Review of Inversion Techniques for
Spaceborne Lidar Systems

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An examination of inversion techniques applicable for a spaceborne lidar system has been made. The purpose has been to recommend a technique that could be used with the lidar system under development by AFGL.

In a Rayleigh atmosphere with no or minimal gaseous absorption, a power law relationship between backscattering and extinction exists in which $k=1$. In an atmosphere with aerosols and clouds, the assumption of a power law relationship is tenuous at best. An examination of the literature leads us to conclude that the power law relationship has no physical basis to support its extensive use. For any application probing different regions of the atmosphere and different types of aerosols, one cannot assume a single set of power law parameters for the entire atmosphere.

**Title**: Review of Inversion Techniques for Spaceborne Lidar Systems

**Authors**: Derek W. Gallon and John R. Hummel

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**Address**: 121 Middlesex Turnpike, Burlington, MA 01803

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An alternate inversion approach has been presented based on fundamental radiative transfer characteristics of the atmosphere. The approach is rooted in fundamental physics but, still, does not eliminate the problem of more unknowns than equations. For the approach to be used, one must still make an assumption about the type of aerosol responsible for the lidar signal.
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1. INTRODUCTION

Laser radars, or lidars, are powerful tools for probing the atmosphere. They have been used, for example, in studies to probe for the presence of aerosols in the atmosphere, to obtain the profiles of absorbing gases such as water vapor and as a wind measuring tool. Theoretical studies have also been done to investigate the possibility of using lidars to invert the temperature structure of the atmosphere. In the past, the lidars have been typically used from ground-based platforms looking upwards or horizontally. A few studies have also utilized lidars mounted in aircraft or on balloons. Some of these latter studies have focused on preparing lidars for eventual use on space shuttles or satellites.

The Air Force Geophysics Laboratory (AFGL) is developing a lidar system to be flown on the Defense Meteorological Satellite Platforms (DMSP) series of meteorological satellites. The lidar system is being designed to study the optical properties of the atmosphere as well as to measure wind and composition.

The reflected laser energy can be used to invert information about the optical properties of the atmosphere. The techniques used to invert this information are the subject of this report. Chapter 2 examines the mathematical formulation of the inversion problem and presents the inversion techniques commonly used by researchers. Chapter 3 presents and compares results from a simulated spaceborne lidar
system utilizing one of the commonly used inversion tech-
niques. Chapter 4 presents a proposal for an inversion
 technique that addresses the needs and goals of a space-
borne lidar system. Finally, chapter 5 summarizes our re-
sults and conclusions.
2. FORMULATION OF THE LIDAR INVERSION PROBLEM

2.1 The Lidar Equation

The received signal for a single wavelength lidar system is described by the equation

\[ P(r) = \frac{P_0 r c}{2r^2} A \beta(r) \exp \left[ -2 \int_0^r \sigma(r') \, dr' \right] \]  

(1)

where \( P(r) \) is the instantaneous received power from the range \( r \), \( P_0 \) is the transmitted power of the laser over the pulse length \( r \), \( c \) is the speed of light, \( A \) is the effective system receiver area, \( \beta(r) \) and \( \sigma(r) \) are the backscatter and extinction coefficients, respectively. The equation (1) is commonly known as the lidar equation. In the above, the backscatter and extinction coefficients include contributions from all sources, molecular and particulate. The equation also assumes a single scattering atmosphere. Although not done at this time, (1) can be broken down to include the separate contributions. A more manageable form of the lidar equation can be established by defining a quantity, \( S(r) \), that is the logarithmic range-adjusted power

\[ S(r) = \ln[r^2 P(r)] \]  

(2)

Utilizing this quantity eliminates system dependent parameters from the lidar equation. The new form of the lidar equation then becomes
\[ S - S_0 = \ln[\beta/\beta_0] - 2 \int_{r_0}^{r} \sigma \, dr' \]  
\[ (3) \]

where \( S_0 = S(r_0) \), \( \beta_0 = \beta(r_0) \) and \( r_0 \) is a constant reference range. The differential equation corresponding to equation (3) is

\[ \frac{dS}{dr} = \frac{1}{\beta} \left( \frac{d\beta}{dr} \right) - 2\sigma \]  
\[ (4) \]

One of the goals of using lidars to probe the atmosphere is to invert information about the optical parameters, \( \sigma \) and \( \beta \). However, a crucial problem is faced when inverting the lidar equation in that it contains two unknown variables, the backscatter and extinction coefficients, and only one equation. In order to eliminate the problem of one equation and two unknowns, researchers have made assumptions about the relationship between \( \sigma \) and \( \beta \). We shall now examine some of the assumptions made concerning \( \sigma \) and \( \beta \) to put the lidar equation into a form that can be inverted. The implications of the assumptions will be discussed along with the advantages and disadvantages of each approach.

