Quantum Monte Carlo Study of the MIES Associated with $H_2(X^1Σ_g^+)$ and $H_2(B^1Σ_u^+)$

October 1987
Author: W. A. Lester
Lawrence Berkeley Laboratory
University of California
Berkeley, CA 94720

AFRPL 69022

Approved for Public Release
Distribution is unlimited. The AFAL Technical Services Office has reviewed this report, and it is releasable to the National Technical Information Service, where it will be available to the general public, including foreign nationals.

prepared for the: Air Force Astronautics Laboratory
Air Force Space Technology Center
Space Division, Air Force Systems Command
Edwards Air Force Base,
California 93523-5000
NOTICE

When U.S. Government drawings, specifications, or other data are used for any purpose other than a definitely related government procurement operation, the government thereby incurs no responsibility nor any obligation whatsoever, and the fact that the government may have formulated, furnished, or in any way supplied the said drawings, specifications, or other data, is not to be regarded by implication or otherwise, or conveying any rights or permission to manufacture, use, or sell any patented invention that may in any way be related thereto.

FOREWORD

This final report was prepared by the Lawrence Berkeley Laboratory in completion of procurement instrument number AFRPL 69022 with the Air Force Astronautics Laboratory (AFAL), Edwards Air Force Base, CA. The period of the report is 1 July 1985 to 30 June 1987. AFAL Project Manager was Dr Steve Rodgers.

This technical report has been reviewed and is approved for distribution in accordance with the the distribution statement on the cover and on the DD Form 1473.

Stephen L. Rodgers
STEVEN L. RODGERS
Project Manager

FOR THE COMMANDER

ROBERT C. CORLEY
Chief, ARIES Office
Initial efforts to characterize the electronically excited state of H_4^+ that correlates in one asymptotic limit to H_2^+(X 1^12S) + H_2(B 1^2S_u) are described. Multiconfiguration self consistent field (MCSCF) and fixed-node quantum Monte Carlo (FNQMC) methods have been used to explore the suggested region of stability. The minimum on the excited-state potential energy surface for pyramidal geometries is found to have an isoclinic base. Initial estimates of nonadiabatic coupling matrix elements needed for the determination of lifetimes and stability are determined using normal mode distortions.

**Title**: Quantum Monte Carlo Study of the MIES Associated with H_2(X 1^12S)

**Author**: Lester, William A.

**Type of Report**: Final

**Date of Report**: 87/10

**Page Count**: 30
11. (continued) and $H_2$ ($B \ ^1\Sigma_u^+$).
<table>
<thead>
<tr>
<th>SECTION</th>
<th>PAGE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Introduction</td>
<td>1</td>
</tr>
<tr>
<td>Quantum Monte Carlo</td>
<td>3</td>
</tr>
<tr>
<td>MCSCF Study of Trigonal Pyramidal</td>
<td>6</td>
</tr>
<tr>
<td>Ground-and Excited-State H₄</td>
<td></td>
</tr>
<tr>
<td>FNQMC Study of the Ground-State Pyramidal</td>
<td>11</td>
</tr>
<tr>
<td>Structure for C₃ᵥ Symmetry</td>
<td></td>
</tr>
<tr>
<td>FNQMC Study of the Excited State</td>
<td>11</td>
</tr>
<tr>
<td>SA-MCSCF Study of Distorted Geometries</td>
<td>15</td>
</tr>
<tr>
<td>Nonadiabatic Coupling</td>
<td>20</td>
</tr>
<tr>
<td>References</td>
<td>24</td>
</tr>
</tbody>
</table>
List of Figures

