MODELING ITEM RESPONSES WHEN DIFFERENT SUBJECTS
EMPLOY DIFFERENT SOLUTION STRATEGIES

Robert J. Mislevy
and
Norman Verhelst
CITO
(National Institute for Educational Measurement)
Arnhem, The Netherlands

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Robert J. Mislevy, Principal Investigator
Educational Testing Service
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A model is presented for item responses when different examinees employ different strategies to arrive at their answers, and when only those answers, not choice of strategy or subtask results, can be observed. Using substantive theory to differentiate the likelihoods of response vectors under a fixed set of solution strategies, we model responses in terms of item parameters associated with each strategy, proportions of the population employing each, and the distributions of examinee parameters within each. Posterior distributions can then be obtained for each examinee, giving the probabilities that they employed each of the strategies and their proficiency under each. The ideas are illustrated with a conceptual example about response strategies for spatial rotation items, and a numerical example resolving a population of examinees into subpopulations of valid responders and random guessers.
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Educational Testing Service

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Abstract

A model is presented for item responses when different examinees employ different strategies to arrive at their answers, and when only those answers, not choice of strategy or subtask results, can be observed. Using substantive theory to differentiate the likelihoods of response vectors under a fixed set of solution strategies, we model responses in terms of item parameters associated with each strategy, proportions of the population employing each, and the distributions of examinee parameters within each. Posterior distributions can then be obtained for each examinee, giving the probabilities that they employed each of the strategies and their proficiency under each. The ideas are illustrated with a conceptual example about response strategies for spatial rotation items, and a numerical example resolving a population of examinees into subpopulations of valid responders and random guessers.

Key Words: Differential strategies, Item response theory, Linear logistic test model, Mixture models
Different Strategies

Introduction

The standard models of item response theory (IRT), such as the 1-, 2-, and 3-parameter normal and logistic models, characterize examinees in terms of their propensities to make correct responses. Consequently, examinee parameter estimates are strongly related to simple percent-correct scores (adjusted for the average item difficulties, if not all examinees have been presented the same items). Item parameters characterize the regression of a correct response on this overall propensity toward correctness.

These models lend themselves well to tests in which all examinees employ the same strategy to solve the items. Comparisons among estimates of examinees' ability parameters are meaningful comparisons of their degrees of success in implementing the strategy. Item parameters reflect the number or complexity of the operations needed to solve a given item (Fischer, 1973).

The same models can prove less satisfactory when different examinees employ different strategies. The validity of using scores that convey little more than percent-correct to compare examinees who have used different strategies must first be called into question. And item parameters keyed only to a generalized propensity toward correctness will not reveal how a particular kind of item might be easy for examinees who follow one line of attack, but difficult for those who follow another.
Extensions of IRT to multiple strategies have several potential uses. In psychology, such a model would provide a rigorous analytic framework for testing alternative theories about cognitive processing (e.g., Carter, Pazak, and Kail, 1983). In education, estimates of how students solve problems could be more valuable than how many they solve, for the purposes of diagnosis, remediation, and curriculum revision (Messick, 1984). And even when a standard IRT model would provide reasonable summaries and meaningful comparisons for most examinees, an extended model allowing for departures along predetermined lines (e.g., malingering) would reduce estimation biases for the parameters in the standard model.

In contrast to standard IRT models, and, for that matter, to the "true score" models of classical test theory, a model that accommodates alternative strategies must begin with explicit statements about the processes by which examinees arrive at their answers. For example, items may be characterized in terms of the nature, number, and complexity of the operations required for their solution under each strategy that is posited.

The recent psychometric literature contains a few implementations of these ideas. Tatsuoka (1983) has studied performance on mathematics items in terms of the application of correct and incorrect rules, locating response vectors in a two-dimensional space based on an ability parameter from a standard IRT model and an index of lack of fit from that model. Paulson
different strategies (1985), analyzing similar data but with fewer rules, uses latent class models to relate the probability of correct responses on an item to the features it exhibits and the rules that examinees might be following. Yamamoto (1987) combines aspects of both of these models, positing subpopulations of IRT respondents and of non-scalable respondents associated with particular expected response patterns. Samejima’s (1983) and Embretson’s (1985) models for alternative strategies are expressed in terms of subtasks whose results are observed, in addition to the overall correctness or incorrectness of the item.

The present paper describes a family of multiple-strategy IRT models that apply when each examinee belongs to one of a number of exhaustive and mutually-exclusive classes that correspond to an item-solving strategy, and the responses from all examinees in a given class are in accordance with a standard IRT model. It is further assumed that for each item, its parameters under the IRT model for each strategy class can be related to known features of the item through psychological or pedagogical theory.

The next section of the paper gives a general description of the model. It is followed by a conceptual example that illustrates the key ideas. A two-stage estimation procedure is then presented. The first stage estimates structural parameters: basic parameters for test items, examinee population distributions, and proportions of examinees following each
strategy. The second stage estimates posterior distributions for individual examinees: the probability that they belong to each strategy class and the conditional distribution of their ability for each class. A numerical example resolves examinees into classes of valid responders and random guessers. The final section discusses some implications of the approach for educational and psychological testing.

The Response Model

This section lays out the basic structure for a mixture of constrained item response models. Discussion will be limited to dichotomous items for notational convenience, but the extensions to polytomous and continuous observations are straightforward.

We begin by briefly reviewing the general form of an IRT model. The probability of response \( x_{ij} \) (1 if correct, 0 if not) from person \( i \) to item \( j \) is given by an IRT model as

\[
p(x_{ij} | \theta_i, \beta_j) = \left[ f(\theta_i, \beta_j) \right]^{x_{ij}} \left[ 1 - f(\theta_i, \beta_j) \right]^{1 - x_{ij}}
\]  (1)

where \( \theta_i \) and \( \beta_j \) are real (and possibly vector-valued) parameters associated with person \( i \) and item \( j \) respectively, and \( f \) is a known, twice-differentiable, function whose range is the unit interval. Under the usual IRT assumption of local independence, the conditional probability of the response pattern \( \mathbf{x}_i = (x_{i1}, \ldots, x_{in}) \) of person \( i \) to \( n \) items is the product of \( n \) expressions like (1):
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\[ p(x_i | \theta_i, \beta) = \prod_{j=1}^{n} p(x_{ij} | \theta_i, \beta_j). \]

It may be possible to express item parameters as functions of some smaller number of more basic parameters \( \alpha = (\alpha_1, \ldots, \alpha_M) \) that reflect the effects of \( M \) salient characteristics of items; i.e., \( \beta_j = \beta_j(\alpha) \). An important example of this type is the Linear Logistic Test Model (LLTM; Fischer, 1973, Schieblechner, 1972). Under the LLTM, the item response function is the one-parameter logistic (Rasch) model, or

\[ p(x_{ij} | \theta_i, \beta_j(\alpha)) = \frac{\exp[\theta_i (\theta_i - \beta_j)]}{1 + \exp(\theta_i - \beta_j)}, \]

and the model for item parameters is linear:

\[ \beta_j(\alpha) = \sum_{m=1}^{M} Q_{jm} \alpha_m = Q_j' \alpha. \]

The elements of \( \alpha \) are contributions to item difficulty associated with the \( M \) characteristics of items, presumably related to the number or nature of processes required to solve them. The elements of the known vector \( Q_j \) indicate the extent to which item \( j \) exhibits each characteristic. Fischer (1973), for example, models the difficulty of the items in a calculus test in terms of the number of times an item requires the application of each of seven differentiation rules. \( Q_{jm} \) is the number of times that rule \( m \) must be employed in order to solve Item \( j \).
Consider now a set of items that may be answered by means of $K$ different strategies. It need not be the case that all are equally effective, nor even that all generally lead to correct responses. Not all strategies need be available to all examinees. We make the following assumptions.

