A PRACTICAL GUIDE TO THE DESIGN OF RHOMBIC AERIALS

by

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SUMMARY

This Memorandum meets the need for practical advice on the design and assessment of the likely performance of the rhombic aerial. No new theoretical approach has been made but basic principles of the rhombic are explained and design charts of a new type are presented. These, compiled using two simple computer programs, offer a considerable time saving over design procedures used until now.

Examples of aerials designed using these charts are given together with polar diagrams of their theoretical performance.
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INTRODUCTION

Much has been published on the subject of rhombic aerials. The original paper appeared in 1935 \(^1\), and it is not difficult to locate ten textbooks published subsequently that deal with the subject in greater or lesser detail. Few of these sources assist a potential user to decide whether a rhombic or some alternative form of aerial would be the most suitable to construct and use in a particular application. Confusion and some frustration then arise from a number of areas: the large number of different symbols, and even names, employed by various authors for the parameters involved: the very involved and complicated approach of some sources of data: and the lack of simple, straightforward means of determining the expected performance, particularly polar diagrams and gains, of an aerial subsequent to its design. In general it is true to say that for many of the formulae used easily made approximations are not possible; calculations are tedious and the graphical methods suggested\(^2,3\) difficult to follow.

Work within RN4 on a proposed meteor scatter link led the author to investigate the design and performance of rhombic aerials and to write computer programs to overcome the drawbacks and deficiencies noted above, and so quickly arrive at a suitable design of a known performance. It then became clear that a simple guide to the design of rhombic aerials to meet specific needs would be useful; no new theoretical approach was necessary, only the understanding of the basic working of rhombics, culled and condensed from as much literature as was found helpful. This Memorandum is the result. Practical construction details can rely on standard rigging practice and are not dealt with here.

2 RHOMBIC AERIAL DESIGN

2.1 Principles

2.1.1 The rhombic is usually employed in the HF and low VHF bands where good directivity (beamwidth less than 20\(^\circ\)) and high gain (greater than 10 dB) are required, and where open ground of moderate to good conductivity is available. Rhombics are wide band, covering a 2:1 range of frequencies. Less stringent requirements or lack of space would indicate the use of simple half wave dipoles, log periodics or vertical monopoles at HF; above about 70 MHz Yagi aerials are often more suitable.

A rhombic aerial is based on the use of four long wires terminated so that the current distribution is non-resonant. The polar diagram of such a non-resonant wire in free space depends on its length; two examples for wires of two and eight wavelengths and of negligible attenuation are given in Fig 1. (In these diagrams the very small lobes that exist to the left of the Y axis have been omitted for clarity.) Note that the main (and secondary) lobes shown are directed away from the source and towards the terminating load. Note also that the longer the wire the sharper the main lobe, and that the total number of lobes on either side of the wire is equal to the length of the wire in wavelengths. Because the largest secondary lobe is only some 5-6 dB below that of the main lobe the usefulness of the long wire aerial is very restricted. The angle which the main lobe is tilted with respect to the wire varies with the length, in the manner summarised in Fig 2. This effect has led to the name 'tilt angle' being used to refer to half the angle formed at the broad apex of a rhombic aerial. The tilt angle is not the angle which the aerial makes with the ground, to which it is normally made parallel.
The diamond (or rhombus) arrangement ensures that the directional characteristics of the four individual wires reinforce each other (Fig 3), so reducing the relative magnitudes of the secondary lobes and giving greater directivity and gain, as well as providing a convenient means of terminating the system to obtain the non-resonant condition. The free space vertical polar diagram through the major axis resulting from such an arrangement, in a typical case, is in Fig 4 (dashed line). Only the upper half of this has been drawn; it is symmetrical about the x axis.

In practice the presence of an earth, having conductivity and dielectric constant which differ from place to place and perhaps with season and weather conditions, will extensively modify the aerial's polar diagram. For simplicity the assumption is often made that the earth is perfect, i.e. it is smooth, plane and of infinite extent and conductivity. This is valid for horizontal polarization over fairly good terrain or better; any error introduced by this assumption is less at the lower elevation angles. Since rhombics have no response to vertical polarisation for signals on the major axis, and little response in other directions, throughout this Memorandum the presence of a perfect earth is assumed.

The effect of a perfect earth on the directional characteristics in the vertical plane of an aerial can be studied by the method of images. For a horizontal aerial, energy radiated (or arriving) is reflected as shown in Fig 5, so that the total field in any direction is the vector sum of the direct and reflected wave. The field actually existing in the presence of a perfect earth is exactly the same field that would be produced by the joint action of the actual aerial and its image with the earth removed. This is of great importance, for if the reflected wave is of comparable intensity to the direct ray there will be a doubling of field strength (6 dB increase) in those directions where the two rays are in phase and a cancellation (zero field) in those directions where they are in anti-phase. For horizontally polarised aerials the field in the presence of a perfect earth is modified by the factor

\[ 2 \sin \left( 2\pi \frac{H}{\lambda} \sin \theta \right) \]

where \( H \) is the aerial's height

\( \theta \) is the angle of elevation above the horizontal.

The form of this expression is also shown in Fig 4 (solid line) together with the total resultant field of the aerial (chain line).

If the presence of an imperfect earth is required to be taken into account, Smith gives an expression for this. However, as is demonstrated in Appendix A, for horizontal polarisation very little difference is found between perfect, good and poor ground, so that the assumption throughout this Memorandum that a perfect ground exists is justified.

