One of the most attractive structural configurations for large space structures (LSS) for outer space applications is the repetitive lattice concept. Achieving the operational requirements of such structures will necessitate considerable knowledge of the dynamics, control, materials and nondestructive evaluation (NDE) of these structural systems. Wave propagation analyses provide potentially valuable perspectives from which to consider this broad range of analysis, design and synthesis issues.

The theoretical and experimental results of a two-year research program on the wave propagation and dynamics of LSS are briefly reviewed. Potential benefits of wave propagation analyses in the vibration, parameter identification, dynamic failure, control and NDE of lattice structures have been identified and are summarized in this report.
ACKNOWLEDGMENTS

The Air Force Office of Scientific Research (Project Monitor, Dr. Anthony K. Amos) is gratefully acknowledged for its support of this research.

NOTICE

This document was prepared under the sponsorship of the Air Force. Neither the US Government nor any person acting on behalf of the US Government assumes any liability resulting from the use of the information contained in this document. This notice is intended to cover WEA as well.
INTRODUCTION

Many papers and reports may be found in the literature on the concepts, design, analysis and potential uses of lattice structures in outer space. Such structures include large antennae, solar power systems and habitable stations for support of space colonies.

Currently, both deployable and erectable concepts are being investigated for the implementation of lattice structures. Also, investigations of size considerations indicate that small antennae ranging from tens of meters in span to solar power collectors ranging up to several thousand meters have been proposed. Such structural sizes along with stringent operational requirements will require considerable information of dynamics, control, materials, nondestructive evaluation (NDE), environmental effects and wave propagation relating to their design and analysis.

Much has been written on the theoretical aspects of the control of such structures. Also, a large number of vibration analyses have been undertaken. However, despite a distinct recognition of the importance of wave propagation in many of the control, vibration and NDE investigations, very little can be found on wave propagation in large space structures (LSS). The major goal of this program was to pursue the development of several aspects of wave propagation analyses in LSS.
ACCOMPLISHMENTS

A number of theoretical and experimental analyses have been undertaken. Some of these efforts have been documented in reports and some as yet remain undocumented in formal reports. The formally documented accomplishments will be described briefly.

As indicated above, the focus of our investigations has been the wave propagation of broadband and narrow-band signals in lattice structures for outer space applications. Theoretical and experimental analyses which have resulted in formal reports are listed here.

1. Theoretical formulations of the input-output relations for arbitrary pairs of locations on periodic structures have been initiated. In this formulation, we have used (and are continuing to use) a transfer matrix technique for analyzing periodic structures which can be modeled as one-dimensional continua, two-dimensional rectangular trusses and three-dimensional tetrahedral trusses. The elements in these structures are capable of transmitting longitudinal, shear and flexural waves. Our efforts on this topic have resulted in reports* as follows:


* Reports written on this topic and other topics that were published on an earlier contract are relevant but are not listed here. For the listing of those reports, refer to J.H. Williams, Jr., "Wave Propagation and Dynamics of Lattice Structures", AFOSR/WEA Final Report, October 1985.
2. The dynamic properties of two two-dimensional lattice structures have been investigated both analytically, using the NASTRAN finite element code and transfer matrices, and experimentally, using an HP Fourier analyzer. The natural frequencies obtained via the three approaches were compared and shown to agree within seven percent of each other. These results are reported in a document as follows:


More recently, this work has been extended to consider the nondestructive detection of damage and is reported as follows:


Further, in these efforts we have collaborated with colleagues at Stanford University and Virginia Polytechnic Institute and State University. In fact, both 5-bay and 22-bay lattice structures which we have fabricated and tested have been sent to colleagues at Stanford for testing.

