors are:

factor 1 form: \((1 + aZ^{-1} + bZ^{-2} + cZ^{-3} + dZ^{-4})\),
the possible number = \(3^3 \times 2 = 54\)

d factor 2 form: \((1 + aZ^{-1})(1 + bZ^{-1} + cZ^{-2} + dZ^{-3})\),
the possible number = \(3^2 \times 2^2 = 36\)

: \((1 + aZ^{-1} + bZ^{-2})(1 + cZ^{-1} + dZ^{-2})\),
the possible number = \(3^2 \times 2^2 = 36\)

factor 3 form: \((1 + aZ^{-1})(1 + bZ^{-1})(1 + cZ^{-1} + dZ^{-2})\),
the possible number = \(3 \times 2^2 = 12\)

d factor 4 form: \((1 + aZ^{-1})(1 + bZ^{-1})(1 + cZ^{-1})(1 + dZ^{-1})\),
the possible number = \(2^4 = 16\)

Total number = 166.

where \(a=b=c=d=\pm 1, -1\) or 0.

![Diagram](image-url)

Figure 47 Direct hardware implementation of equation (4.1).
The total number of possible decompositions as enumerated is 166, while the number of distinct decompositions is equal to 148. The difference is due to the fact that when factor forms 2, 3, and 4 are expanded, some are identical. The possible decomposition factors of this example filter transfer function are:

\[(1 - z^{-1} + z^{-2})(1 + z^{-1} + z^{-2}) = 1 + z^{-2} + z^{-4}\]
\[(1 + z^{-1} + z^{-2} + z^{-3})(1 - z^{-1}) = 1 - z^{-4}\]
\[(1 - z^{-1} + z^{-2} - z^{-3})(1 + z^{-1}) = 1 - z^{-4}\]
\[(1 + z^{-1} - z^{-2} - z^{-3})(1 - z^{-1}) = 1 - 2z^{-2} + z^{-4}\]
\[(1 + z^{-1})(1 + z^{-3}) = 1 + z^{-1} + z^{-3} + z^{-4}\]
\[(1 + z^{-1})(1 - z^{-3}) = 1 + z^{-1} - z^{-3} - z^{-4}\]
\[(1 - z^{-1} + z^{-3}) = 1 + z^{-1} + z^{-3} - z^{-4}\]
\[1 = 1 - z^{-1} - z^{-3} + z^{-4}\]
\[(1 + z^{-2})(1 - z^{-2}) = 1 - z^{-4}\]
\[(1 + z^{-4})\]
\[(1 + z^{-1} + z^{-2})(1 - z^{-2}) = 1 + z^{-1} - z^{-3} - z^{-4}\]
\[(1 - z^{-1} + z^{-2})(1 - z^{-2}) = 1 - z^{-1} + z^{-3} - z^{-4}\]
\[(1 + z^{-1} + z^{-2})(1 - z^{-2}) = 1 + z^{-2} + z^{-4}\]
\[(1 + z^{-1} - z^{-2})(1 + z^{-2}) = 1 + z^{-1} + z^{-3} + z^{-4}\]
\[(1 - z^{-1} - z^{-2})(1 + z^{-2}) = 1 - z^{-1} - z^{-3} + z^{-4}\]
\[(1 - z^{-1} + z^{-2})(1 + z^{-1} + z^{-2}) = 1 + z^{-2} + z^{-4}\]
\[(1 + z^{-1} + z^{-2} + z^{-3})(1 + z^{-1}) = 1 - z^{-4}\]
Using CAD tools, the frequency responses of these decomposed approximation filters were plotted, and examined. Figure 48 shows the best filter resembling the original filter whose decomposed transfer function is:

$$H(Z) = 1 - Z^{-2} + Z^{-4}$$  \hfill (4.2)

Figure 49 shows the hardware implementation of this decomposed result. Comparing the decomposed expression in equation (4.2) with the original equation (4.1), it is clear that 2 less multiplies and 2 less adders are required. Thus, zero-one decomposition saves a lot of hardware in the implementation.

Figure 48 Decomposed filter frequency response that most closely resembles original filter frequency response.
Besides the brute force enumeration method, is there any other method for obtaining the decomposed form of a desired filter? Is it possible to use CAD tools to find the answer?

To answer these questions, a quantitative evaluation between Figure 46, the desired filter, and Figure 48, the designed filter, is necessary. As is well known, the minimum points in the frequency response plots of a filter indicate the places where the zeros of the filter transfer function occur, i.e., the roots of the numerator of the filter's transfer function. Thus, it's possible to plot the zeros of the original filter transfer function along with the zeros of the decomposed filter transfer function. This is shown in Figure 50.
It can be seen that there is a direct relationship between the distance of the zeros of the original and the decomposed filters and how well the decomposed filter’s frequency response approximates that of the original. The relationship is that, the further apart the zeros are, the less accurate the approximating decomposed filter is. This measurement can be used as an error function to select the best decomposition approximation from several available ones for a particular filter transfer function.

