AVSCOM


GENERATION OF SPECIFIC CHROMATICITIES ON CRT MONITORS

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September 1987

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Research and Development Technical Report
Aviation Systems Command
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The increased use of CRT displays for presenting the aviator with flight information has generated an increasing interest in the use of color for representing CRT display symbology. Critical to the implementation of color for this purpose is the development of standardized chromaticities for coding displayed features.

Generation of specific chromaticities on CRT displays applied color matching principles using the chromaticities of CRT phosphors as the new set of primaries, and transforming their ratios in the generated color to those of a desired standard color. A procedure to implement this transformation was proposed by Gutmann and Rogers (1982), and further developed by Spiker and Rogers (1984). While their procedure was theoretically sound with respect to color mixing principles, it failed to produce predicted chromaticities when used in conjunction with a variety of commercial and aircraft CRT monitors. This result raised some perplexing issues with respect to the relationship between color display (contd)
19. ABSTRACT (contd)

technology, the science of color mixing, and the psychophysics of color perception. This report attempts to clarify the problem of direct application of color mixing to CRT's, and proposes a modified procedure for generating specific chromaticities on CRT displays suitable for use in a field environment.
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1. INTRODUCTION

The use of CRT displays for presenting the aviator with flight information has generated an increasing interest in the use of color for representing CRT display symbology. Critical to the implementation of color for this purpose, particularly where redundant coding is not feasible, is the development of standardized colors for coding displayed features. This requires that a color consistently represent the same feature, and that it be perceptually identifiable regardless of background color, observer, display device, or ambient illumination conditions.

While color generally refers to a perceptual attribute such as red, blue, yellow, etc., it can be more precisely specified in terms of its chromaticity. Theoretically, robust methods of generating specific chromaticities have been formally validated and utilized in industrial and scientific environments since the 1930's. These methods rely on replicating with a new set of primaries the precise ratios of the primary mixture present in the desired color or standard. Such techniques require a mathematical transformation of the new primaries to values equivalent to those present in the standard.

Generation of specific chromaticities and luminances on CRT displays applies color matching principles using the CRT phosphors as the new set of primaries, and generating ratios of phosphor luminances which are equal to the known luminance ratios present in a standard color. A procedure to implement this transformation on any CRT, regardless of phosphor characteristics, was proposed by Gutman and Rogers\(^1\), and further developed by Spiker and Rogers\(^2\). Their objective was to enable the generation of a set of colors on aircraft displays for coding digital topographic information\(^3\).

The procedure, while theoretically sound with respect to color mixing principles, failed to produce predicted chromaticities when used in conjunction with a variety of commercial and aircraft CRT monitors. The result raised some perplexing issues with respect to the relationship between color display technology and the science of color mixing.

The present report defines the problem of direct application of color mixing principles to CRT's, presents an experimental evaluation of parameters relevant to color mixing on CRT's, and proposes a modified procedure for generating specific chromaticities suitable for use in a field environment.

2. BACKGROUND

a. Color Theory and Color Perception. While color is frequently assumed to be that physical characteristic of an object which produces a perceptual response iso-morphic with a physical attribute, the perceptual experience of color is never a direct reflection of the absolute properties of its physical composition. Rather, perceived color is the result of an interaction between the physical components of the intensity and wavelength of emitted or reflected light, and the information.


\(^2\) Spiker and Rogers, "Procedure and Guidelines for Producing Anacapa's Topographic Map Colors on Other Displays," Anacapa Sciences, Inc., TR 566-2, July 1984, Santa Barbara, CA.

processing characteristics of the eye and brain. Therefore, precise specification of color requires a definition of the nature of light and of the processing characteristics of the visual system.

b. Light, Luminosity, and Luminance. All illumination sources generate radiant energy. Some of this energy is lost by heat conduction, convection, or absorption, while the remainder is emitted. Emitted radiant energy is referred to as RADIANT FLUX, and, of this, only a fraction lies within the wavelength range to which the eye is sensitive. That portion, between 380 and 770 nanometers, is referred to as LUMINOUS FLUX and is the visible light which forms the basis for our perception of color and brightness.

An international measure is typically used to quantify the LUMINOUS FLUX emitted by a light source. This standard source consists of a fused thoria tube surrounded by pure platinum at its melting point. The LUMINOUS FLUX, emitted by the source in this configuration, is proportional to the cone of radiation measured as a solid angle in units called steradians. One steradian represents a solid angle that subtends an area on a sphere surrounding the source of area equal to the radius of the sphere squared. The LUMINOUS FLUX issuing from one sixteenth of a square centimeter of the standard source, included within the solid angle of one steradian, is referred to as a LUMEN.

The LUMINOUS FLUX striking a flat surface is referred to as ILLUMINANCE and the luminance produced by a point source is called LUMINOUS INTENSITY. A single unit of LUMINOUS INTENSITY is referred to as a CANDLE and is equal to one LUMEN per steradian. The term LUMINANCE is used to define an extended light source and is the LUMINOUS INTENSITY measured in candles per square meter or foot lamberts (FL). This is the unit most frequently used to quantify the light output of self-luminous CRT screens.

A summary of conversion factors for the various measures of emitted light and the derivation of the measure of ILLUMINANCE produced by a point source are presented in Appendix A.

c. Psychophysical Properties of Color. As noted above, the eye does not respond to all frequencies of radiant energy but only to the narrow range of energy from 380 to 770 nanometers. Two kinds of receptors in the human retina are capable of absorbing spectral light, rods, and cones. Rods which are more sensitive to light are responsible for perception in low ambient illumination. Cones, while less sensitive than rods, are capable of distinguishing among various wavelengths of light, and are responsible for color vision.

There are three types of cones, each of which has peak sensitivity to short, medium, and long wavelengths, respectively. Relative sensitivities of the three types of cones are shown in Figure 1. As can be seen from the plot, cones are not exclusively sensitive to specific wavelengths, but are only maximally responsive to specific wavelengths. This is important in color vision since it permits different wavelengths of light from a common source to be absorbed by the eye in varying ratios. This results in a physiological corollary to color mixing and accounts for the perception of the wide variety of spectral and non-spectral colors.

The physiological responses produced by the cones can be perceptually described in terms of the psychophysical attributes of color, HUE, SATURATION, and BRIGHTNESS. HUE refers to the perceived color of reflected or emitted light (e.g., red, yellow, blue, green, etc.) and is generally related to the physical dimension of wavelength.

*Sears and Zemansky, College Physics, Addison-Wesley, Reading, MA, 1952.*
Figure 1. Sensitivity of the Three Types of Cones to Various Wavelengths.
SATURATION refers to the perceived purity of a color. Greater saturation corresponds to a greater perceived distance from white, and HUE and SATURATION together represent the CHROMATICITY of the perceived color.

BRIGHTNESS refers to the perceived luminosity of self-emissive color sources and is related to the physical dimension of intensity or luminous flux.

3. COLOR MATCHING EXPERIMENTS

a. 1924 Standard Observer. Standardization of luminosity measures provided the ability to specify the relative luminances generated by sources emitting different wavelengths of light in terms of perceived brightness. The psychophysical procedure used to compare the luminances of different wavelengths consisted of a test stimulus of a specific chromaticity and luminance, and a variable stimulus which could be manipulated until a perceptual match was achieved with the test stimulus.

In the earliest color matching experiments, observers were asked to compare the relative BRIGHTNESS of two lights which differed in wavelength. Intensity of the variable light (measured in watts) was adjusted until it matched the perceived brightness of the test stimulus. The results of these experiments demonstrated that when wavelength is varied, equal amounts of radiant flux do not produce equal sensations of brightness. (For example, one watt of 555 μm green light has a luminous flux of 680 lumens, whereas one watt of yellow 600 μm has a luminous flux of 411 lumens. The ratio of luminous flux to radiant flux is known as luminous efficiency and is measured in lumens per watt.) In addition, the perception of relative brightness was found to vary as a function of ambient illumination, or more specifically, as a function of whether the absorption of light occurred primarily in rods or cones.

The data first formally taken in 1924, and plotted as wavelength by the intensity required to achieve equal brightness, is known as the standard luminosity curve or the luminance efficiency function. The curve is shown in Figure 2 with engineering units as ordinate, and in Figure 3 as a relative luminosity function.

b. The Standard Observer. In the 1924 color matching experiments, the observer's judgement of equivalent brightness was determined by manipulating the intensity of single wavelength illumination sources. These experiments resulted in the spectral sensitivity function. In 1931, a similar color matching technique was used to determine the ratios of primary wavelength mixtures required to produce equivalent chromaticities.

In this technique, three narrow band wavelengths were chosen such that a mixture of unit amounts of these frequencies produced an equal energy stimulus for the range of colors throughout the visual spectrum. That is, when mixed, the selected primaries produced a composite equivalent to that produced by mixing all wavelengths of the spectrum. The primaries selected for this experiment were (R) = 700.0 nm, lambda (G) = 546.1 nm, (B) = 435.8 nm, yielding an R:G:B ratio of 72.1:1.4:1.0. Relative intensities of the individual primaries were adjusted by the observer until the mixture perceptually matched the chromaticity of a single wavelength test stimulus, where the area covered by the test stimulus was 2 degrees, and was isomorphic with the retinal field of view maximally sensitive to spectral variation.

The results of the experiment produced a set of r, g, b tristimulus values specifying the relative proportions of R, G, and B primaries which precisely defined each of the single wavelength test stimuli used in the experiment. Data from this experiment is presented in Figure 1 and has been adopted as defining the 1931 CIE standard observer.
Figure 2. CIE Standard Luminosity Curve with Engineering Units.
Figure 3. CIE Relative Luminosity Function for Day and Night Vision.
More recently, a standard observer color matching function was developed for defining primary mixtures for areas subtending a 10-degree field of view, this including some absorption by rod receptors. This data, presented in Figure 4, has been adopted as the 1964 CIE 10-degree standard observer data and is used for generating colors in areas subtending a visual field greater than 4 degrees.

c. The Tristimulus Color System. Data from the standard observer experiments revealed that any single wavelength color was achievable by a specific mixture of the red, green, and blue primaries.

\[
S(R+G+B) = rR + gG + bB
\]

where \( S \) is the color of the test stimulus, \( R, G, \) and \( B \) are the values of the primaries, and \( r, g, \) and \( b \) are the proportional or tristimulus values of the primaries required to match the test stimulus.

