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COMPUTATIONAL FLUID DYNAMICS METHODS FOR LOW REYNOLDS NUMBER PRECESSING/ SPINNING INCOMPRESSIBLE FLOWS

MICHAEL J. NUSCA

APRIL 1988

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Three dimensional, steady-state, laminar, fully viscous Navier-Stokes simulations were used to predict the behavior of incompressible liquids that were undergoing steady spin and steady precession at a fixed precession angle. These numerical simulations can predict steady viscous and pressure moments. These moments tend to increase the precession angle and reduce the spin rate of the container system. For a completely filled cylinder, liquid-induced roll and side (yaw) moments were computed as functions of cylinder height to diameter, ($1 < c/a < 5.2$), Reynolds number ($1 < Re < 300$), ratio of precession to spin rate ($0.03 < \tau < 1.0$), and precession angle (α_c (deg) < 20).

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I. INTRODUCTION

The flight stability of liquid-filled, spin-stabilized projectiles has been considered for a wide variety of conditions. Originally, theories and experiments were centered about the case of large Reynolds number ($Re = a^2\dot{\phi}/\nu$).[1,2] Many complicated non-steady effects must be considered for practical applications if the Reynolds number is large, for example, spin-up.[3,4,5] However, if the Reynolds number is very low ($Re < 50$), the effects of these unsteady processes may be neglected. It is then possible to employ an incompressible, fully viscous finite-difference solution to the Navier-Stokes equations for a fixed precession angle and steady rates of spin and precession. Pressures have been measured under these conditions and can be used to validate numerical simulations.[6,7]

This report reviews available methods for calculating the behavior of low Re payloads. Detailed comparisons between two finite-difference simulations are made. Conclusions and recommendations as to the accuracy and computing time for all methods are presented.

II. BACKGROUND

Two such steady state, incompressible Navier-Stokes (SS-INS) codes have been developed. Initially, Vaughn and co-workers[8] developed an explicit code for the steady-state solution of the dimensional Navier-Stokes equations using an iterative finite-difference procedure and Chorin's method of artificial compressibility. This code (SAND) employs uniform numerical grids in the radial, axial, and azimuthal directions and central finite-differences. Recently, Strikwerda and co-workers[9] have developed an implicit code for the steady state solution of the non-dimensional Navier-Stokes equations using an iterative finite-difference method based on modified line successive-over-relaxation (LSOR) and a pressure update from the gradient of the velocity field. This code (UWISC) employs non-uniform grids to better resolve the velocity and pressure near the cylinder walls, central finite-differences in

[1] *Murphy, C.H., "Angular Motion of a Spinning Projectile with a Viscous Liquid Payload," ARBRL-MR-03194, U.S. Army Ballistic Research Laboratory, Aberdeen Proving Ground, Maryland, August 1982. (AD A118676) Also Journal of Guidance, Control, and Dynamics, Vol. 6, pp.280-286, July-August 1983.*

[2] *Gerber, N. and Sedney, R. "Moment on a Liquid-Filled Spinning and Nutating Projectile: Solid Body Rotation," ARBRL-TR-02470, U.S. Army Ballistic Research Laboratory, Aberdeen Proving Ground, Maryland, Februcry 1983. (AD A125332)*

[3] *Murphy, C. H., "Moment Induced by a Liquid Payload During Spin-Up Without a Critical Layer," ARBRL-TR-02581, U.S. Army Ballistic Research Laboratory, Aberdeen Proving Ground, Maryland, August 1984. (AD A145716) Also Journal of Guidance, Control, and Dynamics, Vol. 8, No. 3, pp.354-359, May-June 1985.*

[4] *Gerber N., "Liquid Moment on a Filled Coning Cylinder During Spin-Up: Ad Hoc Model," ARBRL-TR-2528, U.S. Army Ballistic Research Laboratory, Aberdeen Proving Ground, Maryland, December 1984. (AD 150280)*

[5] *D'Amico W. P., "Flight Data on Liquid-Filled Shell for Spin-Up Instabilities," ARBRL-MR-03334, U.S. Army Ballistic Research Laboratory, Aberdeen Proving Ground, Maryland, February 1984. (AD 139136) Also AIAA Paper 83-2143, August 1983.*

[6] *Nusca M. J., D'Amico W. P., and Beima, W. G., "Pressure Measurements in a Rapidly Rotating and Coning, Highly Viscous Fluid," ARBRL-MR-03325, U.S. Army Ballistic Research Laboratory, Aberdeen Proving Ground, Maryland, November 1983. (AD A136224)*

[7] *Hepner, D. J., et. al. "Internal Pressure Measurements for a Liquid Payload at Low Reynolds Numbers." U.S. Army Ballistic Research Laboratory, Aberdeen Proving Ground, Maryland, BRL Memorandum Report in preparation.*

[8] *Vaughn, H. R., Oberkampf, W., and Wolfe, W. R., "Fluid Motion Inside a Spinning Nutating Cylinder," Journal of Fluid Mechanics Vol. 150, pp. 121-138, 1985. Also "Numerical Solution for a Spinning, Nutating Fluid-Filled Cylinder," Sandia Report SAND 83-1789, December 1983.*

[9] *Strikwerda, J. C., and Nagel, Y. M., "A Numerical Study of Flow in Spinning and Coning Cylinders," CRDC-SP-86007, Proceedings of the 1985 Scientific Conference on Chemical Defense Research, Aberdeen Proving Ground, Maryland, April 1986.*

the radial and axial directions and pseudo-spectral differencing to represent the azimuthal dependence. These codes have been used in the present effort to obtain solutions for a wide range of dimensional and non-dimensional parameters.

In addition to finite-difference solutions of the incompressible Navier-Stokes equations, analytical solutions to the linearized equations are available. Herbert[10] has developed a simple model of the viscous flow in a cylinder of infinite length that is spinning and nutating at a small yaw angle. Due to the infinite cylinder assumption of Herbert, the theory has been found to agree with UWISC in liquid roll moment coefficient for aspect ratios in excess of 3 (Figure 1). For the liquid side moment coefficient, Herbert's assumption of zero pressure force in the linearized equations has yielded inaccurate results. However, since a relationship between liquid roll and side moment coefficients has been established,[11] Herbert's theory can be used as a quick and effective tool for initial estimates. Recently, Herbert has developed a spectral collocation method[12] for the Navier-Stokes equations and a finite length cylinder. Results for liquid yaw (side) moment are in good agreement with the UWISC code (see Figure 12 of Reference 12).