<table>
<thead>
<tr>
<th>Figure</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Correlation diagram for $H_1$.</td>
<td>2</td>
</tr>
<tr>
<td>2</td>
<td>MIES coordinate system (assuming $C_{3v}$ symmetry).</td>
<td>7</td>
</tr>
<tr>
<td>3</td>
<td>Ground- and excited-state $H_1$ potential energy curves for trigonal pyramidal geometry.</td>
<td>8</td>
</tr>
<tr>
<td>4</td>
<td>Potential energy curves from MCSCF trial functions for trigonal pyramidal geometry and $C_{3v}$ symmetry ($r = 1.70$ a.u.).</td>
<td>9</td>
</tr>
<tr>
<td>5</td>
<td>Geometries used to calculate an approach of $H_2(B)$ to $H_2(X)$.</td>
<td>10</td>
</tr>
<tr>
<td>6</td>
<td>Plot of total energies corresponding to the pathway presented in Fig. 5.</td>
<td>12</td>
</tr>
<tr>
<td>7</td>
<td>Change of charge distribution between $H_4$ and $H_1$, as a function of $d$, the displacement of the midpoint of $H_2(B)$ above the plane of $H_2(X)$.</td>
<td>13</td>
</tr>
<tr>
<td>8</td>
<td>Potential energy of trigonal pyramidal $H_4$ versus $R$.</td>
<td>14</td>
</tr>
<tr>
<td>9</td>
<td>MIES coordinate system used for geometry variation.</td>
<td>16</td>
</tr>
<tr>
<td>10</td>
<td>Potential energy surfaces for the ground- and excited-state separately for easy visualization and in computed relative positioning.</td>
<td>17</td>
</tr>
<tr>
<td>11</td>
<td>Potential energy contour map for $R = 1.0$ a.u. as a function of $a$ and $b$.</td>
<td>18</td>
</tr>
<tr>
<td>12</td>
<td>Same as Fig. 11 for $R = 3.8$ a.u.</td>
<td>19</td>
</tr>
<tr>
<td>13</td>
<td>Potential energy curves for selected $R$ (in a.u.) vs $d$.</td>
<td>20</td>
</tr>
<tr>
<td>14</td>
<td>Normal mode displacement of $H_1$ in $C_{3v}$ symmetry.</td>
<td>21</td>
</tr>
<tr>
<td>15</td>
<td>CI contribution to coupling matrix elements for normal mode displacement.</td>
<td>24</td>
</tr>
</tbody>
</table>
INTRODUCTION

Excited states of interacting systems, that are bound in the ground state only by van der Waals forces, can interact to form strongly bound species. This is found even for systems in which only one of the fragments is excited, and is exemplified by the excimer states He-He*, Ar-Ar*, and H₂-H₂*. Here the asterisk denotes the first excited state of the same spin symmetry as the ground state. Interest in the latter system has increased recently with the study of Nicolaides, Theodorakopoulos, and Petsalakis (NTP)¹ of the H₂(X ¹Σ⁺ g⁻) - H₂(B ¹Σ⁺ u⁺) system because of the ionic character of the B state which these authors label at 4.0a₀, where charge transfer occurs, a maximum ionicity excited states (MIES); see Fig. 1 for the correlation diagram. This state is of special interest because of its strong electrostatic binding.

A model¹-³ based on MIES properties suggests that bound excited states of polyatomic systems can be formed in regions characterized by an avoided crossing with the ground state, if one of the interacting molecules can exist in a MIES. The description for H₂ H₂*, where H₂* denotes H₂(B ¹Σ⁺ u⁺) is one in which a positive ion complex H₃⁺ is formed, and interacts in its ground state equilibrium geometry (equilateral triangle with r = 1.65a₀) with H. The MIES geometry corresponding to H⁺ H⁺ for the parent H₂(B ¹Σ⁺ u⁺) is the charge transfer region at 4.0a₀. The mechanism supported by NTP with CI computations is one of H₃⁺, which is electron deficient in the center of the triangle, interacting with H at a distance of roughly 4.0a₀ above the plane of the H₃⁺ triangle.
Figure 1. Correlation diagram for $H_4$. 

\[ \text{ENERGY (e.V.)} \]

- $H_2(B) + H_2(X)$
- $H_3(A) + H$
- $H + H + H + H$
- $H_2 + H + H$

\[ H_2(X) + H_2(X) \]
During the contract period studies of the H$_2^+$-H$_2$ system have been pursued in this laboratory using the fixed-node quantum Monte Carlo (FNQMC) method. A brief introduction to the method is presented and followed by a description of FNQMC and other calculations of the subject system.

