The conference on Maximum Principle and Eigenvalue Problems in Partial Differential Equations (June 15-19, 1987, University of Tenn.) brought together more than 30 researchers to interact and listen to lectures by Dr. Protter (Main Speaker) and Drs. Levine, Weinberger, Cosner and Payne. New results in the extension of maximum principles to systems using "best possible" estimates were the highlight topic. These apply to equations in continuum mechanics.
AFOSR-TK-87-1685

FINAL REPORT

Conference on Maximum Principles and Eigenvalue Problems in Partial Differential Equations

TIME: June 15-19, 1987

PLACE: University of Tennessee
Knoxville, TN 37996-1300

CONFERENCE DIRECTOR: Dr. Philip W. Schaefer
Department of Mathematics
University of Tennessee
Knoxville, TN 37996-1300

FUNDING: The Tennessee Science Alliance and the Air Force Office of Scientific Research (AFOSR-87-0275)

REPORT: The Conference on Maximum Principles and Eigenvalue Problems in Partial Differential Equations was held during the week of June 15-19, 1987, and was attended by forty-four (44) participants, including five conferees from foreign countries.

Professor Murray H. Protter of the University of California, Berkeley, presented a series of five lectures which provided the focus and direction of topical matter of the conference. His initial lecture brought the participants from the original maximum principle for a single equation to some principles for systems and to some recent applications of maximum principles. This was further developed by the lectures of Professors Payne and Philippin who introduced the notion of "best possible" maximum principles which lead to isoperimetric inequalities for the solution and gradient of the solution to linear and nonlinear elliptic boundary value problems. Professor Horgan then applied maximum principle techniques to determine exponential decay estimates for the solution and its gradient for problems in static elasticity theory and to estimate stress concentration factors in the theory of finite anti-plane shear.

After his lecture devoted to overdetermined first order elliptic systems and the question of maximum principles for such, Professor Protter shifted his attention to the second main topic of the conference. He discussed previously obtained lower bounds for the spectrum of second order elliptic equations and introduced the notion of the generalized spectrum for elliptic systems. This concept was then amplified by Professor Cosner in his supplementary lecture in which he compared and contrasted the properties
of eigenvalues and eigenfunctions for second order elliptic systems with those for a single elliptic equation.

In his fourth lecture, Professor Protter discussed universal inequalities -valid for all domains- for eigenvalues and some improvements of such in some classical second order and higher ordered elliptic boundary value problems. This included estimates for the ratio or sum of discrete eigenvalues. Professor Levine then presented some recent results which compare Dirichlet and Neumann eigenvalues for the membrane equation.

A mini-series of lectures were devoted specifically to the Schrödinger operator. Professor Ashbaugh reported on some joint work which resulted in an optimal estimate for the "gap" between the first two eigenvalues for the Schrödinger operator in a ball. More generally, Professor Harrell developed general bounds for the spectrum for Schrödinger equations while Professor Gurarie presented an averaging method for the inverse problem which consists of determining a perturbation operator from knowledge of asymptotic formulae for the perturbed eigenvalues.

In his fifth lecture, Professor Protter discussed the asymptotic behavior of the spectrum for elliptic equations. He recalled early results in this area, discussed some recent improvements, and extended the notion to second order systems. Professor Weinberger then utilized the strong maximum principle and some invariant set analysis to extend a recent result of Pucci. He also discussed the characterization of matrices which are similar to matrices with off-diagonal elements nonnegative. This latter idea was motivated by a recent maximum principle for systems which resulted from a decoupling of the system.

In the two lectures by Professors Siegel and Pinchover, the maximum principle was employed to deduce results applicable to unbounded domains. Professor Siegel introduced a new Phragmen-Lindelöf result while Professor Pinchover discussed the structure of the set of all positive solutions for a class of elliptic operators in an unbounded domain.

The proceedings of the conference will appear as a volume in the Pitman Research Notes in Mathematics series which is published by Longman UK Limited. A copy will be provided upon publication. It is anticipated that the volume will be available early in 1988.

The program for the conference is included in appendix 1. The list of official participants in the conference and their university affiliation is provided in appendix 2. In appendix 3 we include copies of some letters which give a general assessment of the conference.
A financial report will be prepared by the Controller's Office of the University of Tennessee. It is noted that we were able to fund thirty participants instead of the proposed twenty-five as a result of the use of super-saver air fares and a special rate for lodging.
PROGRAM

Conference on Maximum Principles and Eigenvalue Problems in Partial Differential Equations

sponsored by

The Tennessee Science Alliance and The Air Force Office of Scientific Research

MONDAY, June 15, 1987

8:30-8:55  Registration, Ayres Hall, Room 119.