While some colors were unrealizable with primary mixes alone, they could be generated by adding a specific amount of one of the primaries to the test stimulus. This resulted in some negative tristimulus values such that:

\[
S + rR = gG + bB
\]

The tristimulus color system was derived from the 1931 standard observer experiments and can be more formally described in terms of a red \((R)\), green \((G)\), and blue \((B)\) coordinate system defining a three-dimensional color space bounding the range of colors realizable by a mixture of the primaries (Fig. 5). Since the coordinate system is a theoretical construct, the units of the axes may be scaled in any desired increments. The conventional choice for scaling the three axes of the tristimulus coordinate system is units of luminance where the axes \( R, G, \) and \( B \) are given by \((1,0,0)\), \((0,1,0)\), and \((0,0,1)\), respectively, and \( R + G + B = 1 \).

The advantage of the \( R, G, \) and \( B \) coordinate system is that any color may be described as a vector within a unit plane, where the location at which the vector intersects the plane defines chromaticity, and the vectors length defines the color's luminance:

\[
L = 1rR + 1gG + 1bB
\]

where \( L \) is the luminance of the new color and \( 1r, 1g, 1b \) are the luminances at the unit amounts.

Within this system, equal amounts of red, green, and blue (when mixed) yield a neutral, typically specified as a color temperature. (See Appendix B for a description of color temperature.) If the unit plane is an equilateral triangle, the neutral color vector \( N \) will intersect the unit plane of a location one-third the distance

---


Figure 5. Tristimulus Axis System with Unit Plane.
from any side of the unit plane triangle to its opposite side. In this case, the neutral vector $N$ is perpendicular to the unit plane. Colors which pass through the plane near the neutral point will be desaturated, and colors which intersect the unit plane near the primaries will approach the color of the primary.

Relationships among chromaticities in the R, G, B system can be represented by transforming the primaries to chromaticity coordinates $(r, g, b)$ defining the color's relative position within the coordinate system. Chromaticity coordinates of the R, G, B system are determined by:

\[
\begin{align*}
    r &= \frac{R}{R+G+B} \\
    g &= \frac{G}{R+G+B} \\
    b &= \frac{B}{R+G+B}
\end{align*}
\]

(2)

where,

\[
r + g + b = 1
\]

(3)

The derivation of the above expressions and their portrayal in the $x, y$ chromaticity diagram is shown in Appendix D.

Fundamental to the tristimulus color system is the assumption that combining primaries to generate a new color is a linear process such that an additive mixture of two identical colors (i.e., the same hue, saturation, and luminance) with two other identical colors will yield identical mixtures regardless of the tristimulus components of the primaries themselves. Conversely, subtracting two identical colors from identical colors will result in a match regardless of the primaries comprising the subtracted colors. Finally, if a single unit of one color matches a unit of another color, increasing both colors by equivalent units will preserve the color match. These principles, known as Grassman's Laws, have remained both the theoretical and empirical basis of modern color science.

The most significant implication of these principles is that absolute increases in the amount of primaries will not affect chromaticity as long as the relative proportions of the primaries remain constant.

4. TRANSFER OF PRIMARY COORDINATES

Given the assumptions of Grassman's Laws, any color can be generated if the chromaticity coordinates, the luminance of the desired color, and the chromaticity coordinates of the set of primaries to be used in the mixture are known.

The chromaticity of the desired color can be described in terms of the tristimulus functions with the relationships:
\[ x = \frac{X}{X+Y+Z} \]
\[ y = \frac{Y}{X+Y+Z} \]
\[ z = \frac{Z}{X+Y+Z} \]  \hspace{1cm} (4)

where, \(X+Y+Z=1\). Thus,
\[ x = X \]  \hspace{1cm} (5)
\[ y = Y \]  \hspace{1cm} (6)
\[ z = Z \]  \hspace{1cm} (7)

Conventionally, desired brightness (L) is set as equivalent to the \(Y\) tristimulus value. Thus,
\[ \frac{x}{Y} = \frac{y}{Y} \]  \hspace{1cm} (8)
\[ Y = L \]  \hspace{1cm} (9)
\[ X = \frac{(xL)}{y} \]  \hspace{1cm} (10)
\[ Z = (1-x-y) \left(\frac{Y}{y}\right) \]  \hspace{1cm} (11)

where, the resulting \(X, Y,\) and \(Z\) tristimulus values represent the relative amounts of CIE primaries required to generate a color with known CIE chromaticity coordinates.

Chromaticity coordinates for the new primaries can be converted to tristimulus values using the green (\(y\)) parameter of each of the primaries as a unity reference. Therefore,
\[ X_{cier} = \frac{x_r}{y_r} \quad Y_{cier} = \frac{y_r}{y_r} \quad Z_{cier} = \frac{z_r}{y_r} \]  \hspace{1cm} (12)
\[ X_{cieg} = \frac{x_g}{y_g} \quad Y_{cieg} = \frac{y_g}{y_g} \quad Z_{cieg} = \frac{z_g}{y_g} \]  \hspace{1cm} (13)
\[ X_{cieb} = \frac{x_b}{y_b} \quad Y_{cieb} = \frac{y_b}{y_b} \quad Z_{cieg} = \frac{z_b}{y_b} \]  \hspace{1cm} (14)

where the resulting set of constants are the tristimulus values of the new set of primaries.
Finally, the two sets of primaries are related to each other in the following homogeneous linear relations\(^7\):

\[
\begin{align*}
R' &= a_{11} R + a_{21} G + a_{31} B \\
G' &= a_{12} R + a_{22} G + a_{32} B \\
B' &= a_{13} R + a_{23} G + a_{33} B
\end{align*}
\]

For a given color, one set of primary values is \( R, G, B \) and the second set is \( B', G', B' \), and \( a_{11}, a_{12}, a_{13} \) are the magnitudes of the second set of primaries required to match the red component \( (R) \) of the original color. The second column coefficients \( a_{21}, a_{22}, a_{23} \) are the amounts of the second set of primaries required to match the green component of the original color. The third column \( a_{31}, a_{32}, a_{33} \) is the amount of the second set of primaries required to match the blue of the original color.

A numerical example of the transformation of primaries is given in Appendix E.

5. COLOR GENERATION ON CRT'S

Color is generated on a self-emissive source such as a cathode ray tube (CRT), by the additive mixture of radiant energy from three primaries, defined by the chromaticity coordinates of the three phosphors specific to the given monitor. While the phosphors are typically perceived as red, green, and blue for all CRT's, the precise chromaticity coordinates of phosphors vary from monitor to monitor. As in other applications of additive color mixing, the chromaticity coordinates of the primaries form the boundaries of the gamut of colors that can be generated by any combination of those primaries.

When the chromaticity coordinates of the desired color are known, and are within the gamut of colors bounded by the coordinates defined by a monitor's red, green, and blue phosphors, the color can be generated by an additive mixture of luminances from the three guns. First, the chromaticity coordinates of the monitor phosphors must be transformed to yield their relationship to the desired color. This transformation can be accomplished by computing the tristimulus values of CRT phosphors with calculations based on Grassman's Laws. Once the ratios of the red, green, and blue luminances have been determined, the bit values corresponding to those ratios must be computed. Following is the mathematical solution to the generation of a specific chromaticity using CRT primaries.

a. Calculation of Tristimulus and Luminance Values for CRT's. The tristimulus values of CRT phosphors are determined for each gun by the integration:

\[
\begin{align*}
X &= \int \lambda E(\lambda) \lambda^2 d\lambda \\
Y &= \int \lambda E(\lambda) \lambda^2 d\lambda \\
Z &= \int \lambda E(\lambda) \lambda^2 d\lambda
\end{align*}
\]
Where \( x(p) \), \( y(p) \), \( z(p) \) are the color matching functions of the standard observer and \( E(p) \) is the emission function of the gun in the units of energy per unit frequency. From these tristimulus values, the chromaticity coordinates of the phosphors can be found by the usual calculation.

\[
x = \frac{X}{X+Y+Z}
\]

\[
y = \frac{Y}{X+Y+Z}
\]

\[
z = \frac{Z}{X+Y+Z}
\] (4)

(It is a noteworthy property of all the above equations that, due to the linearity and form of the equations, dimensions are readily factored out.)

Unknown are the tristimulus values \((R, G, B)\) or drive signals required for the monitor to generate the specific color. The CRT gun parameters give the amount of CIE primary per unit of gun primary. Therefore, equation set (15), (16), (17) can be re-expressed:

\[
R' = Xcier \times Rcrt + Xcieg \times Gcrt + Xcieb \times Bcrt
\] (21)

\[
G' = Ycier \times Rcrt + Ycieg \times Gcrt + Ycieb \times Bcrt
\] (22)

\[
B' = Zcier \times Rcrt + Zcieg \times Gcrt + Zcieb \times Bcrt
\] (23)

Where \( Xcier \) is the gun parameter which gives the CIE tristimulus value of the red gun per unit of red gun output (etc.).

\( Rcrt \), \( Gcrt \), \( Bcrt \) are the amounts of red gun, green gun, and blue gun required to produce the desired color \( R' \), \( G' \), and \( B' \). \( Rcrt \), \( Gcrt \), \( Bcrt \) are the unknowns. It should be noted that \( Xcier \), \( Ycier \), and \( Zcier \) form a column in this matrix. These constants are normally shown in a row format when provided by the CRT manufacturer, and in the pure matrix sense are transposed in this set of equations.

The values of \( Rcrt \), \( Gcrt \), and \( Bcrt \) are most readily found by the determinant method for the solution of simultaneous linear equations. In this method, derived from the inversion of matrices, the simultaneous equations are put in the standard polynomial form:

\[
a_1 X + b_1 Y + c_1 Z = d_1 \quad (24)
\]

\[
a_2 X + b_2 Y + c_2 Z = d_2 \quad (25)
\]

\[
a_3 X + b_3 Y + c_3 Z = d_3 \quad (26)
\]

In order to solve for the variable \( X \), the column \( d_1 \), \( d_2 \), \( d_3 \) is substituted for the desired variable column of constants in the numerator of the following expression:
For completeness, the solution of the determinants for $X$ is given in general form.

$$X = \begin{vmatrix} d_1 & b_1 & c_1 \\ d_2 & b_2 & c_2 \\ d_3 & b_3 & c_3 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} \quad (27)$$

$$X = \frac{d_1b_2c_3 + b_1c_2d_3 + c_1d_2b_3 - c_1b_2d_3 - c_2b_3d_1 - c_3d_2b_1}{a_1b_2c_3 + b_1c_2a_3 + c_1a_2b_3 - a_3b_2c_1 - c_2b_3a_1 - c_3a_2b_1} \quad (28)$$

$X$ and $Z$ are found using the same technique for the $d_1$, $d_2$, $d_3$ column substituted for $b_1$, $b_2$, $b_3$ or $c_1$, $c_2$, $c_3$ in the numerator of (29).