2.1.2 The prospective designer of a rhombic aerial has at his disposal the choice of leg length, \( L \), the tilt angle, \( \gamma \), and the height, \( H \), of the aerial above the ground. These dimensions may be chosen so that one of three types of design results:
(i) The maximum possible output is obtained at the desired elevation angle, $\theta$.

(ii) The peak of the main lobe in the vertical plane is aligned at the desired elevation angle.

(iii) Some compromise is made, either in choice of $L$ or $H$, or perhaps both. Some worked examples later in the Memorandum will illustrate these points.

The alignment design has the peak of its main lobe at the desired elevation angle; a greater output at that elevation angle is obtained from the maximum output design, even though its peak output is at some other elevation angle always lower than the elevation angle designed for. No other rhombic aerial will give as great an output as this latter design. Fig 6 shows a comparison of maximum output and alignment designs.

The question of whether a maximum output or alignment design is the more appropriate deserves careful consideration. If for any reason the wave direction is unstable— as is usually the case for signals propagated by the ionosphere—then alignment of the principle lobe with the mean wave direction becomes necessary. On the other hand, it may be that, at the higher frequencies, the need to override receiver noise by the largest possible margin is paramount, in which case a maximum output design is indicated. Note that a rhombic aerial used on a path over which ionospheric propagation predominates provides some measure of space diversity due to its size of a few wavelengths.

2.2 Design equations

2.2.1 Equations for selecting values for the first two cases referred to above are, from Kraus:

Maximum output

- $H = \frac{1}{4 \sin \alpha} \text{ wavelengths}$ (1)
- $L = \frac{0.5}{2 \sin^2 \alpha} \text{ wavelengths}$ (2)
- $\gamma = 90 - \alpha \text{ degrees}$ (3)

Alignment

- $H = \frac{1}{4 \sin \alpha} \text{ wavelengths}$ (4)
- $L = \frac{0.371}{\sin^2 \alpha} \text{ wavelengths}$ (5)
- $\gamma = 90 - \alpha \text{ degrees}$ (6)

Neither of the first two designs are likely to be suitable for use at elevation angles of less than about $12^\circ$ for several reasons. Firstly, equations (2) and (5) indicate that at low elevation angles $L$ will be very large; and it is undesirable to use leg lengths of greater than eight wavelengths since this can give rise to a large number of side lobes and a rather sharp main lobe in both planes as Fig 7 illustrates. This is probably too narrow for any ionospheric path whose reflection point is varying.
Very large tracts of ground are often also required. There will be practical problems too in constructing an aerial at the large height called for by equations (1) and (4) for the lower elevation angles at the low (long wavelength) end of the HF band. Finally, although gain increases with leg length, the rate of such increase diminishes as legs become longer than about ten wavelengths; section 3.1.2 deals with this last point in greater detail. Fortunately, it is possible to compensate for reduced leg length by changing the tilt angle, and for reduced height by increasing the leg length (if there is room), or again, by changing the tilt angle. If both length and height are a problem, it may be possible to change the tilt angle to produce alignment. The modifications give a compromise design with reduced gain and large amplitude secondary lobes as the penalties, as will be demonstrated and discussed in section 4.

2.2.2 Equations for these compromise designs, again from Kraus, are:

**Reduced height**

\[
\gamma = 90 - \alpha \text{ degrees}
\]

\[
L' = \frac{\tan(L' \sin^2 \alpha)}{\sin \alpha} \left[ \frac{1}{2 \pi \sin \alpha \tan(2\pi H' \sin \alpha)} \right] \text{ wavelengths} \tag{7}
\]

**Reduced length**

\[
H' = \frac{1}{4 \sin \alpha} \text{ wavelengths} \tag{8}
\]

\[
\gamma = \sin^{-1} \left[ \frac{L' - 0.371}{L' \cos \alpha} \right] \tag{9}
\]

where \( L' \) is in wavelengths.

**Reduced length, reduced height.** Solve for \( \gamma \):

\[
\frac{H'}{\sin \gamma \tan \alpha \tan(2\pi H' \sin \alpha)} = \frac{1}{2\pi \psi} - \frac{L'}{\tan(\psi L')} \tag{10}
\]

where \( H' \) and \( L' \) are in wavelengths

and \( \psi = 1 - \sin \gamma \cos \alpha \).

Care should be exercised that the appropriate distinction is drawn between those quantities normally expressed in degrees and those taken to be in radians. Examples of the use of some of these equations are in the Appendices. A rhombic aerial so designed will have the main lobe of its vertical polar diagram aligned with the desired elevation angle.

2.3 Design charts for reduced length aerials

A saving in time and effort can result if published design charts such as those in Refs 7 and 8 are used. The aerial height is decided solely from the elevation angle requirement; the values given by the charts agree with equation (1), that is \( H = 1/(4 \sin \alpha) \).
No compromise height design is possible using these charts. Normally a leg length thought to be suitable is chosen and the corresponding tilt angle read off. Thus there is an infinite variety of choices available; leg length should usually be not more than about eight wavelengths (for the reasons discussed previously in section 2.2.1), and not less than about two wavelengths since this produces inadequate directivity of the main lobe. The tilt angles so obtained for a particular leg length will be found to be different from that given by the equations of section 2.2.2. This is because the published charts generate a maximum output not an alignment design.

