Low Frequency Ocean Ambient Noise: Measurements and Theory

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**Low Frequency Ocean Ambient Noise: Measurements and Theory**

**Abstract**

Low frequency ocean ambient noise data are reviewed and summarized. The experimental data, both omnidirectional and directional, when not dominated by shipping noise, are shown to suggest wind dependent noise at the low frequencies (<500 Hz). Candidate mechanisms are examined with the result that wave-turbulence interaction at low sea states and collective bubble oscillations at high sea states are identified as possible sources of this sound. A description of the sonic properties of bubbly water is presented for low void fractions consistent with those observed in bubble clouds and plumes produced by breaking waves. A description of the collective bubble-water mixture as the resonant oscillation of a flexible volume with a sonic speed determined by the properties of the mixture is presented.

**Subject Terms**
- Measurements and Theory
- Low Frequency
- Ocean Ambient Noise
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ABSTRACT. Low frequency ocean ambient noise data are reviewed and summarized. The experimental data, both omnidirectional and directional, when not dominated by shipping noise, are shown to suggest wind dependent noise at the low frequencies (<500 Hz). Candidate mechanisms are examined with the result that wave-turbulence interaction at low sea states and collective bubble oscillations at high sea states are identified as possible sources of this sound. A description of the sonic properties of bubbly water is presented for low void fractions consistent with those observed in bubble clouds and plumes produced by breaking waves. A description of the collective bubble-water mixture as the resonant oscillation of a flexible volume with a sonic speed determined by the properties of the mixture is presented.

INTRODUCTION

The interaction of the wind with the ocean surface has long been recognized as a major source of acoustic noise (Knudsen (1948), Wenz (1962)). Measurements of the omnidirectional noise at the higher frequencies (>200 Hz) have been found to exhibit wind-dependent characteristics; and, when not dominated by shipping noise, the most likely mechanisms are related to bubbles, spray, and splashes associated with white caps, as well as capillary wave/wave interactions (Urick (1984)). Furduev (1966) has proposed that the characteristic broad maxima in the ocean ambient noise spectrum between 0.2 kHz and 1 kHz be attributed to cavitating bubbles. Kerman (1984) discusses these mechanisms in detail (also see Fitzpatrick (1959)), but stresses the noise generated by the non-resonant oscillation of entrained gas bubbles which result from wave breaking and which are forced by intense velocity of the gravity-capillary waves. For wind speeds with a friction velocity greater than this critical velocity, Kerman concludes that sound is produced with a velocity to the 3/2 power, frequency to the -2 power, and intensity proportional to the number of bubbles. However, in the absence of white caps, since noise persists, capillary wave/wave or non-linear wave interactions may be important (Mellen (1987), Kuo (1968)).

At the other extreme of the spectrum (<2-5 Hz), ambient noise associated with ocean microseisms dominates. Recently, this noise has been
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shown by Nichols (1981) and by Kibblewhite and Evans (1984) to be due to wave/wave interaction. The microseismic effect was postulated by Longuet-Higgins (1950) and confirmed by several authors, including Latham and Nowroozi (1968). Several authors have studied the generation of sound through the second-order pressure effect (Brekhovskikh (1967), Goncharov (1970), Hughes (1976), Lloyd (1981)). Kibblewhite and Evans concludes with theoretical arguments and measurements that the dominant noise source in the 0.1 to 5 Hz range is the non-linear wave interaction. Although difficulties were found in predicting absolute levels, both data and theory showed a frequency dependence to the \(-6\) power.

In the very low frequency (VLF, 2-20 Hz) and low frequency (LF, 20-200 Hz), signals from surface shipping are a significant contributor to the measured noise and have been observed to extend to 500 Hz. In this region, noise contributors can be a great distance from the observation point, and consequently the noise field exhibits the effects of sound propagation in both the horizontal and vertical directions (Carey (1986), Von Winkle (1985)). Wagstaff (1981) showed that, if one knows the locations and types of ships, then one can describe the characteristic of the horizontal noise field. Although the vertical noise distribution, including the broad horizontal maxima, could be qualitatively explained, several discrepancies were observed. Wind-driven noise could explain these differences, and the sources of this noise are the subject of this paper.