Applying this technique to the expression for $R'$, $G'$, and $B'$ and solving for $R_{crt}$, $G_{crt}$, and $B_{crt}$ the lengthy algebraic groupings of Figure 6 are obtained. The symbol delta ($\Delta$) is used to represent the determinant common to all three solutions.

The values of $R_{crt}$, $G_{crt}$, and $B_{crt}$ are in foot-lamberts, if that is the brightness unit selected for the desired color. This appears to be a mathematical slight-of-hand without the following substantiating discussion.

It has been well established that tristimulus values can be scaled up or down as long as the relative magnitude ratio is preserved. As mentioned above, the desired color is scaled in order to set the $Y$ tristimulus value to the desired luminance of the final color. The chromaticity coordinates of each gun are scaled so that the $Y$ value is unity. For the single gun case, the equations are:

$$R' = X_{cier} \cdot R_{crt} \quad (29)$$

$$G' = Y_{cier} \cdot R_{crt} \quad (30)$$

$$B' = Z_{cier} \cdot R_{crt} \quad (31)$$

If $G' = L$ and $Y_{cier} = 1$, $R_{crt}$ (the setting of the red gun is found equal to the desired brightness in the proper units). Since all the $Y_{cie}$ values are unity, by Grassman's Laws of linearity, the $L$ value is also found by summation of the three gun components in the middle row.

$$L = G' = R_{crt} + G_{crt} + B_{crt} \quad (32)$$

Of course, the solution for $R_{crt}$, etc., must be done by either the matrix inversion method shown in Appendix C or the determinant method shown above for the three gun case. A numerical example of the above manipulations is given in Appendix E.

6. EFFECTS OF CRT NON-LINEARITIES ON COLOR MIXING

The theoretical foundation for transforming primaries outlined above is well documented and empirically robust for all primary mixtures. However, CRT's as devices for generating color evidence characteristics which violate the assumptions upon which additive color mixing is based. Therefore, while the theoretical formulation remains unchallenged, precise prediction of specific chromaticities using the formulation may fail
\[
\begin{align*}
R_{cr} &= (Y_{cieb} Z_{cieb} - Y_{cieb} Z_{cieb}) R' + (X_{cieb} Z_{cieb} - X_{cieb} Z_{cieb}) G' + (X_{cieb} Y_{cieb} - X_{cieb} Y_{cieb}) B' \\
\Lambda \\
G_{cr} &= (Y_{cieb} Z_{cieb} - Y_{cieb} Z_{cieb}) R' + (X_{cieb} Z_{cieb} - X_{cieb} Z_{cieb}) G' + (X_{cieb} Y_{cieb} - X_{cieb} Y_{cieb}) B' \\
\Lambda \\
B_{cr} &= (X_{cieb} Z_{cieb} - X_{cieb} Z_{cieb}) R' + (X_{cieb} Z_{cieb} - X_{cieb} Z_{cieb}) G' + (X_{cieb} Y_{cieb} - X_{cieb} Y_{cieb}) B' \\
\Lambda \\
\Lambda &= X_{cieb} (Y_{cieb} Z_{cieb} - Y_{cieb} Z_{cieb}) + X_{cieb} (Y_{cieb} Z_{cieb} - Y_{cieb} Z_{cieb}) \\
&+ X_{cieb} (Y_{cieb} Z_{cieb} - Y_{cieb} Z_{cieb})
\end{align*}
\]

Figure 6. Expressions for the Determination of CRT Gun Brightness in Foot-Lamberts.
when applied to color mixtures generated on CRT's. Shadow mask heating, power drive, and the Gaussian mix of adjacent pixel elements, particularly at high brightness settings, differentially affect the luminance output of each gun. This non-linearity implies a change in the ratio of the red, green, and blue components of the color mixture and suggests the potential for a chromaticity shift in addition to luminance shifts when a monitor's brightness setting is changed.

In order to quantify the effects of CRT non-linearities on generating chromaticities, an experimental test was conducted in which CIE measurements were collected for two different monitors at linear and non-linear brightness settings. For the purpose of this experiment, linear was defined as the maximum brightness setting at which the combined luminances (in foot lamberts) of the three guns at maximum output (255 bits each) was equal to the summed luminances of the individual guns at maximum output. Conversely, non-linearity was defined as the brightness setting where the combined luminance at the maximum output of the three guns was less than the summed luminances of the individual guns. It was predicted that under non-linear conditions, a shift away from reference chromaticities would be found.

a. Experimental Method (Apparatus). Two CRT monitors were selected for evaluation. One monitor was a CONRAC 7211 model No. 499110 and the other was a Collins Rockwell with neutral density filter. Monitor parameters are shown in Table 1.

| TABLE 1. PARAMETERS OF THE CONRAC AND THE COLLINS-ROCKWELL MONITORS SAMPLED FOR TEST |
|---------------------------------|-----------------|----------------|
|                                  | CONRAC          | COLLINS        |
| red phosphor x, y               | .635, .325      | .653, .323     |
| green phosphor x, y             | .220, .680      | .300, .590     |
| blue phosphor x, y              | .152, .063      | .151, .060     |
| max linear lum (white)          | 123 Fl          | 26 Fl          |
| max linear lum (r+g+b)          | 125.1 Fl        | 286.7 Fl       |
| non-linear lum (white)          | 182 Fl          | 36.8 Fl        |
| non-linear lum (r+g+b)          | 191.8 Fl        | 51.6 Fl        |

To control display colors, a program was written which generated a 125 by 125 pixel square (foreground) with a surrounding area (background). Foreground and background could independently receive from 0-255 bits of red, green, and blue, in 16 bit increments. Thus, any foreground/background color combination could be generated.

A Minolta color analyzer tested for accuracy and reliability against chromaticities measured with a Gamma Scientific spectroradiometer, was used as the CIE measurement device. This device represented the portability and compactness required for flight test environments.
Reference CIE's were the chromaticities of the ANACAPA recommended color set as measured within a 125 by 125 pixel square on a black background. Since the background colors included 16 shades of aqua and 16 shades of gray, a sample representing each of the three points (low, medium, and high brightness) on the luminance curve was selected for each of the two colors. Chromaticities and luminances of the color set used in the experiment are represented by the asterisked colors in Table 2.

**TABLE 2. AVRADA COLORS FOR DIGITAL MAP PROGRAM**

<table>
<thead>
<tr>
<th>COLOR</th>
<th>CIEX</th>
<th>CIEY</th>
<th>BRIGHTNESS (REF. 33.7 FL)</th>
<th>POSITION</th>
</tr>
</thead>
<tbody>
<tr>
<td><em>RED</em></td>
<td>.588</td>
<td>.322</td>
<td>7.2</td>
<td>Foreground</td>
</tr>
<tr>
<td><em>MAGENTA</em></td>
<td>.257</td>
<td>.143</td>
<td>3.6</td>
<td>Background</td>
</tr>
<tr>
<td><em>BLUE</em></td>
<td>.171</td>
<td>.100</td>
<td>5.5</td>
<td>Background</td>
</tr>
<tr>
<td><em>YELLOW</em></td>
<td>.422</td>
<td>.425</td>
<td>20.2</td>
<td>Foreground</td>
</tr>
<tr>
<td><em>WHITE</em></td>
<td>.291</td>
<td>.306</td>
<td>33.7</td>
<td>Foreground</td>
</tr>
<tr>
<td><em>GREEN</em></td>
<td>.284</td>
<td>.524</td>
<td>28.6</td>
<td>Foreground</td>
</tr>
<tr>
<td><em>PINK</em></td>
<td>.329</td>
<td>.259</td>
<td>25.0</td>
<td>Foreground</td>
</tr>
<tr>
<td><em>CYAN</em></td>
<td>.187</td>
<td>.220</td>
<td>20.0</td>
<td>Foreground</td>
</tr>
<tr>
<td><em>GREY0</em></td>
<td>.289</td>
<td>.302</td>
<td>0.9</td>
<td>Background</td>
</tr>
<tr>
<td>GREY1</td>
<td>.289</td>
<td>.302</td>
<td>1.0</td>
<td>Background</td>
</tr>
<tr>
<td>GREY 2</td>
<td>.289</td>
<td>.302</td>
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<td>.302</td>
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</tr>
<tr>
<td>GREY 4</td>
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<td>Background</td>
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<td>GREY 6</td>
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<tr>
<td>GREY9</td>
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</tr>
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<td>10.5</td>
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<td>.302</td>
<td>20.0</td>
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<td><em>AQUAO</em></td>
<td>.30</td>
<td>.307</td>
<td>0.9</td>
<td>Background</td>
</tr>
<tr>
<td>AQUA1</td>
<td>.230</td>
<td>.307</td>
<td>1.0</td>
<td>Background</td>
</tr>
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<td>AQUA2</td>
<td>.230</td>
<td>.307</td>
<td>1.2</td>
<td>Background</td>
</tr>
<tr>
<td>AQUA3</td>
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<td>1.5</td>
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<td>AQUA4</td>
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<td>AQUA5</td>
<td>.230</td>
<td>.307</td>
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<td>AQUA6</td>
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<td>.307</td>
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<tr>
<td>AQUA9</td>
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</tr>
<tr>
<td>AQUA12</td>
<td>.230</td>
<td>.307</td>
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<tr>
<td>AQUA13</td>
<td>.230</td>
<td>.307</td>
<td>13.0</td>
<td>Background</td>
</tr>
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<td>.230</td>
<td>.307</td>
<td>16.2</td>
<td>Background</td>
</tr>
<tr>
<td><em>AQUA15</em></td>
<td>.230</td>
<td>.307</td>
<td>20.0</td>
<td>Background</td>
</tr>
</tbody>
</table>
During the initial generation of colors, a tolerable random deviation due to measurement error was determined. Selected deviation tolerance was set at +0.01 of the reference CIE. This value, therefore, represented the expected deviation from reference CIE if no shifts occurred as a function of the experimental manipulation.

b. Experimental Design and Procedure. Since reference colors were previously specified as codes for foreground and background map features, and, since our primary interest was in describing the behavior of the recommended chromaticities in the context of the digital map application, colors designated for coding foreground features were measured only as foreground colors and those designated for background features were measured only in the background or surrounding area.

For each of the two monitors and two brightness settings, six foreground and eight background colors were generated in all combinations. This yielded a 2 by 2 by 6 by 8 repeated measures design with foreground and background CIE measurements for each monitor at each setting. Since the measured $x$ and $y$ could independently deviate from the reference $x$ and $y$ for each color tested, an absolute deviation score was calculated by:

$$\sqrt{\text{Sum} \ [\text{dev} \ x^2 + \text{dev} \ y^2]}$$

where $\text{dev} \ x = \text{abs}[\text{reference} \ x - \text{measured} \ x]$ and $\text{dev} \ y = \text{abs}[\text{reference} \ y - \text{measured} \ y]$.