2.4 Design chart for reduced height, increased length aerials

This section introduces a design chart in which the tilt angle is kept at the complement of the elevation angle (as in maximum output and alignment designs) and a reduction in height compensated for by an increase in leg length. The equations given under 'reduced height' in section 2.2.2 apply in this case, and although the elevation angle, $\alpha$, appears in them, the resultant design graph, Fig 8, applies to all elevation angles. The principal curve is a plot of fractional length against fractional height, where these two terms are defined as that fraction of the length, or height, of the full length, full height, alignment design. Also shown in the figure is variation in gain, relative to that for a full height, full length alignment design aerial. Although a 3 dB reduction does not occur until the height has been reduced to about 40% of the full height, large amplitude secondary lobes may appear with smaller height reductions.

The use of this design chart is dealt with in section 4.2.

Aerials designed for low elevation angles using this procedure may prove difficult or impossible to construct for two reasons. Firstly, leg lengths greater than eight wavelengths may be called for; this is undesirable for the reasons given in section 2.2.1, and secondly, the large tilt angles required result in long, narrow aerials with a very sharp apex at either end, which could prove awkward to fabricate.

2.5 Design charts for reduced height, reduced length aerials

The equations for these designs, given in section 2.2.2, can lead to many wearisome calculations before suitable dimensions are arrived at; the gain may then be found to be insufficient and some relaxation in one of the constraints (length or height) becomes necessary. The process is then repeated. So far as is known at present, no easily used design charts exist to permit the prospective user to shorten this tedious procedure. Accordingly, the computer program written for rapidly arriving at polar diagrams (to be referred to in section 3.2) was used, in a slightly modified form, to generate for a particular fractional height*, curves of tilt angle against fractional length*, with elevation angle as parameter. A number of such graphs deal with different fractional heights. Figs 9, 10 and 11 are, respectively, for fractional heights of 0.6, 0.8 and, for completeness, 1.0. This last graph is, of course, for full height reduced length designs. These are not the same however as those produced by the published design charts referred to in section 2.3, which were for maximum output designs; the curves introduced here are for alignment designs.

* As defined in section 2.4.
Note that this is a new class of reduced height designs, which does not, as in designs given by Fig 8, make the tilt angle the complement of the elevation angle and then increases the length to compensate; rather, both the length and the height are reduced, compared to a full alignment design, with the tilt angle reduced to compensate. Absence of increased length and more easily fabricated tilt angles may make this type of reduced height aerial easier to construct, for small elevation angles, than the more usual reduced height design dealt with in the previous section; thus the charts, introduced here, provide for all compromise designs for alignment of the main lobe.

Each set of curves has superimposed upon it the 3 dB contour; this is the locus of values of L' at which the gain has fallen by 3 dB compared to a full length design. It is perhaps though surprising that gain falls so little as the length reduces. However, as will be dealt with later (section 3.1), the gain of a rhombic is proportional to its breadth (Fig 3); from the curves of Figs 9 to 11, as the length is reduced, so the tilt angle falls, maintaining the breadth almost constant until short lengths are reached. The curves have been drawn dotted where secondary lobes (in either plane) are within 6 dB of the amplitude of the main lobe. With this dual indication of those areas in which the compromises made give rise to possible undesirable characteristics, the designer is guided within the considerable latitude granted to him by the charts in his choice of height and leg length.

An example of aerials designed using these charts is dealt with in sections 4.1 and 4.3 with relevant discussion in section 4.4.

2.6 Radiation resistance; feeders

2.6.1 Radiation resistance is defined by Terman as "that quantity which, when multiplied by the square of the average current in the wire will equal the radiated power". For a rhombic aerial with length and breadth greater than a wavelength, this is

$$R_R = 240 \left[ \log_e (4\pi L \cos^2 \gamma) + 0.577 \right]$$

(11)

where $L$ is in wavelengths.

This gives values of between 570 and 800 ohms in most practical cases; the appropriate value of non-reactive resistor should be used as a terminating resistor. Terman also notes that it is possible to modify minor lobes in the rear of the directional pattern by altering the magnitude or phase angle, or both, of the termination. However, since $R_R$ does not appear in any of the expressions for field strength (except as a constant multiplying factor), it has not been possible to verify this by calculation.

Lewin in giving his derivation of this simplified expression for $R_R$ (as well as that for the somewhat complicated full expression where all lengths and breadths are considered), notes that a horizontal long wire aerial has a radiation resistance close to its free space value only when its height is greater than about 0.35\lambda. He goes on to say (without proof) that it is reasonable to expect a rhombic to be similar in this respect. Caution must be exercised, nevertheless, on this account in the calculation of radiation resistance, and any value of gain derived from it.
Losses in an imperfect earth do not appreciably affect the radiation resistance.  

2.6.2 The characteristic impedance of a rhombic is not quite constant with frequency due to the varying separation between the wires. This variation can be reduced by constructing the rhombic of two or three separate wires for each side, with the wires spread at the side corners and converging at the input apex and the opposite apex. The advantages that accrue include a reduction in the change of characteristic impedance along the length of the aerial, a slight reduction in the value of the characteristic impedance, a slight reduction in termination loss and a smoothing out of impedance variations across the frequency band. These effects are probably more important for transmission than for reception.  

2.6.3 It is often convenient to assume that a rhombic has a characteristic resistance of 600 ohms, so that open wire transmission line may be used as a feeder. Such open wire line, however well designed, constructed and maintained will inevitably radiate (or pick up) unwanted signals. Consequently good quality co-axial cable, preferably buried, should be used where circumstances permit, as in reception and low power transmission (eg less than about 1 kW). When such a co-axial feeder is employed, then some form of balun will be needed; these are available commercially. For reception there are now available low cost high performance integrated circuit RF amplifiers capable of providing input resistances of hundreds of ohms and output resistances suitable for feeding coaxial cable. Since it is fairly easy to wind low loss baluns, using ferrite toroids, of 1:1 or 4:1 impedance ratio, it should be possible to include a balun plus amplifier combination as a compact wideband head amplifier. This will enable the effect of any feeder losses to be counteracted.