EXPERIMENTAL EVIDENCE

Omnidirectional noise data at low frequencies which are free from flow and flow-induced vibrations (Strasberg (1984)) are very difficult to obtain. Several investigators (figure 1) measured the spectrum between 2 Hz and 2000 Hz in the deep sound channel or near the bottom. However, most of this data from the relatively heavily trafficked northern hemisphere reflect distant shipping noise in the 2 to 200 Hz range and, consequently, little local wind speed dependence is observed such as shown in figure 1. VLF/LF ambient noise experiments must be carefully examined to ensure that the results are either from distant or local sources.

Wittenborn (1976) (figure 2) performed an experiment with hydrophones that spanned the water column. Hydrophones within the sound channel showed little dependence on local wind speed between 10 Hz and 200 Hz. However, the hydrophone below critical depth showed an inferred local wind speed dependence (10 to 500 Hz) for wind speeds between 5 and 15 kns with levels of 47 dB and 56 dB re 1\mu Pa @10 Hz. The 15 kn spectra showed a slowly varying broad band characteristic between 56 dB at 10 Hz and 65 dB at 500 Hz (f^{\frac{1}{2}}). Wittenborn cites an earlier experiment with noise levels of 69 dB for 100 Hz at 30 kns, compared to the 300 Hz levels of 44 dB at 5 kns, 51 dB at 10 kns, and 63 dB at 15 kns. These results suggest two wind noise mechanisms for the cases of low and high sea states with the intensity having a squared velocity dependence \((U^2)\). The abrupt transition between 10 and 15 kns \((U^2\), based on the levels at 10 and 15 kns and not considered a true velocity dependence), as shown in figure 2, may be a threshold characteristic associated with
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Figure 1. Selected Low Frequency Ambient Noise Measurements. The region less than 5 Hz is dominated by wave/wave interaction. The measurements between 5 Hz and 300 Hz show little local wind speed dependence, but, rather, the effects of distant shipping and other distant noise sources.

Figure 2. Ambient Noise Level vs. Frequency for the Wittenborn Experiment. The 4850 m deep hydrophone shows the local wind speed dependence (≈200 Hz) with the influence of distant noise sources less than 100 Hz. The 3960 m deep hydrophone is dominated by distant noise sources less than 100 Hz.

breaking waves. (These results agree with the observations of Worley (1982), insofar as his data show a threshold-type behavior between the
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\[ NL = NL_1 + 20n \log(WS) \]

<table>
<thead>
<tr>
<th>INVESTIGATOR</th>
<th>( f )</th>
<th>( n )</th>
<th>( WS (m \text{ sec}) )</th>
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<tr>
<td>PIGGOTT JASA 36(11)</td>
<td>13</td>
<td>2.1</td>
<td>10-20</td>
</tr>
<tr>
<td></td>
<td>13</td>
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<td>1.5</td>
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<td></td>
<td>141</td>
<td>1.53</td>
<td>3.5-20</td>
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<tr>
<td></td>
<td>&lt; 50</td>
<td>2.1</td>
<td></td>
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<td></td>
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<tr>
<td>WHITTENBORN (1976)</td>
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<td>1.65</td>
<td>2.5</td>
</tr>
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<td>5-7.5</td>
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<td></td>
<td>1</td>
<td>7.5-15</td>
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<td></td>
<td>28</td>
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<td></td>
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<td></td>
<td>150</td>
<td>1.1-1.32</td>
<td>10-15</td>
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<td></td>
<td>177</td>
<td>1.36-1.57</td>
<td>8.1</td>
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<td>10-15</td>
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<tr>
<td></td>
<td>200</td>
<td>1.65-2.0</td>
<td>5-10</td>
</tr>
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<td>WILSON JASA 73(1)-83</td>
<td>10</td>
<td>2.07</td>
<td>5-10</td>
</tr>
<tr>
<td>BURGESS JASA 73(1)-83</td>
<td>37</td>
<td>1.66</td>
<td>5-15</td>
</tr>
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</table>

Factors affecting wind speed dependence are 1. distant sources. 2. overlapping wind speed regions. 3. sound propagation factors.