2.2 Homogeneous Atmosphere

If one assumes that the atmosphere is homogeneous, then \( \frac{d\beta}{dr} = 0 \) and equation (4) reduces to
The assumption of $\frac{d\theta}{dr} = 0$ gives rise to what is called the slope method of inversion. The value of the homogeneous extinction coefficient is evaluated as the slope of the least squares fit of the $S(r)$ data. An obvious requirement for the use of the slope method is that $\frac{dS}{dr}$ be negative otherwise negative values of the extinction coefficient will be obtained.

The slope method has been used by Murray\(^1\) with an infrared lidar system. In their study they used a CO\(_2\) lidar that operated at four wavelengths near 10.3 microns. The system operated over horizontal paths and was conceived as a way to measure visibility for aircraft landing operations. Their study indicated that a single-ended measurement approach yielded results that agreed with other measurements for ranges up to 10 km. For additional details on the slope method, the interested reader is referred to the papers by Collis\(^2\) and Viezee\(^3\).


2.3 The Ratio Method

The atmosphere is unlikely to be homogeneous over the entire path being probed by a lidar, especially a spaceborne platform. However, the atmosphere may be nearly homogeneous over small intervals of the path. In this case, one can apply a form of the slope method over the small, nearly homogeneous layers and then build up an approximation of the profile of \( \sigma(r) \). This method, known as the ratio or slice method of inversion assumes homogeneity within each layer, yet relaxes the condition that the extinction coefficient is constant. The extinction coefficient is then given as

\[
\sigma(r_i) = \frac{1}{2\Delta r_i} \ln \left[ \frac{\pi_i^2 P(r_i)}{\pi_{i+1}^2 P(r_{i+1})} \right]
\]

where the subscripts \( i \) and \( i+1 \) refer to the small, nearly homogeneous layers and \( \Delta r_i \) is the distance between the layers.

The assumption of a homogeneous or near homogeneous atmosphere suits this method primarily to horizontal viewing paths, such as would be used for aircraft visibility operations. The technique could also be used for low elevation


slant paths if it was known beforehand that the medium being studied did not have sharp boundaries below any critical altitude levels. However, before applying the ratio or slope techniques, one must be reasonably sure that homogeneity exists. Unfortunately, for many interesting situations such as dense cloud, fog, smoke and dust, these methods cannot be used. Even under stable conditions in fogs, local heterogeneities occur, thus invalidating the inversion process.

Looking down from space, the atmospheric density is exponentially increasing. The slope method will then be, by definition, inappropriate. The ratio method, however, could be utilized if the data were devoid of large fluctuations. Unfortunately, the large fluctuations can result from scientifically interesting phenomena such as aerosol layers or clouds. (The presence of these "scientifically interesting phenomena" can further complicate the inversion process by introducing multiple scattering effects for which there is not current accounting.)

2.4 Power Law Relationship Between Scattering & Extinction Coefficients

It has been suggested that under certain conditions the backscatter and extinction coefficients can be related by a power law expression of the form

\[ \beta = c \sigma^k \]  

(7)
where $c$ is a constant and $k$ depends upon the wavelength of the lidar and the properties of the obscuring medium. Assuming that the power law relationship is valid, the lidar equation can be given as

$$\frac{dS}{dr} = \frac{(k/\sigma)}{dr} - 2\sigma \quad (8)$$

This is a form of the Bernoulli or homogeneous Ricatti equation which has the general solution:

$$\sigma^{-1} = \exp \left[ -\int_{1}^{r} \frac{dS}{dr} \right]$$

$$\times \left[ c - 2/k \int_{1}^{r} \exp \left( -\int_{1}^{r} \frac{dS}{k dr} \right) dr \right] \quad (9)$$

Assuming $k$ to be constant, the solution may be simplified to the following:

$$\sigma = \frac{\exp \left[ (S - S_o)/k \right]}{\left[ \sigma_o^{-1} - 2/k \int_{r_o}^{r} \exp \left[ (S - S_o)/k \right] dr \right]} \quad (10)$$

This is sometimes known as the near-field solution. This solution is not a new one. It has been known for a number of years but, for horizontal or upward looking paths, the solution can be plagued with instability problems that result in questionable and/or unrealistic results. This

---

happens despite the fact that both the numerator and the denominator approach zero at nearly the same rate. The denominator tends to zero as the difference of two large numbers, creating a highly unstable situation. (A more detailed discussion of the instabilities inherent in (10) can be found in Klett\textsuperscript{7}.)