9:00-9:15  Formal Opening, Buehler Hall, Room 414 (All talks).


COFFEE AND DOUGHNUTS, Ayres Hall, Room 119

11:00-12:00  L.E. Payne, "Some Applications of Maximum Principles to Isoperimetric Inequalities."

2:00-3:00  G.A. Philippin, "Applications of the Maximum Principle to a Variety of Problems Involving Elliptic Differential Equations."

REFRESHMENTS, Ayres Hall, Room 119

TUESDAY, June 16, 1987

9:00-10:00  M.H. Protter, "Overdetermined First Order Elliptic Systems."

COFFEE AND DOUGHNUTS, Ayres Hall, Room 119

11:00-12:00  C.O. Horgan, "Some Applications of Maximum Principles in Linear and Nonlinear Elasticity."

2:00-2:25  D. Siegel, "The Dirichlet Problem in a Half-Space and a New Phragmén-Lindelöf Principle."


REFRESHMENTS, Ayres Hall, Room 119

4:00-4:25  D. Gurarie, "Spectra of Anharmonic Oscillators."

**WEDNESDAY, June 17, 1987**

9:00-10:00  M.H. Protter, "Lower Bounds for the Spectrum of Second Order Elliptic Equations."

COFFEE AND DOUGHNUTS, Ayres Hall, Room 119

11:00-12:00  G.C. Cosner, "Spectrum, Generalized Spectrum and Eigenfunctions for Second Order Systems."

1:30-10:00  Smoky Mountains excursion-dinner on your own in Gatlinburg and time for shopping and/or sightseeing.

**THURSDAY, June 18, 1987**

9:00-10:00  M.H. Protter, "Universal Inequalities for Eigenvalues."

COFFEE AND DOUGHNUTS, Ayres Hall, Room 119

11:00-12:00  H.A. Levine, "Inequalities Between Dirichlet and Neumann Eigenvalues."

2:00-3:00  E.M. Harrell, "Generalized Spectral Bounds for Schrodinger Equations."

REFRESHMENTS, Ayres Hall, Room 119

7:00-8:30  Banquet, Holiday Inn, Cumberland Suite

**FRIDAY, June 19, 1987**

9:00-10:00  M.H. Protter, "Asymptotic Behavior of the Spectrum for Elliptic Equations."

COFFEE AND DOUGHNUTS, Ayres Hall, Room 119

11:00-12:00  H.F. Weinberger, "Some Remarks on Maximum Principles for Systems."

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UT Library is located across Cumberland Ave. from Ayres Hall and is open from 8:00 a.m. to 6:00 p.m.

The seminar rooms 209A, 209B, 309A, 309B in Ayres Hall are available Monday through Thursday for participants for study and research.
Conference on Maximum Principles and Eigenvalue Problems in Partial Differential Equations

List of Participants

Principal Lecturer
Murray H. Protter
University of California, Berkeley
Berkeley, California 94720