3. In lattice structures where the signal propagation is dominated by longitudinal waves, we hope to develop general closed-form expressions for the input-output relations of arbitrary pairs of locations of the structure. This type of analysis requires the reckoning of each wave front which leaves joint i and arrives at joint j, having undergone a series of reflections and transmissions at each intervening joint encountered along the way. Such an approach may be called a "wave summation analysis". We have been able to duplicate these results
using wave-mode coordinates. Our efforts thus far on this topic have resulted in a report as follows:


4. The wave propagation characteristics of a 5-bay aluminum planar lattice were studied experimentally. Wave propagation speeds and frequency spectra were obtained, and wave propagation reciprocity was observed. Wave propagation attenuation was quantitatively measured and an attenuation parameter expressed on a per-bay basis was defined. Similar results for a 22-bay structure are reported as follows:


5. One of the most distinctive uses of wave propagation versus modal analysis lies in the description of nonlinear behavior. Our work on dynamic failure and techniques for arresting such failure has provided a fascinating application of wave propagation analyses in LSS. This work is reported as follows:

- J.H. Williams, Jr. and S.S. Lee, "Failure Propagation in Continuum

6. When the wavelength is long compared with the length of lattice elements, continuum models may be used to provide computationally efficient analyses. Wave propagation in a tetrahedral lattice, which was modeled as a transversely isotropic continuum, was conducted to provide some insight into the nonintuitive behavior of long wavelength dynamics of LSS. This work is reported as follows:
SUMMARY

While there continue to be misconceptions and misrepresentations of the wave propagation perspective of dynamics of LSS, the value of wave propagation analyses of dynamic phenomena in LSS is no longer an anomalous curiosity. In an attempt to assist in the enlightenment of the technical community regarding wave propagation in LSS, a presentation entitled "Wave Propagation in Large Space Structures" was presented at the Fifth AFOSR Forum on Space Structures in Monterey in August 1987. The abstract and the transparencies for that presentation are given as the appendix of this report.
APPENDIX

Wave Propagation in Large Space Structures

James H. Williams, Jr.
Raymond J. Nagem

Department of Mechanical Engineering
Massachusetts Institute of Technology
Cambridge, MA 02139

ABSTRACT

Waves and modes are discussed in the context of dynamics of large space structures. It is observed that wave propagation analysis is much more general than modal analysis, and that wave analysis can be applied to a much broader class of dynamic phenomena. Thus, some phenomena which can be conveniently described by wave propagation have no equivalent modal representation. In linear analysis, when wave propagation analysis and modal analysis are theoretically equivalent, the preference between a wave analysis and a modal analysis is assessed in accordance with the ratio of the wavelength of a typical disturbance function to the characteristic structural length. A definition of a large space structure is given in terms of the characteristic structural length and the typical disturbance wavelength. Recent research results in continuum modeling, scattering theory, control system design and dynamic failure are given to illustrate applications of wave propagation analysis in the dynamics of large space structures.

Sponsor: AFOSR
Program Manager: Anthony K. Amos
WAVE PROPAGATION IN
LARGE SPACE STRUCTURES *

James H. Williams, Jr.
and
Raymond J. Nagem

M.I.T.

(AFOSR: Anthony K. Amos)

* Because the original transparencies were multicolored, the quality of these copies is relatively poor.
Recent Titles on
WAVE PROPAGATION IN LSS

- Wave Propagation and Dynamics of Lattice Structures.
- Dynamic Analyses of Two-Dimensional Lattices.
- Wave Propagation Through T and L Joints.
- Nondispersive Wave Propagation in Periodic Structures.
- Feedforward Control of Waves in Lattice Elements.
- Wave Propagation Measurements on Two-Dimensional Lattice.
- Wave Measurements on Truss Model.
- Wave Propagation and Dynamics of Lattice Structures.
- Dynamic Failure of a Periodic Lattice Structure.
- Failure Propagation in Continuum Models of LSS, Part I.
- Dynamic Failure Arrest in Lattice Structures Via Wave Deflectors.
- Failure Propagation in Continuum Models of LSS.
- Joint Coupling Matrices for Wave Propagation in Large Space Structures.
- Dynamic Failure Arrest in LSS Via Active Control.
- Wave-Mode Coordinates and Scattering Matrices for Wave Propagation in Large Space Structures.
- Comparison of Wave-Mode Coordinate and Pulse Summation Methods.
- Wave Propagation in Transversely Isotropic Continuum Models of LSS.
- Pattern Recognition of LSS States Via Wave Propagation.
- Wave Propagation Measurements on 22-Bay Lattice.
- Computation of Natural Frequencies of Planar Lattice Structures.
- Natural Frequencies and Structural Integrity Assessment of Damaged Lattice Structure.
Topics of Current LSS Research Activities

