Annual Report

Fluxons and Order in Long Josephson Junctions

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This report presents the research that was conducted in the year 1987 and some of the results that were achieved. Low frequency fluctuations were studied at metastable states of long Josephson junctions biased at Fiske steps in a magnetic field. The fluctuations exhibit telegraph type of noise whose origin is the thermal noise of the internal resistance of the junction. Temperature dependence of these fluctuations shows that a thermal activation process is responsible for the behavior. At lower temperatures, quantum mechanical tunneling is expected to contribute to the fluctuations and experiments are presently being prepared for such studies down to 10 MK in a He-He dilution refrigerator.
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Abstract

This report presents the research that was conducted in the year 1987 and some of the results that were achieved. Low frequency fluctuations were studied at metastable states of long Josephson junctions biased at Fiske steps in a magnetic field. The fluctuations exhibit telegraph type of noise whose origin is the thermal noise of the internal resistance of the junction. Temperature dependence of these fluctuations shows that a thermal activation process is responsible for the behavior. At lower temperatures, quantum mechanical tunneling is expected to contribute to the fluctuations and experiments are presently being prepared for such studies down to 10 mK in a $^3$He-$^4$He dilution refrigerator. The I-V curves for our junctions show a regular step structure due to cavity resonances and they also show subharmonic generation of period three due to the strong non-linearities of the dissipative system. Such subharmonic voltage steps are important because they normally precede chaotic behavior. A study was also made of the effects of geometry on fluxon motion and the results show that the device characteristics depend strongly on the bias current distribution in the junction. Studies for the next period are presented; they consist of experiments on fluxon quantum tunneling down to temperatures of 10 mK and effects of dissipation on such tunneling; conductance fluctuations in the junction will also be investigated.
1. **Goals of Project**

This project deals with the behavior of quantized current vortices, fluxons, in long Josephson Junctions. The research consists of investigating the fundamental aspects of fluxons in junctions and related phenomena, keeping an eye for possible applications. Specifically the project deals with the following topics:

(a) noise and fluctuations in long Josephson junctions
(b) dynamics of fluxons
(c) effects of disorder inside junction on fluxon behavior
(d) instabilities in fluxon motion
(e) fluxon tunneling - thermal and quantum mechanical aspects
(f) effects of dissipation on fluxon motion

The long Josephson junction was chosen as the basic system in such studies for the following reasons:

(i) it is a relatively simple system which can be considered as the analog of the semiconductor FET, with fluxons being the carriers of information.

(ii) its geometry is much more compact than a SQUID (Superconducting Quantum Interference Device) and hence it has good self-shielding, making it ideal for the fluctuation studies presented here.

(iii) the sin $\phi$ term in the perturbed sine-Gordon equation (to be introduced in the next section) leads to very strong non-linear behavior making it an excellent system for studying non linear physics (it is much more non-linear than optical systems).

(iv) because the long Josephson junction is the dual of a FET there are many possible applications for high speed electronics.
2. **Research and Results**

A. **Introduction**

In a bulk superconductor, the interior is shielded from external magnetic fields by induced currents flowing within the London penetration depth $\lambda_L$. This parameter is inversely proportional to the density of superconducting electrons and it has a fixed value for a given superconductor. In a Josephson junction similar behavior occurs and shielding takes place within a characteristic length $\lambda_J$, the Josephson penetration depth. It depends inversely on the current density $J_c$ and is given by

$$\lambda_J = \left[ \frac{\phi_0}{(2\pi) \frac{1}{\mu_0} \frac{1}{dJ_c}} \right]^\frac{1}{4}$$  

(1)

where $d$ is the magnetic thickness. This parameter can be changed by modifying the critical current density of the junction.

A Josephson junction is considered long when one of its dimensions $L$ is larger than $\lambda_J$. In that case, penetration of magnetic flux occurs above a critical field $B_{c1}$ which then produces magnetic flux quanta inside the junction. Such quanta, known as fluxons, have dimensions of the order of $\pi \lambda_J$. The free energy of a fluxon is given by

$$E = \left( \frac{\hbar}{2e} \right) 8 \lambda_J$$  

(2)

Thus the length $\lambda_J$ sets the scale of the device and also the energies involved. Neglecting demagnetizing effects, the formation of fluxons inside the junction occurs when the external magnetic field or electrical currents satisfy the condition

$$B_{c1} = \frac{\hbar}{4\pi} J_c \lambda_J$$  

(3)
In the presence of an electric current, such as the bias current through the
junction, there will be a Lorentz force on the fluxons causing them to move.
However, due to friction, a constant fluxon velocity is achieved in a
distance of $-\lambda_J \beta_c^2$ (the McCumber parameter $\beta_c = (2e^2/\hbar) I_c R_N^2 C$ is a measure
of damping of the junction; for example, when $\beta_c >> 1$ the junction has low
damping). The dissipation due to friction comes from the quasiparticles
present and the high frequency surface losses in the electrodes.