A spatial eigenvalue method has recently been developed by Hall, Sedney, and Gerber.[13] The Navier-Stokes equations are written in an inertial reference frame and reduced to a set of linear partial differential equations. The angle of coning motion is assumed small and only linear departures from solid body rotation are considered. To obtain boundary conditions, the motion of the cylinder walls is imposed from the projectile motion which is proportional to $e^{i(ft-\theta)}$ where t is time and f is τ, the non-dimensional coning frequency for pure coning motion. The flow variables (perturbed velocity components and pressure) are then proportional to $e^{i(ft-\theta)}$. A particular solution was employed which satisfies axial and lateral wall boundary conditions but not endwall conditions. The eigenvalue problem is defined using a separation of variables technique from which an infinite sequence of complex eigenvalues is generated. The eigenvalues are determined by an iterative process for which sufficiently accurate initial estimates are required for convergence. The flow variables are expressed as eigenfunction expansions with the coefficients determined by satisfying the endwall boundary conditions; a least squares and collocation method have been used for this purpose.

Comparisons of measured liquid moment coefficients with spatial eigenvalue and UWISC results have shown the consistency of both methods. However, since spatial eigenvalue methods yield results in significantly less computer run time, they are perhaps the preferred scheme. Figure 2 shows a comparison of endwall pressure coefficient from UWISC, SAND and the spatial eigenvalue method with experimental values.[7] Considerable discrepancy is noted for the SAND code while the UWISC and spatial eigenvalue results fall within experimental accuracy.

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[10] *Herbert, T., "On the Viscous Roll Moment in a Spinning and Nutating Cylinder," CRDC-SP-86007, Proceedings of the 1984 Scientific Conference on Chemical Defense Research, Aberdeen Proving Ground, Maryland, April 1985.*

[11] *Murphy, C. H., "A Relation Between Liquid Roll Moment and Liquid Side Moment," Journal of Guidance, Control and Dynamics, Vol. 8, No. 2, pp. 287-288, March-April 1985. (See also ARBRL-MR-03347, U.S. Army Ballistic Research Laboratory, Aberdeen Proving Ground, Maryland, April 1984. (AD A140658))*

[12] *Herbert, T., "Numerical Study of the Flow in a Spinning and Nutating Cylinder," AIAA-87-1445, Proceeding of the 19th AIAA Fluid Dynamics, Plasma Dynamics and Lasers Conference, Honolulu, Hawaii, 8-10 June 1987.*

[13] *Hall, P., Sedney, R., and Gerber, N., "Fluid Motion in Spinning, Coning Cylinder via Spatial Eigenfunction Expansions," ARBRL-TR-2818, U.S. Army Ballistic Research Laboratory, Aberdeen Proving Ground, Maryland, August 1987.*

A finite element method has recently been applied to the steady three-dimensional flow in a spinning and nutating cylinder[14] using the software package FIDAP. FIDAP is a commerically available, general purpose code for the solution of incompressible fluid flow problems governed by the Navier-Stokes equations. As is the case for finite-difference codes, the incompressible continuity equation is augmented by the pressure, which has a small coefficient ($\sim 10^{-6}$) called a penalty parameter. The study in Reference 14, used approximately 2200 elements over the entire cylinder with 16 nodes on the sidewall boundary. In the general finite element method the velocity vector is approximated on each element by a simple polynomial function, but in this case the velocities were approximated by linear interpolating functions. A Galerkin method (or weighted residuals) was used to reduce the Navier-Stokes and continuity equations, together with the boundary conditions, to a system of nonlinear algebraic equations. These were solved by a quasi-Newton iterative technique. The code used the aeroballistic reference frame (non-inertial) for steady flow computations.

Figure 3 shows a series of comparisons between UWISC and FIDAP for $c/a = 1.042$, a coning angle of one degree, coning frequencies ranging from 0.0385 to 0.0909, and Reynolds numbers ranging from 5 to 25. FIDAP data were obtained from Dr. Simon Rosenblat of Fluid Dynamics International. Proper conversions have been employed to express all dimensionless quantities in the inertial reference frame. For Reynolds numbers ranging from 5 to 25, both the UWISC and FIDAP codes show consistency between roll and side moment coefficients, and code to code agreement is good. A constant difference of about 9.2% in the results of UWISC and FIDAP was found over all Reynolds numbers and coning frequencies considered.

A reasonable range of dimensionless parameters must be prescribed to relate the computed outputs for code to code comparisons. For the present work, the physical geometry is restricted to that of a completely filled, right circular cylinder. Dimensional analysis and linear theories [1,2] indicate that the liquid moment coefficients (roll and yaw (normally called side)) will depend upon the following dimensionless groups:

$$Linear\ Liquid\ Moment\ Coeff. = F[Re, c/a, \tau, \dot{K}_c/\dot{\phi}] \tag{1}$$

Given the use of a SS-INS code that retains the nonlinear terms, then the liquid moment coefficient will also depend upon the precession angle ($K_c = \sin\alpha_c$). However, a steady state code would require $\dot{K}_c = 0$. Hence, the present codes would yield a dependence as follows:

$$SS - INS\ Nonlinear\ Liquid\ Moment\ Coeff. = F[Re, c/a, \tau, K_c] \tag{2}$$

The case for low Re should also follow this formulation and will be examined using the dimensionless groups as guides. It is highly possible that the liquid moment coefficients are linearly related to α_c for $\alpha_c < 20$ degrees. If this is the case, then two of the remaining

[14] *Rosenblat, S., Gooding, A., and Engleman, M. S., "Finite Element Calculations of Viscoelastic Fluid Flow in a Spinning and Nutating Cylinder," CRDEC-CR-87021, Chemical Research, Development and Engineering Center, Aberdeen Proving Ground, Maryland, December 1986.*

three parameters can be held constant, while the behavior of the liquid moment coefficient upon the third parameter can be explicitly shown.

