Determination of the Commutation Angle and Load Response for a Continuous-Current, Single Phase Controlled Rectifier

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Determination of the Commutation Angle and Load Response for a Continuous-Current, Single-Phase Controlled Rectifier

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A method has been developed to quantitatively determine the commutation angle and load response for a continuous-current, single-phase controlled rectifier. An analytical model was developed and solved numerically. The results, in the form of load current plots, predict the commutation time and the shape and magnitude of the response. The underlying assumptions used in the development of the model, the limitations of the model, and the interpretation of the results are discussed.
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INTRODUCTION

Controlled rectifiers employing power semiconductors are used to vary the average power supplied by a constant voltage a.c. source to a load circuit. One of the problems inherent in the design of such a system is the determination of the commutation or overlap angle. Knowledge of the commutation angle gives the designer information about the rate of change of current in the power semiconductor and allows him to accurately model the load current.

A short review of the literature failed to produce a rigorous determination of the commutation angle. Common methods of analysis either ignore the commutation reactance contained in any rectifier circuit or assume constant load current.

The model developed here represents the circuits depicted in figures 1 and 2. A full wave rectifier employing a transformer connected across the single-phase source and a bridge-rectifier circuit are shown in figure 1. The circuit shown in figure 2 also gives full wave rectification and consists of a center-tapped transformer and controlled rectifiers. While the center-tapped transformer has the effect of transforming the single-phase source into a two-phase source, the circuit is commonly referred to as a single-phase transformer.

The load current obtained from either circuit is the same; the choice of which circuit should be employed is application dependant. Factors to be considered in such a selection include cost, required load voltage, and the available voltage supply.

DESCRIPTION OF MODEL AND BOUNDARY CONDITIONS

The method used to calculate the commutation angle and load current is dictated by the fact that no a priori knowledge of the magnitude of the current at the boundaries exists. The current at the boundary points depends on the firing and commutation angles as well as the circuit parameters. As a result, the conventional Laplace transformation approach is not applicable.

To solve the problem, a set of equations with undetermined constants of integration were developed. The equations represent the current in the load during commutation and conduction and the phase "a" current during commutation. Through successive application of the boundary conditions, four independent transcendental equations containing the constants of integration and commutation angle were generated. The equations were then solved simultaneously through an iterative process resulting in numerical values valid for the input circuit values and firing angle.

Continuity of load current was assumed. The method of application of the boundary conditions forces continuous load current and cases resulting in a discontinuous response will not lead to accurate results. Fortunately, it is not difficult to determine if a discontinuous current situation exists. The procedure to determine continuity of current is discussed later.

During the conduction phase of operation, either circuit may be represented by the single phase equivalent shown in figure 3. Since the response of both "phases" to the load is symmetrical, it is only required to calculate the phase "a" response for one half cycle. The response during the second half cycle is simply a time shifted duplication of the phase "a" current.

The commutation circuit (fig. 4) represents the circuit while SCR 1 is coming on and SCR 2 is going off. The voltage sources are sinusoidal so \( v_a = V \sin \omega t \) and \( v_b = V \sin \omega t \). The time reference is established so that \( t = 0 \) when the phase "a" voltage becomes positive. SCR 1 is fired at \( \omega t = \alpha \). It is assumed that SCR 1 is forward biased at this point. During commutation, both phases conduct simultaneously and the load current, \( i_d \), is the sum of the phase currents \( i_a \) and \( i_b \).

The expected load response is illustrated in figure 5. The following boundary conditions used in the formulation of the load current equations are readily apparent from reference to the figure:

\[
\begin{align*}
    i_a(\alpha) &= 0 \\
    i_d(\alpha + \mu) &= i_a(\alpha + \mu) \\
    i_d(\alpha + \mu) &= i_a c(\alpha + \mu) \\
    i_a c(\alpha + \pi) - i_d(\alpha) &= 0
\end{align*}
\]
DEVELOPMENT OF EQUATIONS FOR LOAD CURRENT AND CONSTANTS