The resonant motion of fluxons in a long junction leads to zero field
steps in the I-V curves and such behavior is attributed to the particle
nature of the fluxons. In the presences of a magnetic field the particle
nature changes to an electromagnetic wave character; this combined with the
asymmetry of the magnetic field relative to the junction, leads to
electromagnetic cavity resonances which are observed as Fiske steps in the
I-V curves. The degree to which all these effects occur depends on the
junction damping and thus on $\beta_c$.

When the junction is well damped, the I-V curves show no hysteresis;
such a device is very useful since it has no latching, it has very high
speeds (essentially the transit time for a fluxon across the junction
travelling at a fraction of the speed of light), and it needs only an
unipolar power supply (since it does not latch). There are applications of
such a device to amplifiers, switches, oscillators, and logic.

The motion of fluxons is governed by a perturbed sine-Gordon equation
of the form

$$\phi_{tt} - \phi_{xx} + \sin \phi - \eta - \alpha \phi_t + \beta \phi_{xtt}$$

where $\phi$ is the pair phase difference across the junction. The respective
derivatives with respect to time $t$ and position $x$ are normalized to the
junction plasma frequency and the Josephson penetration depth $\lambda_J$. The right
hand side of equation 4 shows the bias drive term, the damping term and a
surface loss term due to the normal electrons in the penetration layer.
This equation represents well the behavior of a long junction and we will
use it to analyze our results on the dynamics of fluxons and the effects of
disorder.

B. Low Frequency Fluctuations at Metastable States in Long Josephson
Junctions
When an external magnetic field is applied, the I-V curves of a
Josephson junction show current steps, known as Fiske steps. These are due
to the resonant excitation of the cavity modes in the tunnel junction.
Fluxons can then be considered as an electromagnetic wave which can excite a
cavity mode thus inducing an ac current. This current can interfere with
the original ac current due to the ac Josephson effect causing a dc
tunneling current. Such current steps can be considered as metastable
states and we have studied fluxon fluctuations at such states. Noise-
activated escape from metastable states was treated a long time ago by
Kramers and has since been applied to many systems. We are extending this
to the behavior of fluxons in a long Josephson junction.

Earlier work by us on this problem has shown that, when biased as such
metastable states, fluctuations lead to telegraph noise due to switching
between adjacent states. We have extended this study to investigate the
origins of such fluctuations and to learn about some of the details of this
behavior.

The junctions were Nb-Al-Al2O3-Al-Nb with a fairly uniform bias current
injection; the $\beta_c$ of the junctions ranged from 200 to 50. Fluxons in the
junction were created by an external control line and partly by the bias
current itself. The control line was used to produce a magnetic field and
the junction was biased at current steps; the fluctuations at such steps were studied for different bias conditions and external magnetic fields. Noise power spectra were analyzed to extract the mean lifetime of each state and its dependence on the current step width $\Delta I$. Figure 1 shows part of the I-V curve where the junctions were typically biased and the meaning of the current step width $\Delta I$. Figure 2 shows how the mean lifetime varies with the step width $\Delta I$ for a given junction. Indeed, the narrower the transition region $\Delta I$, the shorter is the mean-lifetime of the step. Fits of our data to a simple model, presented below, give a fluxon characteristic frequency of $2 \times 10^{10}$ Hz corresponding to the transit time across the junction which is 130 $\mu$m long.

Figure 3 shows the probability per unit time of a transition between one state and an adjacent one for different current step widths $\Delta I$. This width is determined experimentally by fine adjustment of the bias current through the junction.