For example, the original transfer function equation (4.1) can be divided into four factored forms as follows:

\[(Z+0.8925+j0.45128),\]
\[(Z+0.8925-j0.45128),\]
\[(Z-0.9664+j0.2576),\]
\[(Z-0.9664-j0.2576).\]

The best decomposed approximation in Equation (4.2) is found to have the following factors:

\[(Z+0.886+j0.5), (Z+0.886-j0.5),\]
\[(Z-0.866+j0.49), (Z-0.866-j0.49).\]

A plot of these two sets of roots is shown in Figure 50.
The error, that is, the distance between the decomposed approximating filter roots and the original filter roots, is determined as follows:
\[ d_1 = \sqrt{(0.2571)^2 + (0.966)^2} - 1 \]
\[ d_2 = \frac{\pi}{7} \]
\[ \text{error 1} = d_1 - d_2 = 0.26229 \]
\[ d_3 = \frac{\sqrt{(0.866)^2 + (0.5)^2} - 1}{\sqrt{(0.892547)^2 + (0.451289)^2} - 1} \]
\[ d_4 = \frac{\pi}{12} \]
\[ \text{error 2} = d_3 + d_4 = 0.5579 \]

The total error for this decomposed approximation filter is:
\[ \text{error 1} + \text{error 2} = 0.318091 \]

This calculation technique is applied to all the possible decomposed approximation filters. The frequency response plots are in Figures 51 to 62.

**TABLE 4.1 APPROXIMATE ERRORS FOR THE DECOMPOSED APPROXIMATION FILTER**

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</tr>
<tr>
<td>60</td>
<td>1.2909</td>
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<td>61</td>
<td>1.13178</td>
</tr>
<tr>
<td>62</td>
<td>0.78507</td>
</tr>
<tr>
<td>48</td>
<td>0.31809</td>
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</table>
Figure 51  Decomposed filter for the error 2.4938.

Figure 52  Decomposed filter for the error 2.3579.
Figure 53 Decomposed filter for the error 2.175.

Figure 54 Decomposed filter for the error 2.1496.
Figure 55  Decomposed filter for the error 1.86178.

Figure 56  Decomposed filter for the error 1.74975.
Figure 57 Decomposed filter for the error 1.46814.

Figure 58 Decomposed filter for the error 1.38558.
Figure 59  Decomposed filter for the error 1.3677.

Figure 60  Decomposed filter for the error 1.2909.
Figure 61 Decomposed filter for the error 1.13178.

Figure 62 Decomposed filter for the error 0.78507.
Why is it necessary to establish this error evaluation? With a valid error evaluation it is possible to determine the best decomposed approximation filter by finding the one with the smallest error. This technique lends itself to computer automation, and a flowchart illustrating a possible computer algorithm is proposed in Figure 63.

Figure 63 Algorithm for finding the best decomposed approximation filter of coefficients 0,+1,-1.
The use of the computer to determine the best decomposition formula is far superior than the original brute force enumeration method.

From the above study, it is shown that it is possible to evaluate the decomposed approximation filter and automate the search procedures. The actual implementation of this procedure is not within the scope of this thesis study, however, it is proposed that another CAD tool could be devised to obtain the best decomposed approximation filter for any order filter.
V. CONCLUDING REMARKS

This thesis has concentrated on procedures in FIR digital filter design using CAD tools. We saw that the filter length, N, has to be large enough so that the transition band will sharp enough for it to meet design specifications. The relationships among \( \Delta F \) (transition band), \( N \) (filter length) and B. R. (band rejection) in the Remez exchange algorithm were also analyzed clearly, and it was determined optimizing one variable has a trade-off effect on the others. Specifically, when \( W \) (weighting function) and \( N \) (filter length) are fixed, the band rejection is proportional to the \( \Delta F \) (transition band). When the \( \Delta F \) (transition band) and \( W \) (weighting function) are fixed, the band rejection is proportional to \( N \) (filter length). While when \( W \) (weighting function) is fixed, to satisfy a certain band rejection, the \( \Delta F \) is inversely proportional to the \( N \) (filter length). These results can be used to guide us in future filter design.

In order to achieve the goal of designing an equivalent FIR digital filter that corresponds to an analog I. F. filter, direct and multiple stage filters were examined. There are two methods in multiple stage design: cascade and parallel. In the cascade design, it is not possible to make the digital filter passband flat and wide enough, but the
band rejection is improved. In the parallel design, it seems we can more closely approximate the desired characteristics. Finally, it was shown that direct design is still better than any one of the multiple stage design methods when considering hardware requirements.

Another reason why direct filter design is better than using the multiple stage design is that, although the Remez exchange algorithm is a very good tool for the design of FIR digital filters, its Chebyshev approximation characteristics yields ripples in the passband and the stopband. The exact 3 db cutoff frequency and stopoff frequency are dependent on the filter length N and other specifications. Also, for the multiple stage design, there are difficulties in setting the cutoff and stopoff frequency.

Finally, the coefficient zero-one decomposition approximation method was presented as a means to decompose any desired filter transfer function into a manner that saves multiplier hardware. In order to do that, the brute force enumeration method was used to investigate the problem. A simple filter was used as an example to search for the best approximation. Additionally an approach based on error evaluation between original filter and decomposed filter was proposed. In summary, it is possible to conceive an algorithm that will search for a good decomposed approximation filter, an idea that has potential for future work.

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LIST OF REFERENCES


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