2.4.1 The "Klett Method"

Klett\textsuperscript{7} has proposed an alternate solution to (10) in which the integration constant in (9) is evaluated in terms of a range $r_m$ far from the lidar. Under this assumption, the solution is generated for ranges less than $r_m$ rather than ranges greater than $r_0$ as in the case of (10). Klett's approach, that utilizes what is known as the far-field solution, is given as

$$\sigma = \frac{\exp \left[ \frac{(S - S_m)}{k} \right]}{\left[ \sigma_m^{-1} + \frac{2}{k} \int_{r_m}^{r_m} \exp \left[ \frac{(S - S_m)}{k} \right] \, dr \right]}$$

This solution form of the Bernoulli equation is what is often known as "the Klett method." In a further modification to his solution, Klett\textsuperscript{8} allows for a variable backscatter-extinction ratio and the inclusion of Rayleigh scattering. Although similar in form to the near-field solution, it has

been found by many researchers\(^9,10\) to be more stable than the near-field solution. However, the majority of studies that have utilized the "Klett method" have employed horizontal or upward looking orientations.

2.4.2 Mid-Field Solution

Ferguson and Stephens\(^11\) have proposed a further alternative to the near-field solution that utilizes the stability of the Klett algorithm to calculate a boundary value at some middle range. In their original proposal, an iterative approach was employed to calculate an accurate far-field boundary condition. Mulders\(^12\) later pointed out that the iterative approach could be replaced with an analytic solution.

2.4.3 The Validity of the Power Law Relationship

The power law relationship allows the researcher to work with an equation with a known solution. The use of the "Klett method" offers mathematical stability. However, use of the power law presupposes knowledge of the attenuating medium being studied. In addition to "throwing the baby out


with the bath water", the power law relationship is not valid for all types of media that might be encountered by a spaceborne lidar system.

In all fairness to Klett, he stated in his original paper (see page 212 of Klett\textsuperscript{7}) that the use of the power law relationship is not appropriate for all aerosol types or wavelengths. Considering that so many researchers have been utilizing the power law relationship, it seems reasonable to examine the history of the power law relationship between backscattering and extinction. The results of that examination are summarized in Table 1.

Curcio and Knestrick\textsuperscript{13} are one of the earliest sources for the power law relationship between backscattered radiation and attenuation. In their paper, they correlated backscattering and atmospheric transmission from a series of horizontal measurements through fog, rain, drizzle, snow and clear air and found that an empirical relationship of the form $\beta = c \sigma^{0.66}$ fit their data. Their data were taken for white light conditions rather than for monochromatic radiation, such as one would have with a laser. The authors also note that their relationship will not hold in "industrially contaminated air."

There is no a priori reason to assume that the backscatter and extinction coefficients would be related by a

Table 1. An Examination of the History of the Power Law Assumption Relating Backscatter and Extinction

<table>
<thead>
<tr>
<th>AUTHOR(s)</th>
<th>FINDINGS</th>
</tr>
</thead>
</table>
| Curcio and Knestrick | From Measurements of Visibility in Clean Air  
A Relationship of the Form  
Range = Constant x Backscatter \(^{1.5}\)  
Was Found to Be Valid Within About 20\% |
| Twomey and Howell    | From a Theoretical Study Using Four Types of Size Distributions:  
1. A Relationship, but not An Unique One, Exists Between Backscattering & Extinction  
2. The Relationship Holds Better For White Light Rather Than Monochromatic Radiation |
| Penn                 | From A Theoretical Examination Concludes That No General Relationship Exists Between Backscatter and Extinction - The Variety of Aerosol Formation and Decay Processes Leads to Different Relationships |
| Pinnick et. al.      | From A Theoretical Examination of Water Clouds Conclude That An Approximate Relationship Can Be Found Between Backscatter and Extinction for Visible and Near-infrared Radiation |
| Mulders              | Measurements Indicated Rapid Changes Can Occur in Backscatter and Extinction Relationships |

unique power law relationship. Fenn\textsuperscript{15} has pointed out that only with a combination of changes in number densities and size distributions can one get a power law relationship between backscattering and extinction. If the aerosol size distributions, complex indices of refraction and particle shapes were all constant then one could assume a power law relationship. However, aerosols and clouds are highly variable phenomena with strong shape, altitude, humidity, composition and meteorological (e.g. wind) dependencies. Even if power law relationships did exist for the aerosols and clouds, one would not know which set of values, k and c, to use because clouds and aerosols can be found at the same altitudes, often at the same times. Also, Mulders\textsuperscript{12} has shown that power law relationships, if they can be obtained, can be fitted to data but that the values of the relevant parameters can change as a function of time. Mulders concludes from a series of horizontal measurements over the ocean that the power law parameters are not constant for more than a few hours.