**QUANTUM MONTE CARLO**

Monte Carlo approaches to solving problems with many degrees of freedom are a class of statistical methods having in common the generation of "random" numbers. In the past few years, Monte Carlo approaches have seen increased application in a number of diverse fields. What we mean here by quantum Monte Carlo (QMC) is a Monte Carlo procedure which solves the Schrödinger equation. This is to be distinguished from so-called variational Monte Carlo, in which one obtains expectation values for a given trial wave function.

This ability to stochastically solve the Schrödinger equation provides an alternative to conventional techniques of quantum chemistry. Early work has shown that highly accurate total energies and correlation energies can be obtained by QMC. In fact, in a procedurally simple manner, accuracies exceeding those of the best *ab initio* configuration interaction calculations have been obtained.

The essence of the procedure is to simulate a quantum system by allowing it (and an ensemble of differently prepared systems) to evolve under the time-dependent Schrödinger equation in imaginary time. Excited states of the same symmetry as a lower state can also be computed with the method.
By writing the imaginary-time Schrödinger equation with a shift in the zero of energy as

$$\frac{\partial \Psi(R,t)}{\partial t} = D \nabla^2 \Psi(R,t) + [E_T - V(R)]\Psi(R,t),$$

(1)

we see that it may be interpreted as a generalized diffusion equation. The first term on the right-hand side is the ordinary diffusion term, while the second term is a position-dependent rate (or branching) term. For an electronic system, 

$$D = \frac{1}{2m}, \quad R$$ is the three-N dimensional coordinate vector of the N electrons, and $$V(R)$$ is the Coulomb potential. Since diffusion is the continuum limit of a random walk, one may simulate Eq. (1) with the function $$\Psi$$ (note, not $$\Psi^2$$) as the density of "walks". The walks undergo an exponential birth and death as given by the rate term.

The steady-state solution to Eq. (1) is the time-independent Schrödinger equation. Thus we have $$\Psi(R,t) \rightarrow \phi(R),$$ where $$\phi$$ is an energy eigenstate. The value of $$E_T$$ at which the population of walkers is asymptotically constant gives the energy eigenvalue. Early calculations employing Eq. (1) in this way were done by Anderson on a number of one- to four-electron systems.

In order to treat systems larger than two electrons, the Fermi nature of the electrons must be taken into account. The antisymmetry of the eigenfunction implies that $$\Psi$$ must change sign; however, a density (e.g., of walkers) cannot be negative. The method which imposes antisymmetry, and at the same time provides efficient sampling (thereby reducing the statistical "noise"), is importance sampling with an antisymmetric trial function $$\Psi_T$$. The zeros (nodes) of $$\Psi_T$$ become absorbing boundaries for the diffusion process, which maintains the
antisymmetry. The additional boundary condition that \( \Psi \) vanish at the nodes of \( \Psi_T \) is the fixed-node approximation. The magnitude of the error thus introduced depends on the accuracy of the nodes of \( \Psi_T(R) \), and vanishes as \( \Psi_T \) approaches the true eigenfunction. To the extent that \( \Psi_T \) is a good approximation of the wave function, the true eigenfunction is almost certainly quite small near the nodes of \( \Psi_T \). Thus one expects the fixed-node error to be small for reasonable choices of \( \Psi_T \).