1. Each examinee is applying the same one of these strategies for all the items in the set. (In the final section, we discuss prospects for relaxing this assumption to allow for strategy-switching).

2. The responses of an examinee are observed but the strategy he or she has employed is not.

3. The responses of examinees following Strategy $k$ conform to an item response model of a known form.

4. Substantive theory posits relationships between observable features of items and the probabilities of success enjoyed by members of each strategy class. The relationships may be known either fully or only partially (as when the Q matrices in LLTM-type models are known but the basic parameters are not).
Let the k'th element in the K-dimensional vector \( \phi_i \) take the value one if examinee i follows Strategy k, and zero if not. Extending the notation introduced above, we may write the conditional probability of response pattern \( x_i \) as

\[
p(x_i | \phi_i, \theta_i, \alpha) = \prod_{k,j} f_k(\theta_{ik}, \beta_{jk})^{x_{ij}} [1 - f_k(\theta_{ik}, \beta_{jk})]^{1 - x_{ij}} \phi_{ik}
\]

where \( \beta_{jk} = \beta_j(k) \) gives the item parameter(s) for Item j under Strategy k.

It will be natural in certain applications to partition basic parameters for items in accordance with strategy classes; that is, \( \alpha = (\alpha_1, \ldots, \alpha_K) \). When there are K versions of the LLTM, for example, differences among strategies are incorporated into the model by K different vectors \( Q_{jk}, k=1,\ldots,K \), that relate Item j to each of the strategies:

\[
\beta_{jk} = \sum_m Q_{jkm} \alpha_{km} = Q'_{jk} \alpha_k.
\]

The item difficulty parameter for Item j under Strategy k, then, is a weighted sum of elements in \( \alpha_k \), the basic parameter vector associated with Strategy k; the weights \( Q_{jkm} \) indicate the degree to which each of the features m, as relevant under Strategy k, are present in Item j. This situation will be illustrated in the following example.
Example 1: Alternative strategies for spatial tasks

The items of certain tests intended to measure spatial visualization abilities admit to solution by nonspatial analytic strategies (French, 1965; Kyllonen, Lohman, and Snow, 1984; Pelligrino, Mumaw, and Shute, 1985). Consider items in which subjects are shown a drawing of a three-dimensional target object, and asked whether a stimulus drawing could be the same object after rotation in the plane of the picture. In addition to rotation, one or more key features of the stimulus may differ from the those of target. A subject may solve the item either by rotating the target mentally the required degree and recognizing the match (Strategy 1), or by employing analytic reasoning to detect feature matches without performing rotation (Strategy 2).

Consider further a hypothetical three-item test comprised of such items. Each item will be characterized by (1) rotational displacement, of 60, 120, or 180 degrees, and by (2) the number of features that must be matched. Table 1 gives the features of the items in the hypothetical test.

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Insert Table 1 about here

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Each subject i will be characterized by two vectors. In the first, \( \phi_i = (\phi_{i1}, \phi_{i2}) \), \( \phi_{ik} \) takes the value 1 if Subject i employs
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Strategy $k$ and 0 if not. In the second, $\theta_i = (\theta_{i1}, \theta_{i2})$, $\theta_{ik}$ characterizes the proficiency of Subject $i$ if he employs Strategy $k$. Only one of the elements of $\theta_i$ is involved in producing Subject $i$'s responses, but we do not know which one.

Suppose that for subjects employing a rotational strategy, probability of success is given by the one-parameter logistic (Rasch) item response model:

$$P(X_{ij} | \theta_{i1}, \beta_{jl}, \phi_{j} - 1) = \frac{\exp[x_{ij}(\theta_{i1} - \beta_{jl})]}{[1 + \exp(\theta_{i1} - \beta_{jl})]} .$$

Here $\theta_{i1}$ is the proficiency of Subject $i$ at solving tasks by means of the rotational strategy, and $\beta_{jl}$ is the difficulty of Item $j$ under the rotational strategy.

It is usually found that the time required to solve mental rotation tasks is linearly related to rotational displacement. To an approximation, so are log-odds of success (Tapley and Bryden, 1977). We assume that under the rotational strategy, item parameters take the following form:

$$\beta_{jl} = Q_{jll} \alpha_{11} + \alpha_{12} ,$$

where $Q_{jll}$ encodes the rotational displacement of Item $j$--1 for 60 degrees, 2 for 120 degrees, and 3 for 180 degrees--and $\alpha_{11}$ is the incremental increase in difficulty for each increment in rotation; and $\alpha_{12}$ is a constant term, for which a coefficient $Q_{jll}^{-1}$ is implied for all items. If $\alpha_{11} = 1$ and $\alpha_{12} = 2$, the item parameters
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$\beta_{j1}$ that are in effect under Strategy 1 are as shown in the second column of Table 2.

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Insert Table 2 about here
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A Rasch model will also be assumed for subjects employing Strategy 2, the analytic strategy, but here the item parameters depend on the number of features that must be matched:

$$\beta_{j2} = Q_{j21} \alpha_{21} + \alpha_{22},$$

where $Q_{j21}$ is the number of salient features, $\alpha_{21}$ is the incremental contribution to item difficulty of an additional feature, $\alpha_{22}$ is a constant term, and $Q_{j22}=1$ implicitly for all items. If $\alpha_{21}=1.5$ and $\alpha_{22}=-2.5$, we obtain the item parameters that are in effect under Strategy 2. They appear in the third column of Table 2.

Note that the items have been constructed so that items that are relatively hard under one strategy are easy under the other. Strategy choice cannot be inferred from observed response patterns unless patterns are more likely under some strategies and less likely under others.

The response pattern 011, for example, has a correct answer to an item that is easy under the Strategy 2 but hard under Strategy 1, and an incorrect answer to an item that is hard under
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Strategy 2 but easy under Strategy 1. Figure 1 plots the likelihood function for the response vector 011 under both strategies; that is, 

\[ p(\mathbf{x}=(011)|\theta_k,\phi_k=1) \] for \( k=1,2 \) as a function of \( \theta_1 \) and \( \theta_2 \) respectively. The maximum of the likelihood under Strategy 2 is about eight times as high as the maximum attained under Strategy 1.

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Insert Figure 1 about here

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We can make probabilistic statements about individual subjects if we know the proportions of people who choose each strategy, or \( \pi_k = p(\phi_k=1) \), and the distributions of proficiency of those using each strategy class, or \( g_k(\theta_k) = p(\theta_k|\phi_k=1) \). Suppose that (i) \( \theta_1 \) and \( \theta_2 \) both follow standard normal distributions among the subjects that have chosen to follow them, and (ii) three times as many subjects follow Strategy 1 as follow Strategy 2—i.e., \( \pi_1 = 3/4 \) and \( \pi_2 = 1/4 \). This joint prior distribution is pictured in Figure 2.