Pinnick et. al.\textsuperscript{16} have presented results from calculations of backscatter and extinction through stratus and cumulus clouds. Their zero order solution (their term) calculations were performed for 156 measured cloud droplet


distributions for visible, near IR and IR wavelengths. They found that a linear relationship exists between backscatter and extinction at 0.6328 µm but that none exists at 10.6 µm.

Shettle\textsuperscript{17} has presented results of backscatter and extinction coefficient comparisons for the tropospheric aerosol models in LOWTRAN 6 as a function of relative humidity. His calculated results for the wavelengths 0.53 and 10.59 µm are reproduced as Figures 1 (a.) and (b.), respectively. Each point on the curves represents a different relative humidity. Table 2 gives the corresponding exponents required to fit the data in Figure 1 (a.) with a power law over the given relative humidity ranges. (The attenuation coefficients increase with relative humidity.) His results demonstrate that a power law relationship is not appropriate for tropospheric aerosols except over very limited relative humidity ranges and for given wavelengths. Figure 2 shows similar results for the AFGL stratospheric aerosol models. The three points refer to representative background stratospheric conditions, aged volcanic aerosols and fresh volcanic aerosols [Shettle, private communication, 1986]. Again, a unique power law relationship does not exist for stratospheric aerosols. In a realistic situation, the stratosphere could contain all three types of aerosols with three different characteristic indices of refraction and size distributions.

Figure 1. Backscatter and Extinction Coefficients for Different AFGL Aerosol Models At (a.) 0.53 microns and (b.) 10.59 microns
Table 2. Values of the Exponent $k$ Required to Fit the APGL Boundary Layer Aerosol Models Shown in Figure 1 (a.) With A Power Law Over the Given Relative Humidity Ranges

<table>
<thead>
<tr>
<th>Aerosol Type</th>
<th>Relative Humidity Range (%)</th>
<th>0 - 70</th>
<th>70 - 80</th>
<th>80 - 99</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rural</td>
<td></td>
<td>-0.4</td>
<td>0.09</td>
<td>0.9</td>
</tr>
<tr>
<td>Maritime</td>
<td></td>
<td>0.7</td>
<td>0.9</td>
<td>1.2</td>
</tr>
<tr>
<td>Urban</td>
<td></td>
<td>1.2</td>
<td>1.2</td>
<td>1.2</td>
</tr>
<tr>
<td>Tropospheric</td>
<td></td>
<td>0.05</td>
<td>0.1</td>
<td>0.7</td>
</tr>
</tbody>
</table>
Figure 2. Backscatter and Extinction Coefficients At 0.55 microns for Representative Background Stratospheric Aerosols, Aged Volcanic Aerosols and Fresh Volcanic Aerosols (Shettle, private communication)
Evans\textsuperscript{18} has confirmed the results of Shettle\textsuperscript{17} and extended the analysis to other aerosol types. Evans performed calculations of the backscatter/extinction ratio using Mie theory for a large range of values of the real and imaginary components of the index of refraction and the modal size parameter. His results are summarized in Table 3 and can be used by researchers to determine for what wavelengths and size distributions one could utilize a power law relationship.

2.5 Linear Wavelength Dependence of Aerosol Scattering

The use of more than one laser wavelength increases the amount of information about the medium being studied. Utilizing frequency doubling and tripling techniques, one can, for instance, use one laser line to provide information about the molecular components of the atmosphere, thereby helping to define the background atmospheric signal, and use a second line to study aerosol contributions.

