AD-A187 086 COMPARISON OF THE ROUGH SURFACE REFLECTION COEFFICIENT WITH SPECULARLY SCATTERED ACOUSTIC DATA(U) NAVAL RESEARCH LAB WASHINGTON DC A R MILLER ET AL. 06 NOV 87 1/1

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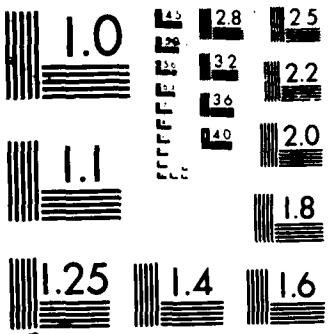
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NRL Report 9063

AD-A187 086

Comparison of the Rough Surface Reflection Coefficient with Specularly Scattered Acoustic Data

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November 6, 1987

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SECURITY CLASSIFICATION OF THIS PAGE

REPORT DOCUMENTATION PAGE

1a. REPORT SECURITY CLASSIFICATION UNCLASSIFIED	1b. RESTRICTIVE MARKINGS A187 086
2a. SECURITY CLASSIFICATION AUTHORITY	3. DISTRIBUTION / AVAILABILITY OF REPORT Approved for public release; distribution unlimited.
2b. DECLASSIFICATION / DOWNGRADING SCHEDULE	
4. PERFORMING ORGANIZATION REPORT NUMBER(S) NRL Report 9063	5. MONITORING ORGANIZATION REPORT NUMBER(S)
6a. NAME OF PERFORMING ORGANIZATION Naval Research Laboratory 6b. OFFICE SYMBOL (If applicable) Code 5352	7a. NAME OF MONITORING ORGANIZATION
6c. ADDRESS (City, State, and ZIP Code) Washington, DC 20375-5000	7b. ADDRESS (City, State, and ZIP Code)
8a. NAME OF FUNDING / SPONSORING ORGANIZATION Naval Air Systems Command 8b. OFFICE SYMBOL (If applicable) APC-209	9. PROCUREMENT INSTRUMENT IDENTIFICATION NUMBER
8c. ADDRESS (City, State, and ZIP Code) Washington, DC 20361	10. SOURCE OF FUNDING NUMBERS PROGRAM ELEMENT NO. 64211N; PROJECT NO.; TASK NO. W1253; WORK UNIT ACCESSION NO. DN180-248

11. TITLE (Include Security Classification)
Comparison of the Rough Surface Reflection Coefficient with Specularly Scattered Acoustic Data

12. PERSONAL AUTHOR(S)
Miller, Allen R. and Vegh, Emanuel

13a. TYPE OF REPORT	13b. TIME COVERED	14. DATE OF REPORT (Year, Month, Day)	15. PAGE COUNT
Final	FROM Aug 86 TO Feb 87	1987 November 6	10

16. SUPPLEMENTARY NOTATION

17. COSATI CODES			18. SUBJECT TERMS (Continue on reverse if necessary and identify by block number)
FIELD	GROUP	SUB-GROUP	Reflection coefficient Acoustic propagation

19. ABSTRACT (Continue on reverse if necessary and identify by block number)

A comparison is made between a theoretically derived family of rough surface reflection coefficients and specularly scattered acoustic data.

20. DISTRIBUTION / AVAILABILITY OF ABSTRACT ☒ UNCLASSIFIED/UNLIMITED ☐ SAME AS RPT. ☐ DTIC USERS	21. ABSTRACT SECURITY CLASSIFICATION UNCLASSIFIED
22a. NAME OF RESPONSIBLE INDIVIDUAL Emanuel Vegh	22b. TELEPHONE (Include Area Code) (202) 767-3481 22c. OFFICE SYMBOL Code 5350

DD FORM 1473, 84 MAR — 83 APR edition may be used until exhausted. All other editions are obsolete.