Contributory Lecturers - One hour Lectures

G. Chris Cosner
University of Miami
Coral Gables, FL 33124

Evans Harrell
Georgia Institute of Technology
Atlanta, GA 30332

Cornelius O. Horgan
Michigan State University
East Lansing, MI 48824

Howard A. Levine
Iowa State University
Ames, Iowa 50010

Lawrence E. Payne
Cornell University
Ithaca, NY 14853

Gerard A. Philippin
Universite' Laval
Quebec, G1K 7P4, Canada

Hans F. Weinberger
University of Minnesota
Minneapolis, MN 55455

Contributory Lecturers - Half Hour Lectures

Mark S. Ashbaugh
University Of Missouri
Columbia, MO 65211

David Gurarie
Case Western Reserve University
Cleveland, OH 44106

Yehuda Pinchover
University of California
Los Angeles, CA 90024

David Siegel
University of Waterloo
Waterloo, Ontario N2L 3G1, Canada

Participants

David Adams
University of Kentucky
Lexington, KY 40506

Karen Ames
Iowa State University
Ames, IA 50010

Richard Brown
University of Alabama
Tuscaloosa, AL 35487-1416

Steve Cox
Rensselaer Polytechnic Institute
Troy, NY 12180
<table>
<thead>
<tr>
<th>Name</th>
<th>Affiliation</th>
<th>Location</th>
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<tr>
<td>Philip Crooke</td>
<td>Vanderbilt University</td>
<td>Nashville, TN 37235</td>
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<td>George Dassios</td>
<td>University of Patros</td>
<td>Patros, Greece</td>
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<td>Anne Dow</td>
<td>Maharishi Int'l University</td>
<td>Fairfield, IA 52556</td>
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<td>John Drake</td>
<td>Oak Ridge National Laboratory</td>
<td>Oak Ridge, TN 37830</td>
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<tr>
<td>Alan Elcrat</td>
<td>Wichita State University</td>
<td>Wichita, KS 67203</td>
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<tr>
<td>Vinod Goyal</td>
<td>Tuskegee Institute</td>
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<tr>
<td>M. Grillakis</td>
<td>Courant Institute</td>
<td>New York, NY 10012</td>
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<td>Don Hinton</td>
<td>University of Tennessee</td>
<td>Knoxville, TN 37996-1300</td>
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<td>Martin Klaus</td>
<td>Virginia Polytechnic Inst.</td>
<td>Blacksburg, VA</td>
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<td>Ian Knowles</td>
<td>University of Alabama-B'ham</td>
<td>Birmingham, AL 35294</td>
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<td>Philip Korman</td>
<td>University of Cincinnati</td>
<td>Cincinnati, OH 45221</td>
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<td>Mikhail Kovalyov</td>
<td>University of Alberta</td>
<td>Edmonton, Canada</td>
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<td>James Kuttler</td>
<td>JHU Applied Physics Lab.</td>
<td>Laurel, MD 20707</td>
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<td>Suzanne Lenhart</td>
<td>University of Tennessee</td>
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<td>Roger Lewis</td>
<td>University of Alabama-B'ham</td>
<td>Birmingham, AL 35294</td>
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<td>Gary Lieberman</td>
<td>Iowa State University</td>
<td>Ames, IA 50010</td>
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<td>Roger Lui</td>
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<td>Worcester, MA 01609</td>
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<td>Edward C. Nichols</td>
<td>University of Tennessee-Chattanooga</td>
<td>Chattanooga, TN 37403</td>
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<td>George Paulik</td>
<td>University of Iowa</td>
<td>Iowa City, IA</td>
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<td>Yoshimi Saito</td>
<td>University of Alabama-B'ham</td>
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<tr>
<td>Francoise Schremmer</td>
<td>West Chester University</td>
<td>West Chester, PA 19383</td>
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<td>J. C. Song</td>
<td>Cornell University</td>
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<td>Raj P. Soni</td>
<td>University of Tennessee</td>
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<td>Brian Straughan</td>
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<td>Roman Svirsky</td>
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<tr>
<td>Andrew Vogel</td>
<td>University of Kentucky</td>
<td>Lexington, KY 40506</td>
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Richard Weinacht  
University of Delaware  
Newark, Delaware  19716

Philip W. Schaefer, Conference Director  
University of Tennessee  
Knoxville, TN  37916
June 30, 1987

Professor Philip W. Schaefer
Mathematics Department
University of Tennessee
Knoxville, TN 37916

Dear Phil,

Enclosed is my contribution to the conference proceedings. I hope it is satisfactory.

The conference was absolutely super. It was certainly the most successful conference I have been to in a long, long time. You did a superb job of managing the whole affair. Thank you once again for inviting me.

Regards,

Howard A. Levine

HAL: jkv

Enc.
June 30, 1987

Professor John S. Bradley
Department of Mathematics
University of Tennessee
Knoxville, TN 37996

Dear Spud,

I wanted to thank you, your Department, and the Tennessee Science Alliance for the opportunity of spending an enjoyable and fruitful six months at the University of Tennessee. You even arranged for great weather most of the time.

I also wanted to comment on the Conference on Maximum Principles and Eigenvalue Problems. It was one of the best organized, smoothly run, and mathematically interesting conferences I have ever attended. A lot of questions were thrown out and almost all of the participants were involved in either formal or informal mathematical discussion. Phil deserves almost all of the credit for making the conference such a great success, an opinion voiced by many of the participants in stating how much they enjoyed and learned from the conference. The funding from the Tennessee Science Alliance was certainly money well spent.

Again let me thank you and your colleagues for making our stay in Knoxville both profitable and enjoyable.

Sincerely,

Lawrence E. Payne

aah
Dear Professor Bradley:

I am writing to you to express the high regard of the Department (and in particular to Professor D. Schaefer) for the organization of the conference on "Recent Advances in Problems in Partial Differential Equations". Your enthusiasm for the topic, and for the field in general, was apparent to all. It was an excellent conference for the Department and the University of Tennessee.

Sincerely,

[Signature]

[Name]
Professor
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