DeLuisi et al.\textsuperscript{19} have proposed using a dual or triple wavelength lidar system in which the aerosol scattering is


Table 3. Summary of Results of Evans\textsuperscript{18} Relating to Power Law Relationships Between Backscatter and Extinction Coefficients

<table>
<thead>
<tr>
<th>The Ratio of Backscatter to Extinction Coefficients Is <strong>Insensitive</strong> to Changes in the Size Distribution:</th>
</tr>
</thead>
<tbody>
<tr>
<td>For Small Non-absorbing Particles</td>
</tr>
<tr>
<td>For Water Droplets with a Modal Particle Size Greater than 1 micron and Visible &amp; Near-infrared Wavelengths</td>
</tr>
<tr>
<td>For Water Droplets with a Modal Particle Size Greater than 10 microns and Wavelengths in the Range 6 - 12 microns</td>
</tr>
<tr>
<td>Real Part of Index of Refraction = 1.5, Imaginary Part $\leq 10^{-4}$ and the Modal Size Parameter, $X_m \geq 6$</td>
</tr>
<tr>
<td>$2kX_m \geq 1$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>The Ratio of Backscatter to Extinction Coefficients Is <strong>Sensitive</strong> to Changes in the Size Distribution:</th>
</tr>
</thead>
<tbody>
<tr>
<td>For Water Droplets with Modal Size Parameters $\leq 2$ and Wavelengths in the Range 0.2 - 12 microns</td>
</tr>
<tr>
<td>For Rural or Maritime Type Aerosols at Any Wavelength in the Visible &amp; Infrared</td>
</tr>
<tr>
<td>For Water Droplets with Particle Sizes on the Order of Several Microns &amp; Far-infrared Wavelengths</td>
</tr>
<tr>
<td>Hygroscopic Aerosols and Relative Humidity Above 70%</td>
</tr>
</tbody>
</table>

\[ X_m = \left( \frac{2 \pi R_m}{\lambda} \right) \]

$k$ = Imaginary Part of Index of Refraction
linearly related. With their formulation, the scattering coefficient for wavelength \( i \) is given as

\[
\beta(r)_i = [1 + c (\lambda_1 - \lambda_i)/(\lambda_1 - \lambda_3)] \beta(r)_1
\]

where \( \lambda \) is the wavelength. Utilizing this assumption DeLuisi et al. were able to develop an inversion technique with equal numbers of equations and unknowns that would allow one to obtain a separate inversion of molecular and aerosol optical properties.

The key to the DeLuisi et al. approach is the assumption that a linear relationship exists relating the aerosol extinction properties at different wavelengths. Figure 3 displays the wavelength dependence of the aerosol extinction and scattering coefficients for the background stratospheric, aged volcanic and fresh volcanic aerosol models used in the AFGL model atmospheres (Shettle, private communication). (For the background stratospheric conditions the extinction and backscatter coefficients overlap.) The figures show that the wavelength dependences are not necessarily linear and that they differ from one aerosol type to another. This being the case, one would have to guess the type of aerosol being probed in order to incorporate the appropriate wavelength dependence.
Figure 3. Backscatter and Extinction Coefficients As A Function of Wavelength for (a.) Background Stratospheric Aerosols, (b.) Aged Volcanic Aerosols and (c.) Fresh Volcanic Aerosols.
2.6 Applicability of the Available Inversion Techniques for Spaceborne Applications

Table 4 summarizes the results from the examination of the various inversion techniques from the perspective of a spaceborne lidar system. The table lists the fundamental assumptions of each of the techniques, advantages and disadvantages.

The slope and ratio methods offer simplicity but are not appropriate for a spaceborne system. The slope method cannot be used because the viewing path is inhomogeneous (i.e. exponentially increasing). The ratio method also cannot be used because the degree of inhomogeneity gives rise to negative attenuation coefficients.