Separate analyses of variance were performed for shifts of foreground and background chromaticities. Analyses of interest were setting, foreground color, and background color main effects, and setting by foreground, setting by background, and foreground by background interactions.

7. RESULTS

Mean deviations for the two monitors at each of the two settings are presented in Tables 3 and 4, for foreground and background colors, respectively. While the Collins monitor tended to have an overall greater sensitivity to chromaticity shifts than the Conrac, the direction of the deviations as a function of setting were not markedly different. Thus, the chromaticity data from the two monitors was combined and analyzed for effects of setting linearity on chromaticity deviations.

<table>
<thead>
<tr>
<th>TABLE 3. MEAN DEVIATIONS OF FOREGROUND CHROMATICITIES AS A FUNCTION OF MONITOR NON-LINEARITY</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>LINEAR</strong></td>
</tr>
<tr>
<td>CONRAC</td>
</tr>
<tr>
<td>COLLINS</td>
</tr>
<tr>
<td>Total Deviation</td>
</tr>
</tbody>
</table>
TABLE 4. MEAN DEVIATIONS OF BACKGROUND CHROMATICITIES AS A FUNCTION OF MONITOR NON-LINEARITY

<table>
<thead>
<tr>
<th></th>
<th>LINEAR</th>
<th>NON-LINEAR</th>
<th>TOTAL</th>
</tr>
</thead>
<tbody>
<tr>
<td>CONRAC</td>
<td>.010</td>
<td>.029</td>
<td>.039</td>
</tr>
<tr>
<td>COLLINS</td>
<td>.012</td>
<td>.030</td>
<td>.042</td>
</tr>
<tr>
<td>Total</td>
<td></td>
<td></td>
<td>.022</td>
</tr>
<tr>
<td>Deviation</td>
<td></td>
<td></td>
<td>.059</td>
</tr>
</tbody>
</table>

a. Foreground Chromaticity Shifts. Mean deviations from reference color as a function of setting and foreground color are presented in Figure 7. Significant main effects were found for setting, F(1,2)=27.15 p < .03, and foreground color, F(5,6)=4.24 p < .05. Scheffe's least squares difference test yielded significance at the .05 level among three groups, RED, GREEN, and YELLOW; and, CYAN, WHITE, and PINK. Means were .054, .034, and .010 for each group, respectively.

Significant interactions were also found for setting by foreground color, F(5,115)=81.58 p < .001 (Fig. 8), and for foreground color by background color, F(35,115)=1.52, p < .05 (Fig. 9). A post-hoc paired means comparison showed that deviations from foreground reference chromaticity were significant for RED, GREEN, and YELLOW, while shifts due to the foreground by background interaction were primarily due to the BLUE background.

b. Background Chromaticity Shifts. Chromaticity data pertaining to background shifts are presented in Figure 10. An analysis of variance revealed significant main effects for setting F(1,2)=197.03, p < .005, and for background color F(7,8)=4.48, p < .03. The post hoc comparison revealed the following Scheffe groupings: BLUE, AQUA0, and MAGENTA: GRAY0, GRAY8, AQUA8, and AQUA15; MAGENTA and GRAY0; and AQUA8, GRAY8, AQUA15, and GRAY15. Also significant was the setting by background color interaction, F(7.115)=57.01, p < .001.

8. DISCUSSION

Chromatic shifts are of practical interest given their potential impact on the identification of the features they represent. They are also of empirical interest since they provide a means to evaluate the performance of specific devices for generating and presenting color coded information.

Data from the present investigation confirmed our prediction that nonlinear brightness settings produce significant chromaticity shifts, and that the severity of the shift is primarily determined by the specific color being generated. For our color set, the most severe shifts were associated with RED, GREEN, and YELLOW foreground colors and with BLUE, AQUA0, and MAGENTA background colors. Results also revealed some evidence of a foreground color by background color interaction for specific colors. This indicates a potential problem in maintaining the integrity of certain colors on multi-color displays.

While the data cannot establish definitively which underlying aspects of those colors result in chromaticity shifts on CRT's, certain patterns seem to emerge. One is that the most degraded of the foreground colors are also the most saturated. Shifts found in these colors all tend toward the center of the CIE color space, or...
Figure 7. Graph of Foreground Shifts as a Function of Foreground Color and Setting.
*Foreground Deviations*

![Graph of Foreground Shifts as a Function of Foreground by Background Interaction.](image)

**Figure 8.** Graph of Foreground Shifts as a Function of Foreground by Background Interaction.
Figure 9. Background Chromaticity Shifts as a Function of Foreground by Background Interaction.
neutral. A second is that the most degraded of the background colors are all low luminance colors. Saturation and luminance, thus, are good candidate parameters for a systematic study of the relationship between color characteristics and chromaticity shifts.

Another implication of the present study is that in choosing a brightness setting for a monitor, one must take into consideration the linearity characteristics of the monitor, both when initially generating a color, and when adjusting monitor brightness for compatibility with ambient illumination. That is, chromaticity shifts may occur as a direct result of increasing the brightness of the monitor to achieve a luminance level compatible with viewing the display in high ambient illumination.

One way to control shifts in chromaticity that result from adjusting brightness setting is to avoid a violation of the linearity criteria as defined in the experiment. Thus, adjustments to brightness levels where the sum of the individual luminances of the RED, GREEN, and BLUE guns exceed the luminance of the WHITE generated by a simultaneous activation of the three guns should be prohibited. This solution, however, may be problematic for certain monitors in which the linearity criteria results in an overall luminance level below that required for viewing under high ambient conditions.

9. PROCEDURE FOR GENERATING SPECIFIC CHROMATICITIES ON A COLOR CRT

Given the interactions between monitor characteristics and color, a direct application of color mixing principles to CRT color generation is not feasible. To address this problem, an iterative procedure was developed based on a modification of the procedure reported by Spiker and Rogers. Step-by-step instructions detailing the procedure are presented below.

a. Prepare Monitor-Display Generator Configuration. Figure 11 illustrates a typical configuration. The display system must be properly coupled to the monitor being calibrated. Present day equipment uses coaxial cable usually with 75-ohm termination. Coaxial lead length should be recorded for future reference and the monitor should be set to coaxial cable input impedance. In addition, the monitor should be degaussed initially and periodically as testing proceeds.

b. Prepare Program to Generate Colors. A program should be developed to generate a 125 to 175 pixel square in the center of the monitor screen. The program should also permit the full range of the display generator for each of the three guns. For conventional equipment, this is typically 255 bits of RED, GREEN, and BLUE. The area surrounding the square should be full off or black.

c. Determine Linear Luminance Range of Monitor. It is essential that the luminance of a color be equal to the sum of the luminances of the individual primaries that make up the color. In all monitors tested at AVRADA, the luminance of the neutral comprised by the three primaries has generally been less than the sum of the individual RGB components at high levels of monitor brightness. A typical degradation of linearity is shown in Table 5 for the CONRAC 5711. Such a table can be generated for any monitor, and can be used to determine the highest brightness and contrast monitor setting not resulting in a luminance "droop." To accomplish this, the following steps should be followed:

Spiker and Rogers, "Procedure and Guidelines for Producing Anacapa's Topographic Map Colors on Other Displays," Anacapa Sciences, Inc., TR 566-2, July 1984, Santa Barbara, CA.
Background Deviations

Figure 10. Background Chromaticity Shifts as a Function of Background Color and Setting.
Figure 11. Typical Monitor-Display Generator Configuration.
TABLE 5. CRT MONITOR LINEARITY TEST-CONRAC 7211 A/N 470693

<table>
<thead>
<tr>
<th>Color</th>
<th>Brightness (FL)</th>
<th>Brightness (FL)</th>
<th>Brightness (FL)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>AT 255 BITS</td>
<td>AT 128 BITS</td>
<td>AT 84 BITS</td>
</tr>
<tr>
<td>RED</td>
<td>24.0</td>
<td>2.5</td>
<td>.05</td>
</tr>
<tr>
<td>GREEN</td>
<td>71.0</td>
<td>12.5</td>
<td>1.3</td>
</tr>
<tr>
<td>BLUE</td>
<td>12.1</td>
<td>2.6</td>
<td>.42</td>
</tr>
</tbody>
</table>

TOTAL OF LINEAR: 109.0 18.1 1.89
TOTAL MEASURED: 107.1 17.6 1.77

<table>
<thead>
<tr>
<th>Color</th>
<th>Linear Measured</th>
<th>Linear Measured</th>
<th>Linear Measured</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>AT 255 BITS</td>
<td>AT 128 BITS</td>
<td>AT 64 BITS</td>
</tr>
<tr>
<td>RED+GREEN</td>
<td>95.0</td>
<td>96.0</td>
<td>15.0 15.4 1.36 1.4</td>
</tr>
<tr>
<td>GREEN+BLUE</td>
<td>83.1</td>
<td>84.0</td>
<td>15.1 15.3 1.72 1.83</td>
</tr>
<tr>
<td>BLUE+RED</td>
<td>36.1</td>
<td>37.0</td>
<td>5.1 5.2 .47 .48</td>
</tr>
</tbody>
</table>

(1) Generate a color bar test pattern on the monitor at a low brightness level.
(2) Increase the contrast to a setting just below where the bars begin to bloom.
(3) Perform the linearity test shown in Table 5.
(4) If the sum of the RGB luminances is 5 percent greater than the luminance of white produced when all guns are on, go back to (2) above.
(5) Proceed in this manner until at least a 5 percent linearity is achieved.
(6) Record this setting and conduct all further measurements at this level of brightness and contrast.