Table I. Low Frequency Ambient Noise Wind Speed Dependence

data corresponding to wind speeds between 2.5 and 5 kns and between 5 and 10 kns at 200 Hz. This effect was especially pronounced at 400 Hz.

Although Wittenborn made use of both refractive effects and bathymetric blockage, noise from distant sources was still found to influence his results (for example, see figure 2 between 10 and 100 Hz). The corrupting influence of distant noise sources (ships, whales, volcanoes, etc.) has the effect of obscuring the low-frequency local wind speed dependence. Consequently, the literature reveals a variety of estimated wind speed dependencies; i.e., the estimate of a parameter \( n \), where \( NL = NL_1 + 20n \log (WS) \). (The mean square pressure would increase with \( 2^n \) power of wind speed.) Table 1 lists several of these estimates of \( n \), ranging from 0.85 to 2.0 for wind speeds between 10 and 20 m/sec. The problem with these estimates also lies in the fact that the data clearly show a region of no wind dependence, a threshold-type behavior, and region with a wind dependence of \( n=2.0 \).

Figure 3 illustrates this trend with the data of Piggott (1964). One observes the frequency dependent cross-over between the low wind speed and higher winds regions. Furthermore, the lower the frequency, the higher the wind speed will be at which the wind speed dependence point is observed.

Distant noise sources influence vertical noise directionality (Von Winkle (1985), Browning (1982), Bannister (1986)). This influence of the distant source produces a broad maximum in the vertical noise intensity centered on the horizontal. This phenomenon results from the con-
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Figure 3. Ambient Noise Spectrum Level vs. Wind Speed, Piggott (1964)

version of higher angle rays to lower angle rays by either reflections from the basin boundaries and seamounts or refractive effects due to shallowing sound channels at the higher latitudes. Wagstaff attributed this effect to surface ships. Since the spectral variation of the horizontal noise is generally smooth, and since ship signatures are narrow-band in this frequency range, wind-produced noise over seamounts, slopes, and at high latitudes was speculated to be an important contributor. The broad maximum along the horizontal has been observed in varied geographical locations, such as the sparsely shipped Southern Hemisphere waters of the South Fiji Basin (shown in figure 4). At the lower frequency the data clearly show a broad maximum. At 105 Hz one observes the influence of a single ship. These results are similar to data obtained in the North Pacific and the North Atlantic (Carey (1986)).

The experimental data were examined to obtain measured levels useful in the estimation of the source level of wind-produced noise at the sea surface. These results are shown in Table II, primarily at 50 Hz. The estimated levels based in the Wittenborn data are shown in the table to be between 43 dB at 5 kns and 51 dB at 15 kns, consistent with the estimates by Wilson and Kewley using the same data. Vertical noise cannot be used for local wind-driven noise; however, estimates for a cylindrical basin with sloping sides yields levels in the 50 dB range. Kewley has carefully estimated source levels, and his curves are shown in figure 5.

In summary, we have presented data which indicate the presence of a wind-driven noise in the 10 to 200 Hz region of the spectrum. The low wind speed range (<8-10 m/sec) appears to have a weak dependence on the wind speed, 0 < n < 1; the high wind speed region (>7.5 to 15 m/sec) appears to have a dependence of 0.85 < n < 2. These estimates point to
Figure 4. Vertical Noise Spectrum Level versus Angle from the Horizontal, Browning (1986)

The uncertainty in our knowledge of wind speed dependence and spectral characteristics.