SECURITY CLASSIFICATION OF THIS PAGE

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Availability Codes
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COMPARISON OF THE ROUGH SURFACE REFLECTION COEFFICIENT WITH SPECULARLY SCATTERED ACOUSTIC DATA

INTRODUCTION

Miller and Vegh [1] in treating reflection from the rough surface of the sea derived a one-parameter family of curves for the rough surface reflection coefficient or roughness factor R given by

$$R(g,\epsilon) = \epsilon^2 \exp[-2\epsilon^2\eta^2(2\pi g)^2]\, I_0[2\epsilon^2\eta^2(2\pi g)^2]$$

$$+ (1-\epsilon^2)^{1/2} \exp[-4\eta^2(2\pi g)^2]$$

$$- \frac{1}{2}\epsilon^2(1-\epsilon^2)\,\Phi_1[\frac{3}{2}, 1; 2; \epsilon^2, -4\epsilon^2\eta^2(2\pi g)^2] \tag{1}$$

where

$$g \equiv (\sigma/\lambda)\sin\psi$$

and

$$\eta \equiv [1 + \frac{\pi}{2}(1-\epsilon^2)]^{-1/2}$$

Here g is a measure of the effective surface roughness or simply surface roughness, $\epsilon\,(0 \leqslant \epsilon \leqslant 1)$ is the spectral width parameter, σ is the standard deviation of the water surface elevation, ψ is the grazing angle for specular reflection, λ is the wavelength of the incident radiation, and $I_0(x)$ is the modified Bessel function of order zero. The function $\Phi_1[\alpha, \beta; \gamma; x, y]$ is a confluent hypergeometric function in two variables first defined in 1920 by P. Humbert [2, p. 58]. In the Appendix we derive an integral representation for Φ_1 that may be used for numerical computation.

$R(g,\epsilon)$, given by Eq. (1), is essentially the Fourier transform of the probability density $D(y,\epsilon)$ for surface elevation y where

$$D(y,\epsilon) = \frac{\epsilon}{2\pi^{3/2}\eta\sigma} \exp\left(\frac{-y^2}{8\epsilon^2\eta^2\sigma^2}\right) K_0\left(\frac{y^2}{8\epsilon^2\eta^2\sigma^2}\right)$$

$$+ \frac{(1-\epsilon^2)^{1/2}}{\pi^{3/2}\eta\sigma} \exp\left(\frac{-y^2}{4\eta^2\sigma^2}\right) \{\cos^{-1}\epsilon + \epsilon\,(1-\epsilon^2)^{1/2} K_{e_0}(2\epsilon^2 - 1, y^2/8\epsilon^2\eta^2\sigma^2)\} \tag{2}$$

Here $K_0(x)$ is the MacDonald function or Bessel function of imaginary argument of order zero. $K_{e_0}(a, x)$ is an incomplete Lipschitz-Hankel integral of $K_0(x)$ and may be written in closed form either in terms of incomplete cylindrical functions [3] or in various ways in terms of Kampé de Fériet functions [4,5]; e.g.

$$K_{e_0}(a, z) = z\, K_0(z)\, A_1(a, z) + z^2\, K_1(z)\, A_0(a, z)$$

Manuscript approved April 9, 1987.

where

$$A_1(a,z) \equiv F^{0:1;1}_{2:0;0}\left[\begin{matrix} -: 1/2;\ 1; \\ 1/2,3/2:\ -;\ -; \end{matrix}\ \frac{a^2z^2}{4},\ \frac{z^2}{4}\right]$$

$$+\ \frac{1}{2}az\ F^{0:2;1}_{2:1;0}\left[\begin{matrix} -: 1,1;\ 1; \\ 1,2:\ 3/2;\ -; \end{matrix}\ \frac{a^2z^2}{4},\ \frac{z^2}{4}\right]$$

$$A_0(a,z) \equiv F^{0:1;1}_{2:0;0}\left[\begin{matrix} -: 1/2;\ 1; \\ 3/2,3/2:\ -;\ -; \end{matrix}\ \frac{a^2z^2}{4},\ \frac{z^2}{4}\right]$$

$$+\ \frac{1}{4}az\ F^{0:2;1}_{2:1;0}\left[\begin{matrix} -: 1,1;\ 1; \\ 2,2:\ 3/2;\ -; \end{matrix}\ \frac{a^2z^2}{4},\ \frac{z^2}{4}\right]$$

$D(y,\epsilon)$, given by Eq. (2), was derived in Ref. 1 by assuming that the water surface could be described locally by sinusoids with uniform phase distribution whose amplitude distribution is given by a density function derived by Rice [6] and by Cartwright and Longuet-Higgins [7]. Figure 1 gives graphs for $D(y,\epsilon)$, for various values of ϵ.