Beginning with the method described by Hoft$^2$, the voltage balance around the outside and lower paths in figure 4 is written

\[-V \sin \omega t + L_a \frac{d}{dt} i_a + L \frac{d}{dt} (i_a + i_b) + R(i_a + i_b) + V_c = 0\]

\[V \sin \omega t + L_t \frac{d}{dt} i_b + L \frac{d}{dt} (i_a + i_b) + R(i_a + i_b) + V_c = 0\]

Adding these two equations and defining a load current during commutation, $i_d = i_a + i_b$, leads to an equation without a forcing term

\[\frac{d}{dt} i_d + \frac{2Ri_d}{L_a + 2L} + \frac{2V_c}{L_t + 2L} = 0\]  \hspace{1cm} (1)

Equation 1 is a linear first order differential equation and has the solution

\[i_d = k_1 e^{-(\tau_1 - \omega t)} \frac{V_c}{R}\]  \hspace{1cm} (2)

where

\[\tau_1 = \frac{L_t + 2L}{2R}\]

and $k_1$ is to be determined from the boundary conditions. $\tau_1$ is the time constant of the circuit during commutation.

In order to obtain an independent expression for the phase "a" current during commutation, consider the voltage balance around the outer loop of figure 4

\[-V \sin \omega t + L_a \frac{d}{dt} i_a + V_0 = 0\]  \hspace{1cm} (3)

where

\[V_0 = L \frac{d}{dt} i_d + R i_d + V_c\]

Taking the derivative of equation 2 and substituting for $V_0$ and $i_d$ in equation 3 yields a separable differential equation for $i_a$ which can be integrated as

$$i_a \int \left\{ \frac{V}{L_t} \sin \omega t - \frac{k_1}{2 \tau_1} \right\} e^{-(\text{to} - \alpha)/\omega t} dt$$

$$= - \frac{V}{\omega L_t} \cos \omega t + \frac{k_1}{2} e^{-(\text{to} - \alpha)/\omega t} + k_2$$

(4)

To determine the constant $k_2$, apply the first boundary condition ($i_a(\alpha) = 0$) to equation 4. Then,

$$k_2 = \frac{V}{\omega L_t} \cos \alpha - \frac{k_1}{2}$$

(5)

Applying the second boundary condition [$i_d(\alpha + \mu) = i_a(\alpha + \mu)$] in equations 2 and 4 and substituting for $k_2$ from above results in an expression for $k_1$,

$$k_1 = \frac{2}{1 + e^{\mu/\omega \alpha}} \left\{ \frac{V}{\omega L_t} [\cos \alpha - \cos(\alpha + \mu)] + \frac{V}{R} \right\}$$

(6)

Now consider the conduction period for phase "a" ($\alpha + \mu < \omega t < \alpha + \pi$). Referring to figure 3, the voltage around the circuit can be expressed as

$$-V \sin \omega t + (L_t + L) \frac{di_a}{dt} + Ri_a + V_i = 0$$

The solution for the conduction current is

$$i_a = \frac{V}{Z} \sin(\omega t - \Phi) + k_1 e^{-(\text{to} - \alpha - \mu)/\omega t} - \frac{V_i}{R}$$

(7)

where

$$Z = \sqrt{\left(\omega L_t\right)^2 + R^2}$$

$$\Phi = \tan^{-1} \left( \frac{\omega L_t}{R} \right)$$

and

$$\tau = \frac{L_t + l}{R}$$
\( \tau_2 \) is referred to as the conduction time constant and \( k_3 \) is to be determined from the third boundary condition.