POSSIBLE MECHANISMS

The fundamental mechanisms for the production of sound in turbulent regions may be derived from first principles. The basic procedure can be found in several treatments on hydrodynamic noise, most notably Light- hill (1979), Ffowcs Williams (1969), Dowling (1983), and Ross (1976). We have rederived the inhomogeneous wave equation with source terms in appendix A for the purpose of ranking the various mechanisms capable of the production of sound at the surface of the sea in the 10 to 200 Hz range. The basic approach is to write the equations governing the conservation of mass and momentum with source terms. The equation of state is specified, fluctuation quantities assumed, linearization employed, and the inhomogeneous wave equation is formed. The integral solutions to this equation are formulated by use of the Kirchoff Method (Stratton (1941), Jackson (1962)) and of the divergence theorem. The derivation
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- OMNIDIRECTIONAL MEASUREMENTS WITH HYDROPHONE BELOW CRITICAL DEPTH

<table>
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<tr>
<th>INVESTIGATOR/FREQ</th>
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<th>50 Hz</th>
<th>100 Hz</th>
<th>w.s (kns)</th>
<th>Location</th>
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<tr>
<td>WHITTENBORNE (1982)</td>
<td>48 dB*</td>
<td>50</td>
<td>44</td>
<td>5</td>
<td>N.E. PACIFIC</td>
</tr>
<tr>
<td></td>
<td>50</td>
<td>51</td>
<td>47</td>
<td>10</td>
<td>N.E. PACIFIC</td>
</tr>
<tr>
<td></td>
<td>57</td>
<td>58</td>
<td>60</td>
<td>15</td>
<td>N.E. PACIFIC</td>
</tr>
<tr>
<td>MORRIS (1978)</td>
<td>-</td>
<td>70</td>
<td>63</td>
<td>10</td>
<td>N.E. PACIFIC</td>
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- VERTICAL NOISE MEASUREMENTS ALONG THE HORIZONTAL

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<th>100 Hz</th>
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<tr>
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<td>67 dB</td>
<td>70</td>
<td>65</td>
<td>FIJI BASIN</td>
</tr>
<tr>
<td>WAGSTAFF (1981)</td>
<td>-</td>
<td>-</td>
<td>82</td>
<td>N.W. ATLANTIC</td>
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<tr>
<td>WALES (1981)</td>
<td>95</td>
<td>87</td>
<td>65-69</td>
<td>N.W. ATLANTIC</td>
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<tr>
<td>AXELROD (1985)</td>
<td>-</td>
<td>-</td>
<td>60-65</td>
<td>N.W. ATLANTIC</td>
</tr>
<tr>
<td>FDX (1964)</td>
<td>-</td>
<td>-</td>
<td>60</td>
<td>E PACIFIC</td>
</tr>
<tr>
<td>FISHER (1986)</td>
<td>74</td>
<td>-</td>
<td>65-69</td>
<td>N PACIFIC</td>
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<tr>
<td>ANDERSON (1979)</td>
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- SEMI-EMPIRICAL ESTIMATES.

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<tr>
<td>WILSON (1983)</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>BURGESS &amp; KEWLEY (1983)</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

- SOURCE LEVEL ESTIMATES BASED ON OMNI MEASUREMENTS.

- VERTICAL LEVELS YIELD FOR A MEAN WIND SPEED OF 10kns 50dB@50Hz AND 54-56dB@100Hz

Table II. Ambient Noise Source Levels

in appendix A is similar to those of Huon-Li (1981) and Yen (1979), and the basic result is the following:

\[
4\pi C_0(p-p_0) = 4\pi P = \frac{1}{2} \int [\varphi] dV/R - \frac{1}{2} \int [T_{i}] dV/R - \int [\varphi \delta U \delta t] dS + \frac{1}{2} \int [2\rho_0 U u_i + \rho_0 u_i u_j + P_{ij}] dS/R
\]

The first term, \[\frac{1}{2} \int [\varphi] dV/R\], represents a monopole term. \(\varphi\) represents mass addition rate per unit volume. The second term represents an external force acting on the volume and has a dipole character. These two terms could be important in the incorporation of entrained bubble oscillation and translation. The third term is the Lighthill turbulence stress tensor and is known to represent an acoustic quadrupole. The term \[\frac{1}{2} \int [\varphi \delta U \delta t] dS\] involves the motion of the boundary and can act as a monopole. The final integral involves the turbulent and compressive stresses acting on the boundary and is seen to have a dipole character. In particular, the term \[\frac{1}{2} \int [2\rho_0 U u_i + \rho_0 u_i u_j + P_{ij}] dS/R\] represents the wave turbulence interaction and is dominant since it represents a product of a first order \(U\); and second order term \(u\).