3 RHOMBIC AERIAL PERFORMANCE  

3.1 Gain  

3.1.1 Rhombics are usually employed in preference to half-wave dipoles and similar simple aerials because of their gain and directivity. Typically, gains relative to isotropic of 12 to 22 dB can be obtained.  

The field intensity given by a rhombic aerial over a perfectly conducting flat earth, at a distance of 1 km is:

\[ F_R = 15081 \sin \frac{\sin \alpha \sin \beta}{\sqrt{m_1}} \sqrt{m_2} \]  

where \( m_1 \) and \( m_2 \) are \( \pi L [1 - \cos \alpha \sin (\gamma + \phi)] \) and the other symbols are defined in Fig 3 and section 2.1.2.  

The field intensity given by a half-wave dipole in its equatorial plane in free space is:

\[ F_D = \frac{601 D}{r} \]  

where \( r \) is distance in metres.
For a distance of 1 km this becomes

\[ F_D = 60 I_D \text{ mV/m} \]

For the same input power to the two aerials the ratio of their currents is:

\[ \frac{I_R}{I_D} = \left( \frac{R_D}{R_R} \right)^{\frac{1}{4}} = \left( \frac{73}{R_R} \right)^{\frac{1}{4}} = \frac{8.544}{\sqrt{8R_R}} \]  

(12)

where \( R_R \) is given by equation (12).

Thus the voltage gain of a rhombic above a perfect earth relative to a half-wave dipole in free space is:

\[ G = \frac{1508}{60} \frac{8.544}{\sqrt{R_R}} \left( L \sin(2\pi H \sin \alpha) \cos \gamma \frac{\sin m_1}{\sqrt{m_1}} \frac{\sin m_2}{\sqrt{m_2}} \right) \]

or in decibels

\[ G = 20 \log_{10} \left[ \frac{214.74}{\sqrt{R_R}} \left( L \sin(2\pi H \sin \alpha) \cos \gamma \frac{\sin m_1}{\sqrt{m_1}} \frac{\sin m_2}{\sqrt{m_2}} \right) \right]. \]  

(13)

This expression ignores losses in the terminating resistor; such losses are of the order of 2-3 dB. Aerial gains are conventionally quoted relative to an isotropic radiator, and since a half-wave dipole has a gain, on this basis, of 2.2 dB, the gain equation derived above can, with little error, be considered to be the gain of a rhombic aerial over a perfectly conducting earth with respect to an isotropic radiator. It will often be sufficiently accurate to assume \( R_R \) is 600 ohms to reduce the labour of calculation.

3.1.2 In any compromise design, there is a very large number of choices available for the values of \( L \) and \( \gamma \), as the design charts show, with the smaller values of \( L \) corresponding to smaller \( \gamma \). However, it is interesting to note that for a rhombic at a given height, its gain is proportional to its breadth, \( B \), and that, from Fig 3, \( B = 2L \cos \gamma \). Fig 12 illustrates the effect of this with a typical full height, reduced length rhombic designed using the charts introduced in section 2.5, with fractional height \( H' = 1.0 \). Gain over isotropic, \( G \), in decibels and the breadth \( B \) (plotted logarithmically for compatibility) are plotted against leg length, \( L \), in wavelengths. As \( L \) is increased, so \( G \) and \( B \) increase synchronously. Note how little the gain increases as \( L \) is increased beyond about 10\( \lambda \); section 2.2.1 gave other reasons for restricting leg lengths to about eight wavelengths or less. This graph is applicable to the aerial with length 4\( \lambda \) referred to in section 3.3.1; it has a gain of 20.1 dBi.

The preceding paragraph applies, it should be remembered, to aerials at a fixed height. In general if the height is reduced then gain will fall, despite increases made in leg lengths to compensate. Conversely, increase of height, up to a maximum of \( H = 1/(4 \sin \alpha) \) will generally give greater gain. Of course factors other than gain, such
as secondary lobe amplitude and available land and masts, may be of overriding importance.

3.2 Directivity

3.2.1 The shape of the vertical polar diagram of any rhombic aerial is a very important consideration in assessing its performance and suitability. While it is a relatively simple matter to arrange for the horizontal pattern to be aligned with the desired direction (by suitably locating the four supporting masts), the direction and magnitude of lobes in the vertical plane depends on height, leg length and tilt angle. Kraus⁶ (quoting the original authors¹), gives for the relative field intensity of a rhombic aerial over a perfectly conducting flat earth:

\[
F_R = \cos \gamma \left[ \sin(2\pi H \sin \alpha) \right] \frac{\left[ \sin^2(2\pi L) \right]}{2\psi}
\]

where \( \psi = 1 - \sin \gamma \cos \alpha \).