The third boundary condition deserves some explanation. Note that 
\[
i_d(\alpha + \mu) = i_a(\alpha + \mu) + i_b(\alpha + \mu) \quad \text{and} \quad i_b(\alpha + \mu) = 0.
\]
In addition, 
\[
i_a(\alpha + \mu) = i_{ac}(\alpha + \mu);
\]

Substituting this condition into equations 7 and 2 leads to an expression for \( k_3 \)
\[
k_3 = k_1 e^{-\mu/\omega t_1} - \frac{V}{Z} \sin(\alpha + \mu - \Phi) \tag{8}
\]

One more independent equation is required to completely specify the four unknowns \((k_1, k_2, k_3, \text{and} \mu)\). To obtain the last equation, the remaining boundary condition is used. Note first that for steady state operation, 
\[
i_{ac}(\alpha + \pi) = i_b(\alpha).
\]
Using reasoning similar to that above, since 
\[
i_a(\alpha) = 0, \quad i_{ac}(\alpha + \pi) = i_d(\alpha).
\]
Using this information in equations 2 and 4 yields
\[
k_1 = -\frac{V}{Z} \sin(\alpha - \Phi) + k_3 e^{-(\pi - \mu)/\omega t_1} \tag{9}
\]

Equations 5, 6, 8, and 9 are four independent equations defining \( k_1, k_2, k_3, \text{and} \mu \).

Substituting equations 6 and 8 into equation 9 yields a single (albeit cumbersome) transcendental equation for \( \mu \) in terms of circuit parameters. Numerical solution to this equation then allows calculation of \( k_1, k_2, \text{and} k_3 \). Once these constants are known, \( i_{ac}, i_d, \text{and} i_a \) can be readily calculated.

**DETERMINATION OF MINIMUM PHASE CONTROL ANGLE**

As mentioned previously, it is assumed that SCR 1 is forward biased at \( \omega t = \alpha \).

Once \( \mu \) is numerically determined, it is a simple matter to check this assumption. Just prior to the instant SCR 1 is fired, Kirchoff’s voltage law can be written for the top loop in figure 4
\[
-2V \sin \omega t + V_{ak} - L \frac{d i_b}{dt} = 0
\]
Since $i_a = 0$, $i_b = i_d$ and, using equation 2, the voltage on the SCR is

$$V_{AK1} = -\frac{L_i k_1}{\tau_1} + 2V\sin\omega t$$

If $V_{AK1}$ is less than zero, then the assumption was violated and the SCR could not be forwarded biased at $\omega t = \alpha$.

### DETERMINATION OF CONTINUITY OF CURRENT

In the previous derivations, it was presumed that operation was under steady-state continuous-current conditions (i.e., $i_d$ is never zero). There are two indicators which can be examined to determine if this assumption is violated.

Some combinations of circuit parameters and firing angle always produce a discontinuous current. These can be determined by assuming a discontinuous current (so there is no commutation) and writing Kirchoff’s law for phase “a” conduction

$$V\sin\omega t = (L_i + L) \frac{di_a}{dt} + R i_a + V_c$$

This equation applies for $\alpha < \omega t < \beta$ where $\beta$ is the electrical angle at which $i_a$ returns to zero. Solving the voltage equation for $i_a$ and writing $i_a(\omega t = \beta)$ produces a transcendental equation for $\beta$.

$$\left\{ \frac{E}{R} - \frac{V}{Z} \sin(\alpha - \Phi) \right\} e^{-R(\alpha - \beta)/\omega L} + \frac{V}{Z} \sin(\beta - \Phi) - \frac{E}{R} = 0$$

---


Since \( i_a \) is always positive while SCR 1 conducts, it is not necessary to solve this equation. Substitution of values for \( \beta \) which are less than the true value will produce a positive number from the above equation, while substitution of values for \( \beta \) which are greater than the true value will produce a negative number. If substitution of the value \( \beta = \alpha + \pi \) results in a negative value, then the true \( \beta \) is less than \( \alpha + \pi \) and the circuit shuts off completely prior to firing SCR 2. In this case, the current is discontinuous.

The above criterion for discontinuity is not sufficient in all cases. Some cases which pass that test still result in negative values for phase "a" current (indicating a discontinuous case) for some value of \( \omega t \) between \( \alpha \) and \( \alpha + \pi \). These cases can only be found by calculating \( \mu \) and the circuit response for the range of values \( \alpha \leq \omega t \leq \alpha + \pi \).

