A SIMULATION OF VOID LINING DURING DUCTILE FRACTURE

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P. E. Magnusen,\textsuperscript{*} D. J. Srolovitz,\textsuperscript{\textdegree} D. A. Koss\textsuperscript{*}

\textsuperscript{*}Department of Metallurgical Engineering
Michigan Tech. Univ., Houghton, MI
(Currently: Alcoa Laboratories)
Alcoa Center, PA 15069

\textsuperscript{\textdegree}Currently: Department of Materials Science and Engineering
University of Michigan
Ann Arbor, MI 48109

\textsuperscript{*}Department of Materials Science and Engineering
The Pennsylvania State University
University Park, PA 16802

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A SIMULATION OF VOID LINKING DURING DUCTILE FRACTURE

P. E. Magnusen, * D. J. Srolovitz, o D. A. Koss

*Department of Metallurgical Engineering
Michigan Technological University
Houghton, MI 49931
(Currently: Alcoa Laboratories, Alcoa Center, PA 15069)

oLos Alamos National Laboratory
Los Alamos, NM 87545
(Currently: Dept. Mat. Sci. Eng., U. of Michigan, Ann Arbor, MI 48109)

+Department of Materials Science and Engineering
The Pennsylvania State University
University Park, PA 16802

ABSTRACT

A computer model has been developed for the simulation of low temperature ductile fracture of metals which contain random distributions of voids. Void distributions are modeled in two dimensions as arrays of circular holes which are characterized by area fraction, hole size, and a minimum interhold hole spacing parameter which controls hole clustering. Experimentally measured local plasticity near holes provide the basis for the simulation, and predictions are made without the use of parameters to force a fit. The simulation illustrates the process of void linking with strain and correctly predicts the failure strains of aluminum and alpha-brass specimens over a range of hole/void microstructures.
I. INTRODUCTION

It has been well documented for many years that ductile fracture occurs by the process of void nucleation, void growth and void linking. However, nearly all previous studies of the low temperature fracture of materials containing voids have only considered regular distributions of voids \(^1\)–\(^9\) or assumed an infinitely long band of voids, \(^10\)–\(^11\) neither of which correlates well with actual microstructures. A continuum analysis of fracture in a solid containing disc-shaped clusters of voids was recently performed, \(^12\) but this also is difficult to correlate with a realistic void distribution. A more recent analysis applies a constitutive relation for a porous solid to a material containing a nonuniform distribution of porosity; the inhomogeneity consists of cells, each of which is characterized by the local void volume fraction. \(^13\) The results indicate a failure process which depend on a critical volume fraction of voids but do not account for local void linking on a microstructural level. As a consequence, accurate prediction of the failure of complex materials is not possible. This is clearly a consequence of having neglected the stochastic nature of void distributions and in particular void linking within a random distribution of voids of differing sizes and shapes. Recent experimental studies have modeled void linking during the low temperature ductile fracture of materials containing random, equi-sized void distributions \(^14\)–\(^15\) and have also compared the flow and fracture behavior of specimens containing regular vs. random distributions. \(^16\) Specifically, tensile tests were performed on sheet and plate specimens containing random or regular arrays of equi-sized, cylindrical, through-thickness holes. These data clearly show that the character of the void distribution (i.e. area fraction of voids, spacing of voids, and void size) has a strong influence on the strains to cause void linking. For example, specimens containing random hole distributions exhibit significantly less extension to
failure than their regular array counterparts. Furthermore, the magnitude of this effect depends on minimum spacing between holes, hole size, and strain hardening of the material. Based on the above observations and as a basis for understanding void linking during microvoid fracture, a step-wise sequence of void (or pore) linking during ductile fracture has been proposed. This process is appropriate to a low temperature failure process in which void linking is triggered by instabilities between voids prior to void coalescence by the impingement of growing cavities in a nearly uniform strain field (such as superplastic failure). In the latter condition, the material's resistance to instabilities (usually due to a very high strain-rate hardening) precludes significant flow instabilities prior to the coalescence of adjacent, growing voids during deformation. The "low temperature" void linking process may be considered in four stages:

1. Local strain gradients are developed near individual holes/voids at small macroscopic strains.
2. Further macroscopic straining results in strain localization between adjacent, favorably located holes; this causes failure of the ligaments between holes. Upon failure of a ligament, an elongated hole is created, which due to its eccentric shape and size, tends also to localize flow along its major axis.
3. Stage 3 involves successive linking with additional holes. In the case of a third hole which is located appropriately close to a linked pair, additional strain localization and linking occurs with the previously linked pair of holes. This stage is controlled by the statistical distribution of holes such that if a third hole is not favorably located, deformation proceeds until another pair of holes link and a favorably located third hole is found. As deformation continues, the linking of adjacent holes repeats and results in increased strain concentration, which in turn increases the probability of
successive linking.