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Insert Figure 2 about here

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Routine application of Bayes theorem then yields the joint posterior density function for \( \phi \) and \( \theta_k|\phi_k=1 \) for \( k=1,\ldots,K \):

\[ p(\theta_k=\theta,\phi_k=1|\mathbf{x},\pi,\alpha) \propto p(\mathbf{x}|\phi_k=1,\theta_k,\beta_k(\alpha)) \pi_k g_k(\theta). \] (3)
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where

\[ p[x|\phi_k=1, \theta, \beta_k(\alpha)] = \prod_{j} \exp[ij(\theta - \beta_jk(\alpha))]/[1 + (\theta - \beta_jk(\alpha))]. \]

The reciprocal of the constant of proportionality required to normalize (3) is the marginalization of the right side, or

\[ \sum_{k} \pi_k \int p[x|\phi_k=1, \theta, \beta_k(\alpha)] g_k(\theta) \, d\theta. \]

The posterior distribution induced by (011) is shown in Figure 3. Marginalizing with respect to \( \theta_k \) amounts to summing the area under the curve for Strategy \( k \), and gives the posterior probability that \( \phi_k=1 \)—that is, that the subject has employed Strategy \( k \). The resulting values for this response pattern are \( P(\phi_1=1|x=011)=.28 \) and \( P(\phi_2=1|x=011)=.72 \). The prior probabilities favoring Strategy 1 have been revised substantially in favor of Strategy 2. The conditional posterior for \( \theta_1 \) given \( \phi_1=1 \) has a mean and standard deviation of about .32 and .80. Corresponding values for the distribution of \( \theta_2 \) given \( \phi_2=1 \) are .50 and .81.

Insert Figure 3 about here

Parameter Estimation

This section discusses estimation procedures for mixtures of IRT models. A two-stage procedure is described. The first stage
integrates over $\theta$ and $\phi$ distributions to obtain a so-called marginal likelihood function for the structural parameters of the problem—the basic parameters for items, the proportions of subjects employing each strategy, and the parameters of the $\theta$ distributions of subjects employing each strategies. Maximum likelihood estimates are obtained by maximizing this likelihood function. If preferred, Bayes modal estimates can be obtained by similar numerical procedures by multiplying the likelihood by prior distributions for the structural parameters. The second stage takes the resulting point estimates of structural parameters as known, and calculates aspects of the posterior distribution of an individual examinee—e.g., $p(\phi_k=1|x)$ and $p(\theta_k|\phi_k=1,x)$.

Stage 1: Estimates of Structural Parameters

Equation 2 gives the conditional probability of the response vector $x$ given $\theta$ and $\phi$, or $p(x|\theta,\phi)$. Consider a population in which strategies are employed in proportions $\pi_k$ and within-strategy proficiencies have densities $g_k(\theta_k|\eta_k)$ among the examinees using them. The marginal probability of $x$ for an examinee selected at random from this population is

$$p(x|\alpha,\pi,\eta) = \sum_k \pi_k \int p(x|\theta_k,\phi_k=1,\alpha) g_k(\theta_k|\eta_k) \, d\theta_k . \quad (4)$$

For brevity, let $\xi$ denote the extended vector of all structural parameters, namely $(\alpha,\pi,\eta)$. The loglikelihood for $\xi$ induced by
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the observation of the response vectors \( \mathbf{X} = (\mathbf{x}_1, \ldots, \mathbf{x}_N) \) of \( N \) subjects is a constant plus the sum of the logs of terms like (4) for each subject:

\[
\lambda = \sum_{i=1}^{N} \log p(\mathbf{x}_i | \xi) = \sum_{i} \sum_{k} \phi_{ik} \log \int p(\mathbf{x}_i | \theta_k, \phi_{k-1}, \beta_k(\alpha)) g_k(\theta_k | \eta_k) d\theta_k + \sum_{i} \sum_{k} \phi_{ik} \log \pi_k .
\] (5)

Let \( \mathbf{S} \) be the vector of first derivatives, and \( \mathbf{H} \) the matrix of second derivatives, of \( \lambda \) with respect to \( \xi \). Under regularity conditions, the maximum likelihood estimates \( \hat{\xi} \) solve the likelihood equation \( \mathbf{S} = 0 \), and a large-sample approximation of the matrix of estimation errors is given by the negative inverse of \( \mathbf{H} \) evaluated at \( \hat{\xi} \).

A standard numerical approach to solving likelihood equations is to use some variation of Newton's method. Newton-Raphson iterations, for example, improve a provisional estimate \( \xi^0 \) by adding the correction term \(-\mathbf{H}^{-1} \mathbf{S} |_{\xi=\xi^0} \). Fletcher-Powell iterations avoid computing and inverting \( \mathbf{H} \) by using an approximation of \( \mathbf{H}^{-1} \) that is built up from changes in \( \mathbf{S} \) from one cycle to the next.

These solutions have the advantage of rapid convergence if starting values are reasonable—often fewer than 10 iterations.
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are necessary. $S$ and $H$ can be difficult to work out, however, and all parameters must be usually be dealt with simultaneously because the off-diagonal elements in $H$ needn't be zero. For these reasons, a computationally simpler but slower-converging solution based on Dempster, Laird, and Rubin's (1977) EM algorithm will now be described as well. The approximation uses discrete representations for the $g_k$s, so the relatively simple "finite mixtures" case obtains (Dempster, Laird, and Rubin, 1977).

Suppose that for each $k$, subject proficiency under Strategy $k$ can take only the $L(k)$ values $\theta_{k1}, \ldots, \theta_{KL(k)}$. The density $g_k$ is thus characterized by these points of support and by the weights associated with each, $g_k(\theta_{k1}, \ldots, \theta_{KL(k)})$. Define the subject variable $\psi_i = (\psi_{i1}, \ldots, \psi_{iKL(K)})$, a vector of length $\sum_k L(k)$ where the element $\psi_{ik}$ is 1 if the proficiency of Subject $i$ under Strategy $k$ is $\theta_{ik}$ and 0 if not. There are a total of $K$ Is in $\psi_i$, one for each strategy—though again, only only of them is involved in producing $x_i$—the one associated with the strategy that Subject $i$ happens to employ. Summations replace integrations in the loglikelihood, which can now be written as

$$
\lambda = \sum_{i} \sum_{k} \phi_{ik} \sum_{l} \psi_{ikl} \log p(x_i | \theta_{ik}, \phi_{ik}, \beta_{ik}(\alpha))
$$

$$
+ \sum_{i} \sum_{k} \phi_{ik} \sum_{l} \psi_{ikl} g_k(\theta_{ik} | \eta_k)
$$

$$
+ \sum_{i} \sum_{k} \phi_{ik} \log \pi_k.
$$

(6)
If values of $\phi$ and $\psi$ were observed along with values of $x$, ML estimation of $\xi$ from (6) would be simpler. The basic parameter $\alpha$ appears only in the first term on the right side of (6), so that maximizing with respect to $\alpha$ need address that term only. When $\alpha$ consists of distinct subvectors for each strategy, even these subvectors lead to distinct maximization problems of lower order. The subpopulation parameters $\eta$ appear in only the second term, separating them in ML estimation; they too lead to even smaller separate subproblems if $\eta$ consists of distinct subvectors for each strategy. The population proportions $\pi$ appear in only the last term. Unless they are further constrained, their ML estimates are simply observed proportions. The values of $\Theta$ may be either specified a priori (as in Mislevy, 1986) or estimated from the data (as in de Leeuw and Verhelst, 1986). In the latter case, their likelihood equations have contributions from both the first and second terms, but the equations for the points of support under Strategy $k$ involve data from only those subjects using Strategy $k$. Their cross second derivatives with points corresponding to other strategies are zero, although their cross derivatives with elements of $\alpha$ and $\eta$ that are involved with the same strategy need not be.

The M-step of an EM solution requires solving a maximization problem of exactly this type, with one exception: the unobserved values of each $\phi_i$ and $\psi_i$ are replaced by their conditional expectations given $x_i$ and provisional estimates of $\xi$, say $\xi^0$. The
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E-step calculates these conditional expectations as follows.