- Scattering of Waves at Joints in LSS
- Nondestructive Evaluation of LSS
- Numerical Efficiency of Transfer Matrix Method for LSS Applications
- Wave Propagation in Anisotropic Continuum Models of LSS
- Control of Wave Propagation in LSS
### Modes vs Waves

**Linear Elasticity vs Nonlinear Elastic-Plastic Continuum Theory**

![Venn Diagram]

<table>
<thead>
<tr>
<th></th>
<th>Modal Analysis</th>
<th>Wave Analysis</th>
</tr>
</thead>
<tbody>
<tr>
<td>Linear Dynamic Response</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>Nonlinear Elastic Dynamic Response</td>
<td>X</td>
<td>✓</td>
</tr>
<tr>
<td>Dynamic Failure</td>
<td>X</td>
<td>✓</td>
</tr>
<tr>
<td>Short Pulse Discontinuous Systems</td>
<td>?</td>
<td>✓</td>
</tr>
<tr>
<td>Wave Control</td>
<td>?</td>
<td>✓</td>
</tr>
</tbody>
</table>
Linear Analysis

\[ \lambda = \text{wavelength of excitation} \]
\[ c = \text{wave speed in structure} \]
\[ T = \text{period of excitation} \]
\[ L = \text{characteristic length of structure} \]

\[ \lambda = cT \]

<table>
<thead>
<tr>
<th>Regime</th>
<th>Probable Preference</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \lambda \gg L )</td>
<td>Statics or Rigid Body Dynamics</td>
</tr>
<tr>
<td>( \lambda \approx L )</td>
<td>Modes</td>
</tr>
<tr>
<td>( \lambda \ll L )</td>
<td>Waves</td>
</tr>
</tbody>
</table>
Long $\lambda$ ($\lambda \gg L$)

Response is Quasistatic Deformation or Rigid Body Dynamics

Continuum models are useful

Structural details are not "seen" by dynamic response

<table>
<thead>
<tr>
<th>System</th>
<th>Excitation Condition</th>
<th>Response</th>
<th>Model</th>
<th>Deformation Response</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\lambda = cT$</td>
<td></td>
<td>Rigid Body</td>
<td>Mass, pAL</td>
</tr>
<tr>
<td>$\rightarrow$</td>
<td>$\lambda \gg L$</td>
<td>Quasistatic Deformation</td>
<td>Spring, $\frac{EA}{L}$</td>
<td>$0 \rightarrow L$</td>
</tr>
<tr>
<td>$\downarrow$</td>
<td>$\rightarrow$ $T$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\downarrow$</td>
<td>$\rightarrow$ $T$</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Intermediate \( \lambda \ (\lambda \approx L) \)

<table>
<thead>
<tr>
<th>System</th>
<th>Excitation Condition</th>
<th>Response</th>
<th>Model</th>
<th>Deformation Response</th>
</tr>
</thead>
<tbody>
<tr>
<td>[ \begin{array}{c} [0.5] \ \rightarrow \ T \end{array} ]</td>
<td>( \lambda \approx 4L )</td>
<td>Dynamic</td>
<td>[ \frac{EA}{L} ]</td>
<td>[ \frac{1}{3} \rho A L \rightarrow T ]</td>
</tr>
</tbody>
</table>

**Exact:** \( \omega_n = 1.571 \sqrt{E/(\rho L^2)} \)

**Approximate:** \( \omega_n = 1.732 \sqrt{E/(\rho L^2)} \)

2\textsuperscript{nd} Model

\[ k = \frac{5EA}{2L} \]

\[ M = \frac{2\rho AL}{5} \]

**Approximate:** \( \omega_n = 1.545 \sqrt{E/(\rho L^2)} \)
Short $\lambda$ ($\lambda \ll L$)

\[ \frac{L_1}{\lambda} \sim 100 \]

How many modes are required to model this disturbance?