Murphy [11] suggested the use of roll and side moment coefficients for small, fixed precession angles, α_c, defined below:

$$Roll\ Moment = m_L a^2 \dot{\phi}^2 [C_{LRM_O} + \tau K_c^2 C_{LRM}] \tag{3}$$

$$Transverse\ Moment = m_L a^2 \dot{\phi}^2 \tau [C_{LSM} + iC_{LIM}] K_c e^{i\phi_c} \tag{4}$$

where,

m_L	is the mass of liquid in a fully-filled container
a	is the maximum radius of the container
$\dot{\phi}$	is the spin rate of the container in the inertial frame
τ	is the ratio of coning rate to spin, $\dot{\phi}_c/\dot{\phi}$
C_{LRM}	is the steady-state liquid roll moment coefficient due to coning motion
C_{LRM_O}	is the liquid roll moment coefficient due to transient liquid spinup
C_{LSM}	is the liquid side moment coefficient
C_{LIM}	is the liquid in-plane moment coefficient
K_c	is $\sin\alpha_c$, where α_c is the precession angle
ϕ_c	is the phase angle of the coning motion

Further, Reference 11 gives a relationship between the moment coefficients for the linearized, viscous Navier-Stokes equations. Hence, for small precession angles and all Reynolds numbers,

$$C_{LRM} = -C_{LSM} \tag{5}$$

These definitions for the liquid moment coefficients will be used to scale computed results and can provide for comparisons with various sources of experimental data.

III. FINITE-DIFFERENCE METHODS

The Navier-Stokes equations are written in a non-inertial coordinate system. Descriptions of the two SS-INS codes (SAND and UWISC) are provided in References 8 and 9 and will not be repeated here in detail. The fundamental approaches of the two codes are similar, however the solution algorithms are different (see Section II). The computed pressure and velocity fields are input to sub-programs that compute the liquid-induced moments. The SAND code computes the pressures and shear stresses (using a second-order finite-difference on velocity) at the lateral and end walls and then integrates these over the entire cylinder. The UWISC code forms an expression for the angular momentum of the liquid. This expression is differentiated with respect to time in order to determine the resultant liquid torques.

The SS-INS codes required the use of a coordinate system where steady solutions would exist. Both UWISC and SAND use the same precessing coordinate system. Equa-

tions 1 thru 5 are defined with respect to the inertial spin rate $\dot{\phi}$. Both the UWISC and SAND codes used components of the intertial spin rate to describe program inputs. The inertial spin, $\dot{\phi}$, is the sum of two component angular velocities: $\dot{\phi} = \dot{\phi}_p + \dot{\phi}_c \cos\alpha_c$. The UWISC and SAND codes define the dimensionless variables using $\dot{\phi}_p$ rather than $\dot{\phi}$; hence, the notation for the dimensionless groups must be augmented with a subscript p for the precessing frame (Re_p, τ_p, C_{LRM_p}, and C_{LSM_p}). For the results presented in this report, proper conversion has been made so that all dimensionless groups are defined with respect to the inertial frame.

Operational differences between the two codes will be discussed. The UWISC code was executed on the Ballistic Research Laboratory (BRL), Launch and Flight Division (LFD) VAX 8600, while due to longer CPU times, the SAND code was run on the BRL CRAY-XMP. TABLE 1 documents nominal running times for "converged" solutions. Although different computing machines were used, run times are expressed in VAX CPU units with 1 hour of CRAY-XMP time equivalenced to 10.6 hours of VAX 8600 CPU units (the code was not altered to take advantage of CRAY vector processing). The UWISC code uses a typical convergence criterion based upon the Laplacian of the solution, while the SAND code simply recommends that additional iterations be computed to assure that the output reflects a "steady state" solution.

Table 1: Typical VAX 8600 CPU Times (Hrs) for Solution Convergence.

Code	Grid Points (r, θ, z)	Total Grid Points	Reynolds Number 10	20	30	40
UWISC	11,12,42	5,544	0.738	0.642	0.621	0.654
*SAND**	11,24,21	5,544	2.100	2.090	2.090	2.090
*Reflects approximately 30,000 iterations per solution for all cases						

One important difference between the codes is that the SAND code utilizes dimensional inputs, whereas, the UWISC code calls for Re_p, τ_p, c/a, and α_c. A series of computer runs were made to establish dimensional consistency for the SAND code. In these test cases, the dimensional variables were chosen such that the dimensionless variables did not change. Hence, the moment coefficients should not change. The results are shown in TABLE 2.

The computed results indicate an inconsistency within C_{LSM}. C_{LRM} involves only the viscous stresses, while C_{LSM} is derived from both viscous and pressure terms. Most likely, the computed pressure values or the integration of these pressures along the cylinder walls are in error in the SAND code.

Both the UWISC and SAND codes have been extensively used to compute C_{LRM} and C_{LSM} values for $.6 < Re < 45$, $.03 < \tau < .5$, and $1 < c/a < 5.2$. Figure 4 shows a typical comparison of C_{LRM} and C_{LSM} between the codes. The C_{LSM} values for SAND have not been included in light of the results of TABLE 2. The UWISC code is consistent for C_{LRM} and C_{LSM} values. Agreement for C_{LRM} values between the codes is shown.

Table 2: Dimensional Consistency Verification for SAND

ϕ (Hz)	ϕ_c (Hz)	Viscosity (cs)	$-C_{LRM}$	C_{LSM}
Case A ($Re = 10$, $c/a = 4.32$, $\tau = 0.091$, $\alpha_e = 2°$)				
50.0	4.55	1.08×10^5	0.0237	0.0417
78.4	7.13	1.70×10^5	0.0237	0.0391
100.0	9.10	2.17×10^5	0.0237	0.0387
200.0	18.20	4.34×10^5	0.0237	0.0310
Case B ($Re = 21.5$, $c/a = 1.042$, $\tau = 0.0797$, $\alpha_e = 2°$)				
50.0	3.99	5.91×10^4	0.0232	0.0306
100.0	7.97	1.18×10^5	0.0232	0.0308
200.0	15.94	2.36×10^5	0.0232	0.0312
Case C ($Re = 20.0$, $c/a = 5.20$, $\tau = 0.087$, $\alpha_e = 2°$)				
50.0	4.35	4.15×10^4	0.0296	0.0422
120.5	10.48	1.00×10^5	0.0296	0.0498
200.0	17.40	1.66×10^5	0.0296	0.0505