Denote by $l_{ikl}^2$ the following term in the marginal likelihood associated with Subject $i$, Strategy $k$, and proficiency value $\theta_{kl}$ within Strategy $k$:

$$l_{ikl}^2 = p[x_i | \theta_k, \phi_k, \gamma_k, \beta_k(\alpha)] g_k(\theta_k | \eta_k) \pi_k.$$

The required conditional expectations are obtained as

$$\psi_{ikl}^0 = E(\psi_{ikl} | x_i, \xi - \xi^0) = l_{ikl}^0 / \sum_{l'} l_{ikl'}^0,$$

and

$$\psi_{ikl}^0 = E(\phi_{ikl} | x_i, \xi = \xi^0) = l_{ikl}^0 / \sum_{k', l'} l_{ikl'}^0.$$

The EM formulation makes it clear how each subject contributes to the estimation of the parameters in all strategy classes, even though it is assumed that only one of them was relevant to the production of his responses. His data contribute to estimation for each strategy class is in the proportion to the
probability that that strategy was the one he employed, given his observed response pattern.

In addition to its simplicity, the EM solution has the advantage of being able to proceed from even very poor starting values. The slowness with which it converges can be a serious drawback, however. Its rate of convergence depends on how well \( x \) determines examinees' \( \theta \) and \( \phi \) values. Accelerating procedures such as those described by Ramsay (1975) and Louis (1982) can be used to hasten convergence.

Stage 2: Posteriors for Individual Examinees

When the population parameters \( \xi \) are accurately estimated, the posterior density of the parameters of examinee \( i \) is approximately

\[
p(\theta_{iK}^{\alpha}, \phi_{iK}^{\alpha} - 1 | x_i, \hat{\xi}) \propto p(x_i | \phi_{iK}^{\alpha} - 1, \beta_k(\alpha)) \pi_k g_k(\theta | \eta_k),
\]

where the reciprocal of the normalizing constant is obtained by first integrating the expression on the right over \( \theta \) within each \( k \), then summing over \( k \). The posterior probability that Subject \( i \) used Strategy \( k \) is approximated by

\[
P(\phi_{iK}^{\alpha} - 1 | x_i, \hat{\xi}) = \int p(\theta_{iK}^{\alpha}, \phi_{iK}^{\alpha} - 1 | x_i, \hat{\xi}) \, d\theta.
\]
The examinee's posterior mean and variance for a given strategy class, given that that was the strategy employed, are approximated by

$$\hat{\theta}_{ik} = \int \theta \, p(\theta, \phi_{ik} = 1 | \hat{x}_i, \hat{\xi}) \, d\theta / P(\phi_{ik} = 1 | \hat{x}_i, \hat{\xi})$$

and

$$\hat{\sigma}_{ik}^2 = \int (\theta - \hat{\theta}_{ik})^2 p(\theta, \phi_{ik} = 1 | \hat{x}_i, \hat{\xi}) \, d\theta / P(\phi_{ik} = 1 | \hat{x}_i, \hat{\xi}) .$$

If the discrete approximation has been employed, (7) and (8) apply.

Example 2: A Mixture of Valid Responders and Random Guessers

Given appropriate instructions, examinees will omit multiple-choice test items when they don't know the answers rather than guess at random. The Rasch model may provide a good fit to such data if omits are treated as incorrect. If a small percentage of examinees responds at random to all items, however, their responses will bias the estimation of the item parameters that pertain to the majority of the examinees.

We may posit a two-class model, under which an examinee responds either in accordance with the Rasch model or guesses totally at random. For examinees in the latter class, probabilities of correct response are constant, e.g., at the reciprocal of the number of response alternatives to each item.
Using the procedures described in the preceding sections, it is possible to free estimates of the item parameters that pertain to the valid responders from biases due to random guessers, even though it is not known with certainty who the guessers are.

A mixture model for the (marginal) probability of response pattern $\mathbf{x}$ in this situation is

$$
P(\mathbf{x} | \xi) = \sum_{k=1}^{2} P(\mathbf{x} | \phi_k = 1, \xi) \, \pi_k .
$$

where Strategy Class 1 is the Rasch model and Class 2 is random guessing. The composition of $\xi$ is now described. It includes first the strategy proportions $\pi_1$ and $\pi_2$. For the Rasch class, the basic parameters $\alpha_1$ are item difficulty parameters $b_j$ for $j = 1, \ldots, n$. Suppose the distribution $g_1$ of proficiencies of subjects following the Rasch model is discrete, with $L$ points of support $\Theta = (\theta_1, \ldots, \theta_L)$ and associated weights $\omega = (\omega_1, \ldots, \omega_L)$. The (marginal) probability of response pattern $\mathbf{x}$ under Strategy 1 is

$$
P(\mathbf{x} | \phi_1 = 1, \alpha_1, \Theta, \omega) = \sum_{\theta_1} \omega_1 \prod_{j} \exp[(\theta_j \cdot b_j)]/[1 + \exp(\theta_j \cdot b_j)] .
$$

Under the random guessing strategy, the basic parameters $\alpha_2$ are the probabilities $c_j$ of responding correctly to each item $j$. All subjects following this strategy are assumed to have the same probabilities of correct response, so no distribution $g_2$ enters
the picture. For such subjects, the probability of response pattern $x$ is simply

$$P(x|\phi_2=-1,\alpha_2) = \prod_j c_j^{x_j} (1-c_j)^{1-x_j}.$$ 

An artificial dataset was created for four items under this model in accordance with the following specifications. Of 1200 simulees in all, 1000 followed the Rasch model and 200 were random guessers, implying $\pi_1=.833$ and $\pi_2=.167$. The Rasch item parameters were $\alpha_1 = (b_1,\ldots,b_4) = (-.511,-.105,.182,.405)$. A discrete density with six points of support was used to create the data for the Rasch class. The points and their corresponding proportions were as follows:

<table>
<thead>
<tr>
<th>Point</th>
<th>Proportion</th>
</tr>
</thead>
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<tr>
<td>-1.204</td>
<td>.08</td>
</tr>
<tr>
<td>-1.357</td>
<td>.17</td>
</tr>
<tr>
<td>-.095</td>
<td>.25</td>
</tr>
<tr>
<td>.262</td>
<td>.25</td>
</tr>
<tr>
<td>.470</td>
<td>.17</td>
</tr>
<tr>
<td>.642</td>
<td>.08</td>
</tr>
</tbody>
</table>

The rates of correct response for the random guessers on the four items were $\alpha_2 = (c_1,\ldots,c_4) = (.30, .35, .20, .15)$. The probability of each of the sixteen possible response patterns was calculated within each class, multiplied by the number of simulees in that class, summed over classes, and rounded to the nearest integer. The resulting data are shown in Table 3.
A standard Rasch model was first fit to the data using the two-step marginal maximum likelihood procedures described by de Leeuw and Verhelst (1986). Conditional maximum likelihood (CML) estimates were first obtained for item parameters. Setting their scale by centering them around zero like the true item parameters for the Rasch class, the resulting values were (-.324, -.053, .127, .252). Note that these values are biased toward their center; the presence of random guessers blurs the distinctions among the differences in item difficulties. A three-point discrete distribution—the greatest number of points leading to an identified model for a four-item test—was next estimated for subjects. The expected counts of response patterns under this model are also shown in Table 3. A chi-square of 7.16 with 8 degrees of freedom results, indicating an acceptable fit for a sample of the size we have employed.