In general, maximum monitor brightness can be determined by manufacturer specifications. It is possible, however, that a particular display generator will not be capable of driving the monitor to its maximum brightness. If this is the case, line drivers can be inserted in the lines from the generator to the monitor and set to increase all power by an equal amount.

d. Selection of Colors to be Generated and Calculation of Brightness. A research effort by Anacapa Sciences, Inc., directed toward AVRADA's digital map program, in conjunction with tests conducted at AVRADA's Tactical Avionics System Simulator (TASS), produced sets of forty colors including shades of GRAY and AQUA, defined in terms of their x, y chromaticity coordinates and luminances. The color set chromaticities and luminances based on a maximum linear display capability of 33.7 for WHITE are shown in Table 6.
<table>
<thead>
<tr>
<th>COLOR</th>
<th>CIEX</th>
<th>CIEY</th>
<th>BRIGHTNESS (REF. 33.7 FL)</th>
</tr>
</thead>
<tbody>
<tr>
<td>RED</td>
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<td>1.5</td>
</tr>
<tr>
<td>AQUA4</td>
<td>.230</td>
<td>.307</td>
<td>1.9</td>
</tr>
<tr>
<td>AQUA5</td>
<td>.230</td>
<td>.307</td>
<td>2.4</td>
</tr>
<tr>
<td>AQUA6</td>
<td>.230</td>
<td>.307</td>
<td>2.9</td>
</tr>
<tr>
<td>AQUA7</td>
<td>.230</td>
<td>.307</td>
<td>3.6</td>
</tr>
<tr>
<td>AQUA8</td>
<td>.230</td>
<td>.307</td>
<td>4.5</td>
</tr>
<tr>
<td>AQUA9</td>
<td>.230</td>
<td>.307</td>
<td>5.5</td>
</tr>
<tr>
<td>AQUA10</td>
<td>.230</td>
<td>.307</td>
<td>6.9</td>
</tr>
<tr>
<td>AQUA11</td>
<td>.230</td>
<td>.307</td>
<td>8.5</td>
</tr>
<tr>
<td>AQUA12</td>
<td>.230</td>
<td>.307</td>
<td>10.5</td>
</tr>
<tr>
<td>AQUA13</td>
<td>.230</td>
<td>.307</td>
<td>13.0</td>
</tr>
<tr>
<td>AQUA14</td>
<td>.230</td>
<td>.307</td>
<td>16.2</td>
</tr>
<tr>
<td>AQUA15</td>
<td>.230</td>
<td>.307</td>
<td>20.0</td>
</tr>
</tbody>
</table>
To compute the luminance values for each color on a new monitor, the maximum luminance of the new monitor is divided by the maximum luminance of the reference monitor (in this case 33.7).

\[ \text{LUMFACTOR} = \frac{\text{MAX LUMINANCE NEW MONITOR (FL)}}{\text{MAX LUMINANCE REFERENCE MONITOR (33.7 FL)}} \]

The quotient is a luminance factor which, when multiplied with the luminance value of each reference color, yields the value of the maximum luminance that the new monitor can generate for that color.

As noted above, monitor linearity has a marked effect on the accuracy of the chromaticities generated. In addition, differences between monitors in the ratios of intensities among the three guns can significantly degrade the accuracy of the pure computation of the luminance factor. Empirical tests revealed that if the factor is reduced by about 16 percent, a more precise scale factor will be yielded. The new scale factor is labeled LUMFACTOR and is computed:

\[ \text{LUMFACTOR} = \left(0.84\right) \left(\text{MAXIMUM LINEAR BRIGHTNESS OF NEW MONITOR}\right) \left(\text{MAXIMUM REFERENCE LUMINANCE}\right) \]

e. Bits to Luminance Curve. The drive signal of the RED, GREEN, and BLUE guns required to generate each desired color can now be determined. To accomplish this, photometric measures are taken every 15 bits from 0 to 255 for each gun. At AVRADA, a Prichard 1980A photometer was used for making luminance measurements. Since luminance values must be determined as accurately as possible, the manufacturer's procedure for zeroing, calibration, selection of lens, and warm-up procedures should be strictly followed. Sample data from one of AVRADA's monitors is shown in Figure 12.

f. Determining RGB Bits for Each Color. When the CIE coordinates and luminance of a desired color are known, the ratio of RED, GREEN, and BLUE guns required to generate the color can be calculated using linear algebra techniques. The mathematics for calculating the relative brightness for a given monitor of the RED, GREEN, and BLUE guns is shown in Appendix E. Once the required luminances of each gun are known, a table lookup based on the photometric measures can be done to determine the bits needed to produce the required brightness. An illustration of such a bits to luminance lookup, with interpolation to the nearest bit, is shown in Appendix E.

The above steps were implemented in a Fortran program LUM4 included in this report (Appendix C). Due to the non-linearity of all monitors tested, an additional feature was added, which permits the interactive adjustment of the RGB values for achieving the desired CIE coordinates within a selected tolerance (i.e., .01). The interactive procedure is depicted in Figure 13. The desired CIE x, y, and Luminance is input to LUM4. The program then calculates RED, GREEN, and BLUE foot lamberts and bit values. The RGB bit values are used to generate the color on the monitor. The actual CIE coordinates achieved are measured using a color measurement device, such as a spectroradiometer, or a Minolta color TV analyzer. The program accepts, from the keyboard, the actual readings and modifies the initial command. When the screen color is within acceptable tolerance, the procedure is complete.
Figure 12. Typical Bits to Luminance Curve.
Figure 13. Iterative Procedure for Determining Color Look-Up Table Values.
The authors would like to express their thanks to Ms. Carole Kortenhaus. Carole spent countless hours developing the skills necessary to make the difficult photometric measurements required in this project. She not only spent much time in a darkened room taking data but also analyzed the data with statistical techniques. Carole is an excellent research technician and a valuable part of the AVRADA technical base program.
APPENDIX A

CONVERSION FACTORS FOR LIGHT MEASURES

A CANDLE IS 1/60 of the intensity of one square centimeter of a black body radiator at the temperature of the solidification of platinum 2046 degrees K.

A FOOT-CANDLE is equal to 1 lumen incident per square foot.

A LAMBERT is the unit of brightness equal to 1/pi candles per square meter.

A FOOT-LAMBERT is the unit of brightness equal to 1/pi candles per square foot.

The ILLUMINANCE produced by a point source (I) on a surface area A is:

\[ E = \frac{I \cos \theta}{r^2} \]

where \( r \) is the distance to the surface being illuminated, \( I \) is the point source intensity in lumens and \( \theta \) is the angle between the normal to the surface and the point source. It is from this expression that the inverse relationship between illuminance and the square of distance to a source is defined. This law is the basis of the quantitative comparison of light brightness as performed in the Lummer Brodhun photometer. Prisms are used in a variable spacing scheme to present two sources of light to an observer. The prism-spacing mechanism is adjusted until the lights appear equally bright to the observer. This technique is quite accurate for sources of similar hue. Different hue sources are compared in similar manner; however, the flicker photometer must be employed as discussed above.

An ideal point source is rarely attainable since most sources are directional. However, theoretically, the total output of a point source whose intensity in all directions is 1 candles would be 4 pi lumens.
In physics, the concept of an ideal radiator is needed to properly treat certain forms of energy transfer. Since a good absorber is also a good emitter, the best emitter will be that surface that is the best absorber and poorest reflector. Black surfaces have been found to be the best in the above respects provided the surface is not at a temperature where it is self luminous. There are various laboratory implementations of devices that are "black body radiators" (Sears and Zemansky, 1957).

The chromaticity coordinates of various temperature black body or Planckian radiators can be plotted as shown in Figure B-1. The spectral radiation distribution is computed to temperatures beyond physical reliability in a furnace using Planck's radiation formula. There is a general agreement as to the points on the Planckian curve which resemble common-white or near-white sources. They are:

<table>
<thead>
<tr>
<th>LIGHT SOURCE</th>
<th>NAME</th>
<th>COLOR TEMPERATURE</th>
<th>1931 CIE CHROMATICITY COORDINATES</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tungsten Gas A</td>
<td>A</td>
<td>2856 Deg. K</td>
<td>X .4476  Y .4074</td>
</tr>
<tr>
<td>Noon Sunlight B</td>
<td>B</td>
<td>4874 Deg. K</td>
<td>X .3484  Y .3516</td>
</tr>
<tr>
<td>Standard Source C</td>
<td>C</td>
<td>6774 Deg. K</td>
<td>X .3101  Y .3162</td>
</tr>
<tr>
<td>Average Daylight</td>
<td>D55</td>
<td>5503 Deg. K</td>
<td>X .3324  Y .3475</td>
</tr>
<tr>
<td>Daylight D65</td>
<td>D65</td>
<td>6504 Deg. K</td>
<td>X .3127  Y .3290</td>
</tr>
<tr>
<td>Fluorescent Lamp</td>
<td>D75</td>
<td>7504 Deg. K</td>
<td>X .2990  Y .3150</td>
</tr>
<tr>
<td>Light Overcast Sky</td>
<td>D75</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

(Judd, 1963)

If a light source is not on the Planckian curve in terms of its chromaticity, the closest point on the curve to this source is called the CORRELATED COLOR TEMPERATURE. Lines perpendicular to the Planckian curve at specific color temperatures are constructed. These lines, known as isotemperature lines when extended, can intercept the chromaticity coordinates of any source, thus facilitating the determination of correlated color temperature.

The specification of color temperature points on the chromaticity diagram leads to the somewhat obsequious definition of color in terms of temperature. For example, a certain TV network might use 6500 degree K white to set a standard for monitors' lines and cameras.
Figure B-1. Chromaticity Coordinates of Various Temperature Planckian Radiators
APPENDIX C

DIGITAL PROGRAM TO DETERMINE BIT VALUES OF COLOR GRAPHIC SYSTEM

LUM 4 is a VAX 11/780 FORTRAN program which performs the computation of bit values required to generate a specific color on a specific monitor. It requires the phosphor x, y, z chromaticity coordinates, luminance factor, and bits-to-luminance data for the guns of the monitor of interest. Using linear algebra techniques, the brightness required for each gun to generate a specific color is determined. Using the piecewise linear bits-to-luminance function of the monitor, the bits required to generate the color are calculated to the nearest bit. An iterative feedback feature is included to modify the bit values when the color generated is not within acceptable tolerance as measured by an external instrument. The program, with appropriate comments, follows.

```fortran
COFP(3,3),COFPT(3,3),P(3,3),L,
1 LUM(3),CIESET(3,38),C7211(3,0:16),C5711(3,0:16),COLLINS(3,0:16),
2 PHOSRED(3,4),PHOSGREEN(3,4),PHOSBLUE(3,4),LUMFACTOR(4),LR,LS,
3 NEW MON(3,0:16)
   INTEGER*2 BUFFER(640)/640*O/,BUFFER2(640)/245*0,150*1,245*0/
   INTEGER*2 IBITS(3,38)
   CHARACTER*30 FILE
   CHARACTER*1 ANS

C IN THIS SECTION THE PHOSPHOR CHROMATICITY COORDINATES FOR THREE
C MONITORS OF INTEREST ARE INPUT, ALSO THE THREE LUMINANCE FACTORS
C ARE INPUT

PHOSRED(XYZ,7211 5711 COLLINS)
PHOSGREEN(XYZ,7211 5711 COLLINS)
PHOSBLUE(XYZ,7211 5711 COLLINS)

DATA PHOSRED/.635,.325,.04,.63,.34,.03,.653,.323,.024,.3*0./
DATA PHOSGREEN/.22,.68,.1,.31,.595,.095,.3,.59,.11,.3*0./
DATA PHOSBLUE/.152,.063,.785,.155,.07,.775,.151,.06,.789,.3*0./
DATA LUMFACTOR/2.7,2...7,0./

C THIS SECTION INPUTS THE CHROMATICITY COORDINATES OF THE DESIRED
C COLOR SET

DATA CIESET/.588,.32,7.2,.257,.143,3.6,.171,.15,.5,.422,.425.
1 20.2,.291,.306,.33,7.284,.524,.236,.329,.259,.5,.7,.289,.302,1.1,
2 .289,.302,1.2,.289,.302,1.5,.289,.302,1.9,.289,.302,2.4,.289,3,
3 .302,2.9,.289,.302,3.6,.289,.302,4.5,.289,.302,5.5,.289,.302,6,
5 6.9,.289,.302,8.5,.289,.302,10.5,.289,.302,13,.289,.302,16.2,
6 .289,.302,20,.23,.307,1,.23,.307,1.2,.23,.307,1.5,.23,.307,2,
7 1.9,.23,.307,2.4,.23,.307,2.9,.23,.307,3.6,.23,.307,4.5,.23,
8 .307,5.5,.23,.307,6.9,.23,.307,8.5,.23,.307,10.5,.23,.307,13,
9 .23,.307,16.2,.23,.307,20,.3*0./
```

