Neutral Beam Propagation Through the Atmosphere

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Abstract:
The problem of Beam Induced Stripping (BIS) process, occurring when a neutral beam propagates through the Earth's atmosphere, has been analyzed. At high current densities the process is important and leads to a rapid disintegration of the beam. At lower current densities currently contemplated for experiments, the effect is probably not significant.
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**Fig. 3**

$T_f$ versus $\theta$: $T_f$ in unit of $1.138952 \times 10^{-7}$ sec, $\theta$ in degree and $n$ for beam energy in unit of MeV.

(a) $n=1$
(b) $n=2$
(c) $n=5$
(d) $n=8$
(e) $n=10$
(f) $n=20$
(g) $n=30$
(h) $n=40$
(i) $n=50$
(j) $n=60$
(k) $n=70$

**Fig. 5**

$T_e$ versus $\theta$: $T_e$ in sec, $\theta$ in radian, and $n$ for beam energy in unit of MeV.

(a) $n=0.1$
(b) $n=1$
(c) $n=2$
(d) $n=10$
(e) $n=20$
(f) $n=40$
(g) $n=50$
(h) $n=70$

**Fig. 6**

$T_o$ versus $\theta$: $T_o$ from Eq. (15) with $\delta=0$ or $\delta=1$; $T_o$ in unit of $1.138952 \times 10^{-7}$ sec, and $\theta$ in degree.

**Fig. 7**

$F(\psi)$ versus $\psi$: $\psi$ in radian.

(a) $\psi$ from 0 to 100 radians.
(b) $\psi$ from 0 to 10 radians.
REFERENCE

T. Li, G. Kalman and P. Pulsifer: "Neutral Beam Propagation Through the Upper Atmosphere II" AFGL-TR-86-0192 (Report II) ADA182601
I. **INTRODUCTION**

Our previous work, hereafter referred to as Report II, concerned the beam attenuation due to "self-stripping" resulting from an "optimum" electric field having the components due to transverse and longitudinal polarization,

\[ E_y = v_b B \sin\theta \]  
\[ E_z = v_b B \cos^2\theta \sin\theta \]  
\[ E_x = -v_b B \cos\theta \sin^2\theta \]

where \( v_b \) is the beam velocity, and \( B = B_x^2 + B_z^2 \), \( B_z = B \sin\theta \), \( B_x = B \cos\theta \) (See Figure 1).

![Figure 1](image)

We assumed a confinement time \( T_1 \) of the form

\[ T_1 = \frac{2R_o}{v_b \sin^2\theta} \]  
with \( \theta < 90^\circ \), and \( R_o \) being the beam radius.

We also assumed another confinement time \( T \) given by

\[ \cos(\omega t) = 1 - \frac{R_o}{R}, \text{ for } \theta < 90^\circ \]

where \( \omega = \frac{eB}{m_e} \) and \( R \) are respectively the electron gyrofrequency and gyroradius.

In the present work, we will study alternative models for the electric field, and the corresponding confinement times. Our purpose is twofold. First, we wish to study the effect of different reasonable models on the confinement time. Second, we wish to combine the confinement time \( T \) with the (energy dependent) collision cross section \( \sigma \) to obtain an estimate for
the total beam-induced stripping probability \( P = \frac{I}{\sigma n v} = \frac{I}{\sigma j} \) as a function of the beam parameters.

II. **ELECTRIC FIELD WITH TRANSVERSE COMPONENT ONLY: CONFINEMENT TIME**

In the first model, the electric field has only one component, due to transverse polarization. More specifically, we have

\[
\begin{align*}
E_y &= f v_b B \sin \theta, \ 0 < f < 1 \\
E_z &= 0 \\
E_x &= 0
\end{align*}
\]  

where \( f \) can be regarded as field strength parameter. \( f = 1 \) corresponds to a fully developed polarization field, while \( f < 1 \) describes reduction due either to ambient plasma screening or to transient situations where the electric field has not fully developed.