(4) With additional straining, the final stage of fracture is triggered: an imperfection-initiated macroscopic instability or propagation of a crack-like defect. In this case sufficient linking has occurred to create an imperfection (either on an extended or local scale) or crack, which triggers a macroscopic shear instability. This results in failure by the generation of large local strains and the percolation of the linking by the void sheet mechanism between large voids/holes. This occurs at strains well below those necessary to coalesce voids by an impingement process without acceleration by micro- and macroscopic instabilities, such as might occur if strain- and/or strain-rate hardening were very high (i.e., high temperature/slow strain-rate fracture).

It is of interest to compare the above model to a continuum approach, such as that employing a specific non-random void distribution. In both cases, strain localization and shear instability control fracture. However, in the present model, discrete stages are identified for the process and no specific void geometry is assumed. Such a situation may be analyzed by computer simulation.

The purpose of this study is to develop a computer model which simulates the void-linking process as described in Steps 1-4 above. Such a model should physically describe the above experimentally observed sequence. It should also be able to accurately predict experimental values of fracture strains without resorting to adjustable parameters. As in the previous experimental modeling and theoretical studies, the simulation assumes that void distributions may be modeled in two dimensions; the present case assumes arrays of initially cylindrical holes. The simulation requires the following input: (a) the dependence of local strain gradients which develop near holes both prior to and
after hole linking on a applied macroscopic strain, (b) a strain criterion for ligament failure during hole linking, and (c) a microstructure consisting of a spatial distribution (e.g. random or regular) of holes.

II. BASIS FOR THE SIMULATION

As is intuitively obvious and is observed in the experimental modeling studies, void linking clearly depends on local plasticity near voids. The simulation therefore relies on an accurate description of such plasticity in a manner consistent with the stepwise linking process described above. Specifically, as noted in Stages 1-3, it is necessary to analytically describe those pertinent features of the plastic deformation which are localized near (a) isolated single holes, (b) pairs of unlinked holes, (c) pairs of linked holes, and (d) multiple holes which have linked (in this case, effects of the changes of hole geometries due to linking must be described). In addition, any comprehensive simulation must also account for the dependence of the local plasticity on material parameters, notably strain and strain-rate hardening. Given that hole linking occurs by failure of the ligament between holes, a realistic ligament failure criterion is also essential. Experimental data for sheet specimens containing holes indicates that ligament failure occurs at a critical thickness strain. This is consistent with a theoretical analyses of flow localization in the form of a localized necking instability or of fracture of sheet occurring prior to a localized necking instability. Finally the simulation must provide an adequate description of a two-dimensional analog of a void/pore microstructure. In this study, the microstructural arrays of equi-sized, circular holes are analyzed in terms of their (a) size, (b) area fraction, and (c) a minimum interhole spacing parameter which defines a circular region around a hole within which no other hole may be located.
a. Localized Deformation Near Holes

Stage I of the void linking process involves the development of local plasticity near isolated holes at small macroscopic strains. Magnusen, Lee, and Koss have made detailed measurements of the development of local strain gradients in a tensile strain field near isolated holes in 1100-0 aluminum sheet (the strain hardening exponent \( n = \frac{d\ln \sigma}{d\ln e} = 0.27 \)) and annealed 70-30 brass sheet (\( n = 0.46 \)). In order that these data may be incorporated into a computer model for simulating void linking during ductile fracture, empirical relations have been developed to describe the local strain profiles near the holes as a function of the macroscopic strain. The significant results are summarized below.