RESULTS

The transcendental equation for \( \mu \) was solved numerically on a CDC Dual Cyber 170/750. The program (Appendices A and B) allowed input of all circuit parameters and a firing angle, \( \alpha \). The commutation angle, \( \mu \), was then calculated using a straightforward method of bisections. If the input \( \alpha \) failed the check, as discussed previously, \( \alpha \) was incremented by 1 degree and \( \mu \) was recalculated. This continued until an acceptable \( \alpha \) was found (to within 1 degree). The current values were then calculated and plotted. In spite of the unsophisticated method of bisections, run times were relatively short.

The plots from several runs are included which show some trends as various parameters are varied (figs. 6 through 11). The input \( \alpha \) for all runs was zero so that the \( \alpha \) indicated was the minimum \( \alpha \) determined iteratively. In general, large values of inductances were selected to clearly display various trends.

Most runs provided current plots with the expected shapes, even for the flat response expected from a very large load inductance (fig. 10). One run in which the load electromotive force is 50 volts (fig. 11) shows a negative phase "a" current indicating the discontinuous case, which violate our initial assumptions. Therefore, these results are invalid. This case is interesting because it did not have a value of \( \beta \) greater than \( \alpha + \pi \).
Figure 1. Bridge rectifier

Figure 2. Two-phase rectifier
Figure 3. Conduction circuit

Figure 4. Commutation circuit
Figure 5. Expected load response

Figure 6. Load response for transformer inductance of 50 mH and load inductance of 326 mH
Figure 7. Load response for transformer inductance increased to 250 mH.

Figure 8. Load response for back EMF of 10 V.
Figure 9. Load response for back EMF increased to 30 V

Figure 10. Flat response for large load inductance
**PHASE A**  
**CURRENT**

<table>
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<tr>
<th>Parameter</th>
<th>Value</th>
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<tr>
<td>Source Voltage (V)</td>
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</tr>
<tr>
<td>Load Resistance (Ω)</td>
<td>100.00</td>
</tr>
<tr>
<td>Source Frequency (Hz)</td>
<td>60.00</td>
</tr>
<tr>
<td>Load EMF (V)</td>
<td>50.00</td>
</tr>
<tr>
<td>Firing Angle (Deg)</td>
<td>11.00</td>
</tr>
<tr>
<td>Conduction Tau (S)</td>
<td>0.0060</td>
</tr>
<tr>
<td>Trans Inductance (H)</td>
<td>0.3000000</td>
</tr>
<tr>
<td>Commutation Tau (S)</td>
<td>0.0045</td>
</tr>
<tr>
<td>Load Inductance (H)</td>
<td>0.3000000</td>
</tr>
<tr>
<td>Commutation Angle (Deg)</td>
<td>6.7505</td>
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**Figure 11.** Invalid response showing negative current swing  
(Current in this case is discontinuous and is not correctly calculated with this model.)
APPENDIX A

FORTRAN SOURCE CODE
PROGRAM SGLFAZ(INPUT, OUTPUT, TAPE5=INPUT, TAPE6=OUTPUT, TAPE7)

* FORTRAN PROGRAM SINGLE PHASE
* JOHN PAPPAS & JOSEPH BENO
* UNIVERSITY OF TEXAS AT AUSTIN
* DEPARTMENT OF COMPUTER AND ELECTRICAL ENGINEERING
* 20 APRIL 1987
* COMPUTES THE COMMUTATION ANGLE AND LOAD RESPONSE
* FOR A SINGLE PHASE FULL-WAVE RECTIFIER. USER
* INPUTS ARE LOAD AND SOURCE CHARACTERISTICS, FIRING
* ANGLE AND TRANSFORMER INDUCTANCE.