1. CODE LIMITATIONS.

The UWISC code was tested over a wide range of non-dimensional coning frequencies and Reynolds numbers to identify code limitations. Figure 5 shows the results for a Reynolds number of 10, cylinder aspect ratio of 3 and a range of coning frequencies. The results from UWISC are compared to results from the spatial eigenvalue method described in section I. The agreement between computational methods for $0 < \tau < 1$ is good. However, the UWISC code is not able to achieve solutions for $\tau = 0$ or $\tau \geq 1$. For $\tau = 0$ the current version of the code is inoperable since the Navier-Stokes equations have been divided by τ. Solutions generated for small values of τ near zero can be extrapolated to $\tau = 0$, but the code can be modified to accept $\tau = 0$ input. For $\tau \geq 1$ the code would not converge for a wide range of relaxation and convergence acceleration parameters. It is not clear that this limitation in the numerical scheme can be corrected.

Figure 6 shows UWISC results for a cylinder aspect ratio of 1.486, $.04 < \tau < .1$, and values of Reynolds number to 300. From a computational standpoint this does not represent a Re limitation in the UWISC code. Solutions for higher Re can be achieved using solutions for lower Re as initial conditions. However, the SAND code is limited to a maximum Re of about 120 [8] and this limitation may be a result of the explicit solution algorithm. The spatial eigenvalue method, described in section I, has been run for these Re on a VAX 8600 mini-computer with run times between 1 and 4 cpu minutes. In addition, the spatial eigenvalue method has been efficiently run for Re as large as 3000, with no apparent Re limitation.

The real limitation in achieving high Re solutions using UWISC is the computational time required to reach a steady state solution. Figure 7 shows the average run time per τ for the cases plotted in the previous figure. For $Re = 300$ the run time on a CRAY XMP/48 (without using vector processing) was 7 hrs. In this case the solution at $Re = 200$ was used as an initial guess. Steady state results for larger Re, while achievable using the solution algorithm, may be prohibitive in light of the computer run time required to reach them. In this regard, the spatial eigenvalue method that runs very efficiently on the VAX 8600

and CRAY computers, would be the preferred choice.

Several additional factors should also be considered. The results of Figure 6 were achieved using a single computational grid for which the grid clustering near the cylinder walls was increased in accordance with the wall viscous gradients (i.e. the Reynolds number). No attempt was made to find the number or distribution of grid points that minimized run time while preserving the accuracy of the solution. In fact, the number of grid points used in each case was chosen to exceed that required for a unique solution. A strict convergence tolerance ($\epsilon = 1\text{x}10^{-6}$) was used so that the computational results could evaluate the predicted relation $-C_{LRM} = C_{LSM}$, to within 1% (on a VAX 8600 minicomputer solutions were achieved with far less run time but with a larger value for ϵ). In addition, fixed values of the relaxation and convergence acceleration parameters were used for each Re: there was no attempt to find the most efficient choice of these parameters. It is highly possible that computer run time for a single case ($Re, \tau, c/a$) could be reduced after a host of preliminary runs had been made to optimize these factors. In practice, the choice of relaxation and acceleration parameters are made based upon experience, since the uniqueness of the solution is independent of these choices. The computer run time, however, is dependent on the particular values of these parameters.

2. NONLINEAR EFFECTS AT LARGE PRECESSION ANGLES.

TABLE 3 shows the ratios of computed values of the moment coefficients for precession angles of 2 and 20 degrees. From computed data, both codes indicate that nonlinear effects for $Re \leq 20$ are relatively weak. Hence, the use of a linear method, such as the spatial eigenvalue approach, can be used with confidence. An additional interpretation can be made with respect to Equation (5): $-C_{LRM} \simeq C_{LSM}$ for precession angles less than 20 degrees if $Re \leq 20$.

Table 3. Importance of Nonlinear Effects

Case	Code	C_{LSM} (20°)/C_{LSM} (2°)	C_{LRM} (20°)/C_{LRM} (2°)
$Re = 20.0$, $c/a = 5.20$, $\tau = .087$	UWISC	0.941	1.000
	SAND	0.965	1.004
$Re = 10.0$, $c/a = 4.32$, $\tau = .091$	UWISC	0.944	1.003
	SAND	0.966	1.004
$Re = 21.5$, $c/a = 1.486$, $\tau = .072$	UWISC	0.941	1.000
	SAND	0.950	1.000
$Re = 21.5$, $c/a = 1.042$, $\tau = .079$	UWISC	0.944	1.004
	SAND	0.958	1.004

IV. CONCLUSIONS

Two incompressible, steady state, fully viscous Navier-Stokes codes (SAND and UWISC) have been compared to each other. The following conclusions have been reached to date:

1. SAND is not dimensionally consistent for yaw moment predictions.
2. UWISC and SAND are consistent for roll moment predictions.
3. UWISC verifies that for small Re and precession angles of 2°, $C_{LRM} \simeq -C_{LSM}$.
4. For coning frequency, UWISC is limited to $\mid \tau \mid < 1$
5. Without optimizing input to the solution algorithm, UWISC code computer run times are prohibitive for steady state solutions above $Re = 300$.
6. Linear methods can be confidently and efficiently used since non-linear effects have been shown to be small.

Acknowledgments

The author is indebted to H. Vaughn, W. Oberkampf, and W. Wolfe of Sandia National Laboratories, Albuquerque, New Mexico, and J. Strikwerda and Y. Nagel of the Center for Mathematical Sciences, University of Wisconsin, for the use of their codes and many discussions. The technical and advisory contributions of Dr. William P. D'Amico of the Launch and Flight Division, USABRL, are also greatly appreciated.