The inversion technique that is commonly used by researchers is based on the power law assumption with the "Klett technique" for the solution. The assumption of a power law relationship yields an equation with a known solution. The "Klett" approach is used rather than the near-field solution because of its supposed improvements in stability. The use of the word "supposed" is deliberate. The studies to-date with the "Klett" technique have not demonstrated that the far-field solution is inherently more stable than the near-field solution. The studies to-date have primarily involved horizontal or upward looking path configurations and it may be that the choice of path configuration determines the stability characteristics of a given form of a Bernoulli equation solution. Kastner and
<table>
<thead>
<tr>
<th>INVERSION TECHNIQUE</th>
<th>FUNDAMENTAL ASSUMPTION(s)</th>
<th>ADVANTAGES</th>
<th>DISADVANTAGES</th>
</tr>
</thead>
<tbody>
<tr>
<td>Slope Method</td>
<td>Homogeneous Atmosphere Over Entire Path</td>
<td>Simple Solution</td>
<td>Not Applicable (Atmosphere is Inhomogeneous)</td>
</tr>
<tr>
<td>Ratio Method</td>
<td>Homogeneous Atmosphere Over Portions of Path</td>
<td>Allows Limited Inhomogeneity</td>
<td>Not Applicable (Gives Negative Values of Extinction)</td>
</tr>
<tr>
<td>Power Law</td>
<td>Backscatter-Extinction Related By A Power Law $\beta = c \sigma k$</td>
<td>Analytic Solution</td>
<td>No Physical Basis</td>
</tr>
<tr>
<td>&quot;Far-field&quot; Solution</td>
<td></td>
<td>Can Be Used to Estimate Optical Properties</td>
<td>No One Power Law for Entire Atmosphere</td>
</tr>
<tr>
<td>&quot;Near-field&quot; Solution</td>
<td></td>
<td>More Stable for Horizontal and Ground Up Paths</td>
<td>Can Change Quickly</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Need to Know the Relationship In Advance</td>
</tr>
</tbody>
</table>
Quenzel\textsuperscript{20} have examined the usefulness of the near- and far-field algorithms in the context of a downward-looking, spaceborne lidar system. Their results indicate that in an atmosphere with low turbidity the near-field solution yields better results than a far-field solution. In an atmosphere with high turbidity, they found that the far-field solution gave better results.

The next chapter presents results from a simulated downward looking spaceborne system that utilizes a power law approach for inversion. Results from both near- and far-field solutions will be presented in order to gauge the superiority, if any, of one solution approach over another.

3. SIMULATION OF INVERSIONS FROM A SPACEBORNE LIDAR SYSTEM

The simulation is intended to be representative of data taken from the lidar system under development by AFGL. A version of the system has been flown on a high altitude balloon from the White Sands Missile Range, New Mexico. The features of the lidar system and the test flight are summarized in Table 5 (Bedo, private communication, 1986).

3.1 Simulated Atmosphere

Lidar signals from a midlatitude summer atmosphere under a variety of conditions are being simulated. A midlatitude summer atmosphere was chosen as being most representative of the conditions at the time of the launch. Tabulated optical parameters at 337 and 514.5 nm were taken from the work of McClatchey et. al. for this study. These wavelengths were assumed to be close enough to the actual lidar lines for the purpose of this study.

The scattering and extinction coefficients for an aerosol-free atmosphere at 337 and 514.5 nm are shown in Figures 4 and 5, respectively. The 337 nm extinction coefficients include molecular absorption from ozone as well as Rayleigh scattering. As shown in Figure 6, the data do not exhibit a linear relationship that corresponds to a power law of k=1. Figure 7 shows the individual components

Table 5. Details of the AFGL Lidar System and the Test Flight From the White Sands Missile Range, NM

<table>
<thead>
<tr>
<th>Description</th>
<th>Details</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wavelengths</td>
<td>355 and 532 nm</td>
</tr>
<tr>
<td>Field of View</td>
<td>2 1/2 milliradians. Lidar looks up, down and 2 degrees above horizon</td>
</tr>
<tr>
<td>Altitude Resolution</td>
<td>150 meters</td>
</tr>
<tr>
<td>Pulse Rate</td>
<td>10 pulses per second. (Data averaged over 10 pulses to smooth out expected 10% deviation between pulses.)</td>
</tr>
<tr>
<td>Date of Flight</td>
<td>24 Aug 1984</td>
</tr>
<tr>
<td>Balloon Float Altitude</td>
<td>33 Km</td>
</tr>
<tr>
<td>Weather Support Data</td>
<td>Radiosonde data taken from Roswell, NM at 2130 LST. No moonlight reported.</td>
</tr>
</tbody>
</table>
Figure 4. (a.) Backscatter and (b.) Extinction Coefficients As A Function of Altitude for a Midlatitude Summer, Aerosol-free Atmosphere at 337 nm
Figure 5. (a.) Backscatter and (b.) Extinction Coefficients As A Function of Altitude for a Midlatitude Summer, Aerosol-free Atmosphere at 514.5 nm
Figure 6. Backscatter Coefficient As A Function of Extinction Coefficients at 337 nm For An Aerosol-free Atmosphere
Figure 7. Extinction, Scattering and Absorption Coefficients As A Function of Altitude at 337 nm
as a function of altitude. The 514.5 nm values consist of Rayleigh scattering only and correspond to a power law relationship in which $k=1$, as shown in Figure 8. At both wavelengths, the scattering coefficients are those at a backscattering angle of 180 degrees.