When the junction is biased at a current step, fluctuations in current will cause the junction to switch from that state to the next one and vice versa. Hence we observe telegraph noise between those 2 states. To explain the source of such fluctuations one can assume that current fluctuations are caused by the thermal noise of the junction resistance $R$ and hence

$$i^2 = \frac{4kT}{R} \Delta f$$

A fit to such a model shows that thermal noise above might be largely responsible for the observed current switching fluctuations. Having investigated this behavior at a fixed temperature, we continued the experiments to study the temperature dependence.
Figure 1. I-V curve for long junction showing current steps. Junction $L = 130 \mu m$, $B_c = 100$. 
Figure 2. Measured mean lifetime as a function of step width ΔI.
Figure 3. Probability per unit for a transition to next state as a function of step width.
The long Josephson junction provides an excellent model for studying the decay of metastable states classically and quantum mechanically as well as for investigating fluxon dynamics.

C. Temperature Dependence of Fluctuations in Long Josephson junctions

We have extended the fluctuations studies at current steps to the temperature range of 4.2K to 1.5K. A special cryostat was constructed for these studies with electrical and magnetic shielding for the devices. Fluctuation measurements similar to the ones described in section B were repeated. Figure 4 shows how the width ΔI at a current step changes with temperature. As expected, the lower the temperature the narrower is the current width, thus leading to a higher transition probability per second for switching to the next state for a junction biased within ΔI. Such results are shown in figure 5 for three different temperatures. A plot of the maximum probability as a function of temperature shows a logarithmic behavior; this is presented in figure 6. Such temperature dependence is characteristic of thermally activated processes causing the escape of fluxons from one state to the next one.

The interpretation of the data is not simple since one has to take into account the change of critical current of the junction with temperature i.e. the change of the junction resistance. That is, in this temperature dependence study of the fluctuations, the degree of hysteresis will change with temperature since βc changes as well. Figure 7 shows a typical critical current temperature dependence of a junction to illustrate this point.
Figure 4. Temperature dependence of step width.
Figure 5. Temperature dependence of transition probability.
Figure 6. Temperature dependence of maximum probability.
Figure 7. Temperature dependence of critical current of long junction.
The probability of escape from a potential well to the next one by thermal hopping over the barrier in a junction with McCumber parameter $\beta_c$ can be written as

$$P = \frac{I_o R w_1 w_2}{2 \phi_0 \beta_c s_1 s_2} \left[ (1 + 4 \beta_c s_2^2)^{1/2} - 1 \right] e^{-\Delta U/kT}$$

where $w_1, w_2, s_1$ and $s_2$ are related to the well and barrier geometry. The depth of the well is given by $\Delta U$. It is important to note that this formula applies to a bias at the middle of the current step and not in the middle of the width $\Delta I$.

The results in figure 6 show that we are still in the thermal activation regime.

D. Fluxon Quantum Tunneling in Long Josephson Junctions

As the temperature is reduced there will be a cross-over from the fluxon thermal activation over the top of the barrier to fluxon tunneling through the barrier. This will occur at some temperature $T_0$ which is given by

$$T_0 = \frac{\hbar \omega_p}{2\pi k} \left[ (1 + 1/4Q^2)^{1/2} - 1/2 Q \right]$$

where $\omega_p$ is the frequency of oscillation inside the well and $Q$ is defined by $Q = \omega_p RC$. From equation 7, $T_0$ for our junction will be less than 1K and hence special refrigeration techniques have to be used to investigate this regime. We have started designing and preparing the apparatus for such experiments. The sample will be mounted inside the mixing chamber of a $^3$He - $^4$He dilution refrigerator to cover the transition from thermal to quantum mechanical tunneling. This is not a simple experiment since special precautions have to be taken to cool the sample and maintain a high degree of shielding. This experiment, once underway, will provide direct
observation of quantum tunneling of fluxons through potential barriers in a long Josephson junction. This is a topic of current interest as it offers the possibility to observe quantum tunneling at a macroscopic level. The effects of dissipation on the tunneling rate will also be studied in this experiment.

E. Effects of Geometry on Characteristics of long Josephson Junctions

The perturbed sine-Gordon equation (eq. 4) shows that the phase $\phi$ has a spatial dependence. The bias drive term $\eta$ assumes a homogeneous current distribution in the junction. Departures from a homogeneous distribution will alter the fluxon response; hence we have investigated this.