The resulting electron velocity equation becomes

\[
\begin{align*}
\dot{x}(t) &= (1-f) v_b \sin^2 \theta \cdot \cos(\omega t) + v_b (f \sin \theta \cdot \cos \theta) \\
\dot{y}(t) &= (1-f) v_b \sin \theta \cdot \sin(\omega t) \\
\dot{z}(t) &= (1-f) v_b \sin \theta \cdot \cos \theta \cdot [1-\cos(\omega t)]
\end{align*}
\]  

For the field-free case, \( f = 0 \), we have

\[
\begin{align*}
\dot{x}_0(t) &= v_b \sin^2 \theta \cdot \cos(\omega t) + v_b \cos^2 \theta \\
\dot{y}_0(t) &= v_b \sin \theta \cdot \sin(\omega t) \\
\dot{z}_0(t) &= v_b \sin \theta \cdot \cos \theta \cdot [1-\cos(\omega t)]
\end{align*}
\]  

For the "maximum" strength field: \( f = 1 \), we have \( \dot{x}(t) = v_b, \dot{y}(t) = 0, \) and \( \dot{z}(t) = 0 \) which describes undisturbed electron motion.
Integrations of Eqs. (8) and (9) give the trajectory equations

\[ y(t) = (1-f) R \sin \theta \left[ 1 - \cos(\omega t) \right] \]  

\[ z(t) = (1-f) R \sin \theta \cdot \cos \theta \left[ \omega t - \sin(\omega t) \right], \]  

where again \( \omega \) and \( R \) are respectively the electron gyrofrequency and gyroradius.

The \( y(t) \) is sinusoidal and confined between 0 and \( 2(1-f) R \sin \theta \), while \( z(t) \) has a secular behavior due to drift.

The confinement time can be considered in the following manner. Looking along the positive \( x \)-axis, we can interpret the interception of \( y-z \) trajectory with the beam (radius \( R_0 \)) circumference as equivalent to confinement time \( T_f \) (See Figure 2)

![Figure 2](image)

We can now write

\[ Y^2(T_f) + Z^2(T_f) = R_0^2 \]

or more specifically

\[ \sin^2 \theta \left[ 1 - \cos(\omega T_f) \right]^2 + \sin^2 \theta \cdot \cos^2 \theta \left[ \omega T_f - \sin(\omega T_f) \right]^2 = \frac{\gamma^2}{(1-f)^2}, \]  

where \( \gamma = R_0/R \). Computer-generated results from Eq. (15) in the form of \( T_f = T_f(\theta, \gamma) \) are given in Figs. 3(a) to 3(n). Assuming \( R_0 \) to have a fixed value of 10cm, we have \( \gamma = \frac{6.34517 \times 10^{-2}}{n} \), with \( n \) being the beam energy \( E_b \) in unit of MeV.
It is worth noting that $T_0$ represents the confinement time for the zero-field case, while $T_1$ represents the confinement time for "optimum" y-component-only field; $T_f$ is infinite, since the optimum y-component-only field results in undisturbed electron drift.

However, $T_f$ can be considered in another way. From Eqs. (13) and (14), we write

$$Y = (1-f) R \sin \theta (1-\cos \omega T_f) = \frac{1}{A} (1-\cos \omega T_f),$$

$$Z = (1-f) R \sin \theta \cdot \cos \theta (\omega T_f - \sin \omega T_f) = \frac{1}{B} (\omega T_f - \sin \omega T_f),$$

where $\frac{1}{A} = (1-f) R \sin \theta$ and $\frac{1}{B} = (1-f) R \sin \theta \cdot \cos \theta$. Rearranging terms and squaring, we obtain a quadratic equation for $T_f$,

$$\omega^2 T_f^2 - 2 \omega B Z T_f + T_f + B^2 Z^2 - 2 A Y = 0,$$

which yields the solution

$$T_f = \frac{B}{\omega} + \frac{(2 A Y - A^2 Y^2)^{1/2}}{\omega}$$

(16)

It is instructive to compare Eq. (16) with Eq. (4). If we let $Y = 0$, we get

$$T_f(Y=0) = \frac{B Z}{\omega} = \frac{B R_0}{\omega} = \frac{2 R_0}{(1-f) \nu_0 \sin \theta} = \frac{T_1}{(1-f)},$$

(17)

where we have made use of the relation,

$$Z^2 + Y^2 = R_0^2$$

i.e., $Z = R_0$ for $Y = 0$.