It is evident from examination of this and other expressions for field strength given previously (section 3.1) that "the work involved in the preparation of polar and power distribution diagrams is extremely tedious and laborious"¹³. The present day ready availability of small computers with graphics capability has considerably improved the position with regard to speed of calculation, presentation and accuracy since that statement was made in 1955 - and indeed Ref 13 is now known to contain some small errors in the location and amplitude of secondary lobes. In connection with the meteor scatter work mentioned in the introduction a computer program, RHUMB2 was written for an HP85 computer; it was based on the equation given above. The computer generates a polar plot of field strength in the vertical plane passing through the main axis of the rhombic, \( \psi \) it takes \( \gamma = 0 \). All the vertical plane polar plots presented here were obtained in this way. Although they have not been scaled in absolute terms, they do provide a simple means of rapid assessment of the vertical pattern, and, if a protractor is employed, of checking that the main lobe lies at the desired vertical angle.

3.2.2 The shape of the horizontal pattern will of course depend on the elevation angle to which it refers. A second computer program, RHOMB4, generates a polar plot of the field strength at a given elevation angle; it was used to obtain all the horizontal plane polar plots presented here.

3.3 Frequency dependence

3.3.1 Where a rhombic is required to work over a band of frequencies it should be designed so that the tilt angle is optimum near the high frequency end of the band. This recommendation is made for two reasons: firstly, at frequencies much greater than the design value, the main lobe in the horizontal plane sharpens and then splits; secondly, at the higher frequencies a small change in tilt angle has a considerable effect on the shape of the vertical polar diagram. The fall in performance as the frequency is reduced is not serious until a frequency nearly half the design frequency is reached. Thus a properly designed rhombic is considered to cover a band of about 2:1.
Fig. 13 and 14 show the performance of a typical rhombic over a wide band of frequencies. The design frequency was 20 MHz for an elevation angle, \( \alpha \), of 80.3°. The aerial was a full height reduced length design obtained by using Fig 11 in the manner dealt with in greater detail in section 4.1. The height required is 1.738\( \lambda \) with a tilt angle of 66°.5 corresponding to the choice of 4\( \lambda \) for the leg length. As the working frequency is reduced, thereby effectively lowering the height in terms of wavelength, the gain falls and and the elevation angle of the main lobe increases, while in the horizontal plane at 80.3° elevation angle, the main lobe broadens. At frequencies greater than 20 MHz the main lobe elevation angle falls, as is to be expected, and a multitude of secondary lobes appear, while in the horizontal plane secondary lobes increase in number and amplitude relative to the main lobe. At twice the design frequencies, i.e. 40 MHz, there is a null at 80.3° elevation angle; a further increase in frequency shows a splitting of the main lobe and a large number of quite narrow lobes appear. The working frequency range of this particular aerial could be considered to be about 12 MHz to 25 MHz.

3.3.2 The increase in elevation angle of the main lobe with reduction in frequency can sometimes be a useful feature in long distance HF working in two ways. Lower frequencies, in use at night, will (because of the increase in the height of the F region during darkness) take off and arrive at somewhat higher elevation angles than the higher, daytime, frequencies. Furthermore, lower frequencies, unlike the higher, may propagate by means of multi-hop reflections by day and by night, so giving rise to greater elevation angles at both transmitter and receiver. Both these effects tend to increase the useful working bandwidth of a rhombic.

4 TYPICAL DESIGNS

4.1 Reduced length design

The meteor scatter link referred to in the introduction called for the use of a high gain aerial in the 70 MHz band over a link giving an elevation angle of 80.3° at the receiver. Appendix B shows the calculations to arrive at the dimensions for an alignment design, and also for a typical reduced length design. This latter and others of the same type are more easily obtained by using the design chart of Fig 11, which is a plot of tilt angle against fractional length for a fractional height of unity, i.e. full height. To use this chart, the leg length of a full, alignment design is needed (already obtained in this example by using equation (5)). By choosing a leg length thought to be convenient, usually between 2\( \lambda \) and 8\( \lambda \), and calculating the corresponding fractional length, the required value of tilt angle can then be read off, interpolating for \( \alpha = 80.3° \). Table I summarises the results in which values of gain have also been included, calculated by the method illustrated in Appendix B.
Table 1

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<td>66°.5</td>
<td>19.9</td>
</tr>
<tr>
<td>0.337</td>
<td>6</td>
<td>71°.5</td>
<td>21.6</td>
</tr>
<tr>
<td>0.449</td>
<td>8</td>
<td>74°.5</td>
<td>22.7</td>
</tr>
<tr>
<td>1.000</td>
<td>17.8</td>
<td>81°.7</td>
<td>25.0</td>
</tr>
</tbody>
</table>

The height, at 1.732$\lambda$ in each case, is 7.37 metres or 24 ft 2 in, which is quite practicable. The figures quoted refer to alignment designs, but as noted in section 2.1 a greater output can be obtained if the tilt angle is changed slightly. The best value of tilt angle in this case was quickly established, using program RHOMB4, as being 64$\degree$ for a leg length of 4$\lambda$. The design charts of Refs 7 and 8 also gave this value. An increase in gain of 0.5 dB results from this move to a maximum output design. The performance of this design in the vertical and horizontal planes is shown in Figs 15 and 16. Fig 16 is a power distribution (or 'onion') diagram, produced as a result of a number of RHOMB4 runs; it shows contours of 3, 6, 9 and 15 dB below maximum output, which occurs at 8°. The shape and size of the main and principal secondary lobe are well illustrated with the dashed lines showing the locus of null response.

4.2 Reduced height design

The authors of Ref 1 give an example of a reduced height rhombic. The elevation angle required is 17°.5 for which the optimum height is 0.831$\lambda$ (see Appendix C). A maximum output design would have a leg length of 5.53$\lambda$ and a tilt angle of 72°.5; the corresponding values for an alignment design would be 4.10$\lambda$ and 72°.5. Can the height be reduced without too much penalty?