(1) The experimentally observed extension of isolated holes along the tensile axis with increasing applied macroscopic tensile strain can be described by the relationship:

\[
\frac{b}{b_0} = 1 + \alpha \varepsilon_1
\]

where \( b_0 \) is the initial hole radius,
\( b \) is the hole radius along the tensile axis,
\( \varepsilon_1 \) is macroscopic tensile strain, and
\( \alpha \) and \( a \) are empirically fit parameters.

For 1100-0 aluminum, \( \alpha = 12.1 \) and \( a = 1.3 \), and for brass \( \alpha = 3.8 \) and \( a = 1.0 \). The magnitude of the measured hole extension rates are approximately twice those predicted by the McClintock analysis. This discrepancy has also been observed by other investigators.

(2) The local strain distribution near an isolated hole includes several
strain components. For the specific case of analyzing hole linking under plane stress (sheet) deformation, the local thickness strain $\varepsilon_{zz}$ measured along a line bisecting the hole and normal to the tensile axis is of particular interest. This is based on the experimental observation that the fracture of sheet specimens containing arrays of holes occurs preferentially on a plane roughly normal to the tensile axis. Thus the extent of strain concentration adjacent to a hole and normal to the tensile axis is of primary importance in controlling the subsequent localization of strain and ligament failure between holes. As has been discussed, ligament failure is also observed to occur at a critical value of the thickness strain for both the Al and brass sheets. Thus, data describing the variation of $\varepsilon_{zz}$ measured along a line normal to the tensile axis and passing through the center of an isolated hole, is shown in Figure 1. These data roughly obey the following empirical relationship which resembles that used to describe the elastic distribution of stresses near a circular hole in a strip stressed in uniaxial tension

$$\varepsilon_{zz} = \alpha \varepsilon_1^c + (\beta/r^2 + \alpha/r^4)\varepsilon_1^c$$

where $\varepsilon_{zz}$ is the local thickness strain, $\varepsilon_1$ is the macroscopic thickness strain, $r$ is the distance from the hole center as measured along a line, normal to the tensile axis, and

$\alpha, \beta, \gamma$ and $c$ are empirically fit parameters whose values for Al and brass are listed in Table II.

It should be noted that $\varepsilon_{zz}$ increases with macroscopic strain $\varepsilon_1$ and at a given position, it increases more rapidly with strain $\varepsilon_1$ in Al (low $n$-value) than in brass (hi $n$-value). Thus, increasing strain hardening diffuses strain, as expected.
Linking of pairs of holes (stage 2 in the process) depends on the manner in which strain localization occurs in the ligaments between closely spaced and preferentially oriented holes. The local thickness strain between a pair of holes prior to linking has been experimentally determined as a function of macroscopic strain and along a line normal to the tensile axis passing through a pair of holes. Most of the data for 1100 Al indicate that for the range of hole spacings tested (ligament widths = 1.0 to 2.5 times the hole diameter), the strain between the holes is observed to be approximately equal to the sum of the strain due to the two individual holes. In the case of the brass specimens, the strain level between the holes is less than that obtained by a superposition assumption.

Failure of the ligament between holes (Stage 2) results in two effects occurring simultaneously: (a) an increase in hole extension rate on an axis parallel to the tensile axis and (b) an abrupt increase in local strain near the holes with no imposed increment in macroscopic strain. For a pair of holes which are linked, experimental data indicate that the rate of hole extension increases over that of single holes. The hole extension for a pair of linked holes may be described by a modification of Eq.1 to include an amplification factor. The amplification factor \( f \) can be represented by the empirically fit function:

\[
f = 1 + G(L/D - 1)\varepsilon_1
\]  

(3)

where \( L \) is the length of the linked hole pair as measured between outer surfaces of the holes (transverse to the tensile axis),

\( D \) is the hole diameter prior to linking, and

\( G \) is an empirically fit parameter.
The value of $G$ was found to be 2.2 for 1100 aluminum and 1.8 for 70-30 brass. As before, hole extension is greater at low strain-hardening values.