A mixture model of the generating form was then fit to the data, with two exceptions. First, the multiplicative form of the Rasch model was employed during calculations. Since maximum likelihood estimates are invariant under transformations, the estimates of the structural parameters obtained under the multiplicative form need merely be transformed back to the usual
additive form shown above. Second, a three-point discrete
distribution was again employed for the Rasch class, with the
lowest point fixed at zero in the multiplicative scale. This
corresponds to $\theta_1 = -\infty$ in the additive scale, implying incorrect
responses to all items with probability one. (As it turns out,
the estimated weight associated with this point will be zero.)
The total number of parameters to be estimated, then, was 13:

- 2 free points in the Rasch distribution: $\theta_2$ and $\theta_3$.
- 2 free values for weights at the three points in the Rasch
distribution: $\omega_1$, $\omega_2$, and $\omega_3$, where $\sum \omega_i = 1$.
- 4 item parameters for the Rasch class: $\alpha_1 = (b_1, \ldots, b_4)$.
- 4 item parameters for the guessing class: $\alpha_2 = (c_1, \ldots, c_4)$.
- 1 relative proportion for class representation: $\pi_2$.

In light of the fact that only 15 degrees of freedom are
available from the data, in the form of 16 response patterns whose
counts that must sum to 1200, an unaccelerated EM solution
converged painfully slowly. Fletcher-Powell iterations were
employed instead, and they converged rapidly. The Rasch-only
estimates described above were used as starting values for the
Rasch class item parameters and population distribution. For the
c's, a common value midway among the true values was used. For
$\pi_2$, starting values of .10, .15, and .20 were used in three
different runs. All runs converged to the same solution:
\[ \alpha_1 = (-.501, -.091, .193, .398); \]
\[ \Theta = (-\infty, -.534, .354); \]
\[ \omega = (\leq 10^{-10}, .319, .681); \]
\[ \alpha_2 = (.287, .230, .179, .139); \]
\[ \pi_2 = .164. \]

Although the c’s are slightly underestimated, the structure of the data has been reconstructed quite well. The expected counts of response patterns are also shown in Table 3. As they should, they yield a nearly perfect fit: a chi-square of .008 on 3 degrees of freedom. The improvement in chi-square is dramatic if not significant—it would be for larger samples or longer tests—but the removal of the bias in the Rasch item parameter estimates is the point of the exercise.

Table 4 shows conditional likelihoods of each response pattern given that an examinee is a guesser, a member of the Rasch class with \( \Theta = -.534 \), and a member of the Rasch class with \( \Theta = .354 \). The estimated proportions of the population in these categories are .164, .267, and .569 respectively. Multiplying these population probabilities times a pattern’s corresponding likelihood terms, then normalizing, gives the posterior probabilities that also appear in the table. Posterior probabilities are given for membership in the guessing class, and for \( \Theta = -.534 \) and \( \Theta = .354 \) given membership in the Rasch class.
Recall from the description of the EM solution that the data from an examinee is effectively distributed among strategy classes to estimate the item parameters within that class. This means that the responses of all examinees play a role in both estimating both b's and c's--but with weights in proportion to the posterior probabilities shown in Table 4. From responses to only four items, we never have overwhelming evidence that a particular examinee is a guesser. Only those with all incorrect responses can be judged more likely than not to have guessed. Had only those respondents been treated as guessers--and that would be the Bayesian modal estimate of strategy class--estimated c's would all have been zero. But employing a proportion of data from all patterns, even those with all items correct, yields estimated c's that essentially recover the generating values.

As a consequence of using the Rasch model for Strategy 1, the conditional posterior distributions given that a subject belongs to this class, or \( p(\theta|x, \phi=1) \), are identical for all response patterns \( x \) with the same total score. The probability that an examinee belongs to the Rasch class vary considerably within patterns with the same score, however. For any given response pattern, the posterior probability of being in the Rasch
class can be inferred from Table 4 as $1 - P(\phi_2 = 1 | x)$. For patterns with exactly one correct response, these probabilities are, for Items 1-4 in turn, .869, .800, .687, and .519.

Discussion

Theories about the processes by which examinees attempt to solve test items play no role in standard applications of test theory, including conventional item response theory. Only a data matrix of correct and incorrect responses is addressed, and items and examinees are parameterized strictly on the basis of propensities toward correct response. When all that is desired is a simple comparison of examinees in terms of a general propensity of this nature, IRT models suffice and in fact offer many advantages over classical true-score test theory.

Situations for which standard IRT models prove less satisfactory involve a desire either to better understand the cognitive processes that underlie item response, or to employ theories about such processes to provide more precise or more valid measurement. Extensions of item response theory in this direction are exemplified by the Linear Logistic Test Model (Schieblechner, 1972; Fischer, 1973), Embretson's (1985) multicomponent models, Samejima's (1983) model for multiple strategies, and Tatsuoka's (1983) "rule space" analyses.

The approach offered in this paper concerns situations in which different persons may choose different strategies from a
Different Strategies

number of known alternatives, but overall proficiencies provide meaningful comparisons among persons employing the same strategy. We suppose that strategy choice is not directly observed but can be inferred (with uncertainty) from response patterns on theoretical bases. Assuming that substantive theory allow us to differentiate our expectations about response patterns under different strategies, and that a subject applies the same strategy on all items, it is possible to estimate the parameters of IRT models for each strategy. It is further possible to calculate the probabilities that a given subject has employed each of the alternative strategies, and estimate his proficiency under each given that that was the one he used.

Assuming that a subject uses the same strategy on all items is obviously undesirable for many important problems. In a technical sense, the approach can be extended to allow for strategy-switching by defining additional strategy classes that are in effect combinations of different strategies for different items. Based on Just and Carpenter's (1985) finding that subjects sometimes apply whichever strategy is easier for a given problem, we might define three strategy classes for items like those in our Example 1:

- Always apply the rotational strategy;
- Always apply the analytic strategy;
- Apply whichever strategy is better suited to an item
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If items were constructed to run from easy to hard under the rotational strategy and hard to easy under the analytic, subjects using the third "mixed" strategy would find them easy, then harder, then easier again.

There are limitations to how far these ideas can be pressed in applications with binary data. Our second example showed that the misspecified Rasch model fit a four-item test acceptably well with a sample of 1200 subjects; in one way or another, more information would be needed to attain a sharper distinction between strategy classes and, correspondingly, more power to differentiate among competing models for the data. One source of information is more binary items. Fifty items rather than four, including some that are very hard under the Rasch strategy, would do. A different source of information available in other settings would be to draw from richer observational possibilities. Examples would include response latencies as well as correctness, eye-fixation patterns, and choices of incorrect alternatives that are differentially likely under different strategies.

Differentiating the likelihood of response patterns under different strategies is the key to successful applications of the approach. Its use would be recommended when identifying strategy classes is of primary importance to the selection or placement decision that must be made, and overall proficiency is of secondary importance. The items in the test must then be constructed to maximize strategy differences, e.g., using items
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that are hard under one strategy but easy under another. Most tests in current use with standard test theory are not constructed with this purpose in mind; indeed, they are constructed so as to minimize differentiation among strategies, since it lowers the reliability of overall-propensity scores. When strategy class decisions are of interest, a conventional test is not likely to provide useful information. (Although a battery of conventional tests might; differences in score profiles are analogous to differential likelihoods of item response patterns, but at a higher level of aggregation.)

In addition to the applications used in the preceding examples, a number of other current topics in educational and psychological research are amenable to expression in terms of mixtures of IRT models. We conclude by mentioning three.