Can a control system designed to control the first twenty modes respond accurately to such an excitation?
**LSS**

"Large" means, by definition, \( l >> \lambda \)

Consequences:

- Time lags are likely to be important
- Structural details become important
- Truncated modal summation may result in significant errors

Some dynamic behavior has no modal description
Wave Propagation in LSS

In steel or aluminum, \( c_{\text{shear}} \approx 10^4 \text{ ft/sec} \)

\[ \lambda = cT \]

<table>
<thead>
<tr>
<th>( T ) (sec)</th>
<th>( \lambda ) (ft)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( 10^{-5} )</td>
<td>( 10^{-1} )</td>
</tr>
<tr>
<td>( 10^{-4} )</td>
<td>( 10^{-2} )</td>
</tr>
<tr>
<td>( 10^{-3} )</td>
<td>( 10^{-1} )</td>
</tr>
<tr>
<td>( 10^{-2} )</td>
<td>( 10^{0} )</td>
</tr>
</tbody>
</table>

For a structure \( 10^3 \text{ ft} \) long, an impact of duration \( 10^{-3} \text{ sec} \) produces a disturbance with

\[ \frac{L}{\lambda} \approx 100 \]

Criterion for Wave Propagation:

If the wavelength produced by a disturbance is much less than the characteristic length of the structure, then a wave analysis is likely justified.
Excitation Source

- CMC Imbalance
- Reaction Wheel Imbalance
- Reaction Wheel Bearing Noise
- Solar Array Drive Torque
Typical Repeating Element

Tetrahedral lattice structure.

<table>
<thead>
<tr>
<th>Cross-sectional Area</th>
<th>Length</th>
<th>Material Density</th>
<th>Symbol</th>
</tr>
</thead>
<tbody>
<tr>
<td>Top Surface Bars</td>
<td>A</td>
<td>$\rho_L$</td>
<td>[ ]</td>
</tr>
<tr>
<td>Bottom Surface Bars</td>
<td>A</td>
<td>$\rho_L$</td>
<td>[ ]</td>
</tr>
<tr>
<td>Bracing Bars</td>
<td>$A_d$</td>
<td>$\rho_d$</td>
<td>[ ]</td>
</tr>
</tbody>
</table>

$A = 80 \times 10^{-6} \text{ m}^2$    
$L = d = 7.5 \text{ m}$

$A_d = 50 \times 10^{-6} \text{ m}^2$    
$E = 71.7 \times 10^9 \text{ N/m}^2$

$\rho_L \cdot \rho_d = 2768 \text{ kg/m}^3$
Phase velocity versus angle of incidence for SH wave travelling in tetrahedral truss plate.
Path of SH-SH-... wave which arrives at receiving transducer after n reflections from bottom boundary.
Delay time versus dimensionless separation ($f/h$) with number of reflections ($n$) from the bottom face of tetrahedral truss plate as parameter.
Model of joint connecting two elastic longitudinal rods.
Nondimensional transmitted force in lattice due to impulse incident upon flexible joint ($\zeta = 0.025$, $\omega_n = 1240$ rad/s) with mass.
Nondimensional reflected force in lattice due to impulse incident upon flexible joint ($\beta = 0.025$, $\omega_n = 1240$ rad/s) with mass.
Single Degree of Freedom (Second-Order) System

Equation of motion:

\[ m\ddot{x} + c\dot{x} + kx = 0 \]

Or

\[ \ddot{x} + 2\delta\omega_n \dot{x} + \omega_n^2 x = 0 \]

where

\[ \delta = \frac{c}{2m} \quad \omega_n = \sqrt{\frac{k}{m}} \]

Assume

\[ x(t) = Ce^{\lambda t} \implies \lambda^2 + 2\delta\omega_n \lambda + \omega_n^2 = 0 \]

\[ \lambda_1, \lambda_2 = \left( -\delta \pm \sqrt{\delta^2 - 1} \right) \omega_n \]

Three Cases:

1. If \( \delta > 1 \), roots are real, negative and distinct.
2. If \( \delta = 1 \), roots are real, negative and equal.
3. If \( \delta < 1 \), roots are complex conjugate with negative real part.
\[ m \ddot{x} + c \dot{x} + kx = 0 \]

\[ \xi = \frac{c}{2\sqrt{km}} \]