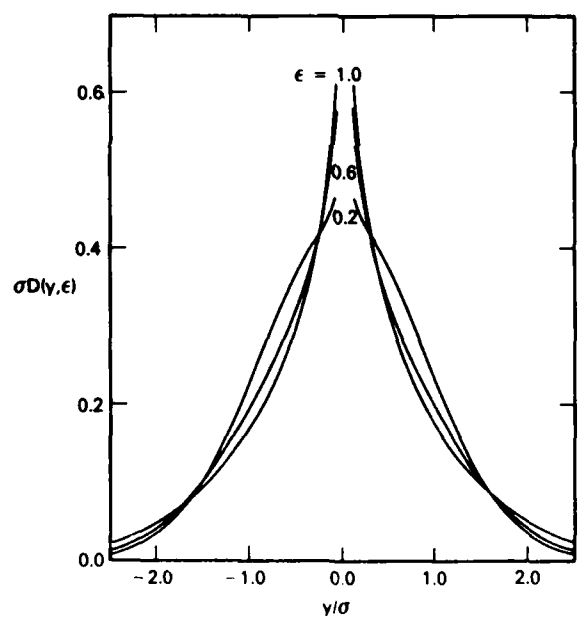

Fig. 1 – Density function $D(y,\epsilon)$ for various values of the spectral width parameter ϵ

COMPARISON OF $R(g,\epsilon)$ WITH ACOUSTIC DATA

In 1980 DeSanto [8, p. 70, Fig. 5] compared $R(g,1)$ with acoustic data from Clay, Medwin, and Wright [9]. Although $R(g,1)$ was first derived in 1974 [10], a mathematically rigorous derivation was not obtained until 1984 [11]. In view of Eq. (1), it now appears appropriate to compare $R(g,\epsilon)$ with

the aforementioned data. Whereas $R(g, 1)$ takes into account only the standard deviation, σ, of surface elevation, $R(g, \epsilon)$ is dependent on ϵ also and hence on the moments of the frequency energy spectrum $\Phi(s)$ of the surface through the equations [12, p. 346]

$$\epsilon^2 = (m_0 m_4 - m_2^2)/m_0 m_4$$

$$m_\nu \equiv \int_0^\infty s^\nu \Phi(s)\, ds \quad (m_0 = \sigma)$$

Figure 2 compares $R^2(g, 1/3)$ with the data given by Clay et al. in Fig. 5 of Ref. 9. $R^2(g, 1/3)$ appears to be in better agreement with this data than the multiple scattering theoretical result given in Fig. 5 of Ref. 8.

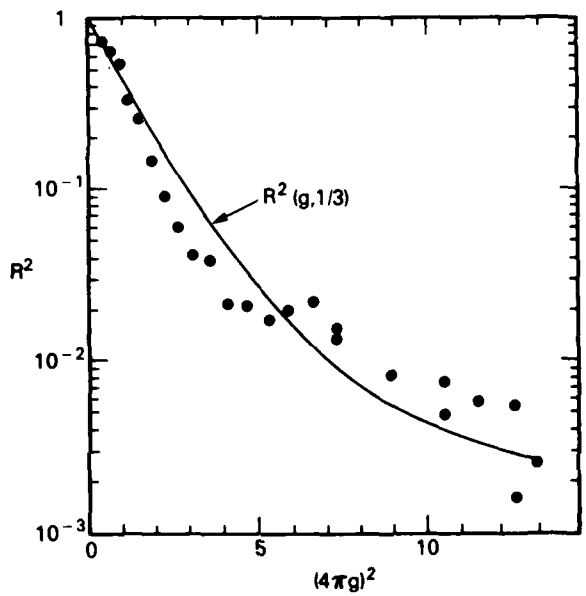

Fig. 2 — Comparison of the theoretical curve $R^2(g, 1/3)$ with experimental data

CONCLUSION

One of the family of rough surface reflection coefficients agrees with acoustic data reasonably well; at least as well as the curve given previously by the multiple scattering model.