The results to be presented are referenced to a spaceborne platform. Therefore, the near-field solution refers to a solution that begins at high altitudes and proceeds toward the surface. The far-field solution begins at or near the surface and proceeds towards the spacecraft.

The simulations will be performed for the eight cases listed in Table 6. It is assumed that the short wavelength laser line would be used primarily to define the molecular components of the atmosphere and that the visible line would be used for aerosol studies. Under this assumption, one would invert the shorter wavelength data first to determine the gaseous components and then invert the visible data to extract any aerosol information.

In the results that follow it is assumed that the near- and far-field boundary conditions are known accurately. Results will also be presented in which the boundary conditions are perturbed by +/- 10 and 40% to determine the effects of inaccuracies in the boundary conditions on the inversion algorithms.
Aerosol-Free Atmosphere - 514.5nm

Figure 8. Backscatter Coefficients As A Function of Extinction Coefficients at 514.5 nm For An Aerosol-free Atmosphere
Table 6. Test Cases Used in the Lidar Simulation.

<table>
<thead>
<tr>
<th>CASE</th>
<th>CONDITIONS</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Aerosol-Free Atmosphere at 337 nm</td>
</tr>
<tr>
<td>2</td>
<td>Aerosol-Free Atmosphere at 514.5 nm</td>
</tr>
<tr>
<td>3</td>
<td>Case 2 Plus Background Stratospheric Aerosols</td>
</tr>
<tr>
<td>4</td>
<td>Case 2 Plus Clear Tropospheric Conditions</td>
</tr>
<tr>
<td>5</td>
<td>Case 2 Plus Combined Stratospheric &amp; Tropospheric Aerosols (Cases 3 &amp; 4)</td>
</tr>
<tr>
<td>6</td>
<td>Case 2 Plus Aged Volcanic Aerosols</td>
</tr>
<tr>
<td>7</td>
<td>Case 2 Plus Fresh Volcanic Aerosols</td>
</tr>
<tr>
<td>8</td>
<td>Case 2 Plus Combined Background Stratospheric &amp; Fresh Volcanic Aerosols</td>
</tr>
</tbody>
</table>
3.1.1 Case 1 - Aerosol-free Atmosphere at 337 nm

Figures 9 (a.) and (b.) show the results for an aerosol-free simulated atmosphere at 337 nm from the near- and far-field inversions, respectively. The results are given for k values of 0.8, 1.0 and 1.2. The differences between the inverted atmospheric data and the simulated atmosphere with k=1.0 are small for both the near- and far-field solutions with the differences being due to the presence of ozone absorption (see Figure 7).

Figure 10 shows the results of the inversions with changes in the boundary conditions considered. The near-field solution results, Figure 10 (a.), show a somewhat greater sensitivity to changes in the boundary conditions than do the far-field solution results, Figure 10 (b).

3.1.2 Case 2 - Aerosol-free Atmosphere at 514.5 nm

Figures 11 (a.) and (b.) show the results for a simulated atmosphere at 514.5 nm from the near- and far-field inversions, respectively. For both solution forms, a value of k=1 yielded the best solutions, as they should, seeing that the simulated data were based on a power law relationship between the extinction and backscattering. The differences shown in the curves are due to the linear fitting of the simulated data. Figures 12 (a.) and (b.) show the results with perturbations in the boundary conditions considered.
Figure 9. Comparison of Simulated and Inverted Data at 337 nm for an Aerosol-free Atmosphere for the (a.) Near-field and (b.) Far-field Solutions.
Figure 10. Inverted Results at 337 nm with Variations in the Boundary Conditions Considered for (a.) Near-field and (b.) Far-field Solutions
Figure 11. Comparison of Simulated and Inverted Data at 514.5 nm for an Aerosol-free Atmosphere for the (a.) Near-field and (b.) Far-field Solutions.
Figure 12. Inverted Results at 514.5 nm with Variations in the Boundary Conditions Considered for (a.) Near-field and (b.) Far-field Solutions
The preceding two sets of results demonstrate that a power law approximation will reproduce an atmosphere consisting of Rayleigh scatterers with little or no molecular absorption. The results to follow will examine the ability of a power law approach to accurately reproduce an atmosphere containing Rayleigh as well as non-Rayleigh components. It is presumed that the non-Rayleigh components are aerosols or clouds. The results will utilize the 514.5 nm laser line.