Hierarchical development. Wilson's (1984, 1985) "saltus" model (Latin for "leap") extends the Rasch model to developmental patterns in which capabilities increase in discrete stages, by including stage parameters as well as abilities for persons, and stage parameters as well as difficulties for items. Examples would include Piaget's (1960) innate developmental stages and Gagne's (1962) learned acquisition of rules. Suppose that \( K \) stages are ordered in terms of increasing and cumulative competence. In our notation, \( \phi \) would indicate the stage membership of a subject. In the highest stage, item responses follow a Rasch model with parameters \( b_j \). Rasch models fit lower
Different Strategies

stages as well, but the item parameters are offset by amounts that depend on which stage the item can first be solved. Our basic parameters $a$ would correspond to the item parameters for the highest stage and the offset parameters for particular item types at particular lower stages. Figure 4 gives a simple illustration in which items associated with higher stages have an additional increment of difficulty for subjects at lower stages. In applications such as Siegler's (1981) balance beam tasks, subjects at selected lower stages tend to answer certain types of higher-stage items correctly for the wrong reasons. In these cases, the offset works to give easier item difficulty parameters to those items in those stages.

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Insert Figure 4 about here

----------------------------------

Mental models for problem solving. In the introduction to their experimental study on mental models for electricity, Gentner and Gentner (1983) state

Analogical comparisons with simple or familiar systems often occur in people's descriptions of complex systems, sometimes as explicit analogical models, and sometimes as implicit analogies, in which the person seems to borrow structure from the base domain without knowing it. Phrases like "current being routed along a conductor" and "stopping the flow" of electricity are examples (p. 99).

Mental models are important as a pedagogical device and as a guide to problem-solving. Inferring which models a person is
Different Strategies

using, based on a knowledge of how conceivable analogues help or hinder the solution of certain types of problems, provides a guide to subsequent training. In Gentner and Gentner's experiment, the problems concerned simple electrical circuits with series and parallel combinations of resistors and batteries. Popular analogies for electricity are flowing waters (Strategy 1) and "teeming crowds" of people entering a stadium through a few narrow turnstiles (Strategy 2). The water flow analogy facilitates battery problems, but does not help with resistor problems: indeed, it suggests an incorrect solution for the current in circuits with parallel resistors. The teeming crowd analogy facilitates problems on the combination of resistors, but is not informative about combinations of batteries. If a Rasch model holds for items within strategies, Gentner and Gentner's hypotheses correspond to constraints on the order of item difficulties with the two strategies. If each item type were replicatated enough times, it would be possible to make inferences about which model a particular examinee was using, in order to plan subsequent instruction.

Changes in Intelligence over age. An important topic in the field of human development is whether, and how, intelligence changes as people age (Birren, Cunningham, and Yamamoto, 1983). Macrae (n.d.) identifies a weakness of most studies that employ psychometric tests to measure aging effects: total scores fail to reflect important differences in the strategies different subjects
Different Strategies

bring to bear on the items they are presented. Total score
differences among age and educational-background groups on Raven's
matrices test were not significant in the study she reports. But
analyses of subjects' introspective reports on how they solved
items revealed that those with academically oriented background
were much more likely to have used the preferred "algorithmic"
strategy over a "holistic" strategy than those with vocationally
oriented backgrounds. Since the use of algorithmic strategies was
found to increase probabilities of success differentially on
distinct item types, this study would be amenable to IRT mixture
modeling. Inferences could then be drawn about problem-solving
approaches without resorting to more expensive and possibly
unreliable introspective evidence.
Different Strategies

References


Gentner, D., and Gentner, D.R. (1983). Flowing waters or teeming...


Macrae, K.S. (n.d.). Strategies underlying psychometric test responses in young and middle-aged adults of varying educational background. La Trobe University, Australia.


Different Strategies


Different Strategies

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Table 1

Item Features

<table>
<thead>
<tr>
<th>Item</th>
<th>Rotational Displacement</th>
<th>Salient Features</th>
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<tbody>
<tr>
<td>1</td>
<td>60 degrees</td>
<td>3</td>
</tr>
<tr>
<td>2</td>
<td>120 degrees</td>
<td>2</td>
</tr>
<tr>
<td>3</td>
<td>180 degrees</td>
<td>1</td>
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</table>

Table 2

Item Difficulty Parameters

<table>
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<tr>
<th>Item</th>
<th>Strategy 1</th>
<th>Strategy 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-1.0</td>
<td>2.0</td>
</tr>
<tr>
<td>2</td>
<td>0.0</td>
<td>0.5</td>
</tr>
<tr>
<td>3</td>
<td>1.0</td>
<td>-1.0</td>
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Different Strategies

Table 3

Observed and Fitted Response Pattern Counts for Example 2

<table>
<thead>
<tr>
<th>x</th>
<th>observed frequencies</th>
<th>expected frequencies (Rasch model only)</th>
<th>expected frequencies (2-class model)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0000</td>
<td>143</td>
<td>143.00</td>
<td>143.08</td>
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<tr>
<td>0001</td>
<td>94</td>
<td>98.66</td>
<td>93.95</td>
</tr>
<tr>
<td>0010</td>
<td>83</td>
<td>87.12</td>
<td>83.11</td>
</tr>
<tr>
<td>0011</td>
<td>101</td>
<td>90.55</td>
<td>101.09</td>
</tr>
<tr>
<td>0100</td>
<td>73</td>
<td>72.75</td>
<td>72.78</td>
</tr>
<tr>
<td>0101</td>
<td>78</td>
<td>76.62</td>
<td>77.75</td>
</tr>
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<td>0110</td>
<td>65</td>
<td>66.77</td>
<td>65.26</td>
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<td>0111</td>
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<td>105.98</td>
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<tr>
<td>1111</td>
<td>83</td>
<td>83.00</td>
<td>83.07</td>
</tr>
</tbody>
</table>
Table 4
Response Pattern Likelihoods and Posterior Probabilities

| x     | L(x|θ₂) | L(x|θ₂,φ₁) | L(x|θ₂,φ₁) | P(φ₂|x) | P(θ₂|x,φ₁) | P(θ₃|x,φ₁) |
|-------|-------|-----------|-----------|---------|------------|------------|
| 0000  | .388  | .150      | .027      | .534    | .719       | .281       |
| 0001  | .063  | .131      | .058      | .131    | .513       | .487       |
| 0010  | .085  | .107      | .047      | .200    | .513       | .487       |
| 0011  | .014  | .093      | .100      | .027    | .303       | .697       |
| 0100  | .116  | .080      | .036      | .313    | .513       | .487       |
| 0101  | .019  | .070      | .076      | .047    | .303       | .697       |
| 0110  | .025  | .057      | .062      | .076    | .303       | .697       |
| 0111  | .004  | .050      | .131      | .008    | .151       | .849       |
| 1000  | .156  | .053      | .024      | .481    | .513       | .487       |
| 1001  | .025  | .047      | .050      | .092    | .303       | .697       |
| 1010  | .034  | .038      | .041      | .143    | .303       | .697       |
| 1011  | .005  | .033      | .087      | .015    | .151       | .849       |
| 1100  | .047  | .029      | .031      | .234    | .303       | .697       |
| 1101  | .008  | .025      | .065      | .027    | .151       | .849       |
| 1110  | .010  | .020      | .053      | .045    | .151       | .849       |
| 1111  | .002  | .018      | .113      | .004    | .068       | .932       |

Note: φ₁ denotes membership in the class of Rasch responders; φ₂ denotes membership in the class of random guessers; θ₂ denotes membership in the class of Rasch responders, with θ = .534; θ₃ denotes membership in the class of Rasch responders, with θ = .354.
Different Strategies

Figure 1

![Graph of likelihood function]

The graph illustrates the likelihood function, with theta on the x-axis and likelihood on the y-axis. Two curves are depicted, representing different strategies:

- **p(θ|θ, p):** The top curve represents the likelihood function for strategy 1.
- **p(θ|θ, p):** The bottom curve represents the likelihood function for strategy 2.

