COMPARISON OF THE ROUGH SURFACE REFLECTION COEFFICIENT WITH SPECULARLY SCATTERED ACOUSTIC DATA U) NAVAL RESEARCH LAB WASHINGTON DC A R MILLER ET AL. 06 NOV 07

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Comparison of the Rough Surface Reflection Coefficient with Specularly Scattered Acoustic Data

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Comparison of the Rough Surface Reflection Coefficient with Specularly Scattered Acoustic Data

A comparison is made between a theoretically derived family of rough surface reflection coefficients and specularly scattered acoustic data.
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COMPARISON OF THE ROUGH SURFACE REFLECTION COEFFICIENT WITH SPECULARLY SCATTERED ACOUSTIC DATA

INTRODUCTION

Miller and Vegh [1] in treating reflection from the rough surface of the sea derived a one-parameter family of curves for the rough surface reflection coefficient or roughness factor $R$ given by

$$R(g, \epsilon) = \epsilon^2 \exp \left[ -2\epsilon^2 \eta^2 (2\pi g)^2 \right] I_0[2\epsilon^2 \eta^2 (2\pi g)^2]$$

$$+ (1 - \epsilon^2)^{1/2} \exp \left[ -4\eta^2 (2\pi g)^2 \right]$$

$$- \frac{1}{2} \epsilon^2 (1 - \epsilon^2) \Phi_1[\frac{3}{2}, 1; 2; \epsilon^2, -4\epsilon^2 \eta^2 (2\pi g)^2]$$

where

$$g \equiv (\sigma/\lambda) \sin \psi$$

and

$$\eta \equiv [1 + \frac{\pi}{2} (1 - \epsilon^2)]^{1/2}$$

Here $g$ is a measure of the effective surface roughness or simply surface roughness, $\epsilon (0 \leq \epsilon \leq 1)$ is the spectral width parameter, $\sigma$ is the standard deviation of the water surface elevation, $\psi$ is the grazing angle for specular reflection, $\lambda$ is the wavelength of the incident radiation, and $I_0(x)$ is the modified Bessel function of order zero. The function $\Phi_1(\alpha, \beta; y; x, y)$ is a confluent hypergeometric function in two variables first defined in 1920 by P. Humbert [2, p. 58]. In the Appendix we derive an integral representation for $\Phi_1$ that may be used for numerical computation.

$$R(g, \epsilon), \text{ given by Eq. (1), is essentially the Fourier transform of the probability density } D(y, \epsilon) \text{ for surface elevation } y \text{ where}$$

$$D(y, \epsilon) = \frac{\epsilon}{2\pi^{3/2} \eta \sigma} \exp \left( -\frac{y^2}{8\epsilon^2 \eta^2 \sigma^2} \right) K_0\left(\frac{y^2}{8\epsilon^2 \eta^2 \sigma^2}\right)$$

$$+ \frac{(1 - \epsilon^2)^{1/2}}{\pi^{3/2} \eta \sigma} \exp \left( -\frac{y^2}{4\eta^2 \sigma^2} \right) \left\{ \cos^{-1} \epsilon + \epsilon (1 - \epsilon^2)^{1/2} K_{\epsilon 0}(2\epsilon^2 - 1, y^2/8\epsilon^2 \eta^2 \sigma^2) \right\}$$

Here $K_0(x)$ is the MacDonald function or Bessel function of imaginary argument of order zero. $K_{\epsilon 0}(a, x)$ is an incomplete Lipschitz-Hankel integral of $K_0(x)$ and may be written in closed form either in terms of incomplete cylindrical functions [3] or in various ways in terms of Kampé de Fériet functions [4,5]; e.g.

$$K_{\epsilon 0}(a, z) = z K_0(z) A_1(a, z) + z^2 K_1(z) A_0(a, z)$$

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where

\[ A_1(a, z) \equiv F \begin{bmatrix} 0:1; & 1 \end{bmatrix}_{2:0;0} \begin{bmatrix} -: & 1/2; \ 1; \ a^2 z^2 \ z^2 \ \end{bmatrix}_{1/2, 3/2; \ -: \ -; \ 4 \ 4} \]

\[ + \frac{1}{2} a z F \begin{bmatrix} 0:2; & 1 \end{bmatrix}_{2:1;0} \begin{bmatrix} -: & 1,1; \ 1; \ a^2 z^2 \ z^2 \ \end{bmatrix}_{1,2; 3/2; \ -: \ -; \ 4 \ 4} \]

\[ A_0(a, z) \equiv F \begin{bmatrix} 0:1; & 1 \end{bmatrix}_{2:0;0} \begin{bmatrix} -: & 1/2; \ 1; \ a^2 z^2 \ z^2 \ \end{bmatrix}_{3/2, 3/2; \ -: \ -; \ 4 \ 4} \]

\[ + \frac{1}{4} a z F \begin{bmatrix} 0:2; & 1 \end{bmatrix}_{2:1;0} \begin{bmatrix} -: & 1,1; \ 1; \ a^2 z^2 \ z^2 \ \end{bmatrix}_{2,2; 3/2; \ -: \ -; \ 4 \ 4} \]

\[ D(y, \epsilon), \text{ given by Eq. (2), was derived in Ref. 1 by assuming that the water surface could be described locally by sinusoids with uniform phase distribution whose amplitude distribution is given by a density function derived by Rice [6] and by Cartwright and Longuet-Higgins [7]. Figure 1 gives graphs for } D(y, \epsilon), \text{ for various values of } \epsilon. \]

![Graph](image-url)

Fig. 1 - Density function \( D(y, \epsilon) \) for various values of the spectral width parameter \( \epsilon \)