For the zero-field case ($f = 0$) we obtain

$$T_f = T_0(Y=0)$$

Thus, we have clarified the approximation involved in the confinement time $T_1$ as used in Report II. In other words, $T_1$ is equal to the zero-field confinement time with the approximation of having $Y = 0$. In the following we will examine closely various types of confinement times and their comparative
merits.

III. ELECTRIC FIELD WITH "OPTIMUM" STRENGTH: CONFINEMENT TIME

The "optimum" electric field as given by Eqs. (1), (2) and (3) results in the following y-z trajectory equations:

\[ y_e(t) = R \sin \theta \cdot \cos \theta \left[ \omega t - \sin(\omega t) \right] \]  
\[ z_e(t) = -R \sin^2 \theta \left[ 1 - \cos(\omega t) \right] \]

where the subscript \( e \) denotes "optimum" field strength.

Recalling the corresponding field-free equations as given in Eqs. (11) and (12), we can write

\[ y_0(t) = z_0(t), \]
\[ z_0(t) = -\cos^2 \theta \cdot y_0(t) \]

which can be graphically described in Fig. 4.

![Figure 4](image-url)
The switching-on of optimum field has the effect of rotating the y-z trajectory about the x-axis through 90° and reducing z projection by a factor of \( \cos^2 \theta \). Specifically, we have

\[
\begin{align*}
    r_0^2(t) &= y_0^2(t) + z_0^2(t) \\
    r_c^2(t) &= y_c^2(t) + z_c^2(t) \\
    &= \cos^4 \theta y_0^2(t) + z_0^2(t)
\end{align*}
\] (23)

which shows that \( r_c(t) \) is less than \( r_0(t) \) except at \( \theta = 0^\circ \). Consequently, the confinement time \( T_c \) from \( r_c(T_c) = R \) is longer than the field-free value \( T_0 \) from \( r_0(T_0) = R \).

To obtain \( T_c \), we set

\[
\gamma_c^2(T_c) = y_c^2(T_c) + z_c^2(T_c) = R_0^2
\] (25)

which can be re-expressed as

\[
\frac{\gamma^2}{\sin^2 \theta \cos^2 \theta} = \omega^2 T_c^2 - 2 \omega T_c \gamma \sin \gamma T_c - 2 \cos \theta \cos \omega T_c \\
+ 2 \cos^2 \theta + \sin^2 \theta \sin^2 \omega T_c
\] (26)

where the dimensionless parameter \( \gamma = \frac{R_0}{R} = 6.34517 \times 10^{-2}/\sqrt{n} \), with \( R_0 = 10 \text{cm} \) and \( n \) is beam energy in unit of Mev.

Computer-generated results for \( T_c \) are shown in Figs. 5(a) through 5(h). It is worth noting that at \( \theta = 45^\circ \), \( T_c \) is about \( 1.2 \times 10^{-7} \text{ sec} \) for beam energies ranging from 1 to 70 Mev.

IV. ELECTRIC FIELD WITH FRACTIONAL STRENGTH: IMPACT ENERGY.

A field of less-than-optimum strength can be expressed as

\[
\begin{align*}
    E_y &= f_v B \sin \theta \\
    E_z &= g_v B \sin \theta \cos^2 \theta
\end{align*}
\] (27)
\[ E_x = -g v_b B \sin^2 \theta \cdot \cos \theta \]  

where both \( f \) and \( g \) are respectively \( 0 < f < 1 \) and \( 0 < g < 1 \). \( f \) and \( g \) can be interpreted as before in Section II, as representing ambient plasma screening and transient effects.