As holes link, the change in effective hole geometry causes an additional, abrupt amplification of the local strain outside of the linked holes [this is the last aspect of Stage 2 of the linking process]. For a pair of holes which are linked, the local thickness strain along a line normal to the tensile axis increases compared to that for a single hole. This increase is adequately described by the following empirical relationship in which the second term of Eq. 2 for a single hole is amplified. Thus for a pair of linked holes, the thickness strain $\varepsilon_{zz}$ is now given by:

$$\varepsilon_{zz} = \alpha \varepsilon_3^C + g(\beta/r^2 + \alpha/r^4) \varepsilon_3^C$$ (4a)

where

$$g = (L/D)^y$$ (4b)

and $y$ is a constant. The value of $y$ is 0.95 for the 1100 Al and 0.86 for the 70-30 brass. Eq. 4 is very important in hole/void linking in that it represents the increase in plasticity with hole linking. Note that the local strain level is greatest when holes spaced far apart link (or several holes in a common line link) in a material of low strain hardening (high $y$-value in Eq. 4b).

b. Criterion for Hole Linking

Any simulation of hole/void linking must adopt a realistic criterion for ligament failure. This can be a result of either a strain-driven flow instability or damage accumulation between neighboring voids (as in a void-sheet linking of large voids). In the case of simulating the two-dimensional modeling of voids using sheet specimens containing holes, the criterion for hole linking has been observed to be a critical thickness strain value. For both 1100-0 aluminum and 70-30 brass sheet, the plastic flow between a pair of holes
localizes at the same value of the local thickness strain: \( \varepsilon_{zz} = -0.250 \). This strain value is independent of the spacing of the holes and the angle of orientation between the holes for the conditions studied (i.e. spacings of 1.0 and 2.5 times the hole diameter, hole diameters = 3.125 mm and orientation angles from 0° to 30° measured from the tensile axis normal). This is consistent with both instability and accumulated damage failure criteria.\(^{18,19}\)

For a localized necking type of instability in sheets deformed in the range of strain paths from uniaxial tension to plane-strain tension, both previous analysis and data show that for a given material, the instability occurs at a critical value of \( \varepsilon_{zz} \), independent of strain path,\(^{18}\), namely \( \varepsilon^* \).

Furthermore, the onset of localized necking occurs at a critical thickness strain, \( \varepsilon_{zz} = -n \frac{d\ln\sigma}{d\ln\varepsilon} \).\(^{17}\) In the present study, the experimentally determined value of \( \varepsilon^* \) (-.25) is in good agreement with hole linking by instability for Al (\( n = 0.27 \)), but it underestimates the instability strain in brass (\( n = 0.46 \)). This indicates that fracture by damage accumulation intervenes prior to localized necking in the brass and also obeys a critical thickness strain\(^{19}\) but with a value less than \( n \). Visual observations of the deforming ligaments between holes just prior to failure show the presence of a localized necking instability in the Al but fracture prior to the necking instability in the brass, as expected.

One of the difficulties of extending the present two dimensional simulation to three dimensions is the identification of a realistic failure criterion for the linking of voids in three dimensions. An obvious analog to the "line-of-no-extension" condition\(^{25}\) for localized necking type instability which occurs in the present study for Al specimens containing holes is a "surface-of-no-extension" condition between voids in three dimensions. This is the basis for fracture by a shear instability process as analyzed by others.\(^{10,11,25}\)
Computer Simulation Procedure

In the present study, voids are modeled in two dimensions as an array of equi-sized cylindrical holes. The initial step in the computer simulation of the ductile fracture of a void-containing material is to generate a random array of holes. The array is subject to the same constraints of area fraction, hole diameter, and minimum interhole spacing as in the experimental model. Furthermore, as in the experimental testing of sheet specimens containing holes, plane-stress deformation and ligament fracture is assumed. This provides a basis for illustrating the step-wise process of hole/void linking, deducing the effects of void/hole microstructure on fracture, and establishing a correlation with experimental results.

In the analysis, a macroscopic strain is applied to the material, and in response, the holes elongate and local strain gradients develop near the holes as in Stage I of the linking process. The thickness strain distribution near holes can be represented by iso-strain contours as in Fig. 2a and are approximated as ellipses in the simulation: see Fig. 2b. These elliptical "zones of influence" have minor axes which lie along the tensile axis and correspond to the elongated hole diameter. The major axis is the measured distance from the hole center, along a line normal to the tensile axis, to the position at which the local strain level equals one half that of the critical thickness strain. The basis for the zones of influence is that ligament failure occurs at a critical value $\epsilon_{zz}^*$ of the local thickness strain. Thus the "zone of influence" approximate the radial distribution of $0.5 \epsilon_{zz}^*$. For simulation purposes, when the $0.5 \epsilon_{zz}^*$ (contours from two holes overlap), the critical value of $\epsilon_{zz}^*$ is attained and the ligament fails. As noted previously, the superposition of strains is a reasonable assumption for closely spaced holes in Al, but it somewhat underestimates the macroscopic strain.
required for ligament failure in the brass and for holes spaced more than two hole diameters apart in Al.