The reduced height selected by the original authors was 0.5$\lambda$, which is 0.6 of the full height; Appendix C gives the calculations to arrive at a value of 5.15$\lambda$ for the leg length. As before (section 4.1) this could much more easily have been obtained from a design chart, in this case the reduced height increased length chart of Fig 8; the original alignment design leg length of 4.10$\lambda$, when multiplied by 1.26, the value of $L'/L$ corresponding to a fractional height of 0.6, gives 5.15$\lambda$. The tilt angle remains at 72°.5.

4.3 Reduced height reduced length design

It is of interest to see how well the design requirement of the last section can be met by a reduced height reduced length design, making use of the design charts introduced in section 2.5.

First, the values of full length, $L$ and full height, $H$ for an alignment design are obtained. In this case they have already been calculated in section 4.2 and
Appendix C; they are 4.10λ and 0.831λ respectively. With a fractional height of 0.8, Fig 10 shows that there is a large number of designs from which to choose, with quite large length reductions possible. A further reduction in height, with fractional height 0.6 (Fig 9), gives a restricted choice with the small values of $L'$ introducing larger secondary lobes. Some designs considered worthwhile are summarised in Table 2.

Table 2
Reduced height, reduced length designs

<table>
<thead>
<tr>
<th>$H'/H$ wavelengths</th>
<th>$L'/L$ wavelengths</th>
<th>$Y$ degrees</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.8</td>
<td>0.66</td>
<td>0.7</td>
</tr>
<tr>
<td>0.8</td>
<td>0.66</td>
<td>0.5</td>
</tr>
<tr>
<td>0.6</td>
<td>0.5</td>
<td>0.9</td>
</tr>
<tr>
<td>0.6</td>
<td>0.5</td>
<td>0.7</td>
</tr>
</tbody>
</table>

4.4 Discussion

4.4.1 Section 4.1 produced a design for a reduced length aerial working at an elevation angle of 8.3°. It is of interest to compare its polar diagrams (Fig 15) with those of a corresponding full size alignment design (Fig 7). (Note that the scale of reproduction of Fig 15 is twice that of Fig 7 to preserve detail.) The beamwidth in each plane for the full design is about half that of the reduced length design and is probably too small for any path relying on a propagation medium such as the ionosphere, where the reflection point is variable. The reduced length design offers other advantages too: there is a considerable saving in land and the angles formed at the corners of the aerial are more easily constructed and maintained. Conversely, secondary lobes are larger and the main lobe gain is about 8 dB less than for the full size aerial.

4.4.2 Figs 17 and 18 contain, respectively, polar diagrams in the vertical and horizontal planes for a range of alternative designs intended for use at an elevation angle of 17°.5. They are shown in the same order as they were presented in sections 4.2 and 4.3, viz full alignment: reduced height, increased length: and four reduced height reduced length designs.

(i) If maximum signal pick-up is called for, and the maximum output design is not appropriate for reasons such as discussed in section 2.1.2, then the alignment design, case (a), is indicated, although the reduced height reduced length design of (c) is almost as good. There is in fact little to choose from the point of view of sensitivity between all the designs illustrated; overall gain variation, from best to worst case, is only 2.4 dB.

(ii) Secondary lobes are always present. The amplitudes of the largest are between 5 dB and 7 dB below the main lobe in the horizontal plane but in
the vertical plane even the worst case is 9.5 dB down, with the best (c) 14.5 dB down and at a very high elevation angle.

(iii) Beamwidths (for 3 dB reduction) in the vertical plane range from $13^\circ$ (b) to $17^\circ$ (d), and in the horizontal plane from $14^\circ$ (e and f) to $19^\circ$ (a).

If interference is likely to be a problem then the direction, in either plane, of secondary lobes and intermediate nulls could be decisive, but otherwise it is unlikely that any of the factors mentioned will be of over-riding importance. The required design could very well be selected from those showing such a worthwhile reduction in height and land area, the final choice depending on practical engineering matters such as availability of appropriate masts and open space.

5 VALIDATION

5.1 Some of the references$^1,2,6$ contain polar diagrams, both in the vertical and horizontal planes, of actual aerials, while other authors$^{10,12,14}$ give examples of power distribution diagrams. Using the computer programs described in section 3.2, checks were made of the performance of all aerials for which sufficient directional data were available. In all cases good agreement was found within the limits of graphical accuracy.

5.2 The opportunity arose to make measurements on an existing rhomic aerial designed some time previously. The requirements were for the main lobe to be at an elevation angle of $28^\circ$ at about 60 MHz. Comparison of the measured vertical polar diagram with that predicted by RHOMB2 showed excellent agreement above about $20^\circ$; below that angle the aerial gave an output greater than expected. This is thought to be due to the proximity of a metal fence, accepted as a potential hazard when the aerial was erected.