The resulting (electron) velocity equations are

\[ \dot{x}(t) = -\delta v_b \sin^2 \theta \cdot [1-\cos(\omega t)] + g v_b \sin^2 \theta \cdot \cos \theta \cdot \sin(\omega t) + v_b \]  

\[ \dot{y}(t) = \delta v_b \sin \theta \cdot \sin(\omega t) + g v_b \sin \theta \cdot \cos \theta \cdot [1-\cos(\omega t)] \]  

\[ \dot{z}(t) = \delta v_b \sin \theta \cdot \cos \theta \cdot [1-\cos(\omega t)] - g v_b \sin \theta \cdot \cos^2 \theta \cdot \sin(\omega t) \]  

where we have written \( \delta = 1-f \).

Integration of Eqs. (31) and (32) gives

\[ y(t) = \delta R \sin \theta \cdot [1-\cos(\omega t)] + g R \sin \theta \cdot \cos \theta \cdot [\omega t - \sin(\omega t)] \]  

\[ z(t) = \delta R \sin \theta \cdot \cos \theta \cdot [\omega t - \sin(\omega t)] - g R \sin \theta \cdot \cos^2 \theta \cdot [1-\cos(\omega t)] \]  

where we have again written \( \frac{v_b}{\omega} = R \), the electron gyroradius.

Now both \( y \) and \( z \) have secular drift components. The relative electron-beam velocity components are

\[ v_x = -\delta v_b \sin^2 \theta \cdot [1-\cos(\omega t)] + g v_b \sin^2 \theta \cdot \cos \theta \cdot \sin(\omega t) \]  

\[ v_y = \delta v_b \sin \theta \cdot \sin(\omega t) + g v_b \sin \theta \cdot \cos \theta \cdot [1-\cos(\omega t)] \]  

\[ v_z = \delta v_b \sin \theta \cdot \cos \theta \cdot [1-\cos(\omega t)] - g v_b \sin \theta \cdot \cos^2 \theta \cdot \sin(\omega t) \]  

Taking the time-average of the above quantities over the confinement time \( T \), we have the time-averaged relative electron-beam velocity components:

\[ <v_x> = -\delta v_b \sin^2 \theta \cdot [1 - \frac{1}{\omega T} \sin(\omega T)] + g v_b \sin^2 \theta \cdot [1 - \frac{1}{\omega T} \sin(\omega T)] \]  

\[ <v_y> = \delta v_b \sin \theta \cdot \frac{1}{\omega T} \cdot [1-\cos(\omega T)] + g v_b \sin \theta \cdot \cos \theta \cdot [1 - \frac{1}{\omega T} \sin(\omega T)] \]  

\[ <v_z> = \delta v_b \sin \theta \cdot \cos \theta \cdot \frac{1}{\omega T} \cdot [1-\cos(\omega T)] - g v_b \sin \theta \cdot \cos^2 \theta \cdot [1 - \frac{1}{\omega T} \sin(\omega T)] \]
As in our preceding work, Report II, we define the electron-beam "impact energy" $E_I$ as

$$E_I = \frac{1}{2}m\langle v^2 \rangle = \frac{1}{2}m [\langle v_x^2 \rangle + \langle v_y^2 \rangle + \langle v_z^2 \rangle]$$

From Eqs. (38), (39) and (40), we obtain

$$E_I = E_b[(1-f)^2 \sin^2 \theta + g^2 \sin^2 \theta \cdot \cos^2 \theta] F(T)$$

where $E_b = \frac{1}{2}mv_b^2$ is the beam energy, $1-f = \delta$ and

$$F(T) = 2\left(\frac{1}{\omega_T}\right)^2[1-\cos(\omega T)] - 2\left(\frac{1}{\omega_T}\right)\sin(\omega T) + 1$$

More specifically, we rewrite Eq. (42) for three types of electric field models, namely, zero-field, $y$-component-only field and optimum field.