* VARIABLE DICTIONARY

* A  FIRING ANGLE IN RADIANS
  ALPHA  FIRING ANGLE IN DEGREES
  DELTA  TIME-ANGLE INCREMENT IN CURRENT CALCULATION
  ESIGN  DETERMINES ZERO CROSSING IN MU ESTIMATION ROUTINE
  EFAC  ACCURACY FACTOR IN METHOD OF BISECTION
  ENEW  VALUE OF FUNCTION DRIVEN TO ZERO IN MU ESTIMATION
  EOLD  VALUE OF FUNCTION DRIVEN TO ZERO IN MU ESTIMATION
  ERROR  NEARNESS TO ZERO REQUIREMENT FOR FUNCTION OF MU IN BISECTION
  F  SOURCE FREQUENCY IN HERTZ
  FX  SUBSCRIPTED F (IE. F1, ETC.). HOLDS VALUES GENERATED IN KX ROUTINE
  ID  LOAD CURRENT DURING COMMUTATION
  IA  SCR#1 CURRENT DURING COMMUTATION
  IAC  SCR#1 AND LOAD CURRENT DURING CONDUCTION
  IAP  ARRAY HOLDING PHASE A CURRENT VALUES FOR ZETA PLOT ROUTINE
  IDP  ARRAY HOLDING LOAD CURRENT DURING COMMUTATION FOR ZETA PLOT ROUTINE
  J  COUNTER
  KX  SINGLE SUBSCRIPTED K. FUNCTION CONTAINING CONSTANTS OF INTEGRATION IN CURRENT EQUATIONS
  KXX  DOUBLE SUBSCRIPTED K. INTERMEDIATE
RESULT

L   LOAD INDUCTANCE IN HENRIES
LL  TRANSFORMER INDUCTANCE IN HENRIES
MU  COMMUTATION ANGLE IN DEGREES
MUST START ESTIMATE OF MU FOR BISECTION
MUSTP STOP ESTIMATE OF MU FOR BISECTION
MUOLD INTERMEDIATE RESULT IN BISECTION
    USED IN ACCURACY CHECK
MUNEW INTERMEDIATE RESULT IN BISECTION
    USED IN ACCURACY CHECK
PHI LOAD IMPEDANCE ANGLE IN RADIANS
PI   PI
R   LOAD RESISTANCE IN OHMS
T1  TIME CONSTANT OF COMMUTATION CIRCUIT
T2  TIME CONSTANT OF CONDUCTION CIRCUIT
TEST1 SIGN DETERMINES SIDE OF ZERO CROSSING
    IN BISECTION
U   COMMUTATION ANGLE IN RADIANS
V   PEAK VALUE OF SOURCE IN VOLTS
VC  LOAD EMF
W   SOURCE FREQUENCY IN R/S
WT  INCREMENT AT LE OF HALF CYCLE
WTP ARRAY HOLDING TIME-ANGLE INFORMATION
    FOR ZETA PLOT ROUTINE
Z   LOAD IMPEDANCE MAGNITUDE IN OHMS

*************************************************************************

HOUSEKEEPING

*************************************************************************

REAL L, LL, K1, K12, K13, K1, K41, K42, K43, K44, K4
REAL MU, K2, K3, MUST, MUSTP, MUOLD, MUNEW, ID, IA, IAC
REAL IAI, IA2, IA3
REAL IAP(182), IDP(182), WTP(182)

*************************************************************************

: INTERACTIVE DATA ENTRY

*************************************************************************

299 FORMAT(/**/
270 FORMAT('INPUT DATA(1) OR USE INTERNAL(0)')
271 FORMAT('DATA MAY BE INPUT IN REAL, INTGEO UT OR
    EXPONENTIAL (XX.XE XX) FORM')
272 FORMAT('PEAK SOURCE VOLTAGE (V)')
273 FORMAT('SOURCE FREQUENCY (HZ)')
274 FORMAT('TRANSFORMER INDUCTANCE (H)')
275 FORMAT('LOAD INDUCTANCE (H)')
276 FORMAT('LOAD EMF, VC (V)')
277 FORMAT('LOAD RESISTANCE (OHM)')
278 FORMAT('FIRING ANGLE (DEG)')