Noise generation by the interaction of surface waves and turbulence near the surface was suggested by Concharov (1970). He calculated levels of 80 dB at 10 Hz and 40 dB at 100 Hz by assuming a Pierson-Moskowitz surface wave spectrum and Kolmogorov's similarity hypothesis. His expression can be shown to be equivalent to the above integral. However, instead of using velocities, he employs the displacement spectrum for the surface wave and turbulence. His expression is \(\rho(\omega) = 40\pi^2/\omega^2\).
Yen and Perrone (1979) derived expressions yielding the frequency-dependent radiation characteristics for the wave/wave, wind/turbulence, and wave/turbulence interaction mechanisms. Their results for the wave/turbulence interaction (70 dB at 10 Hz and 50 dB at 100 Hz) show a linear dependence on surface wave velocity \( U \) and an inverse square dependence on frequency \( \omega \):

\[
P(\omega, k) = 2 \cdot 10^{-2} \rho^2 \cos^4 \theta \cdot U/\omega^2.
\]

The Yen & Perrone result contains three interesting factors. The linear dependence on surface wave velocity is consistent with the previously discussed experimental results prior to wave breaking. The \( \omega^2 \) dependence is also consistent with the observed behavior at low frequencies; i.e., an overlap region composed of the interaction of the low and higher frequency roll-offs. However, of particular note is the \( \cos^4 \theta \) dependence. This sharply peaked angular dependence would accentuate the role of the ocean bottom and basin boundaries with respect to the vertical noise directionality. Thus, wave/turbulence interaction could be a source of noise in the 2 to 200 Hz region for those sea states low enough that breaking waves do not occur, due to the fact that it appears as a physically realizable mechanism (considering the uncertainty of the turbulence spectrum).

Kerman (1984) shows that, above a critical wind speed of approximately 10 m/s, small (micrometer (\( \mu m \)) bubbles are produced and can be a source of higher frequency sound. Thorpe (1986, 1982) has performed interesting experiments which demonstrate the existence of bubble plumes and layers composed of \( \mu m \)-size bubbles, (mean bubble size approximately 50 \( \mu m \) with densities between \( 10^4 \) to \( 10^6 \) bubbles/m\(^3\)) extending several
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M.Y. Su (1984) has shown that fresh water breaking waves produced in wave tanks produce bubble plumes which penetrate to depths on the order of significant wave height, with bubbles of centimeter diameters due to coalescence (which is absent in salt water). Several reviews (see bubble references) have been written on the existence and densities of bubbles produced by breaking waves. At high sea states a residual layer is formed of micro-bubbles with a density that decreases exponentially with depth, and bubble plumes which are convected to several meters depth by the vorticity beneath the wave. Even though individual bubble oscillations with these micron-size bubbles could not produce VLF/LF noise, collective oscillations of the bubbly mixture driven by the hydrodynamic pressure field could produce sound.

It is well known (see appendix B) that a small amount of bubbles in water significantly changes the bulk compressibility while not drastically changing the density. Wood has shown that the sonic velocity ($C_m$) can be described by the following relationship between void friction ($x$), density ($\rho_m$), and bulk compressibilities ($K$):

$$C_m^2 = \frac{[(1-x) \rho + x \rho_0] [(1-x) K + x K_g]}{[(1-x) \rho + x \rho_0]}.$$