(i) Zero-Field:
We have $f = 0$ and $g = 0$

$$E_{IO} = E_b \sin^2 \theta \cdot F(T_0)$$

(ii) Optimum Field:
We have $f = 1$ and $g = 1$

$$E_{Io} = E_b \sin^2 \theta \cdot \cos^2 \theta \cdot F(T_o)$$

(iii) $y$-Component-Only Field:
We have $g = 0$,

$$E_{Iy} = E_b(1-f)^2 \cdot \sin^2 \theta \cdot F(T_f)$$
These three equations represent the field models with their corresponding confinement times, and will be considered in detail in the following section.

V. COMPARISON OF FIELD MODELS AND CONFINEMENT TIMES

In Report II, the zero-field model is represented by Eq. (40) of Report II, i.e.,

\[
\frac{\langle v_{eb} \rangle^2}{v_b^2} = \sin^2 \theta [2 \left( \frac{1}{\psi_e} \right)^2 (1 - \cos(\psi_e))] + 1 - 2 \left( \frac{1}{\psi_e} \right) \sin(\psi_e)
\]

which can be rewritten as

\[
E_{10} = E_b \sin^2 \theta \cdot F(T_u)
\]

where \(E_{10} = \frac{1}{2} \langle v_{eb} \rangle^2 \), \(E_b = \frac{1}{2} m_e v_b^2\), and \(\psi_e = \omega T_u\). We have now used the correct confinement \(T_o\) (as given by Fig. 6) instead of the approximate confinement time \(T_i\) which is obtained from Eq. (15) with \(f = 0\), and re-entry of electron into the beam is not considered. Therefore, at \(\theta = 90^\circ\) and especially for \(R < R_o\), \(T_o\) is, like \(T_i\), not valid.

Let us now compare the two confinement times \(T_i\) and \(T_o\). As pointed out in Report II, \(T_i\) is symmetric with respect to \(\theta = 45^\circ\), but is not valid for \(\theta = 90^\circ\). Both \(T_i\) and \(T_o\) are infinite as \(\theta = 0^\circ\) as they should, however, \(T_o\) approaches asymptotically its lowest value at \(\theta = 90^\circ\). Note that \(T_o\) is obtained from Eq. (15) with \(f = 0\), and re-entry of electron into the beam is not considered. Therefore, at \(\theta = 90^\circ\) and especially for \(R < R_o\), \(T_o\) is, like \(T_i\), not valid.

For numerical comparison, let \(R_o = 10\, \text{cm}\), \(E_b = 1\, \text{MeV}\) and \(\theta = 45^\circ\); we then get \(T_o = 4.45 \times 10^{-8}\, \text{s}\) and \(T_i = 1.44 \times 10^{-8}\, \text{s}\). Note that \(T_o > T_i\) for \(\theta < 80^\circ\).

The result for the optimum field model in Report II was given by Eq. (44) of Report II, which is rewritten in the form

\[
E_{10} = E_b \sin^2 \theta \cdot \cos^2 \theta \cdot F(T_i).
\]
Again, in our present result, as given by Eq. (44),
\[ E_{1c} = E_b \sin^2 \theta \cos^2 \theta F(T_c), \]
we have used the correct confinement time \( T_c \) instead of the approximate \( T_0 \).

Figs. 5(a) through 5(h) show that \( T_c \) is almost constant with a value of about \( 1.25 \times 10^{-7} \) s for \( \theta \) ranging from 45° to 75° and beam energy ranging from 1 MeV to 70 MeV. \( T_c \) increases rapidly as \( \theta \) approaches 0° and 90°. For numerical comparison, we let \( R_0 = 10 \) cm, \( E_b = 1 \) MeV and \( \theta = 45^\circ \); we then get \( T_c = 1.25 \times 10^{-7} \) s, thus we have \( T_c > T_0 > T_b \).