When two or more holes link (Stages 2 and 3), there is an amplification of both the hole extension rates \( \frac{db}{d \varepsilon_1} \) and the local strain concentration near the linked holes. This is a result of the increase in the effective length of the linked cluster and is described by Eqs. 3 and 4. Thus, the sizes of the elliptical zones of influence near linked holes abruptly increase by the amplification factors given in Eqs. 3 and 4. The resulting amplification of the local strain near linked holes may in turn be sufficiently large so that overlap of the \( 0.5 \varepsilon^*_zz \) contours of adjacent holes occurs. This is likely to occur within clusters of holes/voids and can trigger further linking and additional strain amplification near the holes of the linked cluster (Stage 3). Such a process implies that Stage 3 involves a repeated sequence of hole linking, local strain amplification, and additional linking, all of which can be triggered several times at a nearly constant macroscopic strain. It is clear that the statistical nature of the hole distribution will determine what, if any, additional linking will occur with a previously linked pair or cluster of holes (Stage 3). If no holes are favorably located, deformation will continue with additional linking occurring elsewhere within the sample. This process is indicated in Fig. 3.

Fracture of the material occurs when void linking has percolated across the width of the sample (Stage 4). There is no formal introduction of instability theory \(^{10-12,26}\) or of a crack-like defect of critical length in our current analysis. A flow chart of the computer code used for the simulation is given in Fig. 3.

The low temperature ductile fracture of materials containing both random and regular distributions of voids has been previously modeled experimentally
using two-dimensional arrays of through-thickness, equi-sized cylindrical holes in sheet and plate specimens.\textsuperscript{14-16} For a comparison to the simulation predictions, these data are supplemented by additional data on the fracture behavior of 1100-0 aluminum sheet and 70-30 alpha brass sheet (for details of the experimental procedures see ref. 17). These materials were chosen for their contrasting strain-hardening behavior (i.e. $n = 0.27$ for 1100-0 aluminum and $n = 0.46$ for 70-30 brass).\textsuperscript{*} In addition to the previous data base, specimens containing 0.01 to 0.1 are a fraction of holes of either 1.2 or 2.0 mm diameter were tested. The positions of the holes were random but subject to minimum interface spacing $S$. The value of $S$ defines the minimum allowable distance between adjacent hole edges. This parameter controls the degree of clustering within the array. The magnitude of $S$ was varied from 0.5 mm to a value approaching the hole spacing of a regular distribution of holes. The fracture strains for 1100-0 aluminum sheet and 70-30 alpha brass sheet from the present work and previous studies\textsuperscript{14} are given in Table 1.

Predictions of the strain to fracture for the above aluminum and alpha brass specimens containing random arrays of holes have also been made using the computer simulation described. Specifically, predictions for both materials are made for hole area fractions of 0.01 to 0.1, hole diameters of 1.2 and 2.0 mm, and minimum interhole spacings from 0.5 mm to values approaching the hole spacing of a regular hole array. For a given set of area fraction, hole diameter, and minimum interhole spacing, at least ten computer runs were made to

\textsuperscript{*} We recognize that the choice of $\frac{d\ln \sigma}{d\ln e}$ is an overly simplified definition of strain hardening. In the present instance, such a relationship is a reasonable description of the observed strain hardening and a more rigorous definition is not necessary for the purposes of this discussion.
obtain the fracture predictions. An effective sample area of 127x127 mm$^2$ was employed, and free boundary conditions were assumed.

III. SIMULATION RESULTS

The simulated sequence of fracture events resulting in failure of a specimen containing a random array (0.05 area fraction of 1.2 mm holes with a minimum interhole spacing of 0.5 mm) is illustrated in Fig. 4 for 1100 aluminum and Fig. 5 for 70-30 brass. In these figures, the solid ellipses represent the holes which have elongated with increasing macroscopic strain. The dashed ellipses indicate the zones of influence which approximate the isostrain contours at $*\epsilon_{zz} = 0.5 \epsilon_{zz}$. It should be recalled that the overlap of two zones of influence is assumed to result in a superposition of strains sufficient in magnitude to cause linking of the holes; thus zone overlap represents attainment of the ligament failure criterion.