To implement importance sampling and the fixed-node approximation, Eq. (1) is multiplied on both sides by \( \Psi_T \), and rewritten in terms of the new probability density \( f(R,t) = \Psi_T(R)\Psi(T,t) \). The resultant equation for \( f(R,t) \) may be written

\[
\frac{\partial f}{\partial t} = D \nabla^2 f + \left[ E_T - E_L(R)\right] f - D \nabla^2 \left[ f F_Q(R) \right].
\]

The local energy \( E_L(R) \), and the “quantum force” \( F_Q(R) \) are simple functions of \( \Psi_T \) given by

\[
E_L(R) \equiv H\Psi_T(R)/\Psi_T(R),
\]

and

\[
F_Q(R) \equiv 2\nabla \Psi_T(R)/\Psi_T(R).
\]

Equation (2), like Eq. (1) is a generalized diffusion equation, though now with the addition of a drift term due to the presence of \( F_Q \).

In order to perform the random walk implied by Eq. (2) we use a short-time approximation to the Green’s function which is used to evolve \( f(R,t) \) to \( f(R',t+\tau) \). This evolution process is iterated to large \( t \). The Green’s function becomes exact in the limit of vanishing time-step size, \( \tau \).
MCCFC PILOT STUDY OF THE GROUND AND EXCITED STATES OF H$_2$MIES PATHWAYS

Calculations were carried out for the H$_4$MIES (H$_4$MIES) in which these molecules are contained initially in perpendicular planes that bisect each other. Table 5 displays the sequence of calculations, in the accompanying table a sequence of geometry changes labeled I-VIII, that carry the system from the...
Figure 2. MIES coordinate system (assuming $C_{3v}$ symmetry): $R$ is the distance from $H_d$ to plane of $H_b$, $H_c$, $H_d$ equilateral triangle of side $r$. 
Figure 4. Ground and excited state H₂ potential energy curves for trigonal prismatic geometries: upper panel (C₃ᵥ symmetry), lower panel (Cₛ symmetry).
Figure 4. Potential energy curves from MCSCF trial functions for trigonal pyramidal geometry and C₃ᵥ symmetry \( (r = 1.70 \text{ a.u.}) \). State designations for \( C₂ \) symmetry are indicated in parentheses. The lack of coincidence of the degenerate \( E \) curves reflects their calculation in lower symmetry and provides an indication of MCSCF convergence.
Geometries and energies for the approach of $H_2(B)$ to $H_2(X)$

<table>
<thead>
<tr>
<th>step</th>
<th>$R'$</th>
<th>$L_1$</th>
<th>$L_2$</th>
<th>$L_2(\text{+})$</th>
<th>$L_2(\text{-})$</th>
<th>$D$</th>
<th>energy lowering</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>5.22</td>
<td>1.40</td>
<td>2.43</td>
<td>1.215</td>
<td>1.215</td>
<td>0.0</td>
<td>-0.01</td>
</tr>
<tr>
<td>II</td>
<td>2.2</td>
<td>1.40</td>
<td>2.43</td>
<td>1.215-1.93</td>
<td>1.215-0.5</td>
<td>0.715</td>
<td>-0.20</td>
</tr>
<tr>
<td>III</td>
<td>2.2</td>
<td>1.60</td>
<td>2.63</td>
<td>1.93-2.13</td>
<td>0.5</td>
<td>0.865</td>
<td>-0.20</td>
</tr>
<tr>
<td>IV</td>
<td>2.2-1.75</td>
<td>1.60</td>
<td>2.63</td>
<td>2.14</td>
<td>0.5</td>
<td>0.865</td>
<td>-0.20</td>
</tr>
<tr>
<td>V</td>
<td>1.75-1.55</td>
<td>1.60</td>
<td>2.73</td>
<td>2.23</td>
<td>0.5</td>
<td>0.915</td>
<td>-0.46</td>
</tr>
<tr>
<td>VI</td>
<td>1.55</td>
<td>1.60</td>
<td>3.13</td>
<td>2.23-2.63</td>
<td>0.5</td>
<td>1.115</td>
<td>-0.69</td>
</tr>
<tr>
<td>VII</td>
<td>1.50</td>
<td>1.70</td>
<td>3.23</td>
<td>2.63-2.73</td>
<td>0.5</td>
<td>1.165</td>
<td>-0.11</td>
</tr>
</tbody>
</table>

**TOTAL ENERGY LOWERING FOR THE SEVEN STEPS IS 3.75 ev.**

* Distances in a.u.; energies in ev.