**COMPARISON OF R(g, \epsilon) WITH ACOUSTIC DATA**

In 1980 DeSanto [8, p. 70, Fig. 5] compared \( R(g, \epsilon) \) with acoustic data from Clay, Medwin, and Wright [9]. Although \( R(g, \epsilon) \) was first derived in 1974 [10], a mathematically rigorous derivation was not obtained until 1984 [11]. In view of Eq. (1), it now appears appropriate to compare \( R(g, \epsilon) \) with
the aforementioned data. Whereas $R(g, 1)$ takes into account only the standard deviation, $\sigma$, of surface elevation, $R(g, \epsilon)$ is dependent on $\epsilon$ also and hence on the moments of the frequency energy spectrum $\Phi(s)$ of the surface through the equations [12, p. 346]

$$\epsilon^2 = (m_0 m_4 - m_2^2) / m_0 m_4$$

$$m_\nu \equiv \int_0^\infty s^\nu \Phi(s) \, ds \quad (m_0 = \sigma)$$

Figure 2 compares $R^2(g, 1/3)$ with the data given by Clay et al. in Fig. 5 of Ref. 9. $R^2(g, 1/3)$ appears to be in better agreement with this data than the multiple scattering theoretical result given in Fig. 5 of Ref. 8.

![Figure 2 — Comparison of the theoretical curve $R^2(g, 1/3)$ with experimental data](image)

**CONCLUSION**

One of the family of rough surface reflection coefficients agrees with acoustic data reasonably well; at least as well as the curve given previously by the multiple scattering model.

**REFERENCES**


Appendix

INTEGRAL REPRESENTATIONS FOR $\Phi_1[\alpha, \beta; y; x, y]$

The confluent double hypergeometric function $\Phi_1$ is defined by

$$\Phi_1[\alpha, \beta; y; x, y] \equiv \sum_{m,n=0}^{\infty} \frac{(\alpha)_m \beta_n}{(y)_n} \frac{x^m}{m!} \frac{y^n}{n!} \quad |x| < 1, \ |y| < \infty$$

The definition of $\Phi_1$ given in Erdélyi et al. [A1, p. 225] and Gradshteyn et al. [A2, 9.261, Eq. 1] is incorrect.

By using Ref. A3, p. 266

$$\frac{(\alpha)_p}{(\gamma)_p} = \frac{\Gamma(\gamma)}{\Gamma(\alpha) \Gamma(\gamma - \alpha)} \int_0^1 t^{\rho - \alpha - 1} (1 - t)^{-\alpha - 1} dt, \quad \text{Re} \gamma > \text{Re} \alpha > 0$$

with the definition of $\Phi_1$ given above we obtain

$$\Phi_1[\alpha, \beta; y; x, y] = \frac{\Gamma(\gamma)}{\Gamma(\alpha) \Gamma(\gamma - \alpha)} \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} \int_0^1 t^{m+n-\alpha-1} (1 - t)^{-\alpha - 1} (\beta)_m \frac{x^m y^n}{m! n!} dt$$

Now interchanging the integral sign and double sum and noting that

$$\sum_{n=0}^{\infty} \frac{(y)^n}{n!} = e^y, \quad \sum_{m=0}^{\infty} (\beta)_m \frac{(tx)^m}{m!} = (1 - tx)^{-\beta}$$

we obtain for $\text{Re} \gamma > \text{Re} \alpha > 0, \ |x| < 1, \ |y| < \infty$

$$\Phi_1[\alpha, \beta; y; x, y] = \frac{\Gamma(\gamma)}{\Gamma(\alpha) \Gamma(\gamma - \alpha)} \int_0^1 e^y (1 - xt)^{-\beta} (1 - t)^{-\alpha - 1} \rho^{-1} dt$$

In particular,

$$\Phi_1[3/2, 1; 2, x, y] = \frac{2}{\pi} \int_0^{\pi/2} e^{x t} \frac{\sin t}{(1 - xt) (1 - t)^{1/2}} dt$$

Now making the transformation $t = \sin^2 \theta$ and replacing $x$ by $\epsilon^2$ and $y$ by $-\epsilon^2$ we obtain

$$\Phi_1[3/2, 1; 2; \epsilon^2, -\epsilon^2 y^2] = \frac{4}{\pi} \int_0^{\pi/2} \sin^2 \theta \frac{e^{\epsilon^2 \sin^2 \theta}}{1 - \epsilon^2 \sin^2 \theta} d\theta$$

For real $\epsilon, y$ the integrand here is nonnegative on the closed interval $[0, \pi/2]$ and has no singularities for $0 \leq \epsilon < 1$; the integral in Eq. (A1) is therefore suitable for numerical quadrature and $\Phi_1$ may thereby be computed.
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It may also be shown [1, Eq. 15] that

\[ \Phi_1[3/2, 1; 2; \varepsilon^2, -\varepsilon^2y^2] = \frac{2}{\varepsilon^2(1 - \varepsilon^2)^{1/2}} \left\{ e^{-\varepsilon^2} - 2 \int_0^{\infty} t e^{-t^2} J_0(2\varepsilon t) \operatorname{erf} \left[ \frac{(1 - \varepsilon^2)^{1/2}}{\varepsilon} t \right] dt \right\} \]

(A2)

from which it follows that

\[ \lim_{\varepsilon \to 1} \varepsilon^2 (1 - \varepsilon^2) \Phi_1[3/2, 1; 2; \varepsilon^2, -\varepsilon^2y^2] = 0 \]

Hence Eq. (1) is valid in the limit for \( \varepsilon = 1 \).

REFERENCES


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