Now we come to the interesting and important result as represented by Eq. (45),
\[ E_{1y} = E_b [(1-f)^2 \sin^2 \theta] F(T_f). \]

Note that the "strength" parameter \( f \) plays an important role in determining the value of \( E_{1y} \). Especially for \( f > 0.9 \), \( T_f \) increases drastically and, as expected, is infinite at \( f = 1 \). [See Figs. 3(a) through 3(n)].

Again for numerical comparison, we let \( R_0 = 10 \) cm, \( E_b = 1 \) MeV and \( \theta = 45^\circ \); we then get \( T_f = 0.9 = 1.6 \times 10^{-7} \) s, thus we have \( T_f = 0.9 > T_c > T_0 > T_b \).

Comparing the impact energies as given by Eqs. (44), (45) and (46), we have, for \( R_0 = 10 \) cm, \( E_b = 1 \) MeV and \( \theta = 45^\circ \)
\[ E_{10} = E_b \sin^2 \theta F(T_0) = (1 \text{ MeV}) (\frac{1}{\sqrt{2}})^2 (0.2) = 100 \text{ KeV}, \]
\[ E_{1c} = E_b \sin^2 \theta \cos^2 \theta F(T_c) \]
\[ = (1 \text{ MeV}) (\frac{1}{\sqrt{2}})^2 (0.48) = 120 \text{ KeV} \]
\[ E_{1y} = E_b (1-f)^2 \sin^2 \theta F(T_f) \]
\[ = (1 \text{ MeV}) (1-0.9)^2 (\frac{1}{\sqrt{2}})^2 (0.3) = 1.5 \text{ KeV} \]
It is worth noting that while the relative impact energies $E_{I0}$ (for zero-field) and $E_{Ic}$ (for optimum field) are comparable, $E_{Iy}$ (for $y$-component-only field with $f = 0.9$) is two orders of magnitudes smaller. This significant quantitative difference between $E_{Iy}$ and both $E_{I0}$ and $E_{Ic}$ is brought about by the strength parameter $f$, and we will discuss the various consequences resulting from the differences among the three impact energies $E_{Iy}$, $E_{I0}$ and $E_{Ic}$.

VI. DISCUSSION AND CONCLUSION

We start with the confinement-time-dependent function $F(T)$ as defined by

$$F(T) = 2\left(\frac{1}{\omega T}\right)^2 \left[1 - \cos(\omega T)\right] - 2\left(\frac{1}{\omega T}\right)\sin(\omega T) + 1,$$

where the confinement time $T$ is set equal to $T_0$, $T_f$ or $T_c$ in accordance with the respective field model. [In Report II, $T$ was set equal to $T_B$.]

Writing $\psi = \omega T$, we get

$$F(\psi) = 2\left(\frac{1}{\psi}\right)^2 \left[1 - \cos(\psi)\right] - 2\left(\frac{1}{\psi}\right)\sin(\psi) + 1,$$

which is graphically represented by Figs. 7(a) and (b). Note that $F(\psi)$ is an oscillating function with decreasing amplitude and approaches unity as $\psi \to \infty$. It has a maximum value of 1.6 at $\psi = 4.2$, and approaches zero as $\psi = 0$.

In terms of the confinement time $T$, we have $F(T) \to 1$ as $T \to \infty$. Since we have assumed a value of $5 \times 10^{-5}$ Tesla for the geomagnetic field throughout our work, we get $\omega = 8.78 \times 10^6$ rad/sec. Thus, $F$ is maximum at $T = 4.78 \times 10^{-7}$ sec. For $\psi = 0.0878$ or $T = 10^{-8}$ sec, we have the following approximate version of Eq. (43')

$$F(T) \approx \frac{1}{4}(\omega T)^2$$
Rewriting Eqs. (44) and (45) of Report II, together with Eqs. (44), (45) and (46), we have