$R'$ - distance between the midpoint of $H_2(X)$ and the point where $H_2(B)$ meets the $H_2(X)$ plane.

$L_1$ - length of $H_2(X)$.

$L_2$ - length of $H_2(B)$.

$L_2(\text{+})$ - length of $H_2(B)$ above $H_2(X)$ plane.

$L_2(\text{-})$ - length of $H_2(B)$ below $H_2(X)$ plane.

$D$ - magnitude of shift of $H_2(B)$ midpoint (above (+)/below (-)) plane of $H_2(X)$.

---

Figure 5. Geometric data to calculate upper limit of $H_2(B)$ to $H_2(X)$. 
asymptotic region to the neighborhood of the $H_4$ MIES configuration. The table also contains the energy lowering associated with the steps that are plotted in Fig. 6. It is noteworthy that no energy barrier is encountered along this path. Further, Fig. 7 show that when $H(B)$ is displaced from bisecting $H_2(X)$ in step II, charge transfer immediately occurs. (Similar behavior has been found for the related $H_2(B) + He$ system in an independent study by WAL.)

**FNQMC STUDY OF THE GROUND-STATE PYRAMIDAL STRUCTURE FOR $C_{3v}$ SYMMETRY**

FNQMC calculations using the MCSCF trial functions discussed above yield $\approx 0.6-1.0\, \text{eV}$ energy lowering compared to the results of NTP and are presented in Fig. 8. Such a large change was not anticipated and so it was important to test the validity of this finding. To this end a configuration interaction calculation including all single and double excitations (SDCI) using the MCSCF pilot study basis set was carried out at $R = 3.4\, \text{a.u.}$ The energy was 0.32 eV lower than NTP’s value and is consistent with the improvement expected based on studies of other systems. The FNQMC results of Fig. 8 obtained using a new trial function optimization algorithm, mentioned in the next section, are generally an improvement over those of Fig. 8 obtained using MCSCF trial functions.

**FNQMC STUDY OF THE EXCITED STATE**

These calculations provide the severest test of the FNQMC approach because of the lack of knowledge of the accuracy of the excited state trial function needed to provide a nodal description that assures orthogonality to the
Figure 6. Plot of total energies corresponding to the pathway presented in Fig. 5.
Figure 7. Charge of charge distribution between $H_0$ and $H_p$ as a function of $d$, the displacement of the midpoint of $H_2(B)$ above the plane of $H_2(X)$. 
Figure 5: Potential energy of trigonal pyramidal $H_4$ versus $R$. The A (variational) results are determined using a new optimization method for XC total energies.
ground state of the same (A') symmetry (C\textsubscript{3}). MCSCF calculations close to the avoided crossing suffered from root flipping. Despite the use of familiar MCSCF strategies to address the problem, it could not be resolved.

The MCSCF convergence problem had been encountered earlier for He + H\textsubscript{2}(B) but resolved by the use of an \textit{ab initio} CI method. Interest here in using the FNQMC method led us to develop a method for trial function construction\textsuperscript{6} that avoids the MCSCF procedure. The approach amounts to the introduction of parameter optimization in the random walk process. Using group theory, a projection operator is constructed and used to constrain the wave function to have the symmetry properties of the state of interest. This method has been employed in the present effort to generate the A-state FNQMC results, denoted A(FNQMC), of Fig. 8. Calculations using this method are in progress to complete the E-state curve, labeled E(FNQMC), in Fig. 8 for 3.8 < R < 6.0 a.u.