6 CONCLUSIONS

The rhombic is a high gain horizontally polarised aerial used mainly at HF and low VHF having good directivity in both the horizontal and vertical planes over at least an octave bandwidth. It is now a straightforward matter to design an aerial to suit particular circumstances using simple formulae and the design charts introduced here; with the wide variety of designs so generated it is likely that dimensions can be chosen to utilise to the best advantage the land area and supporting masts available. The aid of a small computer or programmable calculator enables the prospective user of a rhombic to quickly verify that the chosen dimensions give an acceptable performance in terms of gain, directivity, beamwidth, characteristic impedance and secondary lobe amplitude. Thus in future a rhombic could frequently be the preferred aerial in substitution for those other types which in the past might have been chosen simply because their characteristics were the better known and understood; this will result in the usual improvements to be expected from the use of a high gain directive aerial.
Appendix A

THE EFFECT OF GROUND CONSTANTS ON AERIAL GAIN

A.1 Using the method of images (section 2.1.1 and Fig 5), if the reflected ray has its amplitude reduced by a factor \(\rho\) and its phase delayed by an amount \(\delta\), then the field \(E\) due to the combined rays is

\[
E = E_0 \left[ 1 + \rho \exp \left( j \left( \frac{2\pi d}{\lambda} + \delta \right) \right) \right]
\]

where \(E_0\) is the free space value.

The magnitude of \(E\) is

\[
|E| = E_0 \left[ 1 + \rho^2 + 2 \rho \cos \left( \frac{2\pi d}{\lambda} + \delta \right) \right]^{\frac{1}{2}}
\]

\[
= E_0 \left[ 1 + \rho^2 + 2 \rho \cos \left( \frac{4\pi H}{\lambda} \sin \alpha + \delta \right) \right]^{\frac{1}{2}}
\]

which, in the case of a perfect earth and horizontal polarisation (where \(\rho = 1\) and \(\delta = 2\pi\)), reduces to:

\[
|E| = E_0 \left[ 2 \sin \left( \frac{2\pi H}{\lambda} \sin \alpha \right) \right].
\]

This is the expression given in section 2.1.1 and illustrated in Fig 4.

A.2 As an example consider a rhombic aerial intended for use at 10 MHz where the main lobe is required to be at 17°. The height, calculated from equation (1), is 0.855\(\lambda\). Over a perfect earth, equation (A-2) gives

\[
|E| = E_0 \left[ 2 \sin(2\pi 0.855 \sin 17^\circ) \right]
\]

Thus there is a 6 dB enhancement of the signal at the peak of the main lobe due to the perfect earth.

Now consider good soil of conductivity \(12.10^{-3}\) S and dielectric constant 15, for which Williams\(^{14}\) gives \(\rho = 0.90\) and \(\delta = 176^\circ\) or 3.072 radians. Using equation (A-1)

\[
|E| = E_0 \left[ 1 + (0.90)^2 + 2 \times 0.90 \cos(4\pi 0.855 \sin 17^\circ + 3.072) \right]^{\frac{1}{2}}
\]

\[
= 1.899E_0 .
\]
This is 0.5 dB below the value for a perfect earth. Similarly poor soil of conductivity \(1.10^{-3}\) S and dielectric constant \(S (\rho = 0.77, \varepsilon = 3.072)\) gives an output 1.1 dB below the perfect earth value.

A.3 Higher frequencies and increases in elevation angle give greater reductions.
Appendix B

REDUCED LENGTH DESIGN CALCULATIONS

A rhombic is required to work at an elevation angle, $\alpha$, of 80.3° (section 4.1).

For an alignment design equations (4) and (5) give:

$$H = \frac{1}{4 \sin \alpha} = 1.732\lambda$$

$$L = \frac{0.371}{\sin^2 \alpha} = 17.80\lambda$$

A reduced length design is required; using equation (9) and choosing $L' = 4\lambda$:

$$\gamma = \sin^{-1} \left[ \frac{L' - 0.371}{L' \cos \alpha} \right]$$

$$= \sin^{-1} \left[ \frac{4 - 0.371}{4 \cos 80.3°} \right]$$

$$= 66°.5$$

The height is 1.732\lambda as before.

For the gain, the value of radiation resistance is first required. Equation (11) gives:

$$R_R = 240 \left[ \log_e (4\pi L \cos^2 \gamma) + 0.577 \right]$$

$$= 240 \left[ \log_e (16 \pi \cos^2 66°.5) + 0.577 \right]$$

$$= 637.3 \text{ ohms}$$

The gain, relative to isotropic is, from equation (13):

$$G = 20 \log_{10} \left[ \frac{214.74}{\sqrt{R_R}} \left( \frac{L \sin (2\pi H \sin \alpha) \cos \gamma}{L \frac{m_1 \sin \alpha}{m_2}} \right) \right]$$

where, for $\phi = 0$, $i.e.$ in the principal vertical plane,

$$m_1 = m_2 = \pi L \left[ 1 - \cos \alpha \sin \gamma \right]$$

$$= 4\pi \left[ 1 - \cos 80.3° \sin 66°.5 \right]$$

$$= 1.1630$$
Therefore \[ \sin m_1 = \sin m_2 = 0.9180. \]

Therefore

\[
G = 20 \log_{10} \left[ \frac{214.74}{\sqrt{637.3}} \left( \frac{\sin(1.732.2 \cdot \sin 8^\circ.3) \cos 66^\circ.5 \cdot (0.9180)^2}{1.1630} \right) \right]
\]

\[= 19.9 \text{ dB} \].
Appendix C

REDUCED HEIGHT DESIGN CALCULATIONS

A rhombic is required to work at an elevation angle, \( \alpha \), of 17\( ^\circ \).5 (section 4.2).