\[ E^{'}_{10} = E_b \sin^2 \theta \cdot F(T_f) \]  \hspace{1cm} (44')
\[ E_{10} = E_b \sin^2 \theta \cdot F(T_o) \]  \hspace{1cm} (44)
\[ E_1 = E_b \sin^2 \theta \cdot \cos^2 \theta \cdot F(T_f) \]  \hspace{1cm} (45')
\[ E_{1c} = E_b \sin^2 \theta \cdot \cos^2 \theta \cdot F(T_c) \]  \hspace{1cm} (45)
\[ E_{1y} = E_b (1-f)2 \cdot \sin^2 \theta \cdot F(T_f) \]  \hspace{1cm} (46)

For the zero-field and the optimum field models, we have used the corresponding "correct" confinement times \( T_o \) and \( T_c \) instead of the approximate confinement time \( T_i \) as was done in Report II.

As shown by Fig. (6), \( T_o \) approaches a lowest value at \( \theta = 90^\circ \), and increases at decreasing \( \theta \) and is infinite at \( \theta = 0^\circ \) while \( T_i \) has a minimum at \( \theta = 45^\circ \), is symmetric with respect to \( \theta = 45^\circ \), and diverges at \( \theta = 0^\circ \) (90°). Figs 5(a) to 5(h) show that \( T_c \) diverges at both ends, is approximately linear with slightly lower value at lower range of \( \theta \), and can be regarded as qualitatively similar to \( T_i \).

The interesting new result obtains from the \( y \)-component-only field model as represented by Eq. (46) and is discussed below.

First of all, it should be noted that all the confinement times, \( T_i, T_o, T_c \) and \( T_f \) are not rigorously valid at \( \theta = 90^\circ \), because re-entry of electron into the beam path is not taken into account. Therefore, if we exclude large \( \theta \), all the \( T \)'s are qualitatively similar, even though we have \( T_f \) (with \( f > 0.9 \)) > \( T_c > T_o > T_i \). It is worth mentioning that all the \( T \)'s decrease very slowly with energy above 5 Mev.

Because of factor \((1-f)^2 \) in Eq. (46), \( E_{1y} \) is at least two orders of magnitude smaller than the other confinement times if \( f \) is assumed to be \( > 0.9 \). Therefore, \( E_{1y} \) results in a comparatively larger ionization cross section, as
indicated by the energy versus cross section curve [See Fig. 8 of Report II.]
Especially at small $\theta$ ($\theta < 10^0$), we have larger $T_f$, and also larger $\sigma$ because of factor $\sin^2 \theta$ in $E_{ly}$.

Regarding $T_f$ and $\sigma$ (respectively, the collision time and collision cross section), we can offer a semi-quantitative argument in favor of the $y$-component-only field model in terms of the relation

$$\tau = \frac{1}{\sigma n v} = \frac{1}{\sigma J}$$

where $\tau$ is the time between collisions, $n$ is concentration per cm$^3$, $v$ is the beam velocity, and $\sigma n v = J$ is the beam current density.

With $J = 1 A/cm^2$, $\sigma = 10^{-16} cm^2$, we have $\tau = 10^{-3}$ sec, which is a rather long time. However, for $f > 0.95$, $\theta < 10^0$, and especially with $E_b < 1$ MeV, we obtain a confinement time $T_f$ comparable to $\tau$.

Another physical consideration, also in favor of the $y$-component-only field model is the following. The time scale of beam operation is probably too short for the build-up of "optimum" space-change field as described in Report II; thus, a partial ($f < 1$) $y$-component-only field is a more likely possibility. In fact, $y$-component-only polarization field was suggested and observed [see Ref. 3 of Report II.]

In conclusion, we can say that our present work suggests that the partial $y$-component-only field model is a physically more reasonable, and thus, is a more likely physical mechanism involved in the self-stripping. Conversely, if the actual polarization field corresponds to the partial $y$-component-only field model, as studied here, we can expect the self-stripping process to be significantly enhanced.
$T_f$ (in unit of $1.138052 \times 10^{-7}$ sec) versus $\alpha$ (in degree).