18
FORMAT('CONVERGENCE FACTOR (START WITH ABOUT 0.1)')
FORMAT('RUN IDENTIFICATION NUMBER')
FORMAT('200 ITERATIONS AND STILL THE FUNCTION DOES
& NOT CONVERGE!!')
FORMAT('THE ERROR FACTOR CURRENTLY IN USE IS: ',G18.8)
FORMAT('THE "ZERO" YOU ARE TRYING TO MATCH IS: ',G18.8)
FORMAT('THE CLOSEST YOU HAVE COME SO FAR IS: ',G18.8)
FORMAT('INPUT A LARGER ERROR FACTOR')
FORMAT('MINIMUM ALPHA REQUIREMENT VIOLATED.'))
FORMAT('ALPHA INCREASED BY 1 DEGREE TO:',F7.3,'DEGREES')
WRITE(9,400)
ACCEPT J
IF(J.EQ.0)GOTO 10
WRITE(9,401)
WRITE(9,402)
ACCEPT V
WRITE(9,403)
ACCEPT F
WRITE(9,404)
ACCEPT LL
WRITE(9,405)
ACCEPT L
WRITE(9,406)
ACCEPT VC
WRITE(9,407)
ACCEPT R
WRITE(9,408)
ACCEPT ALPHA
WRITE(9,409)
ACCEPT EFACT
WRITE(9,410)
WRITE(9,411)
ACCEPT RUN
GOTO 70

-----------------------------------------------------------------------------
* INITIALIZATION AND IDENTIFICATION OF CONSTANTS
-----------------------------------------------------------------------------

EFACT=0.1
R: 100.
L: -1
L*: -128
V: 163.
VC=0.
ALPHA=0.
F=60.
RUN=100.
20 WT=0.
  I=1
  ENEW=0.
  MUSTP=0.
  J=0
  T1=(LL+2.*L)/(2.*R)
  T2=(LL+L)/R
  PI=ACOS(-1.)
  W=2*PI*F
  Z=(R**2.+(LL+L)**2.)*(W**2.)**.5
  PHI=ATAN2(W*(LL+L),R)
30 A=(2.*PI/360.)*ALPHA
  U=0

******************************************************************************
  ITERATIVE CALCULATION TO ESTIMATE THE VALUE OF THE
  COMMUTATION ANGLE
******************************************************************************

110 F1=K1(A,U,T1,W,V,LL,VC,R)
   F3=K3(A,U,T1,W,VC,F1,PHI,Z,R)
   E1=2.*VC*T1
   E2=V/Z*SIN(A-PHI)
   E3=F3*EXP(-(PI-U)/(W*T2))
   E4=VC/R
   EOLD=ENEW
   ENEW=E1-F1-E2+E3-E4
   MUST=MUSTP
   MUSTP=U*180./PI
   E=ENEW-EOLD
   ERROR=ABS(ENEW-EOLD)*EFAC
   IF(E.LT.0) GOTO 100
   U=U+PI/180.
   GOTO 110

******************************************************************************
  METHOD OF BISECTION TO CALCULATE
  COMMUTATION ANGLE
******************************************************************************

120 MUOLD=MUST
   MU=(MUST+MUSTP)/2.
120 IF(J.EQ.200) THEN
   J=0
   WRITE(9,399)
WRITE (9, 440)
WRITE (9, 399)
WRITE (9, 441) EFACT
WRITE (9, 399)
WRITE (9, 442) ERROR
WRITE (9, 399)
WRITE (9, 443) ABS (E)
WRITE (9, 399)
WRITE (9, 444)
ACCEPT EFACT
ERROR = ABS (ENEW - EOLD) * EFACT
ENDIF

J = J + 1
U = MU * PI / 180.
F1 = K1 (A, U, T1, W, V, LL, VC, R)
F3 = K3 (A, U, T1, W, V, VC, F1, PHI, Z, R)
E3 = F3 * EXP (- (PI - U) / (W * T2))
E = E1 - F1 - E2 + E3 - E4
IF (ABS (E) .LE. ERROR) THEN
  GOTO 130
ENDIF

TEST1 = EOLD * E
IF (TEST1 .LT. 0) THEN
  MUNEW = (MU - MUOLD) / 2. + MUOLD
  MUOLD = MU
  MU = MUNEW
  GOTO 120
ENDIF

MUNEW = (MU - MUOLD) * 1.5 + MUOLD
MUOLD = MU
MU = MUNEW
GOTO 120

130 U = MU * PI / 180.
F1 = K1 (A, U, T1, W, V, LL, VC, R)
F2 = K2 (A, T1, W, V, LL, VC, R, F1)
F3 = K3 (A, U, T1, W, V, VC, F1, PHI, Z, R)