The consequence of this result is shown in the figures of appendix B. Small volume fractions result in large changes in the sonic speed when the mixture can be treated as a continuum. For example, the sonic velocity of the bubble mixture with a 0.2% volume fraction is approximately 225 m/s. Ffowcs Williams (1969) describes the efficiency of the radiation from a cloud of bubbly turbulent flow:

$$4\pi C_0^2 (\rho - \rho_0) = 4\pi P = \int [\partial q/\partial t] \, dV/R.$$  

For a compact source with a small gas volume fraction

$$4\pi C_0^2 (\rho - \rho_0) = 4\pi P \equiv \frac{1}{R} \frac{\partial}{\partial t} \int q \, dV,$$

Ffowcs Williams estimates that

$$q = -\rho \frac{D}{Dt} \ln(1-x) - C_m^2 \frac{DP}{Dt}$$

$$dp = C_m^2 \Delta (1-x) \rho = -\rho C_m^2 \Delta x - \rho - \rho_0 - \rho/R \, m^4 (C_0/C_m)^2, m = u/C_0$$

Thus he concludes that "a cloud of bubbly flow radiates very much more efficiently than turbulence alone;" that is, the radiation from such a flow would be $(C/C_m)^4 = 1975$-times larger than the radiation from turbulent flow. However, one must account for the presence of a pressure release surface.

An alternative approach is to consider the bubble cloud as a flexible sphere of radius $a$ with composite mixture properties and to assume it is compact with respect to the acoustic wave length and the vorticity and turbulence scales. Then the forced oscillation of the bubble cloud in absence of a boundary is

$$P(R,t) = \dot{Q}(t) / 4\pi R = \rho_0 \frac{\ddot{V}(t)}{4\pi R} = \frac{3\omega^2 a^3 \rho}{4\pi R} \frac{m^2 (C_0/C_m)^2 \alpha f(\omega)/R}{(1-x)},$$

where $f(\omega)$ represents the simple harmonic oscillator transfer function. This forced oscillation of a bubble cloud can have a resonant behavior.
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(Fitzgerald and Mellen (1982)). The bubble cloud is simply a monopole source and the presence of the boundary can be approximately taken into account via the surface image interference effect. Thus, we find the following for a cloud of micro-bubbles below the pressure release surface driven in forced oscillation by the hydrodynamic forces:

\[ |\rho(R,t)|^2 = \left[ \frac{3\omega^2 a^2 \rho}{(1-x)} \right] \frac{m^2 (C_0/C_m)^2 f(\omega)}{R^2} \left( \frac{2\pi n \lambda}{\lambda} \right)^2 \sin^2 \theta. \]

This expression shows a frequency-dependent efficiency approximately \((z/\lambda)^2\) at a given bubble cloud depth. This term indicates that, as seas pick up, the deeper the plume, the more efficient the radiation at longer \(\lambda\). Furthermore, we note that this monopole has an \(m^2\) improvement over the non-compact bubble cloud.

Thus, the low-frequency noise could be caused by wave turbulence prior to wave-breaking and, thereafter, by aggregate bubble (bubble cloud) oscillations exhibiting a threshold-type behavior and velocity squared dependence for the mean radiated pressure.

APPENDIX A: DERIVATION OF THE SOURCE INTEGRALS

The purpose of this appendix is to briefly outline the derivation of source terms important to the production of sound near the surface of the sea.

Conservation of mass: \(\frac{\partial \rho}{\partial t} + \frac{\partial \rho \mathbf{v}}{\partial x} = q\).

Conservation of momentum: \(\frac{\partial \rho \mathbf{v}}{\partial t} + \frac{\partial \rho \mathbf{v} \mathbf{v}}{\partial x} = -\frac{\partial P_{\text{ref}}}{\partial x} + F_e\),

where \(P_{\text{ref}} = -\rho \delta + \mu D_{\text{stud}} + \mu \theta \delta\) (ref. Hinze, p. 17)

\[ \mu = 2/3 \mu, \quad D_{\text{stud}} = \partial U/\partial x + \partial U/\partial x, \text{ and } \Theta = 1/2 D_{\text{stud}} = \partial U/\partial x. \]