Our long junctions have reasonably homogeneous bias current distributions because the current is injected by means of fingers on the top and bottom electrodes. This is shown in figure 8. The critical current $I_0$ of the junction was measured as a function of external magnetic field coupling into the junction by means of a control line carrying a current $I_s$. Measurements were first made on a junction with 8 current fingers; then a finger was removed at each end and measurements were repeated with 6 current fingers. Two more fingers were removed, etcetera, till finally there were only 2 fingers in the middle of the junction. The results are shown in figure 9. At the same time the experiment was repeated with another similar junction where the fingers were removed progressively by factors of two from the middle of the junction finally leaving only 2 fingers, one at each end. The results for this case are shown in figure 10. Analysis of the date shows that a large number of current injection fingers provides a large current gain ($\frac{\Delta I}{\Delta I_s}$) in the device, i.e. it is important to have a homogeneous current distribution. However, by placing
Figure 8. Geometry of long junction used in this research.
Figure 9. Current gain of long junction as current injection fingers are removed at the ends.
Figure 10. Current gain of long junction as current injection fingers are removed from the center.
the current fingers near the center of junction a sharper current switching
gain characteristic can be achieved. This is because the junction for that
geometry can better shield its inside from the external magnetic field thus
increasing its $B_c$. Indeed, a long Josephson junction will provide its own
shielding currents against external magnetic field penetration.

Examination of figures 9 and 10 shows that the bias current is not
entirely switched off by the external magnetic field. This is possibly due
to pinning of the fluxons at the junction edges. Our results are in good
agreement with theoretical predictions proposed by Likharev.

F. Sub-harmonic Generation in Long Josephson Junction

A long Josephson junction, according to equation 7, is a highly non-
linear system with a few degrees of freedom, each having its own
characteristic frequency and it has dissipation. The frequencies can be
incommensurate or commensurate and hence mixing of these frequencies leads
to a variety of sideband frequencies. Such a junction exhibits for example
current steps which are related to the periodic behavior of the junction.
Recently it has become interesting to study non-linear systems in order to
observe the onset of chaos when the system is driven by an external driving
force. The transition from periodic to chaotic motion can proceed by
subharmonic generation. The Josephson junction with its large non-
linearities and dissipative behavior should show some such effects. Hence,
we have investigated the characteristics of a long junction for indications
of the approach of instabilities. A junction with $\beta_c > 1$ can exhibit
chaotic behavior and thus its onset should be seen in the I-V curve.
To observe possible subharmonic generation, we have investigated the I-V characteristics of a long junction in a magnetic field. At a certain value of the magnetic field, current steps are observed. Figure 11 shows one typical result with current steps characteristic of the resonances inside the junction. All the steps are equally spaced except for the one before the 1st step; that extra step has a voltage which corresponds to a frequency 1/3 of the 1st step, suggesting that this is the one-third subharmonic (period tripling) effect. Presumably other subharmonics are smeared out by noise.

We are currently modeling our junction on the computer to verify the result. However a recent publication for a microwave driven small Josephson junction shows the generation of such subharmonic. Our long junction is driven internally thus exhibiting the subharmonic as a small step on the I-V curve. The significance of this is that between even and odd-harmonic solutions there can be chaotic solutions. The chaotic solutions can be strongly affected by noise and thermal fluctuations.
Figure 11. Current steps in I-V curve showing subharmonic step before 1st step.
3. Publications and Conferences


4. **Personnel**
- O.G. Symko, P.I.
- B. Lee, post-doctoral fellow
- D.J. Zheng, post-doctoral fellow
- L. Baselgia, graduate student.

5. **Proposed Experiments**

The following experiments are planned:

(a) **Fluxon Quantum Tunneling**

Fluctuations of fluxons at metastable states in long junctions will be studied down to 10 mK in a dilution refrigerator.

(b) **Effects of Dissipation on Quantum Tunneling**

These effects have been studied in optical systems. However recent articles by Leggett and co-workers has brought new interest in this field, especially for Josephson junctions. We want to investigate this down to the 10 mK temperature range.

(c) **Conductance Fluctuations in a long Josephson junction.**

Low frequency noise in junctions will be studied using a very sensitive D.C. SQUID voltmeter. The magnitude of the noise can provide information on the fluctuations of the conductance which show changes in the order of $-e^2/h$ in 1 and 2 dimensional systems. [ref. S. Feng, P.A. Lee, and A.D. Stone, Phys. Rev. Lett. 56, 1960 (1986)]. These measurements will provide information on low frequency noise in long junctions as well as information concerning the motion of single scattering centers.
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