Examination of Figs. 4 and 5 show that fracture occurs by the sequential, stepwise process which is consistent with the stages previously described for void/hole linking. In Figs. 4a and 5a, the development of local strain gradients near the holes is evident (Stage 1). With continued straining, flow localization and ligament fracture between closely spaced and favorably oriented holes occurs (Stage 2) and, as shown in Figs. 4b and 5b, is depicted by overlapping strain contours which are larger than for a single hole due to the amplification effects given by Eqs. 3 and 4. It can be seen in Figs. 4c and 5c that if additional holes are not located such that continued linking occurs with the previously linked hole cluster, then deformation proceeds by linking elsewhere in the material (Stage 3). However, when hole linking occurs the increased strain concentration increases the probability of successive linking; the linking of the four holes in the upper right corner of Fig's. 4c and 5c is a
good example of this. Failure of the material occurs at that level of macroscopic strain where the repeated process of hole linking, strain amplification, and additional linking percolates across the width of the specimen (Stage 4). In this simulation, imperfection theory or the notion of a critical crack is not introduced; specimen failure results from the hole linking process just described.

The dependence of the elongation to failure on the area fraction of holes for both the experimental model¹⁴,¹⁵ and the computer simulation is shown in Fig. 6. Several aspects of the fracture behavior are evident. An increase in the area fraction of holes results in a decrease in the elongation to failure in both experiment and the computer simulation as well as normalized data for powder metallurgy alloys containing porosity. In addition, the value of the elongation to failure predicted by the computer simulation agrees well with the values observed in the experimental modeling studies¹⁴-¹⁶, although the rate of decrease of the elongation to failure with increasing area fraction is slightly greater in the simulation (which is in good agreement with the powder metal data). It is important to recognize that the agreement between experiment and computer simulation in Fig. 6 is obtained without any parameters which are adjusted to force a fit. The only parameters employed in the computer simulation used are those required to empirically fit the strain profiles for individual holes and pairs of holes in Figs. 1 and 2 (i.e., the material constants in Eqs. 1-4, such as in Table II). It should further be noted that the statistical error is greater in the experimental modeling data than in the simulation data.

An important characteristic of the hole arrays is the minimum spacing between holes. This parameter has a significant effect on the strains necessary to cause hole linking, especially with regard to the behavior of specimens with
random vs. regular hole arrays. The effect of the minimum interhole spacing on the nature of the hole array is to control the degree of clustering of holes within the array. The influence of this parameter on the elongation to failure from both the experimental model and the computer simulation is shown in Fig. 7. Increasing the minimum interhole spacing increases the ductility at a given area fraction and hole diameter. For example, a specimen which contains clusters of holes will exhibit less ductility than a specimen wherein the holes are uniformly spaced. This is the principal reason why specimens containing random arrays of holes are less ductile than their regular array counterparts. The computer simulation predicts both this trend and the associated experimental data accurately, especially in view of the absence of adjustable parameters which may be used to force a fit.
V. DISCUSSION

Existing models of low temperature ductile fracture which are based on the assumption of a regular distribution of voids\(^1\)
\(^9\) are limited since they rely on an unrealistic void distribution and tend to overestimate fracture strains. While models based on the assumption of an initial imperfection\(^10,11\) are often accurate in predicting fracture and the effects of strain hardening, it is difficult to define either the severity or spatial dimensions of an imperfection in terms of an actual microstructure. A similar difficulty is encountered in correlating actual void microstructures to "assumed" void distributions in continuum models which treat void clustering as either disc-shaped clusters of voids\(^12\) or as cells each of which contain a specified local void fraction which varies with the degree of void clustering.\(^13\) A previous attempt to model the fracture behavior of materials with random distributions of voids using computer simulation\(^27,28\) has also been inadequate since it predicts larger fracture strains for specimens with random void distributions then for those with regular arrays. This is contrary to experimental results.\(^14\)\(^-\)\(^16\)