For a maximum output design, equations (1), (2) and (3) give:

\[
H = \frac{1}{4 \sin \alpha} = 0.831 \lambda
\]

\[
L = \frac{0.5}{\sin^2 \alpha} = 5.53 \lambda
\]

\[
\gamma = 90 - \alpha = 72^\circ.5
\]

For an alignment design, equations (4), (5) and (6) give:

\[
H = \frac{1}{4 \sin \alpha} = 0.831 \lambda
\]

\[
L = \frac{0.371}{\sin^2 \alpha} = 4.10 \lambda
\]

\[
\gamma = 90 - \alpha = 72^\circ.5
\]

For a reduced height, increased length design, try a height, \( H' \), of 0.5\( \lambda \); thus equation (7) gives:

\[
L' = \tan \left[ \pi L' \sin^2 \alpha \right] \frac{1}{\sin \alpha} \left[ 1 - \frac{H'}{\tan(2\pi H' \sin \alpha)} \right]
\]

\[
= \frac{\tan \left[ \pi L' \sin^2 17^\circ.5 \right]}{\sin 17^\circ.5} \left[ 1 - \frac{0.5}{\tan(2\pi 0.5 \sin 17^\circ.5)} \right]
\]

Therefore

\[
L' = 0.5578 \tan(0.2841 L')
\]

This is best solved by trial, giving

\[
L' = 5.15 \lambda
\]

\[
\gamma = 90 - \alpha = 72^\circ.5
\]
<table>
<thead>
<tr>
<th>No.</th>
<th>Author</th>
<th>Title, etc</th>
</tr>
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<tbody>
<tr>
<td>1</td>
<td>E. Bruce, A.C. Beck, L.R. Lowry</td>
<td>Horizontal rhombic antennas. Proc. IRE 23, No.1, 24-48 (1935)</td>
</tr>
<tr>
<td>2</td>
<td>W.R. Piggott</td>
<td>A method of determining the polar diagrams of long-wire and horizontal rhombic aerials. DSIR, 16 (1948)</td>
</tr>
<tr>
<td>3</td>
<td>R.H. Barker</td>
<td>Rhombic aerial design chart. Wireless Engineer, 25, 361-369 (1948)</td>
</tr>
<tr>
<td>4</td>
<td>International Radio Consultative Committee</td>
<td>Handbook on high frequency directional antennae. 16 and 82, Geneva, ITU (1966)</td>
</tr>
<tr>
<td>9</td>
<td>L. Lewin</td>
<td>Rhombic transmitting aerial. Wireless Engineer, 18, 180-187 (1941)</td>
</tr>
<tr>
<td>11</td>
<td>R.A. Smith</td>
<td>Aerials for metre and decimetre wavelengths. Cambridge, University Press (1949)</td>
</tr>
<tr>
<td>13</td>
<td>International Radio Consultative Committee</td>
<td>CCIR antenna diagrams. 41, Geneva, ITU (1953)</td>
</tr>
</tbody>
</table>
Fig 1 Free space polar diagrams of non-resonant wires 2 and 8 wavelengths long
Fig 2. Tilt angle (angle between the wire and the main lobe) for a non-resonant wire plotted against wire length.
Fig 3 Formation of a rhombic radiation from combination of individual wire patterns

Fig 4 Polar diagrams of a typical rhombic in free space (dashed line) and above a perfect earth (chain line), as modified by the height factor (solid line)
Fig 5 A horizontal aerial and its image

\[ \Delta = 2H \sin \alpha \]

Fig 6 Comparison of (a) maximum output and (b) alignment designs
**Vertical Plane Polar Plot**
for a rhombic aerial
- Leg length (m) 75.76
- Frequency (MHz) 70.4875
- Height (m) 7.37
- Tilt angle (deg) 81.7

**Horizontal Plane Polar Plot**
- Leg length (m) 75.76
- Frequency (MHz) 70.4875
- Tilt angle (deg) 81.7
- Elevation angle (deg) 8.3

Fig 7 A rhombic with very long legs - such as 18 wavelengths, illustrated here - has very narrow lobes in both planes.
Fig 8 Design chart for reduced height increased length rhombics
Fig 9 Design chart for reduced height reduced length rhombics, with elevation angle as parameter. Fractional height 0.6
Fig 10 Design chart for reduced length rhombics, with elevation angle as parameter.
Fig 11 Design chart for reduced height reduced length rhombics, with elevation angle as parameter. Fractional height 1.0
Fig 12  Gain and breadth for a typical family of full height rhombics plotted against leg length
Fig 13 Polar diagrams in the vertical plane for a typical rhombic over the band 10-50 MHz. Design frequency was 20 MHz for an elevation angle of 80°.3
Fig 14

Fig 14 Polar diagrams in the horizontal plane at 80.3 elevation angle for a typical rhombic over the band 10-50 MHz. Design frequency was 20 MHz for an elevation angle of 80.3.
Fig 15 Polar diagrams for a typical reduced length rhombic, fractional length 0.225, tilt angle adjusted to give maximum output.
Fig 16 Power distribution ('onion') diagram for a typical reduced length rhombic, fractional length 0.225, tilt angle adjusted to give maximum output.
Fig 17  Polar diagrams in the vertical plane for a range of alternative rhombic designs intended for use at 20 MHz and an elevation angle of 170.5
Fig 18

Polar diagrams in the horizontal plane for a range of alternative rhombic designs intended for use at 20 MHz and an elevation angle of 170.5°.
This Memorandum meets the need for practical advice on the design and assessment of the likely performance of the rhombic aerial. No new theoretical approach has been made but basic principles of the rhombic are explained and design charts of a new type are presented. These, compiled using two simple computer programs, offer a considerable time saving over design procedures used until now.

Examples of aerials design used these charts are given together with polar diagrams of their theoretical performance.