37
THIS SECTION IS THE BITS TO LUMINANCE DATA FOR CONRAC 7211 MONITOR

DATA C7211/O..0..0..
1 0.04, 0.02, 0.02, 0.08, 0.04, 0.04, 0.042, 0.42, 0.18, 0.055, 1.4, 0.43,
2 1.3, 3.3, 8.5, 79.5, 8.1, 35, 1.53, 9.1, 1.98, 2.6, 12.9, 2.66, 4, 17.4, 3.5,
3 5.86, 23.4, 4.54, 7.9, 29.5, 5.5, 10.4, 36.6, 6.7, 13.3, 44.4, 7.9, 16.4
3 53, 9.15, 20.7, 64, 10.8, 24.5, 72, 12.2/

THIS SECTION IS THE BITS TO LUMINANCE DATA FOR CONRAC 5711 MONITOR

DATA C5711/O..0..0..
1 0.02, 0.07, 0.02, 0.06, 0.44, 0.06, 2.1, 1.2, 2, 4, 2, 4, 4, 82, 4, 3,
1 7, 1.26, 6, 6, 1.05, 1.9, 9, 2, 1.59, 2.55, 12.6, 2, 3.4, 16.4, 2.7, 4.55,
2 20, 3.5, 5.75, 25, 4.2, 7.2, 30, 5.2, 8.6, 36, 6.2, 10.5, 42, 7.3, 12.3
2 48, 8.7, 14.1, 55, 9.8/

THIS SECTION IS THE BITS TO LUMINANCE DATA FOR COLLINS MONITOR

DATA COLLINS/O..0..0..
1 0.017, 0.01, 0.05, 0.035, 0.02, 0.01, 12, 12, 0.027, 0.23, 35,
1 0.44, 44, 76, 12, 68, 1.3, 23, 1, 2.1, 4, 1.38, 3, 6, 1.83, 4, 12, 85,
2 2.4, 5, 6, 1.15, 3, 7, 2, 1.46, 3, 62, 9, 03, 1.8, 4, 4, 11, 04, 2.15, 5.1, 13.1
2 2.48, 6, 02, 15, 7, 2.85, 6.8, 17.8, 3.25/

INITIALIZATION ROUTINE FOR LEXIDATA GRAPHICS SYSTEM

IR=4097
IG=8193
IB=12289
ICOUNT=1
CALL DSPON(IERR,ICHAN)
CALL DSPLD(-1)
CALL DSLLU(0,4095,0)
CALL DSLIM(0,639,479)
DO I= 1,165
CALL DSPUT(BUFFER,640)
CALL DSWT
END DO
DO I=1,150
CALL DSPUT(BUFFER2,640)
CALL DSWT
END DO
DO I= 1,165
CALL DSPUT(BUFFER,640)
CALL DSWT
END DO
38
C ASK FOR THE MONITOR TYPE

TYPE *, 'INPUT MONITOR TYPE'
TYPE *, '
TYPE *, '1.  7211'
TYPE *, '2.  5711'
TYPE *, '3.  COLLINS'
TYPE *, '4.  NEW MONITOR'
TYPE *, '
ACCEPT 100, IMON

100 FORMAT(1)
   IF(IMON .LT. 1 .OR. IMON .GT. 4)GO TO 30

C THIS IS PROVISION FOR A NEW MONITOR

C IF(IMON .EQ. 4)THEN
   TYPE *, 'INPUT THE NAME OF THE FILE CONTAINING MONITOR INFORMATION'
   ACCEPT 103, FILE

103 FORMAT(A)

OPEN FILE CONSISTING OF:

C 1. 16 VALUES OF BRIGHTNESS OF RED GUN FROM 0 TO 255 INCREMENTS OF 16
C 2. 16 VALUES OF BRIGHTNESS OF GREEN GUN FROM 0 TO 255 INCREMENTS OF 16
C 3. 16 VALUES OF BRIGHTNESS OF BLUE GUN FROM 0 TO 255 INCREMENTS OF 16

DO NOT PUT IN A VALUE OF FOR ZERO BITS
C
C 4. PHOSPHOR RED X,Y,Z VALUES
C 5. PHOSPHOR GREEN X,Y,Z VALUES
C 6. PHOSPHOR BLUE X,Y,Z VALUES
C 7. LUMINANCE FACTOR
C
OPEN(UNIT=1, NAME=FILE, STATUS='OLD')
READ(1,104)(NEW_MON(1,J), J=1,16)

104 FORMAT(16F7.3)
READ(1,104)(NEW_MON(2,J), J=1,16)
READ(1,104)(NEW_MON(3,J), J=1,16)
READ(1,105)(PHOSRED(J,4), J=1,3)

105 FORMAT(3F6.3)
READ(1,105)(PHOSGREEN(J,4), J=1,3)
READ(1,105)(PHOSBLUE(J,4), J=1,3)
READ(1,106)LUMFACTOR(4)

106 FORMAT(F6.1)
CLOSE(UNIT=1)
END IF
DO I=1,3
   PCC(1,I)=PHOSRED(I,IMON)
   PCC(2,I)=PHOSGREEN(I,IMON)
   PCC(3,I)=PHOSBLUE(I,IMON)
END DO
WITH THE ABOVE DATA THE CALCULATION OF FOOT LAMBERTS FOR THE RED AND BLUE GUNS TO GENERATE A SPECIFIC COLOR CAN BE CALCULATED

PCCT(1,1)=PCC(1,1)
PCCT(1,2)=PCC(2,1)
PCCT(1,3)=PCC(3,1)
PCCT(2,1)=PCC(1,2)
PCCT(2,2)=PCC(2,2)
PCCT(2,3)=PCC(3,2)
PCCT(3,1)=PCC(1,3)
PCCT(3,2)=PCC(2,3)
PCCT(3,3)=PCC(3,3)

CY(1,1)=1./PCCT(2,1)
CY(2,2)=1./PCCT(2,2)
CY(3,3)=1./PCCT(2,3)

P(1,1)=PCCT(1,1)*CY(1,1)
P(1,2)=PCCT(1,2)*CY(2,2)
P(1,3)=PCCT(1,3)*CY(3,3)
P(2,1)=PCCT(2,1)*CY(1,1)
P(2,2)=PCCT(2,2)*CY(2,2)
P(2,3)=PCCT(2,3)*CY(3,3)
P(3,1)=PCCT(3,1)*CY(1,1)
P(3,2)=PCCT(3,2)*CY(2,2)
P(3,3)=PCCT(3,3)*CY(3,3)

DP=\frac{P(1,1)*P(2,2)*P(3,3)+P(1,2)*P(2,3)*P(3,1)+P(1,3)*P(2,1)*P(3,2)}{1}

COFP(1,1)=P(1,2)*P(3,3)-P(2,3)*P(3,2)
COFP(1,2)=(P(2,1)*P(3,3)-P(2,3)*P(3,1))/-1.
COFP(1,3)=P(2,1)*P(3,2)-P(2,2)*P(3,1)
COFP(2,1)=(P(1,2)*P(3,3)-P(1,3)*P(3,2))/-1.
COFP(2,2)=P(1,1)*P(3,3)-P(1,3)*P(3,1)
COFP(2,3)=(P(1,1)*P(3,2)-P(1,2)*P(3,1))/-1.
COFP(3,1)=P(1,2)*P(2,3)-P(1,3)*P(2,2)
COFP(3,2)=(P(1,1)*P(2,3)-P(1,3)*P(2,1))/-1.
COFP(3,3)=P(1,1)*P(2,2)-P(1,2)*P(2,1)

COFPT(1,1)=COFP(1,1)/DP
COFPT(1,2)=COFP(2,1)/DP
COFPT(1,3)=COFP(3,1)/DP
COFPT(2,1)=COFP(1,2)/DP
COFPT(2,2)=COFP(2,2)/DP
COFPT(2,3)=COFP(3,2)/DP
COFPT(3,1)=COFP(1,3)/DP
COFPT(3,2)=COFP(2,3)/DP
COFPT(3,3)=COFP(3,3)/DP

C THIS IS THE COLOR SELECTION PROCESS. COLOR 38 IS FOR A COLOR
C NOT PREVIOUSLY DEFINED
C
31 TYPE *, 'INPUT NUMBER OF COLOR TO DISPLAY OR 39 TO EXIT'
  ACCEPT 100,NUM
  IF(NUM.EQ.39)GO TO 32
  IF(NUM.EQ.38)THEN
    TYPE *, 'INPUT X,Y AND LUMINANCE FOR NEW COLOR'
    ACCEPT 107,(CIESET(J,38),J=1,3)
  END IF

C CALCULATE TRISTIMULUS VALUES
C
C Y IS THE DESIRED LUMINANCE
C 1  XL=CIESET(1,NUM)
    YL=CIESET(2,NUM)
    L=CIESET(3,NUM)*LUMFACTOR(IMON)
    XS=XL
    YS=YL
    LS=L
    Y=L
    X=XL*(Y/YL)
    Z=(1.-XL-YL)*(Y/YL)
    LUM(1)=COFPT(1,1)*X+COFPT(1,2)*Y+COFPT(1,3)*Z
    LUM(2)=COFPT(2,1)*X+COFPT(2,2)*Y+COFPT(2,3)*Z
    LUM(3)=COFPT(3,1)*X+COFPT(3,2)*Y+COFPT(3,3)*Z