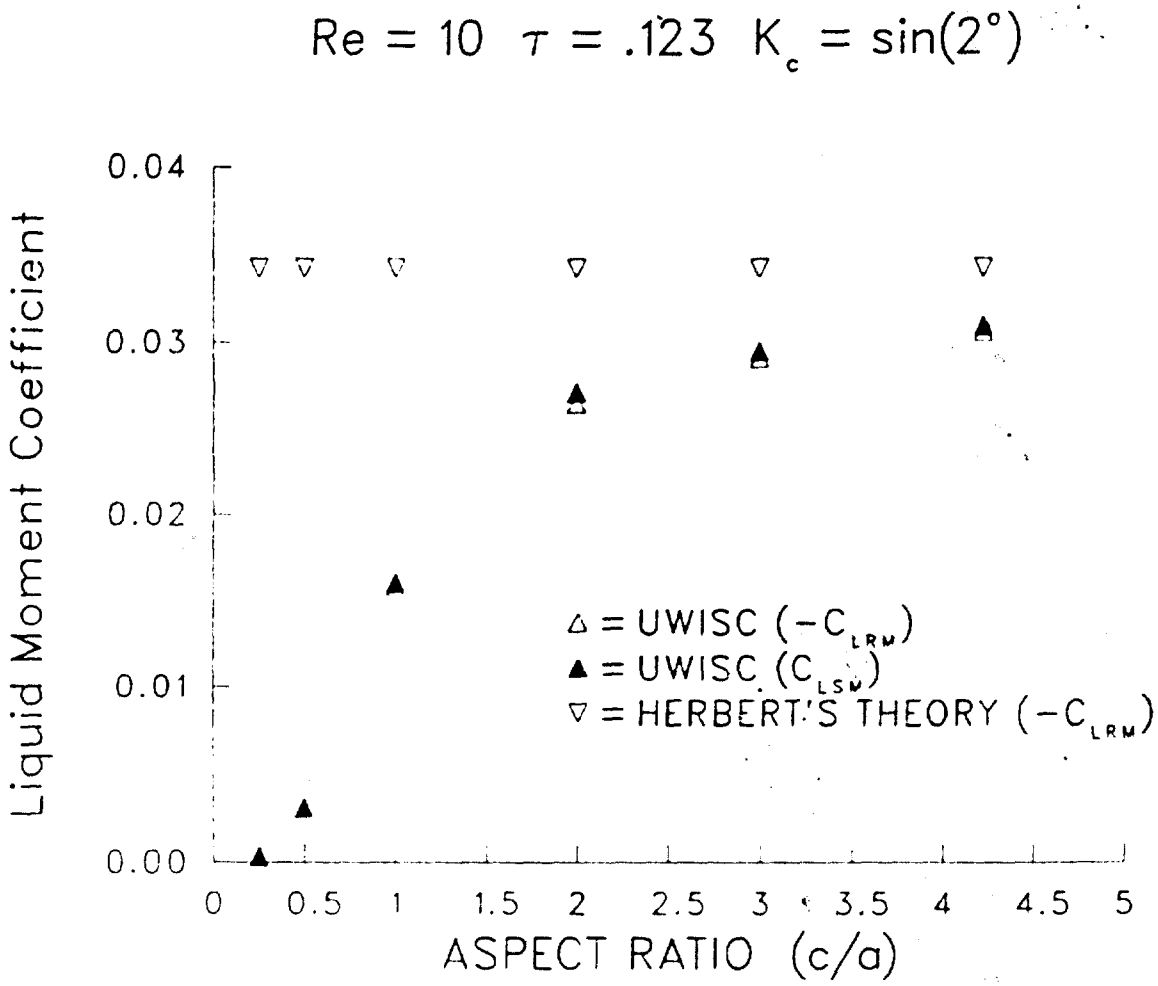

Figure 1: Comparison of C_{LRM} and C_{LSM} from UWISC code with Herbert's theory ($Re = 10$).

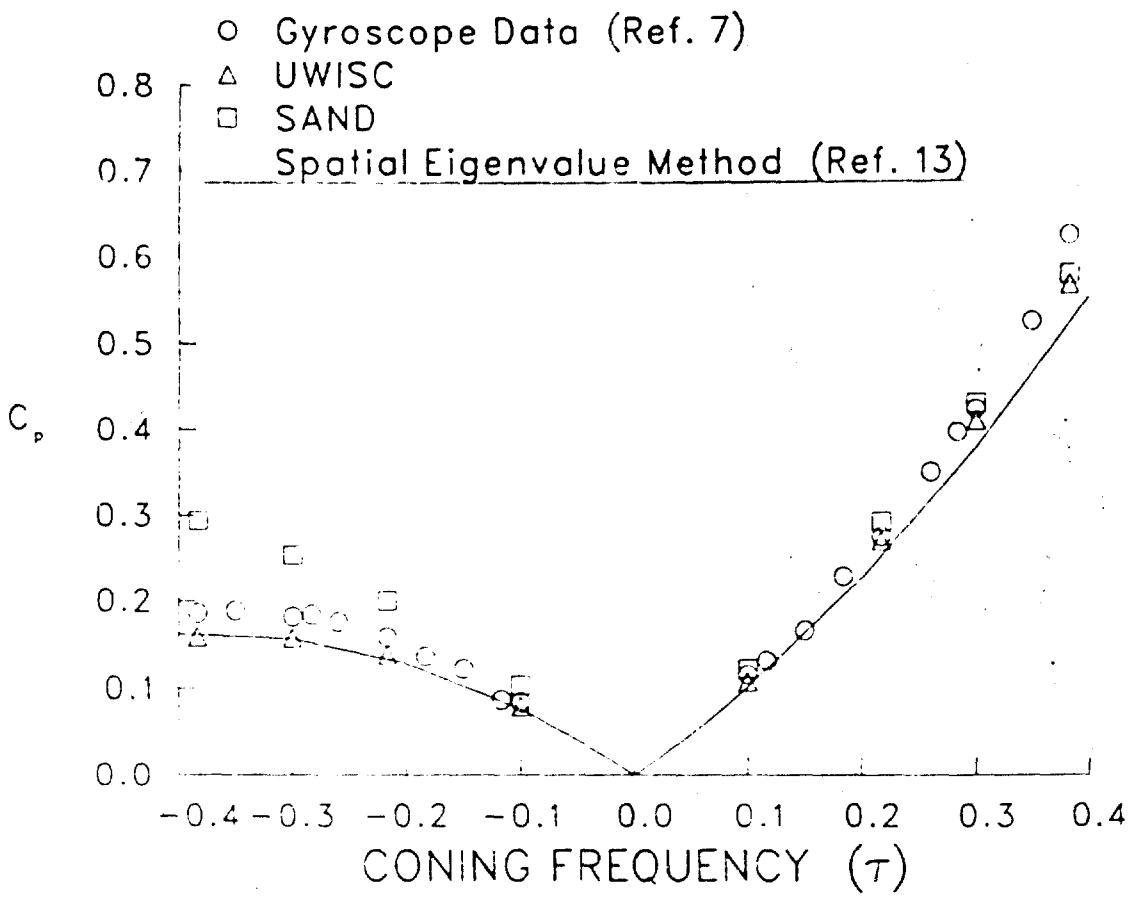

Figure 2: Cylinder endwall pressure coefficient. $Re = 3.1$, $c/a = 3.148$, $\alpha_c = 2^o$, $r = 0.667$.