***********************************************************************

* CHECK FOR MINIMUM FIRING ANGLE
***********************************************************************

ACHECK = 2 * V * SIN (A) - (LL * F1) / T1
IF (ACHECK .LT. 0.) THEN
  ALPHAT = ALPHAT + 1

21
WRITE (9, 445)
WRITE (9, 446) ALPHA
GOTO 30
ENDIF

CALL OUTPUT (F1, F2, F3, T1, T2, V, VC, LL, L, R, ALPHA & , MU, F, EFACT, RUN)

******************************************************************************************************************
* CALCULATE CURRENT RESPONSE THROUGH COMMUTATION AND
* ONE CONDUCTION PERIOD (WT=PI+ALPHA)
******************************************************************************************************************

WT=A
DELTA=PI/180.

140 IDP(I)=F1*EXP(-(WT-A)/(W*T1))-2.*VC*T1
IA1=V/(W*LL)*COS(WT)
IA2=(F1/2)*EXP(-(WT-A)/(W*T1))
IA3=(VC/LL)*(1.-2.*R*T1)*(WT/W)
IAP(I)=-IA1+IA2-IA3+F2
WTP(I)=WT
I=I+1
WT=WT+DELTA

IF(WT.LE.A+U)GOTO 140

150 IAP(I)=(V/Z)*SIN(WT-PHI)+F3*EXP(-(WT-A-U)/(W*T2))
& (WTT2))-VC/R
WTP(I)=WT
I=I+1
WT=WT+DELTA

IF(WT.LE.A+PI)GOTO 150

******************************************************************************************************************
* CREATE PLOT FILE COMPATIBLE WITH ZETA PLOTTER
******************************************************************************************************************

WTP(181)=A
WTP(182)=18.
NPTS=1/DELTA
NPTS2=180.
CALL PLOTS(0,0,L"PLOTS")
CALL PGIN(1.0,4.0,0)
CALL SCALE(IAP,6.,180,1)
IDP(181)=IAP(181)
IDP(182)=IAP(182)
CALL AXIS(0.,0.,'wt',-2,10.,0.,WTP(181),WTP(182))
CALL AXIS(0.,0.,'Current (A)',11,6.,90.,IAP(181))
CALL LINE(WTP,IAP,NPTS2,1,0,2)
CALL LINE(WTP,IDP,NPTS,1,0,2)
CALL PLOT(0.,0.,999)
STOP
END

*****************************************************************************
* DEFINED FUNCTIONS FOR CONSTANT OF INTEGRATION
* IN MU AND CURRENT EQUATIONS
*****************************************************************************

REAL FUNCTION K1(A,U,T1,W,LL,VC,R)
REAL K11,K12,K13,A,U,T1,W,LL,VC,R
K11=2./(1.+EXP(-U/(W*T1)))
K12=V/(W*LL)*(COS(A)-COS(A+U))
K13=VC/(W*LL)*U*(1.-2.*R*T1)
K1=K11*(K12-K13+2.*VC*T1)
RETURN
END

REAL FUNCTION K2(A,T1,W,LL,VC,R,F1)
REAL A,T1,W,LL,VC,R,F1,K21,K22,K23
K21=V/(W*LL)*COS(A)
K22=F1/2.
K23=(VC*A)/(W*LL)*(1.-2.*F1*T1)
K2=K21-K22+K23
RETURN
END

REAL FUNCTION K3(A,U,T1,W,VC,F1,PHI,Z,R)
REAL A,U,T1,W,VC,F1,K31,K32,K34,Z,R,K33
K31=F1*EXP(-U/(W*T1))
K32=2.*VC*T1
K33=VC/R
K34=V/Z*SIN(A+U-PHI)
K3=K31-K32-K33-K34
RETURN
END