The present analysis is significant because it depends on the random characteristics of a void/pore distribution and correctly indicates the step-wise nature of the void linking process. Relying only on empirical fits of strain distributions near single holes and pairs of holes and a critical thickness strain criterion for ligament failure, the computer simulation quantitatively predicts the dependence of fracture strains on microstructural parameters such as the area fraction of holes and minimum hole spacing. Specifically, predictions based on the present computer simulation of specimens containing random distributions of holes agree well with the experimentally observed trend of decreasing tensile ductility with increasing area fraction of holes: see Fig. 6. When compared to both the simulation results and the
behavior of porous metals, the experimental modeling data show a rather abrupt decrease in the fracture strain associated with the presence of only one hole. This is due to crack initiation and propagation from the hole which is the primary cause for the rather low ductility at the very low area fractions of holes in the experimental data in Fig. 6. In contrast, fracture of specimens containing a greater area of fraction of holes occurs not by cracking but by localization of flow between neighboring holes and failure of the ligament between the holes. In the computer simulation (as well as in porous metals with small amounts of porosity), it is presumed that the initial fracture occurs by ligament failure and hole linking even at small area fraction of holes. The cracks which preclude hole linking at very low area fractions do not occur in the simulation, and the predicted fracture strains from the computer simulation are greater than the experimental model at very low area fractions.

Increasing the minimum interhole spacing (decreasing clustering of holes) increases the ductility at constant hole area fraction and hole size. This has been demonstrated previously and is especially obvious in contrasting the behavior of specimens containing random and regular hole arrays. As shown in Fig. 7, this effect is accurately predicted by the simulation. Increasing the minimum interhole spacing delays hole linking as greater macroscopic deformation is necessary to increase the extent of strain concentration near individual holes for sufficient overlap of the local strain gradients such that flow localization and linking between holes is triggered. Thus microstructures containing distributions of uniformly spaced voids/pores should be more ductile than those in which clustering is prevalent.

Previous results from the experimental modeling studies indicate that increasing strain hardening acts to increase the strains required for hole linking. As shown in Figs. 6 and 7, this effect is accurately predicted by
the simulation. It has been shown previously that the local plastic strain distribution near holes is more intense in materials with low work hardening rates 20-29. This is consistent with the present observation that the local strain gradients are more intense for 1100 aluminum than for 70-30 brass [note the values of $\alpha$, $\beta$, $\gamma$, and $C$ in Eqs. 2-4 or the data in Fig. 1]. Thus, the critical thickness strain to trigger ligament fracture between holes occurs at smaller macroscopic strains in materials with low strain-hardening capacity. Thus, materials with low strain hardening should be especially sensitive to small amounts of porosity or voiding. This appears to be the case for high strength alloys as well as the contrasting behavior of 7075-T6 aluminum and alpha brass specimens containing holes.15

The computer model contains several advantages which are valuable for predicting certain trends in the fracture of porous or voided materials. Of primary importance is the fact that it is not computationally intensive. Once the experimental data describing local strain development has been obtained, predictions can easily be made without consuming extensive amounts of computer time. Also, it is possible to make predictions over a wide range of area fractions of voids as well as cases in which hole/void densities increase with strain. While the current predictions have been limited to equi-sized holes, the model could very easily be applied to material containing a distribution of sizes. The model can also be applied to elongated holes/voids which are simulated as elliptical in shape. Finally, the nature of the spatial distribution of the voids is taken into account in predicting fracture strains through the incorporation of the minimum interhole spacing parameter, which controls the degree to which voids cluster. Thus, predictions are not only obtained on the basis of the area fraction and void size(s) but also as a function of the degree of clustering within the hole/void microstructure.
The subsequent application of such a model to a three-dimensional void array is possible in principle but may be difficult in practice. Such an analysis would rely on an approximation of the local strain gradients near voids or spherical cavities in three-dimensions: this is obviously difficult especially when complex interactive effects occur between voids. A realistic criterion for the failure of the ligaments between neighboring voids is also required. The critical thickness strain criterion for hole linking used in the present two-dimensional analysis is not plausible for a three-dimensional array. Its corollary, a surface of no extension is an essential consequence of the shear localization analyses of void materials. 10-12 A further limitation of the present computer model is the inherent stiffness of the material in the simulation. When holes link in the experimental samples, a free surface is created resulting in a relaxation of the local stresses. This prevents hole linking immediately above and below the linked holes. In the present simulation, the material is stiff and no effective relaxation near the linked holes occurs. Thus, continued straining and hole linking can occur directly above and below the free surface created by hole linking.