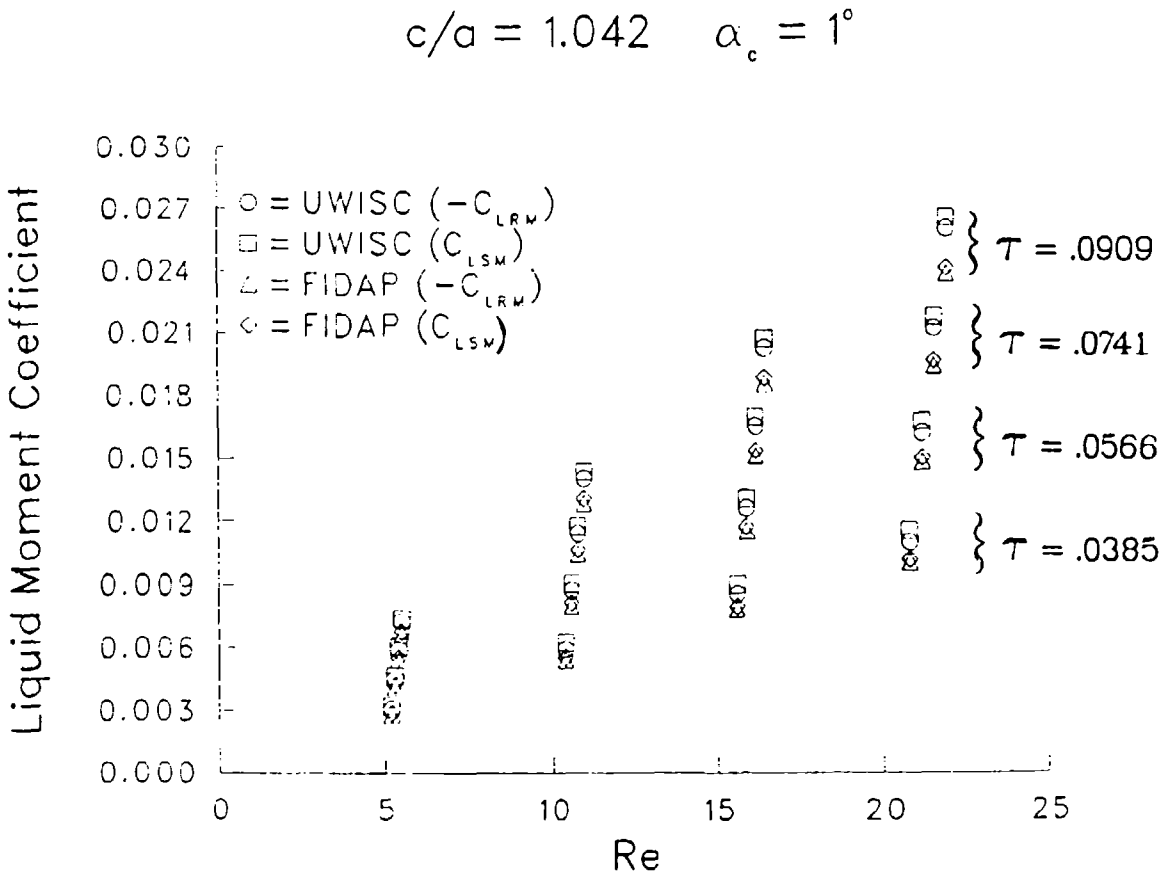

Figure 3: Comparison of C_{LRM} and C_{LSM} from UWISC code with FIDAP code ($c/a = 1.042$).