The graph visually compares the performance of these two strategies, indicating their respective likelihoods across different values of theta.
Different Strategies

prior distribution

Figure 2
Different Strategies

posterior distribution

Figure 3
Figure 4

Saltus example: 3 stages, common offset
Dr. Terry Ackerman  
American College Testing Programs  
P.O. Box 168  
Iowa City, IA 52243

Dr. Robert Ahlers  
Code N711  
Human Factors Laboratory  
Naval Training Systems Center  
Orlando, FL 32813

Dr. James Algina  
University of Florida  
Gainesville, FL 32605

Dr. Erling B. Andersen  
Department of Statistics  
Studiestraede 6  
1455 Copenhagen  
DENMARK

Dr. Eva L. Baker  
UCLA Center for the Study of Evaluation  
145 Moore Hall  
University of California  
Los Angeles, CA 90024

Dr. Isaac Bejar  
Educational Testing Service  
Princeton, NJ 08540

Dr. Menucha Birenbaum  
School of Education  
Tel Aviv University  
Tel Aviv, Ramat Aviv 699/8  
ISRAEL

Dr. Arthur S. Blaiwes  
Code N711  
Naval Training Systems Center  
Orlando, FL 32813

Dr. Bruce Bloxom  
Defense Manpower Data Center  
550 Camino El Estero, Suite 200  
Monterey, CA 93943-3231

Dr. R. Darrell Bock  
University of Chicago  
NORC  
6030 South Ellis  
Chicago, IL 60637

Cdt. Arnold Bohrer  
Sectie Psychologisch Onderzoek  
Rekruterings-En Selectiecentrum  
Kwartier Koningen Astrid  
Bruijnstraat  
1120 Brussels, BELGIUM

Dr. Robert Breaux  
Code N-095R  
Naval Training Systems Center  
Orlando, FL 32813

Dr. Robert Brennan  
American College Testing Programs  
P.O. Box 168  
Iowa City, IA 52243

Dr. Lyle D. Broemeling  
ONR Code 1111SP  
800 North Quincy Street  
Arlington, VA 22217

Mr. James W. Carey  
Commandant (G-PTE)  
U.S. Coast Guard  
2100 Second Street, S.W.  
Washington, DC 20593

Dr. James Carlson  
American College Testing Program  
P.O. Box 168  
Iowa City, IA 52243

Dr. John B. Carroll  
409 Elliott Rd.  
Chapel Hill, NC 27514

Dr. Robert Carroll  
OP 01B7  
Washington, DC 20370

Mr. Raymond E. Christal  
AFHRL/MOE  
Brooks AFB, TX 78235
Dr. Benjamin A. Fairbank
Performance Metrics, Inc.
5825 Callaghan
Suite 225
San Antonio, TX 78228

Dr. Pat Federico
Code 511
NPRDC
San Diego, CA 92152-6800

Dr. Leonard Feldt
Lindquist Center
for Measurement
University of Iowa
Iowa City, IA 52242

Dr. Richard L. Ferguson
American College Testing Program
P.O. Box 168
Iowa City, IA 52240

Dr. Gerhard Fischer
Liebigasse 5/3
A 1010 Vienna
AUSTRIA

Dr. Myron Fischl
Army Research Institute
5001 Eisenhower Avenue
Alexandria, VA 22333

Prof. Donald Fitzgerald
University of New England
Department of Psychology
Armidale, New South Wales 2351
AUSTRALIA

Mr. Paul Foley
Navy Personnel R&D Center
San Diego, CA 92152-6800

Dr. Alfred R. Freely
AFOSR/NL
Bolling AFB, DC 20332

Dr. Robert D. Gibbons
Illinois State Psychiatric Inst.
Rm 529W
1501 W. Taylor Street
Chicago, IL 60612

Dr. Janice Gifford
University of Massachusetts
School of Education
Amherst, MA 01003

Dr. Robert Glaser
Learning Research & Development Center
University of Pittsburgh
3939 O'Hara Street
Pittsburgh, PA 15260

Dr. Bert Green
Johns Hopkins University
Department of Psychology
Charles & 34th Street
Baltimore, MD 21218

Dipl. Pad. Michael W. Habon
Universitat Dusseldorf
Erziehungswissenschaftliches Universitaetsstr. 1
D-4000 Dusseldorf 1
WEST GERMANY

Dr. Ronald K. Hambleton
Prof. of Education & Psychology
University of Massachusetts
at Amherst
Hills House
Amherst, MA 01003

Dr. Delwyn Harnisch
University of Illinois
51 Gerty Drive
Champaign, IL 61820

Ms. Rebecca Hetter
Navy Personnel R&D Center
Code 62
San Diego, CA 92152-6800

Dr. Paul W. Holland
Educational Testing Service
Rosedale Road
Princeton, NJ 08541

Prof. Lutz J. Hornke
Institut fur Psychologie
RWTH Aachen
Jaegerstrasse 17/19
D-5100 Aachen
WEST GERMANY
Dr. Paul Horst  
677 G Street, #184  
Chula Vista, CA 92010

Mr. Dick Hoshaw  
DF-135  
Arlington Annex  
Room 2834  
Washington, DC 20550

Dr. Lloyd Humphreys  
University of Illinois  
Department of Psychology  
603 East Daniel Street  
Champaign, IL 61820

Dr. Steven Hunka  
Department of Education  
University of Alberta  
Edmonton, Alberta  
CANADA

Dr. Huynh Huynh  
College of Education  
Univ. of South Carolina  
Columbia, SC 29708

Dr. Robert Jannarone  
Department of Psychology  
University of South Carolina  
Columbia, SC 29208

Dr. Dennis E. Jennings  
Department of Statistics  
University of Illinois  
1409 West Green Street  
Urbana, IL 61801

Dr. Douglas H. Jones  
Thatcher Jones Associates  
P.O. Box 6640  
10 Trafalgar Court  
Lawrenceville, NJ 08648

Dr. Milton S. Katz  
Army Research Institute  
5001 Eisenhower Avenue  
Alexandria, VA 22333

Prof. John A. Keats  
Department of Psychology  
University of Newcastle  
N.S.W. 2308  
AUSTRALIA

Dr. G. Gage Kingsbury  
Portland Public Schools  
Research and Evaluation Department  
501 North Dixon Street  
P. O. Box 3107  
Portland, OR 97209-3107

Dr. William Koch  
University of Texas-Austin  
Measurement and Evaluation  
Center  
Austin, TX 78703

Dr. James Kraatz  
Computer-based Education  
Research Laboratory  
University of Illinois  
Urbana, IL 61801

Dr. Leonard Kroeker  
Navy Personnel R&D Center  
San Diego, CA 92152-6800

Dr. Daryll Lang  
Navy Personnel R&D Center  
San Diego, CA 92152-6800

Dr. Jerry Lehnus  
Defense Manpower Data Center  
Suite 400  
1600 Wilson Blvd  
Rosslyn, VA 22209

Dr. Thomas Leonard  
University of Wisconsin  
Department of Statistics  
1210 West Dayton Street  
Madison, WI 53705

Dr. Michael Levine  
Educational Psychology  
210 Education Bldg.  
University of Illinois  
Champaign, IL 61801
Educational Testing Service/Mislevy

Director, Manpower and Personnel Laboratory,
NPRDC (Code 06)
San Diego, CA 92152-6800

Director, Human Factors & Organizational Systems Lab,
NPRDC (Code 07)
San Diego, CA 92152-6800