**SA-MCSCF STUDY OF DISTORTED GEOMETRIES**

Calculations have been carried out using the state averaged(SA)-MCSCF method to develop trial functions for a QMC study of the topography of the ground- and excited-state potential energy surface (pes) in the region of the symmetric geometry of the MIES determined by NTP. Figure 9 presents the coordinate system. Fig. 10 provides perspective views, and Fig. 11 (R = 4.0 a.u.) and Fig. 12 (R = 3.8 a.u.) show contour maps of the pes in the MIES region. Figures 11 and 12 show that the ground state has a saddle point at smaller R than the minimum of the excited state and that both features correspond to an isosceles triangle base for the MIES system. Further geometry optimization is
Figure 9. MFS coordinate system used for geometry variation. 
\( d \) is the magnitude of the displacement of \( H_a \) along the 
bisector of the \( H_c-H_d \) side of length \( b \).
Figure 10. Potential energy surfaces for the ground- and excited-state separately for easy visualization and in computed relative positioning. Note the minimum of the excited state and saddle point of the ground state.
Figure 11. Potential energy contour map as a function of $\alpha$ and $\beta$; see Fig. 9.
POTENTIAL ENERGY $V_8$ vs $d$

- $R = 3.7$
- $R = 3.8$
- $R = 3.9$

$d$ (a.u.)

-1.994

-1.996

-1.998

-2.000

-2.002

-2.004

-0.05 0.00 0.05 0.10 0.15 0.20 0.25 0.30
Figure 14. Normal mode displacement of $H_4$ in $C_3v$ symmetry.
explored in Fig. 13, which plots potential energy as a function of \( d \), the displacement of \( H_2 \) towards the base of the \( H_4 \) isoelectronic arrangement. The minimum for the excited state is found for \( d = 0.1 \) a.u.

**NONADIABATIC COUPLING**

The stability of the MIES system is dependent on nonadiabatic coupling (NAC) to the ground state. Although the focus of this study is characterization of the region of pes of the MIES, the need to ascertain first the stability of the excited system is a high priority here because of the effort associated with the trial function optimization method and the expense of QMC calculations with small statistical variances. To calculate the NAC matrix elements we took advantage of the simplifications made possible by the use of normal mode distortions (Fig. 14) from the highly symmetric \( C_3v \) symmetry. Our calculations show that nuclear displacements associated with modes \( Q_4 \) and \( Q_5 \) lead to strong coupling between the \( A \) and \( E \) states. Relatively weaker coupling is found in \( Q_1 \) and \( Q_2 \). The two totally symmetric modes (\( Q_3 \) and \( Q_6 \)) give no contribution.

The size of the NAC matrix elements connecting the \( A' \) state to the \( E \) state components (\( C_3v \) notation is used here to indicate parentage) are tabulated in Fig. 15 for the minimum energy geometry of the \( A \) state. These results establish that there is strong coupling between these states. Further computational study is needed to confirm these predictions and to estimate reliably the excited state lifetime.
Coupling matrix elements $D^{CI}$ by a finite difference method (extrapolated from $\delta Q = 0.002, 0.001, \text{and } 0.0005$).

<table>
<thead>
<tr>
<th>$Q_1$</th>
<th>$-2.0112(64)$</th>
<th>$0.0015(0)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Q_2$</td>
<td>$0.0017(0)$</td>
<td>$2.0127(98)$</td>
</tr>
<tr>
<td>$Q_3$</td>
<td>$0.0000$</td>
<td>$0.0000$</td>
</tr>
<tr>
<td>$Q_4$</td>
<td>$-0.3904(389)$</td>
<td>$-0.0010(3)$</td>
</tr>
<tr>
<td>$Q_5$</td>
<td>$-0.0039(13)$</td>
<td>$-0.3113(196)$</td>
</tr>
<tr>
<td>$Q_6$</td>
<td>$0.0000$</td>
<td>$0.0000$</td>
</tr>
</tbody>
</table>

$E_1$ and $E_2$ are doubly degenerate states where $E_1$ is symmetric and $E_2$ is antisymmetric.

Figure 15. CI contribution to coupling matrix elements for normal mode displacement, see Fig. 14.
References


END
DATE
FILMED
MARCH
1988
DTIC