Taking \(\partial /\partial t\) of the continuity equation and \(\partial /\partial x\) of the momentum equation yields upon subtraction:

\[ \frac{\partial^2 \rho}{\partial t^2} = \frac{\partial^2 \rho \mathbf{v} \mathbf{v}}{\partial x \partial x} + \frac{\partial^2 P_{\text{ref}}}{\partial x \partial x} - \frac{\partial F_e}{\partial x} + \frac{\partial q}{\partial t}. \]

\(v = U + u, (\partial U/\partial x = 0), \rho' = \rho_0 + \rho, \quad \frac{\partial \rho_0}{\partial t} = 0\)

\[ \frac{\partial^2 \rho}{\partial t^2} - C_0^2 \frac{\partial^2 \rho}{\partial x^2} = UU, \quad \frac{\partial^2 \rho}{\partial x \partial x} + \frac{\partial^2 (\rho_0 + \rho) uu}{\partial x \partial x} + \frac{2 \sigma (\rho_0 + \rho) uu}{\partial x \partial x} + \frac{\partial q}{\partial t}. \]
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For the case of incompressible, inviscid flow with no sources or sinks:

\[ \frac{\partial^2 \rho_0 u}{\partial x_i \partial x_i} + 2 \frac{\nu}{\rho_0} \frac{\partial u_i}{\partial x_i} + \partial^2 \rho / \partial x^2 = 0; \]

Compressible fluid:

\[ \rho C_0^2 = P \]

\[ \frac{1}{C^2} \frac{\partial^2 P}{\partial t^2} - \frac{\partial^2 P}{\partial x^2} = \frac{\partial q}{\partial t} - \frac{\partial f}{\partial x} + \frac{\partial^2 T_y}{\partial x_2} = \frac{\partial^2 T_p}{\partial x_1 \partial x_2}; \]

(in most instances \( P_i - C_0^2 \delta_i = 0 \), (Lighthill)).

Since \( P = \rho_0 \frac{\partial }{\partial t} (\rho_0 \psi) = -i \omega \rho_0 \psi \), we finally have the wave equation with the source terms:

\[ \frac{\partial^2 \psi}{\partial x_i^2} - \frac{1}{C_0^2} \frac{\partial^2 \psi}{\partial t^2} = \frac{1}{i \omega \rho_0} \left\{ \frac{\partial q}{\partial t} - \frac{\partial f}{\partial x} + \frac{\partial^2 T_p}{\partial x_2} \right\} = -4\pi f(x,t). \]

This inhomogeneous wave equation can be integrated by use of the Kirchoff method (Stratton (1941) and Jackson (1962)) to yield:

\[ \psi(x,t) = \int \frac{dV}{|x-x'|} \frac{1}{4\pi} \int dS \left[ \frac{1}{R} \frac{\partial \psi}{\partial n} - \frac{1}{C_0} \frac{\partial}{\partial n} (1/R) \psi + \frac{1}{C_0} \frac{\partial R}{\partial n} \frac{\partial \psi}{\partial t} \right]. \]

This solution, when applied to our specific problem with the properties of \( \delta [ ] / \partial x_i \) and \( \delta [ ] / \partial y_i \), as well as the divergence theorem, yields the desired results:

\[ 4\pi C_0^2 (P - P_0) = 4\pi P = \int \left[ \frac{\partial q}{\partial t} \right] dV/R - \frac{\partial T}{\partial x_2} \int [f_e] dV/R + \frac{\partial^2 T_y}{\partial x_1 \partial x_2} \int [T_y] dV/R - \right] l [\partial \psi / \partial t] dS + \partial / \partial x_i \int l [2\rho_0 u_i + \rho_0 u_i + P_0 - C_0^2 \delta_i] dS/R. \]

APPENDIX B: MIXTURE THEORY

A.B.Wood (1932) showed that the sonic speed could be calculated for an air-bubble/water mixture by use of the mixture density (\( \rho_m \)) and the mean compressibility (\( K_m \)). The mixture can be treated as a continuous medium when the bubble diameter (\( d \)) and spacing between the bubbles (\( D \)) are much less than the wavelength of sound. In the case of low frequencies, for the mixture with a volume fraction (\( X \)) of gas we can calculate the mean density and compressibility as follows:

\[ \rho_m = (1 - x) \rho_0 + x \rho_w \]

\[ K_m = \frac{-d v_m}{v_m d P} = \frac{d v}{v d P} + \frac{d v_w}{v_w d P} = \frac{1}{1 - x) K + x K_0} \]