Fleet Support Office,
NPRDC (Code 301)
San Diego, CA 92152-6800

Library, NPRDC
Code P201L
San Diego, CA 92152-6800

Commanding Officer,
Naval Research Laboratory
Code 2627
Washington, DC 20390

Dr. Harold F. O'Neill, Jr.
School of Education - WPH 801
Department of Educational Psychology & Technology
University of Southern California
Los Angeles, CA 90089 0031

Dr. James Olson
WICA1, Inc.
1875 South State Street
Orem, UT 84057

Office of Naval Research,
Code 1142CS
800 N. Quincy Street
Arlington, VA 22217 5000
6 (Copies)

Office of Naval Research,
Code 125
800 N. Quincy Street
Arlington, VA 22217-5000

Assistant for MPrI Research,
Development and Studies
OP 0187
Washington, DC 20310

Dr. Judith Orasanu
Army Research Institute
5001 Eisenhower Avenue
Alexandria, VA 22333

Dr. Jesse Oriansky
Institute for Defense Analyses
1901 N. Beauregard St.
Alexandria, VA 22311

Dr. Randolph Park
Army Research Institute
5001 Eisenhower Blvd.
Alexandria, VA 22333

Wayne M. Patience
American Council on Education
GED Testing Service, Suite 20
One Dupont Circle, NW
Washington, DC 20036

Dr. James Paulson
Department of Psychology
Portland State University
P.O. Box 751
Portland, OR 97207

Administrative Sciences Department,
Naval Postgraduate School
Monterey, CA 93940

Department of Operations Research,
Naval Postgraduate School
Monterey, CA 93940

Dr. Mark D. Reckase
ACT
P. O. Box 168
Iowa City, IA 52243

Dr. Malcolm Ree
AHRL/MP
Brooks AFB, TX 78235

Dr. Barry Riegelhaupt
HumRRO
1100 South Washington Street
Alexandria, VA 22314

Dr. Carl Ross
CNEI-PDCD
Building 90
Great Lakes NLC, IL 60088
Educational Testing Service/Mislevy

Dr. J. Ryan
Department of Education
University of South Carolina
Columbia, SC 29208

Dr. Fumiko Samejima
Department of Psychology
University of Tennessee
3108 Austin Peay Bldg.
Knoxville, TN 37916-0900

Dr. Mary Schratz
Navy Personnel R&D Center
San Diego, CA 92152-6800

Dr. Dan Segall
Navy Personnel R&D Center
San Diego, CA 92152

Dr. W. Steve Sellman
OASD(MR&A&L)
28269 The Pentagon
Washington, DC 20301

Dr. Kazuo Shigemasu
7-9-24 Kugenuma-Kaigan
Fujusawa 251
JAPAN

Dr. William Sims
Center for Naval Analysis
4401 Ford Avenue
P.O. Box 16258
Alexandria, VA 22302-0268

Dr. H. Wallace Sinaiko
Manpower Research
and Advisory Services
Smithsonian Institution
801 North Pitt Street
Alexandria, VA 22314

Dr. Richard E. Snow
Department of Psychology
Stanford University
Stanford, CA 94306

Dr. Richard Sorensen
Navy Personnel R&D Center
San Diego, CA 92152-6800

Dr. Paul Speckman
University of Missouri
Department of Statistics
Columbia, MO 65201

Dr. Judy Spray
ACT
P.O. Box 168
Iowa City, IA 52243

Dr. Martha Stocking
Educational Testing Service
Princeton, NJ 08541

Dr. Peter Stoloff
Center for Naval Analysis
200 North Beauregard Street
Alexandria, VA 22311

Dr. William Stout
University of Illinois
Department of Statistics
101 Illini Hall
725 South Wright St.
Champaign, IL 61820

Maj. Bill Strickland
AF/MPXOA
4E168 Pentagon
Washington, DC 20330

Dr. Hariharan Swaminathan
Laboratory of Psychometric and Evaluation Research
School of Education
University of Massachusetts
Amherst, MA 01003

Mr. Brad Symson
Navy Personnel R&D Center
San Diego, CA 92152-6800
Dr. John Tangney  
AFOSR/NL  
Bolling AFB, DC 20332

Dr. Kikumi Iatsuoka  
CERL  
252 Engineering Research Laboratory  
Urbana, IL 61801

Dr. Maurice Iatsuoka  
220 Education Bldg  
1310 S. Sixth St.  
Champaign, IL 61820

Dr. David Hissen  
Department of Psychology  
University of Kansas  
Lawrence, KS 66044

Mr. Gary Thomasson  
University of Illinois  
Educational Psychology  
Champaign, IL 61820

Dr. Robert Isutakawa  
University of Missouri  
Department of Statistics  
222 Math. Sciences Bldg.  
Columbia, MO 65211

Dr. Ledvard Lucker  
University of Illinois  
Department of Psychology  
603 E. Daniel Street  
Champaign, IL 61820

Dr. Vern W. Urry  
Personnel R&D Center  
Office of Personnel Management  
1900 E. Street, NW  
Washington, DC 20415

Dr. David Vale  
Assessment Systems Corp.  
2233 University Avenue  
Suite 310  
St. Paul, MN 55114

Dr. Frank Vicino  
Naval Personnel R&D Center  
San Diego, CA 92152-6800

Dr. Howard Wainer  
Division of Psychological Studies  
Educational Testing Service  
Princeton, NJ 08541

Dr. Ming Mei Wang  
Lindquist Center for Measurement  
University of Iowa  
Iowa City, IA 52242

Dr. Thomas A. Warm  
Coast Guard Institute  
P.O. Substation 18  
Oklahoma City, OK 73169

Dr. Brian Waters  
Program Manager  
Manpower Analysis Program  
HumRRO  
1100 S. Washington St.  
Alexandria, VA 22314

Dr. David J. Weiss  
N660 Elliott Hall  
University of Minnesota  
75 E. River Road  
Minneapolis, MN 55455

Dr. Ronald A. Weitzman  
NPS, Code 54W  
Monterey, CA 92152-6800

Major John Welsh  
AHRL/MOAN  
Brooks AFB, TX 78223

Dr. Douglas Wetzel  
Code 12  
Navy Personnel R&D Center  
San Diego, CA 92152-6800

Dr. Rand R. Wilcox  
University of Southern California  
Department of Psychology  
Los Angeles, CA 90007
German Military Representative
ATTN: Wolfgang Wildegrube
Streitkraefteamt
D-5300 Bonn 2
4000 Brandywine Street, NW
Washington, DC 20016

Dr. Bruce Williams
Department of Educational Psychology
University of Illinois
Urbana, IL 61801

Dr. Hilda Wing
NRC GF-176
2101 Constitution Ave
Washington, DC 20418

Dr. Martin F. Wiskoff
Navy Personnel R & D Center
San Diego, CA 92152-6800

Mr. John H. Wolfe
Navy Personnel R&D Center
San Diego, CA 92152-6800

Dr. George Wong
Biostatistics Laboratory
Memorial Sloan-Kettering Cancer Center
1275 York Avenue
New York, NY 10021

Dr. Wallace Wulfeck, III
Navy Personnel R&D Center
San Diego, CA 92152-6800

Dr. Kentaro Yamamoto
Computer-based Education Research Laboratory
University of Illinois
Urbana, IL 61801

Dr. Wendy Yen
CTB/McGraw Hill
Del Monte Research Park
Monterey, CA 93940

Dr. Joseph L. Young
Memory & Cognitive Processes
National Science Foundation
Washington, DC 20550

Dr. Anthony R. Zara
National Council of State Boards of Nursing, Inc.
625 North Michigan Ave.
Suite 1544
Chicago, IL 60611
End
Date
Filmed
4-88
Dtic