VI. SUMMARY

The step-wise process of void linking in materials undergoing low temperature ductile fracture is simulated. The analysis is based on two-dimensional modeling of voids as arrays of equisized, cylindrical holes characterized by an area fraction, hole size, and a minimum interhole spacing. Utilizing experimentally determined strain distributions near holes and a critical thickness strain criterion for the failure of ligaments between holes, the simulation depicts the sequential stages of void/pore linking and accurately predicts the experimentally determined fracture strains of specimens.
containing random arrays of holes. It is significant that the predictions are obtained using only empirically represented experimental data without the use of additional parameters which are adjusted to obtain good quantitative agreement with the experimentally determined fracture strain data.

VII. ACKNOWLEDGMENTS

The authors wish to acknowledge valuable discussions with Drs. J. D. Embury, J. K. Lee, J. Pilling and O. Richmond. This research was supported by the Office of Naval Research through Contract N00014-86-K-0381 and by the Department of energy while one of the authors (DJS) was at the Los Alamos National Laboratory.
Table I

Elongation-to-failure data from specimens containing arrays of holes with specified area fractions $A_f$, diameters $D$, and exclusion radii $S_m$.

<table>
<thead>
<tr>
<th>Random Array Parameters</th>
<th>Failure Strains (%)</th>
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<td>$1100\text{ Al}$</td>
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<tr>
<td>$A_f$ (%)</td>
<td>$D$(mm)</td>
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TABLE 2

Values of the parameters in Eq. 2 used to describe the stain profile near a single hole.

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<tr>
<th>Parameter</th>
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<th>70-30 Brass</th>
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<td>ρ</td>
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<td>B</td>
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<td>γ</td>
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<tr>
<td>C</td>
<td>1.6</td>
<td>0.9</td>
</tr>
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</table>
REFERENCES

15. E. M. Dubensky and D. A. Koss, to be published in Metall. Trans. A.


Fig. 1. The thickness strain profiles along a line normal to the tensile axis for (a) 1100 aluminum and (b) 70-30 brass sheet. The dashed curves represent the empirical fit to the data.
Fig. 2. (a) Thickness strain contour map about an isolated hole with the highlighted contour having a value equal to \(0.5(\varepsilon_{\text{th}})\), and (b) approximation of the local thickness strain contour corresponding to \(0.5(\varepsilon_{\text{th}})\) as an elliptical "zone of influence".
Fig. 3. A flow chart of the computer code used for the computer simulation of hole/void linking during ductile fracture.
Fig. 4. The simulated sequence of fracture events in 1100 aluminum with 0.05 area fraction of 1.2mm holes and a minimum hole spacing of 0.5mm. The macroscopic tensile strains are (a) $\varepsilon_1 = 0.06$, (b) $\varepsilon_1 = 0.07$, (c) $\varepsilon_1 = 0.08$, and (d) $\varepsilon_1 = 0.098$. 
Fig. 5. The simulated sequence of fracture events in 70-30 brass with 0.05 area fraction of 1.2mm holes and a minimum hole spacing of 0.5mm. The macroscopic tensile strains are (a) $\varepsilon_1 = 0.13$, (b) $\varepsilon_1 = 0.15$, (c) $\varepsilon_1 = 0.18$, and (d) $\varepsilon_1 = 0.202$. 
Fig. 6. The dependence of the elongation to failure of (a) 7000 aluminum and (b) brass on the area fraction of holes from both the experimental model and the computer simulation. The combinations of hole diameter and minimum hole spacing are (a) d=1.2mm, and s=0.5mm, and d=1.2mm. Data for the aluminum included at are fractions of 0.025 and 0.05 are from ref. 14 and the normalized powder metallurgy PM data are from ref. 30.
Fig. 7. The dependance of the elongation to failure of (a) 1100 Al and (b) 70-30 brass on the minimum hole spacing from both the experimental model and the computer simulation at all area fractions for a hole diameter of d=1.2mm.
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