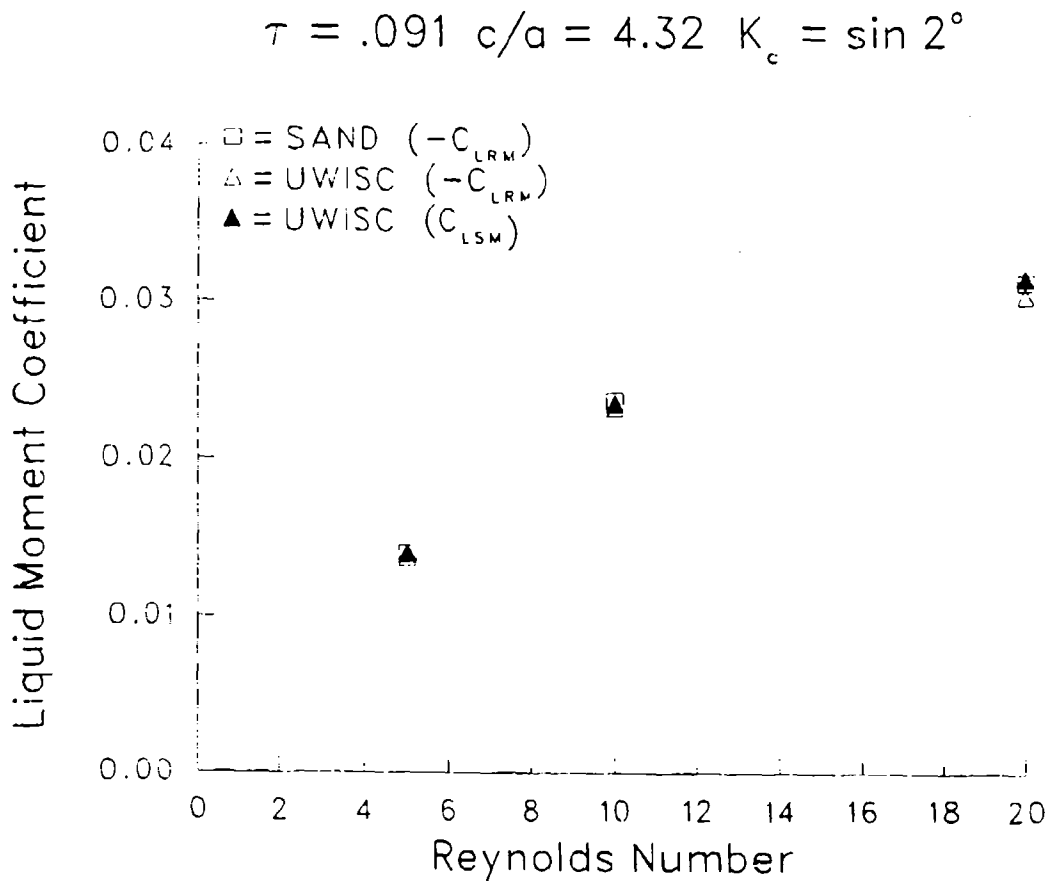

Figure 4: Comparison of C_{LRM} and C_{LSM} from UWISC code with SAND code ($c/a = 4.32$).

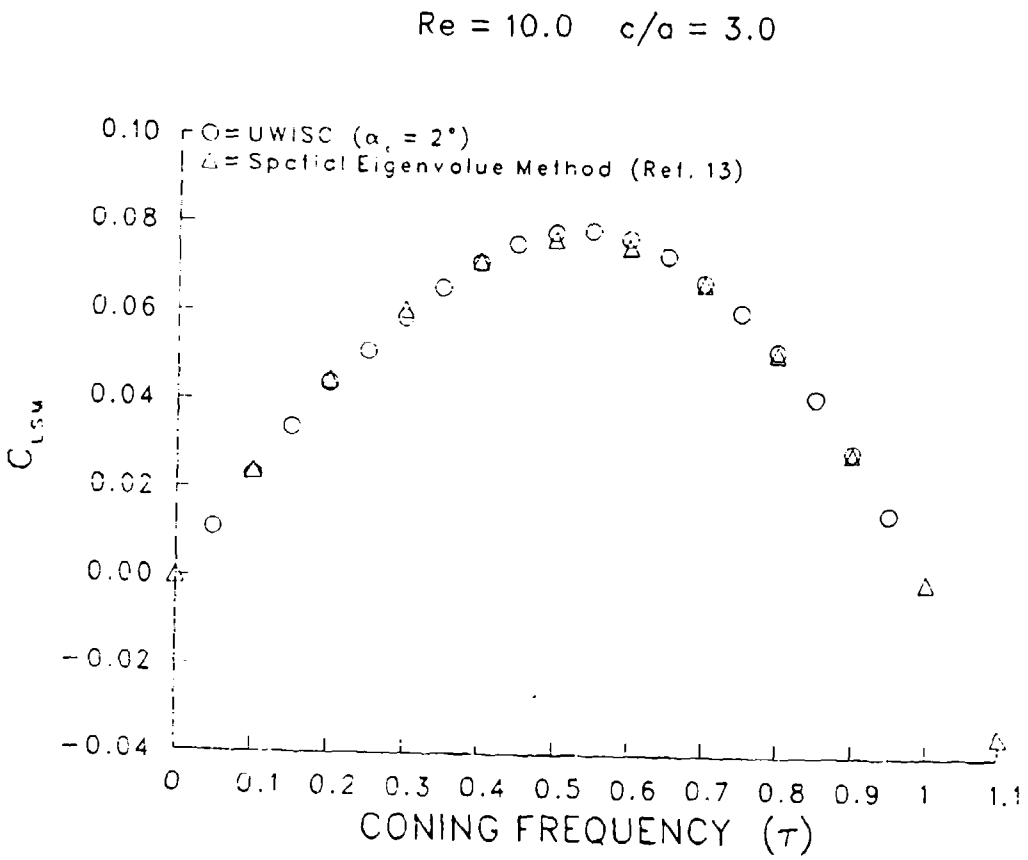

Figure 5: Comparison of C_{LSM} from UWISC code (α_c = 2°) with spatial eigenvalue method, Re = 10, c/a = 3.0.