C OUTPUT OF FOOT LAMBERTS FOR THE THREE GUNS
C
C TYPE*, 'RED= ',LUM(1), ' GREEN= ',LUM(2), ' BLUE= ',LUM(3), ' FOOT LAMBERTS'
  IF(IMON.EQ.1)THEN
    DO 12 I=1,16
    IF(C7211(I,1).GT.LUM(1))GO TO 2
    CONTINUE
  END IF
  TYPE*, 'RED LUMINANCE VALUE NOT IN TABLE'
  STOP

2  RI=I-1
  A=RI*16.
  B=LUM(1)-C7211(1,I-1)
  C=C7211(1,I)-C7211(1,I-1)
  D=B/C
  E=D*16.
  RBITS=A+E
  IF(RBITS.LT.0.)RBITS=0.
  DO 13 I=1,16
  IF(C7211(2,I).GT.LUM(2))GO TO 3
CONTINUE
TYPE *, 'GREEN LUMINANCE VALUE NOT IN TABLE'
STOP
3
RI=I-1
A=RI*16.
B=LUM(2)-C7211(2, I-1)
C=C7211(2, I)-C7211(2, I-1)
D=B/C
E=D*16.
GBITS=A+E
IF(GBITS .LT. 0.)GBITS=0.
DO 14 I=1,16
IF(C7211(3, I) .GT. LUM(3))GO TO 4
CONTINUE
TYPE *, 'BLUE LUMINANCE VALUE NOT IN TABLE'
STOP
4
RI=I-1
A=RI*16.
B=LUM(3)-C7211(3, I-1)
C=C7211(3, I)-C7211(3, I-1)
D=B/C
E=D*16.
BBITS=A+E
IF(BBITS .LT. 0.)BBITS=0.
GO TO 11
END IF
IF(IMON .EQ. 2) THEN
DO 15 I=1,16
IF(C5711(I; I) .GT. LUM(I))GO TO 5
CONTINUE
TYPE *, 'RED LUMINANCE VALUE NOT IN TABLE'
STOP
5
RI=I-1
A=RI*16.
B=LUM(1)-C5711(1, I-1)
C=C5711(1, I)-C5711(1, I-1)
D=B/C
E=D*16.
RBITS=A+E
IF(RBITS .LT. 0.)RBITS=0.
DO 16 I=1,16
IF(C5711(2, I) .GT. LUM(2))GO TO 6
CONTINUE
TYPE *, 'GREEN LUMINANCE VALUE NOT IN TABLE'
STOP
6
RI=I-1
A=RI*16.
B=LUM(2)-C5711(2, I-1)
C=C5711(2, I)-C5711(2, I-1)
D=B/C
E=D*16.
GBITS=A+E
IF(GBITS .LT. 0.)GBITS=0.
DO 17 I=1,16
IF(C5711(3, I) .GT. LUM(3))GO TO 7
CONTINUE
TYPE *,'BLUE LUMINANCE VALUE NOT IN TABLE'
STOP
7
RI=I-1
A=RI*16
B=LUM(3)-C5711(3,I-1)
C=C5711(3,I)-C5711(3,I-1)
D=B/C
E=D*16.
BBITS=A+E
IF(BBITS .LT. 0.)BBITS=0.
GO TO 11
END IF
IF(IIION .EQ. 3) THEN
DO 10 I=1,16
IF(COLLINS(3,1) .GT. LUM(3)) GO TO 8
10 CONTINUE
TYPE *,'RED LUMINANCE VALUE NOT IN TABLE'
STOP
8
RI=I-1
A=RI*16.
B=LUM(1)-COLLINS(1,1-1)
C=COLLINS(1,1)-COLLINS(1,1-1)
D=B/C
E=D*16.
RBITS=A+E
IF(RBITS .LT. 0.)RBITS=0.
DO 19 I=1,16
IF(COLLINS(2,1) .GT. LUM(2)) GO TO 9
19 CONTINUE
TYPE *,'GREEN LUMINANCE VALUE NOT IN TABLE'
STOP
9
RI=I-1
A=RI*16.
B=LUM(2)-COLLINS(2,1-1)
C=COLLINS(2,1)-COLLINS(2,1-1)
D=B/C
E=D*16.
GBITS=A+E
IF(GBITS .LT. 0.)GBITS=0.
DO 20 I=1,16
IF(COLLINS(3,1) .GT. LUM(3)) GO TO 10
20 CONTINUE
TYPE *,'BLUE LUMINANCE VALUE NOT IN TABLE'
STOP
10
RI=I-1
A=RI*16
B=LUM(3)-COLLINS(3,1-1)
C=COLLINS(3,1)-COLLINS(3,1-1)
D=B/C
E=D*16.
BBITS=A+E
IF(BBITS .LT. 0.)BBITS=0.
GO TO 11
END IF
DO 28 I=1,16
IF(NEW_MON(1,I) .GT. LUM(1))GO TO 22
CONTINUE
TYPE *, 'RED LUMINANCE VALUE NOT IN TABLE'
STOP
22
RI=I-1
A=RI*16.
B=LUM(1)-NEW_MON(1,I-1)
C=NEW_MON(1,I)-NEW_MON(1,I-1)
D=B/C-
E=D*16.
RBITS=A+E
IF(RBITS .LT. 0.)RBITS=0.
DO 23 I=1,16
IF(NEW_MON(2,I) .GT. LUM(2))GO TO 24
CONTINUE
TYPE *, 'GREEN LUMINANCE VALUE NOT IN TABLE'
STOP
24
RI=I-1
A=RI*16.
B=LUM(2)-NEW_MON(2,I-1)
C=NEW_MON(2,I)-NEW_MON(2,I-1)
D=B/C-
E=D*16.
GBITS=A+E
IF(GBITS .LT. 0.)GBITS=0.
DO 25 I=1,16
IF(NEW_MON(3,I) .GT. LUM(3))GO TO 26
CONTINUE
TYPE *, 'BLUE LUMINANCE VALUE NOT IN TABLE'
STOP
26
RI=I-1
A=RI*16.
B=LUM(3)-NEW_MON(3,I-1)
C=NEW_MON(3,I)-NEW_MON(3,I-1)
D=B/C-
E=D*16.
BBITS=A+E
IF(BBITS .LT. 0.)BBITS=0.
C
C OUTPUTS RED, GREEN AND BLUE BITS BASED ON TABLE LOOKUP TO THE
C NEAREST BIT
C
11
TYPE *, 'RBITS= ',RBITS,' GBITS= ',GBITS,' BBITS= ',BBITS
IRED=RBITS+.5
IGREEN=GBITS+.5
IBLUE=BBITS+.5
C
C COLOR BASED ON THESE BITS IS OUTPUT TO THE SCREEN
CALL DSWLT(IR,ICOUNT,IRED)
CALL DSWLT(IG,ICOUNT,IGREEN)
CALL DSWLT(IB,ICOUNT,IBLUE)
TYPE *, 'REFERENCE X= ',XS,' Y= ',YS,' L= ',LS

C THIS IS THE ITERATION PROCESS TO GET THE EXACT COLOR
C
TYPE *, 'WOULD YOU LIKE TO CHANGE THE X, Y AND L OR N'
ACCEPT 102,ANS

102 FORMAT(A)
IF(ANS .EQ. 'Y') THEN
TYPE *, 'INPUT THE VALUES READ FOR X, Y AND LUM 3F6.4'
ACCEPT 101,XR,YR,LR

101 FORMAT(3F6.4)
DX=XS-XR
DY=YS-YR
DL=LS-LR
XL=XS+DX*.2+DXO
YL=YS+DY*.2+DYO
L=LS+DL*.2+DLO
DXO=DX
DYO=DY
DLO=DLO
TYPE *, 'NEW XL= ',XL,' YL= ',YL,' L= ',L
GO TO 21
ELSE
IBITS(1,NUM)=IRED
IBITS(2,NUM)=IGREEN
IBITS(3,NUM)=IBLUE
GO TO 31
END IF

32 OPEN(UNIT=1,NAME='COLORLIST.LIS', STATUS='NEW')
WRITE(1,110) IMON
110 FORMAT(' THE FOLLOWING COLOR TABLE IS FOR MONITOR ',I2, //,
1 ' NO. RED GREEN BLUE ')
DO 33 I=1,38
WRITE(1,109)(IBITS(J,I),J=1,3)
109 FORMAT(I3,16,18,17)
33 CONTINUE
TYPE *, 'WOULD YOU LIKE TO LOOK AT THE STORED COLORS Y OR N'
ACCEPT 102,ANS
IF(ANS .EQ. 'Y') THEN
TYPE *, 'INPUT COLOR TO BE DISPLAYED OR 39 TO EXIT'
ACCEPT 100,NUM
IF(NUM .EQ. 39) THEN
CALL DSCLS
CLOSE(UNIT=1)
STOP
END IF
IRED=IBITS(1,NUM)
IGREEN=IBITS(2,NUM)
IBLUE=IBITS(3,NUM)
CALL DSLWT(IR,ICOUNT,IRED)
CALL DSLWT(IG,ICOUNT,IGREEN)
CALL DSLWT(IB,ICOUNT,IBLUE)
GO TO 34
ELSE
CALL OSCLS
CLOSE(UNIT=1)
STOP
END IF
END
END
A convenient system for the numerical display of color values based on the tristimulus concept has been developed. This measurement system which can be made specific to the 1931 CIE standard observer is called the $x, y$ chromaticity diagram. In the development of this diagram, the three primaries, usually chosen as a specific red, green, and blue for additive mixing, are the axes of a non-orthogonal axis system. An arbitrary color $S$ will have components along the red, green, and blue axes $R', G', and B'$ of magnitudes $R, G,$ and $B$.

A unit vector system is established in order to permit the quantification of magnitudes of different colored stimuli in an orderly manner. Amounts of the primaries can be measured in physical terms such as watts of radiant flux or in psychophysical terms such as luminous flux or luminance (Judd and Wyszecki\textsuperscript{5}). The conventional choice is in luminance with one unit of red, green, and blue being that amount that produces a neutral or achromatic color when mixed. A plane may be passed through the three unit points where $R=G=B=1$.

A color vector $S$ represented by its $R, G,$ and $B$ values will pass through the unit plane. If the value of $R, G,$ and $B$ are one and the unit plane is symmetrical, the piercing point will be at a geometrically central point. If, for example, the unit plane is an equilateral triangle, the neutral color vector $N$ will intersect the unit plane at a location one third the distance from any side of the unit plane triangle to its opposite side. The neutral vector $N$ would be perpendicular to the unit plane in this case (Fig. 5).