*****************************************************************************
* SUBROUTINE TO PRINT INPUT DATA AND CALCULATED
* CONSTANTS TO OUTPUT FILE
*****************************************************************************
SUBROUTINE OUTPUT(F1,F2,F3,T1,T2,V,VC,LL,L,R,ALPHA & ,MU,F,EFACT,RUN)
        410 FORMAT(//)
        399 FORMAT(//)
        411 FORMAT(10X,'INPUT DATA & CALCULATED CONSTANTS FOR & PROGRAM SINGLE PHASE')
        412 FORMAT(34X,'RUN #',F6.2)
        413 FORMAT(10X,'PEAK SOURCE VOLTAGE    ',G18.8, ' VOLTS')
        414 FORMAT(10X,'SOURCE FREQUENCY      ',G18.8, ' HERTZ')
        415 FORMAT(10X,'TRANSFORMER INDUCTANCE ',G18.8, ' HENRIES')
        416 FORMAT(10X,'FIRING ANGLE       ',G18.8, ' DEGREES')
        417 FORMAT(10X,'LOAD INDUCTANCE    ',G18.8, ' HENRIES')
        418 FORMAT(10X,'LOAD RESISTANCE     ',G18.8, ' OHMS')
        419 FORMAT(10X,'LOAD EMF           ',G18.8, ' VOLTS')
        420 FORMAT(10X,'COMMUTATION ANGLE ',G18.8, ' DEGREES')
        421 FORMAT(10X,'ACCURACY FACTOR   ',G18.8)
        422 FORMAT(10X,'COMMUTATION TAU   ',G18.8, ' SECONDS')
        423 FORMAT(10X,'CONDUCTION TAU    ',G18.8, ' SECONDS')
        424 FORMAT(10X,'CONSTANTS IN CURRENT EQUATIONS')
        425 FORMAT(15X,'K1=',G18.8)
        426 FORMAT(15X,'K2=',G18.8)
        427 FORMAT(15X,'K3=',G18.8)

WRITE(7,410)
WRITE(7,411)
WRITE(7,412) RUN
WRITE(7,399)
WRITE(7,413) V
WRITE(7,410) F
WRITE(7,416) ALPHA
WRITE(7,410)
WRITE(7,415) LL
WRITE(7,410)
WRITE(7,417) L
WRITE(7,410)
WRITE(7,418) R
WRITE(7,410)
WRITE(7,419) VC
WRITE(7,410)
WRITE(7,422) T1
WRITE(7,410)
WRITE(7,423) T2
WRITE(7,410)
WRITE(7,420) MU
WRITE(7,410)
WRITE(7,421) EFACT
WRITE(7,399)
WRITE(7,424)
WRITE(7,410)
WRITE(7,425) F1
WRITE(7,410)
WRITE(7,426) F2
WRITE(7,410)
WRITE(7,427)F3
RETURN
END
APPENDIX B

INSTRUCTIONS FOR RUNNING THE FORTRAN PROGRAM
INSTRUCTIONS FOR RUNNING PROGRAM SGLFAZ ON THE UT CDC DUAL CYBER 170/750

Log into your account, then type the commands listed below at the appropriate prompt. User inputs are in **boldface** type.

```fortran
.READ PF **** **** SGLFAZ
.FTN5 I=SGLFAZ
.REWALLX
.LDSET LIB=ZETLIBF/
.LGO
```

**** **** is the access code for the permanent file where the program is stored.

At this point, you will be presented with a choice of using input data contained in the source file or entering your own. If you choose the internal data, the program will run and create the plot and output data files.

If you choose to input your own data, the program will prompt you from the screen. Correct units for the data are indicated with each prompt.

After the program has run, the load response can be plotted. At the prompt type:

```
.DISPOSE PLOT ID=**
```

** is the unit identifier for the plotter you are using.

In order to view the input data used and the constants calculated in the program (μ, k1, k2, k3, α, t1, and t2) on the screen, type:

```
.SHOW TAPE7
```

If you want to print the data on a line printer type:

```
.PRINT TAPE7 ID=**
```

** is the unit identifier of the line printer you are using.

The file "OUTPUT" contains a compiled listing of the FORTRAN source code.
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