This implies that a state of equilibrium prevails and the mixture mass is conserved, and the pressure, \( P \), is uniform throughout the mixture (a low frequency assumption). Since the sonic speed is

\[ C^2 = \frac{d P}{d \rho} = \rho K \]

we have

\[ C_m^2 = \frac{C_m^2}{C_0^2} = \frac{[(1 - x) \rho + x \rho_w] [(1 - x) K + x K_0]}{\rho_0 C_0^2 + \rho_0^2 C_0^2} \]

\[ C_m^2 = (1 - x)/C_0^2 + x/C_0^2 + (x)(1 - x) \frac{\rho_0 C_0^2 + \rho_0^2 C_0^2}{\rho_0 C_0^2 + \rho_0^2 C_0^2}. \]
The expression for the sonic speed poses the question of whether the gas compressibility is described by an isothermal or adiabatic process, especially since the single phase sonic speed is known to be adiabatic. However, in the case of an air-bubble/water mixture, the controlling physical factor is the transfer of the heat generated in bubble compression to the surrounding liquid. If the transfer is rapid, then the bubble oscillation is isothermal, \( \gamma_P = \gamma (1/P, K_0 = 1/P) \), as compared to the adiabatic condition \( \gamma_P = \gamma (1/P, K_0 = 1) \). Thus, in use of the above equations one must use either for the adiabatic or isothermal case, \( C_m = C_{gas}/\gamma \). Isothermal conditions are most likely to prevail for air-bubble/water mixtures due to the large thermal capacity of water. Examination of the above expressions shows that as \( x \rightarrow 0 \), \( C_m^2 = C_1^2 \), and as \( x \rightarrow 1 \), \( C_m^2 = C_0^2 \) as one would expect. The striking characteristic revealed by these equations (shown in figure B-1) is the sharp reduction in the sonic velocity at small volume fractions; i.e., \( x = 0.002 + C_m = 225 \text{ m/sec} \). These equations may be approximated for the air/water mixture:

\[
C_m^2 = \frac{\gamma P}{\rho x (1-x)} \quad \text{and} 
\]

\[
C_m (x = 0.5) = 20 \text{ m/sec}
\]

Karplus (1958) used an acoustic tube to determine the standing wave pattern as a function of air volume fraction. His results are shown in figure B-2. Close agreement was found between the inferred sonic speeds.
Figure B-2. Measured and Computed Mixture Sonic Speeds. a) Measured Mixture Speed vs. Volume Fraction at 500 and 1000 Hz, Karplus (1958); b) Measured Dispersive Character of Mixture Speed Below 2000 Hz, Karplus (1958); c) Measured and Computed Mixture Sonic Speed Showing the Behavior Below, At, and Above Resonance, Fox, et al. (1955)
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and results calculated with Wood's expressions. Similar results have also been observed at the low frequencies by Campbell and Pitcher (1955). These results are also observed at the higher frequencies above and below resonance. Several studies and texts have been written on this subject and are listed in the references. An example of the agreement between theory and measurement near the vicinity of bubble resonance is shown in figure B-2c. It is important to note that most calculations performed at these higher frequencies use $K_m = K_1 + xK_g$, rather than the Wood approach $K_m = (1 - x)K_1 + xK_g$. This difference is unimportant near resonance and for small volume fraction but is important as one approaches the low frequencies of interest to this paper. One can show that the correct expression is:

$$\frac{1}{ \omega^2_n} = \frac{1}{C_i^2} \left( \frac{1}{C_m^2} \left( 1 - \omega^2/\omega_0^2 \right) + 2i\omega/\omega_0 \right),$$

when h.f. and l.f. are the high frequency and low frequency values of the sonic speed.

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