The general value of the unit plane is that colors may be described in terms of the location at which they intersect the unit plane. It can be readily seen that vectors which pass through the plane near the neutral point will be essentially neutral, and colors which intersect the unit plane near the primaries will approach the color of the primary. The luminance of the color is related to its length and it is not determined from the unit plane, but may be readily calculated by using the laws of additivity of luminance.

\[ L = lrR + lgG + lbB \]  \hspace{1cm} (1)

Where $lr, lg, lb$ are the luminances at the unit amounts.

In order to numerically define each point on the unit plane, a coordinate system must be established on the unit plane.

Before this can be done, numerical relationships must be established between plane intercept points and the color vector $S$.

The equation of the unit plane which intercepts the red, green, and blue unit vectors may be found by substituting the three known points into the generalized equation of a plane.

\[ ax + by + cz + d = 0 \]  \hspace{1cm} (2)
Using the more general form of coordinate system, X Y Z instead of RGB:

- at one unit of the Red unit vector \( X = 1, Y = 0, Z = 0 \)
- at one unit of the Green unit vector \( X = 0, Y = 1, Z = 0 \)
- at one unit of the Blue unit vector \( X = 0, Y = 0, Z = 1 \)

Solving the three equations for the three knowns, the unit plane is found to be:

\[
X + Y + Z = 1
\] (3)

For a given color \( X, Y, Z \), the color vector may be expressed in the symmetrical form of the equation of a line in space:

\[
\frac{X-a}{1} = \frac{Y-b}{m} = \frac{Z-c}{n}
\] (4)

In this equation, \( a, b, \) and \( c \) are the coordinates of a known point which the line passes through. In this case, the color vector \( S \) passes through the origin of the non-orthogonal axis net. Therefore, \( a, b, \) and \( c \) are 0. The constants \( l, m, \) and \( n \) are the direction ratios of the line. Direction ratios are the same as direction cosines for orthogonal axes. In this case they are defined as:

\[
l = X/r
\] (5)

\[
m = Y/r
\] (6)

\[
n = Z/r
\] (7)

where \( r \) is the length of color vector \( S \). Placing the unit plane defined above in a more general form:

\[
aX + bY + cZ + d = 0
\] (8)

where \( a = 1, b = 1, c = 1, d = 1 \)

Using the direction ratios to determine the \( X, Y, Z \) coordinates of a point on a line through a point \( AO, BO, CO \):

\[
X = AO + lR
\] (9)

\[
Y = BO + mR
\] (10)

\[
Z = CO + nR
\] (11)

For this case where the line intersects the origin \( AO, BO, \) and \( CO \) are zero.

Substituting these expressions in the equation for the plane:

\[
r (a1 + bm + cn) -1 = 0
\] (12)

\[
r = 1.0/(l+m+n)
\] (13)

---

\[
X = \frac{1}{l+m+n} \quad (14)
\]
\[
Y = \frac{m}{l+m+n} \quad (15)
\]
\[
Z = \frac{n}{l+m+n} \quad (16)
\]

Since
\[
l = \frac{x}{r}, \quad m = \frac{y}{r}, \quad \text{and} \quad n = \frac{z}{r}
\]
\[
x = \frac{X}{X+Y+Z} = \frac{R}{R+G+B} \quad (17)
\]
\[
y = \frac{Y}{X+Y+Z} = \frac{G}{R+G+B} \quad (18)
\]
\[
z = \frac{Z}{X+Y+Z} = \frac{B}{R+G+B} \quad (19)
\]

These are the coordinates in the primary axes system of the point of interaction of a color vector of any length and the unit plane. The X coordinate represents travel in the unit plane in a direction toward the X primary intercept of the unit plane. When this coordinate is considered as a distance, it can also be used to represent travel in the unit plane parallel to the X axis. Y and Z may be considered as distance in the unit plane in a direction parallel to the Y and Z axes, respectively.

A new axis system of convenience can now be established. As shown in Figure D-1, the primaries can be reoriented to create a unit plane primary axis intersection in the form of a right angle triangle. The unit vectors are unchanged but the point of origin is moved to a less symmetrical location. Since this location did not appear in the derivation of the intercept points (17), (18), and (19) are unchanged. The blue primary is taken as the right angle of the triangle at \(x = 0, \ y = 0\). Green is at the \(y = 1\) point and the red at the \(x = 1\) point. The coordinates \(x, y,\) and \(z\) are known as the chromaticity coordinates. The right angle configuration of the axes system is most convenient for rapid determination of a plotted color's \(x, y\) value. The \(z\) value is normally found from the relationship:
\[
x + y + z = 1 
\quad (20)
\]

The oblique projection which is the usual presentation form of chromaticity diagrams is shown in Figure D-1. The chromaticity coordinates of the 1931 CIE standard observer are plotted with spectrum frequencies and color temperatures indicated (Fig. D-2).

Several points can be noted. An "equal energy" point "E" can be calculated for (17), (18), and (19). As mentioned above, this equal energy point would, in fact, be the point where \(R, G,\) and \(B\) vectors are unity and produce a neutral color. This point is no longer in the geometric center of the axes triangle as shown in Figure 5. However, the significance of the point is the same.

The intersection of the color vector with the plane is the point of interest with respect to chromaticities. There is no particular significance to a color vector that actually terminates on the unit plane except for the fact that its coordinates would follow the equation \(R+G+B = 1\).
Figure D-1. Tristimulus Axis System with Right Angle Unit Plane.
Figure D-2. Chromaticity Coordinate System with CIE 1931 Standard Observer.
APPENDIX E

GENERATION OF SPECIFIC COLORS BY DETERMINANT METHOD

(NUMERICAL EXAMPLE)

This appendix illustrates a numerical example of the procedure described in Section 3 to determine the RGB bits required to generate a specific color. The determinant method for transfer of primaries from Section 4 is used. With this approach, a computer program is not needed to process the mathematics. An additional advantage is that the user does not need facility with linear algebra techniques. The basic steps of Section 3 are listed as a complete guide to the procedure order. However, the main purpose of this section is to illustrate a rapid hand calculation technique for the determination of the RGB bits needed to generate a specific color on a CRT monitor.

STEP 1. Prepare monitor-display generator as recommended.

STEP 2. Prepare hardware or program needed to generate color squares on monitor.

STEP 3. Determine linear range of monitor. For this example, the linear range of the CONRAC 7211 is 107.1 (Table 5).

STEP 4. The color to be generated is red (cie chromaticity coordinates x = .558 y = .320 L= 7.2). The luminance of 7.2 foot-lamberts is based on a monitor with a total output of 33.7 footlamberts. For the CONRAC 7211, the brightness needed is calculated as follows:

\[
\frac{107.1 \text{ FL}}{33.7 \text{ FL}} \times \frac{.84 \times 7.2 \text{ FL}}{1.0} = 19.2 \text{ FL}
\]

STEP 5. Determine bits to luminance curves (Fig. 12).

STEP 6. This step will determine the RGB bits required at the display generator to obtain the specific color.

a. Converting chromaticity coordinates of the desired color to tristimulus values with \(Y=L\).

\[
X = (x \cdot L) / (1.0) = (0.558)(19.2) / 33.7 = 0.349 = R' \quad (1)
\]

\[
Y = L = 19.2 = G' \quad (2)
\]

\[
Z = (1.0 - 0.558 - 0.320)(19.2 / 33.7) = 5.525 = 8' \quad (3)
\]

b. The manufacturer's values for the phosphor chromaticity coordinates are:

<table>
<thead>
<tr>
<th>Color</th>
<th>x</th>
<th>y</th>
<th>z</th>
</tr>
</thead>
<tbody>
<tr>
<td>Red</td>
<td>.635</td>
<td>.325</td>
<td>.040</td>
</tr>
<tr>
<td>Green</td>
<td>.220</td>
<td>.680</td>
<td>.100</td>
</tr>
<tr>
<td>Blue</td>
<td>.152</td>
<td>.063</td>
<td>.780</td>
</tr>
</tbody>
</table>

53
c. Converting these x,y,z values to tristimulus values, each gun normalized by its green(y) value gives the following parameter set:

<table>
<thead>
<tr>
<th></th>
<th>x</th>
<th>y</th>
<th>z</th>
</tr>
</thead>
<tbody>
<tr>
<td>Red</td>
<td>1.95</td>
<td>1.0</td>
<td>.12</td>
</tr>
<tr>
<td>Green</td>
<td>.32</td>
<td>1.0</td>
<td>.15</td>
</tr>
<tr>
<td>Blue</td>
<td>2.41</td>
<td>1.0</td>
<td>12.38</td>
</tr>
</tbody>
</table>

d. Expressing these values in the notation required for transfer of primary coordinate systems and listing in vertical or column format yields:

\[
\begin{align*}
X_{cier} &= 1.95 & X_{cieg} &= 0.32 & X_{cieb} &= 2.41 \\
Y_{cier} &= 1.00 & Y_{cieg} &= 1.00 & Y_{cieb} &= 1.00 \\
Z_{cier} &= 0.12 & Z_{cieg} &= 0.15 & Z_{cieb} &= 12.38
\end{align*}
\]

e. Using the expressions of Figure 6, the brightness settings of the three guns are found to be:

\[
\begin{align*}
R_{crt} &= 17.43 \text{ FL} \\
G_{crt} &= 1.46 \text{ FL} \\
B_{crt} &= 0.26 \text{ FL}
\end{align*}
\]

f. Using the bits to luminance curves of Figure 12 and the data of Table 10, the command signals are found, after interpolation, to be:

\[
\begin{align*}
R(\text{BITS}) &= 228 \\
G(\text{BITS}) &= 64 \\
B(\text{BITS}) &= 52
\end{align*}
\]

g. The color generated by these bit values should be checked for chromaticity and brightness with laboratory instruments. If the values are not the desired ones, a second iteration should be performed using an adjusted command value. The adjusted value should differ from the original value by a factor of 0.2 times the error of the color achieved as shown below.

\[
\begin{align*}
x & \quad y & \quad L \\
\text{Desired Color} & \quad .588 & \quad .320 & \quad 19.2 \\
\text{Color Achieved} & \quad .600 & \quad .310 & \quad 18.0 \\
\text{Error} & \quad .012 & \quad -.010 & \quad -1.2 \\
0.2 \times \text{Error} & \quad .0024 & \quad -.002 & \quad -.24 \\
\text{New Command} & \quad .586 & \quad .322 & \quad 19.44
\end{align*}
\]

The adjusted commands are the new inputs to Step 6. Convergence is usually achieved within three iterations.
END
Feb.
1988
DTIC