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Appendix

INTEGRAL REPRESENTATIONS FOR $\Phi_1[\alpha, \beta; \gamma; x, y]$

The confluent double hypergeometric function Φ_1 is defined by

$$\Phi_1[\alpha, \beta; \gamma; x, y] \equiv \sum_{m,n=0}^{\infty} \frac{(\alpha)_{m+n} (\beta)_m}{(\gamma)_{m+n}} \frac{x^m}{m!} \frac{y^n}{n!} \qquad |x| < 1, \quad |y| < \infty$$

The definition of Φ_1 given in Erdélyi et al. [A1, p. 225] and Gradshteyn et al. [A2, 9.261, Eq. 1] is incorrect.

By using Ref. A3, p. 266

$$\frac{(\alpha)_p}{(\gamma)_p} = \frac{\Gamma(\gamma)}{\Gamma(\alpha)\Gamma(\gamma - \alpha)} \int_0^1 t^{p+\alpha-1} (1-t)^{\gamma-\alpha-1} dt, \quad \mathrm{Re}\gamma > \mathrm{Re}\alpha > 0$$

with the definition of Φ_1 given above we obtain

$$\Phi_1[\alpha, \beta; \gamma; x, y] = \frac{\Gamma(\gamma)}{\Gamma(\alpha)\Gamma(\gamma - \alpha)} \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} \int_0^1 t^{m+n+\alpha-1} (1-t)^{\gamma-\alpha-1} (\beta)_m \frac{x^m y^n}{m!n!} dt$$

Now interchanging the integral sign and double sum and noting that

$$\sum_{n=0}^{\infty} \frac{(ty)^n}{n!} = e^{ty}, \qquad \sum_{m=0}^{\infty} (\beta)_m \frac{(tx)^m}{m!} = (1 - tx)^{-\beta}$$

we obtain for $\mathrm{Re}\gamma > \mathrm{Re}\alpha > 0$, $|x| < 1$, $|y| < \infty$

$$\Phi_1[\alpha, \beta; \gamma; x, y] = \frac{\Gamma(\gamma)}{\Gamma(\alpha)\Gamma(\gamma - \alpha)} \int_0^1 e^{yt} (1 - xt)^{-\beta} (1-t)^{\gamma-\alpha-1} t^{\alpha-1} dt$$

In particular,

$$\Phi_1[3/2, 1; 2; x, y] = \frac{2}{\pi} \int_0^1 \frac{e^{yt} t^{1/2}}{(1 - xt)(1-t)^{1/2}} dt$$

Now making the transformation $t = \sin^2\theta$ and replacing x by ϵ^2 and y by $-\epsilon^2 y^2$ we obtain

$$\Phi_1[3/2, 1; 2; \epsilon^2, -\epsilon^2 y^2] = \frac{4}{\pi} \int_0^{\pi/2} \frac{\sin^2\theta \, e^{-\epsilon^2 y^2 \sin^2\theta}}{1 - \epsilon^2 \sin^2\theta} d\theta \tag{A1}$$

For real ϵ, y the integrand here is nonnegative on the closed interval $[0, \pi/2]$ and has no singularities for $0 \leqslant \epsilon < 1$; the integral in Eq. (A1) is therefore suitable for numerical quadrature and Φ_1 may thereby be computed.

It may also be shown [1, Eq. 15] that

$$\Phi_1 [3/2, 1; 2; \epsilon^2, -\epsilon^2 y^2] = \frac{2}{\epsilon^2 (1-\epsilon^2)^{1/2}} \left\{ e^{-y^2} - 2 \int_0^\infty t e^{-t^2} J_0(2yt) \, erf \left[\frac{(1-\epsilon^2)^{1/2}}{\epsilon} t \right] dt \right\} \tag{A2}$$

from which it follows that

$$\lim_{\epsilon \to 1} \epsilon^2 (1-\epsilon^2) \Phi_1 [3/2, 1; 2; \epsilon^2, -\epsilon^2 y^2] = 0$$

Hence Eq. (1) is valid in the limit for $\epsilon = 1$.

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