From bottom $\alpha = 0, 0.05, 0.1, 0.15, 0.2, 0.4, 0.5, 0.7, 0.8, 0.9$ at tor
$T_f$ (in unit of $1.138952 \times 10^{-7}$ sec) versus $\theta$ (in degree).

From bottom $f=0, 0.05, 0.01, 0.15, 0.2, 0.4, 0.5, 0.7, 0.8, 0.9$ at top.

FIGURE 3(b)   $n=2$
$T_f$ (in unit of $1.138952 \times 10^{-7}$ sec) versus $\theta$ (in degree).

From bottom $f=0, 0.05, 0.01, 0.15, 0.2, 0.4, 0.5, 0.7, 0.8, 0.9$ at tor

**FIGURE 3(c) n=5**
$T_f$ (unit of $1.138952 \times 10^{-7}$ sec) versus $\theta$ (in degree).

From bottom $f=0, 0.05, 0.01, 0.15, 0.2, 0.4, 0.5, 0.7, 0.8, 0.9$ at top.
$T_f$ (in unit of $1.138952 \times 10^{-7}$ sec) versus $\theta$ (in degree).

From bottom $\theta = 0, 0.05, 0.01, 0.15, 0.2, 0.4, 0.5, 0.7, 0.8, 0.9$ at top.

FIGURE 3(e) $n=10$
$T_f$ (in unit of $1.138052 \times 10^{-7}$ sec) versus $\varphi$ (in degree).

From bottom $\varphi = 0, 0.05, 0.01, 0.13, 0.2, 0.4, 0.5, 0.7, 0.8, 0.9$ at top.
$T_f$ (in unit of $1.138052 \times 10^{-7}$ sec) versus $\theta$ (in degree).

From bottom: $f=0, 0.05, 0.1, 0.15, 0.2, 0.4, 0.5, 0.7, 0.8, 0.9$ at top.
$T_f$ (in unit of $1.138952 \times 10^{-7}$ sec) versus $\alpha$ (in degree).

From bottom $\alpha = 0, 0.05, 0.1, 0.15, 0.2, 0.4, 0.5, 0.7, 0.8, 0.9$ at top.
$T_f$ (in unit of $1.138952 \times 10^{-7}$ sec) versus $\theta$ (in degree).

From bottom $\theta=0, 0.05, 0.1, 0.15, 0.2, 0.4, 0.5, 0.7, 0.8, 0.9$ at top.

FIGURE 3(i) $n=50$
$\tau_f$ (in unit of $1.1389 \times 10^{-7}$ sec) versus $\gamma$ (in degree).

From bottom $f=0, 0.05, 0.1, 0.15, 0.2, 0.4, 0.5, 0.7, 0.8, 0.9$ at ton.

FIGURE 3(a) $n=60$
\( T_f \) (in unit of \( 1.138952 \times 10^{-7} \) sec) versus \( \theta \) (in degree).

From bottom \( f=0, \; 0.05, \; 0.01, \; 0.15, \; 0.2, \; 0.4, \; 0.5, \; 0.7, \; 0.8, \; 0.9 \) at top.
Optimum Field Confinement Time $T_c$ versus $\theta$

$n=0.1$

FIGURE 5(a)
\( T_c \) versus \( i \), \( n=40 \)

**Figure 5(f)**
$T_c$ versus $\theta$, $n=50$

**Figure 5(g)**
$T_c$ versus $\theta$, $n=70$
Zero-Field Confinement Time $T_0$ versus $\theta$

$T_0$ from Eq. (15) with $f=0$ or $\delta=1$

$T_0$ in unit of $1/138952\times10^{-7}\text{sec}$.

**FIGURE 6**
$F(\psi) \text{ versus } \psi$

$F(\psi) = 2\left(\frac{1}{\psi}\right)^2(1-\cos\psi)-2\left(\frac{1}{\psi}\right)\sin\psi + 1$
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