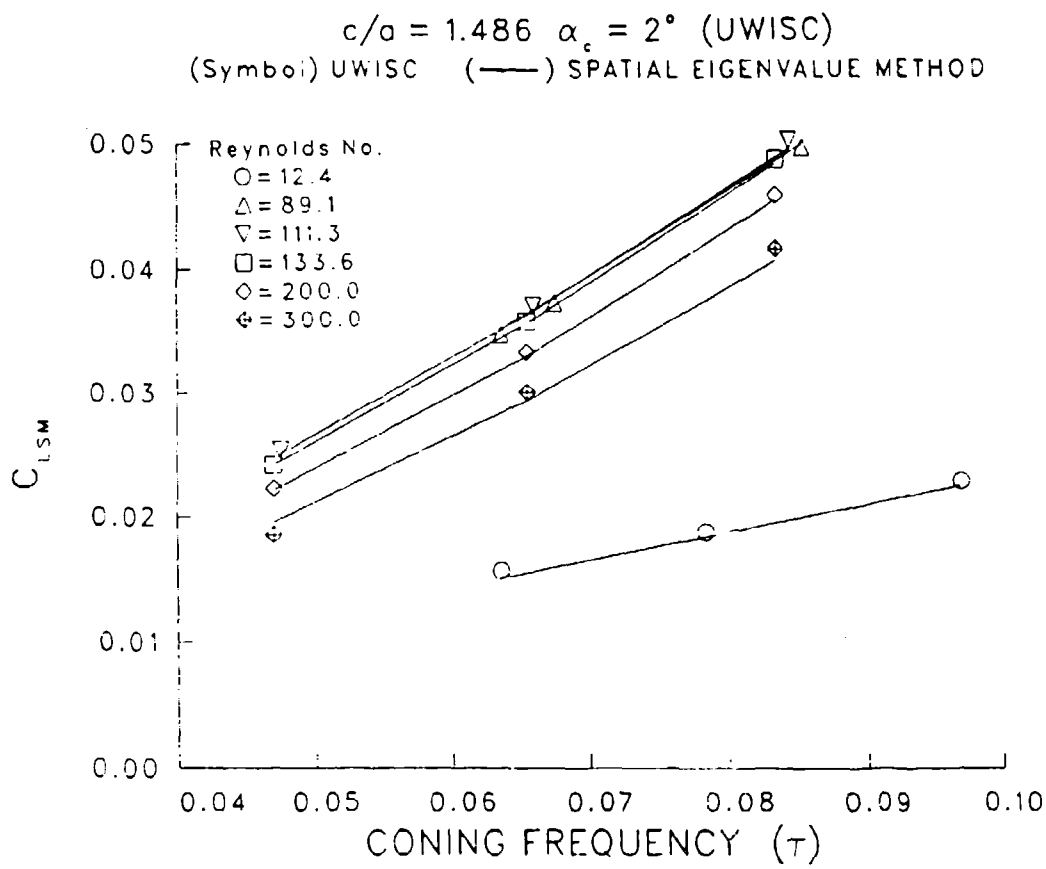

Figure 6: C_{LSM} results from UWISC code ($\alpha_c = 2°$) and Spatial Eigenvalue Method for $c/a = 1.486$, $.04 < \tau < .1$, $12 < Re < 300$.

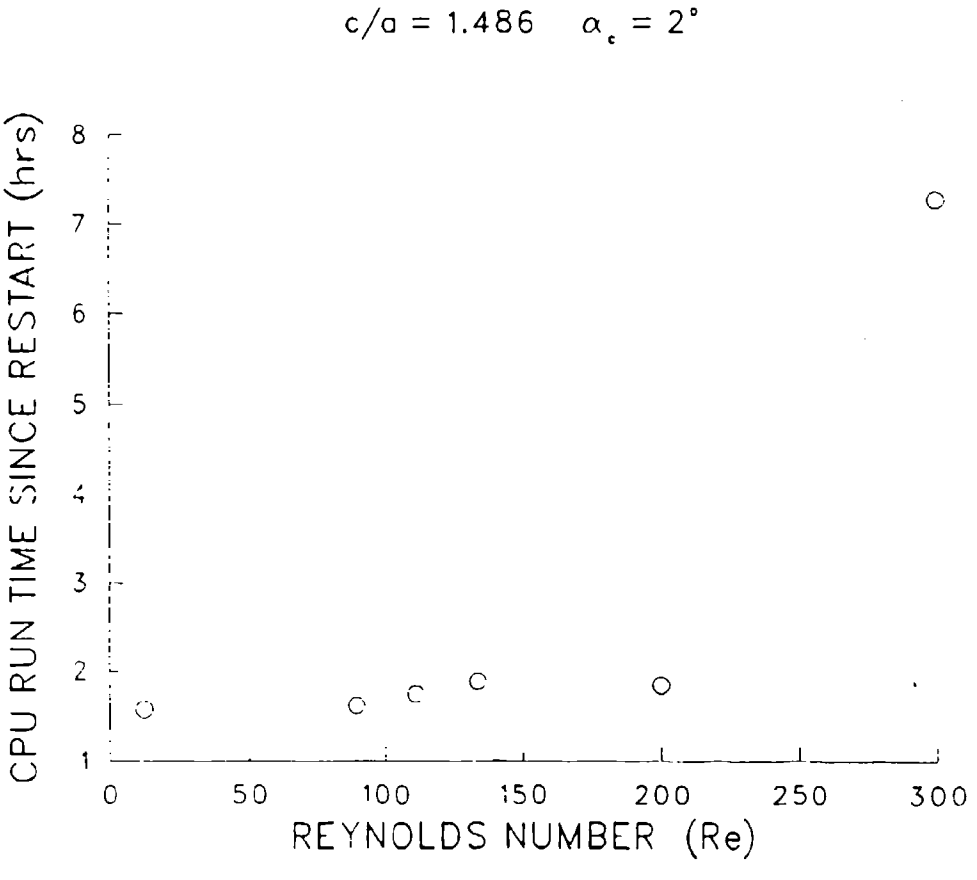

Figure 7: UWISC code computer run time for $c/a = 1.486$, $\alpha_c = 2°$, $12 < Re < 300$.

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List of Symbols

a	Maximum radius of the container
c	Half-height of the container
c/a	Aspect ratio of the container
C_{LIM}	Liquid in-plane moment coefficient (see Eq. 4)
C_{LRM}	Liquid roll moment coefficient (see Eq. 3) using inertial spin rate ($\dot{\phi}$)
C_{LRM_p}	Liquid roll moment coefficient using non-inertial spin rate ($\dot{\phi}_p$)
C_{LRM_O}	Liquid roll moment coefficient due solely to spin (see Eq. 3)
C_{LSM}	Liquid side moment coefficient (see Eq. 3) using inertial spin rate ($\dot{\phi}$)
C_{LSM_p}	Liquid side moment coefficient using non-inertial spin rate ($\dot{\phi}_p$)
C_p	Nondimensional pressure coefficient, $C_p = \frac{P}{\alpha_c \rho a^2 \dot{\phi}^2}$
K_c	$\sin(\alpha_c)$
m_L	Mass of the liquid fill
r	Radial direction normalized by a
Re	Reynolds number ($\dot{\phi}a^2/\nu$) using inertial spin rate
Re_p	Reynolds number using non-inertial spin rate
z	axial direction normalized by a
α_c	Precession angle
ϵ	Convergence tolerance for UWISC code
ν	Kinematic viscosity of the liquid fill
ρ	Density of the liquid fill
τ	Ratio of coning rate to inertial spin rate ($\dot{\phi}_c/\dot{\phi}$)
τ_p	Ratio of coning rate to non-inertial spin rate ($\dot{\phi}_c/\dot{\phi}_p$)
ϕ	Circumferential angle
$\dot{\phi}$	Spin rate of the container in the inertial frame ($\dot{\phi}_p + \dot{\phi}_c \cos\alpha_c$)
$\dot{\phi}_c$	Coning rate of the container
$\dot{\phi}_p$	Eulerian spin rate

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