Root Locus

Stable \( \xi = 1 \)

Unstable \( \xi = -1 \)

Symmetric \( \xi > 1 \)

\( \xi < -1 \)
Wave Control

\[
\begin{align*}
\epsilon_1 &= \epsilon_2 = 0 \\
\epsilon_1 &= 0 \\
\epsilon_2 &= 0
\end{align*}
\]

<table>
<thead>
<tr>
<th>(\tilde{F}(x,t_1))</th>
<th>(F(x,t_1))</th>
<th>(F(x,t_1))</th>
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<th>(F(x,t_3))</th>
<th>(F(x,t_3))</th>
</tr>
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<tbody>
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</table>
Root Locus as a Function of Amplitude

Error $e_1$

\[ y = \text{Im}(s) \quad \text{rad/sec} \]

\[ \chi = \text{Re}(s) \quad \text{sec}^{-1} \]

$e_2 = 0 \quad l_{13} = 10 \text{ m} \quad l_{23} = 6 \text{ m} \quad l_{16} = 30 \text{ m}$

$c = 5000 \quad \text{m/sec}$
Root Locus as a Function of Timing

Error $e_2$

$y = \text{Im}(s) \ (\text{rad/sec})$

$t = 0 \quad h_{13} = 10 \ m \quad h_{23} = 6 \ m \quad h_{16} = 30 \ m$

$c = 5000 \ \text{m/sec}$
Root Locus as a Function of Timing Error $e_2$

for $e_1 = -0.12$

$y = \text{Im}(s)$ (rad/sec)

$x = \text{Re}(s)$ (sec$^{-1}$)

$L_{13} = 10 \text{ m}$  
$L_{23} = 6 \text{ m}$  
$L_{16} = 30 \text{ m}$

$c = 5000 \text{ m/sec}$
FAILURE PROPAGATION IN LSS

Generalized Lattice

Generalized Structure

"Failure" Zone

Failure = \{ Rupture, Buckling, LoJI, Etc. \}

2-D Planar Lattice Model

Failure Propagation and Failure Arrest Criteria

\[ \ell_d = \frac{dW}{dA} - \frac{dV}{dA} - \frac{dT}{dA} \]

\[ \ell_d = R \rightarrow \text{Failure Propagation (Itamno Theory of Fracture Mechanics)} \]

\[ \ell_d < R \rightarrow \text{Failure Arrest} \]

where \( \ell_d \) = Dynamic Energy Release Rate (Driving Force)

\( R \) = Dynamic Failure Toughness
Criteria for Failure Propagation and Failure Arrest

\[ \text{Dynamic Failure Toughness } R \]

\[ \text{Initial Static Energy Release Rate } G_h q^2 \]
Location of Failure Arrest

Failure Arrest Location $n^*$

Nondimensional Failure Speed $\frac{\dot{l}_d}{C_s}$

$\frac{\dot{l}_d}{d} = 100$

$\frac{\dot{l}_d}{d} = 50$

$\frac{\dot{l}_d}{d} = 20$

$\frac{\dot{l}_d}{d} = 5$

$\frac{\dot{l}_d}{d} = 1$
Control of Dynamic Failure

$x = 0$

Active Control Elements

$W(x,t)$  $F(t)$

$l_1$

$l(t)$
Failure Propagation and Failure Arrest Criteria

No Initial Failure Propagation

Violation of Contact Constraint

Failure Arrest Due to Control Force

No Failure Arrest Due to Control Force

\[ \frac{F_0}{\sqrt{gh}} \]

Initial Slope, \( \frac{W_0}{2} \)
Nondimensional Control Force Magnitude, $\frac{F}{gbh}$

- Initial Slope, $\frac{\sqrt{E}}{g}$
- Continued Failure
- Propagation of Failure
- No Initial Failure
- Propagation
- Violation of Constraint
- Remobilization after Control Force is Removed
- Criteria for Continued Failure: Arrest or Remobilization
END

DATE

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