3.1.3 Case 2 Plus Background Stratospheric Aerosols

This case considers an atmosphere with background stratospheric aerosols added to a background Rayleigh atmosphere. The aerosols are added between 10 and 25 km. Figures 13 (a.) and (b.) show, respectively, the backscatter and extinction coefficients for the case. The background aerosol-free values are dashed in for reference. Figure 14 gives the backscatter coefficients plotted against the extinction coefficients.

The near- and far-field inversions are shown in Figures 15 (a.) and (b.), respectively. In both the near- and far-field solutions, a profile based $k=1$ reproduces the simulated data above and below the aerosol layer. Figures 16 (a.) and (b.) show the sensitivity of each solution to changes in boundary conditions with a value of $k=1$. 
Figure 13. (a.) Backscatter and (b.) Extinction Coefficients As A Function of Altitude for a Midlatitude Summer Atmosphere at 514.5 nm With Background Stratospheric Aerosols.
Figure 14. Backscatter Coefficients As A Function of Extinction Coefficients at 514.5 nm For A Midlatitude Summer Atmosphere With Background Stratospheric Aerosols
Figure 15. Comparison of Simulated and Inverted Data at 514.5 nm for Case 3 For Various Values of k for the (a.) Near-field and (b.) Far-field Solutions
Figure 16. Inverted Results at 514.5 nm with Variations in the Boundary Conditions Considered for Case 3 for the (a.) Near-field and (b.) Far-field Solutions.
3.1.4 Case 2 Plus Clear Tropospheric Aerosols

In this example, a layer of tropospheric aerosols is added between the surface and 10 km. The optical conditions correspond to a "clear" atmosphere. Figures 17 (a.) and (b.) show the backscatter and extinction coefficients as a function of altitude. Figure 18 shows the plot of the backscatter versus the extinction coefficients.

The results from the inversion with various values of k are shown in Figures 19 (a.) and (b.). Both approaches give good results with a k=1 power law for altitudes above 10 km. The near-field solution looks like a value of k between 0.8 and 1.0 will give a reasonable fit. The far-field solution has reasonable fit with k=0.8. Figures 20 (a.) and (b.) show the results with variations in the boundary condition considered. The results assumed k=1.

3.1.5 Case 2 Plus Combined Stratospheric and Tropospheric Aerosols

This case corresponds to the combination of the background stratospheric aerosols with the clear tropospheric aerosols from the preceding case. Figures 21 (a.) and (b.) show the backscatter and extinction coefficients, respectively, as a function of altitude. Figure 22 displays the backscatter coefficients plotted against the extinction coefficients.

Figures 23 (a.) and (b.) show the results of the inversion with k varied and Figures 24 (a.) and (b.) show the results of the variation of the boundary conditions.
Rayleigh + Tropospheric Aerosols

Wavelength - 514.5 nm

Figure 17. (a.) Backscatter and (b.) Extinction Coefficients As A Function of Altitude for a Midlatitude Summer Atmosphere at 514.5 nm With "Clear" Tropospheric Aerosols
Figure 18. Backscatter Coefficients As A Function of Extinction Coefficients at 514.5 nm For A Midlatitude Summer Atmosphere With "Clear" Tropospheric Aerosols
Figure 19. Comparison of Simulated and Inverted Data at 514.5 nm for Case 4 for Various Values of k for the (a.) Near-field and (b.) Far-field Solutions
Figure 20. Inverted Results at 514.5 nm with Variations in the Boundary Conditions Considered for Case 4 for the (a.) Near-field and (b.) Far-field Solutions.
Figure 21. (a.) Backscatter and (b.) Extinction Coefficients As A Function of Altitude for a Midlatitude Summer Atmosphere at 514.5 nm With Background Stratospheric and "Clear" Tropospheric Aerosols

Tropospheric + Stratospheric Aerosols

Wavelength - 514.5 nm

Atmospheric Background

Background Stratospheric Aerosols Added

"Clear" Tropospheric Aerosols Added

Atmospheric Background

Background Stratospheric Aerosols Added

"Clear" Tropospheric Aerosols Added

Extinction Coefficient (km$^{-1}$)
Figure 22. Backscatter Coefficients As A Function of Extinction Coefficients at 514.5 nm For A Midlatitude Summer Atmosphere With Background Stratospheric Aerosols and "Clear" Tropospheric Aerosols
Figure 23. Comparison of Simulated and Inverted Data at 514.5 nm for Case 5 For Various Values of k for the (a.) Near-field and (b.) Far-field Solutions
Figure 24. Inverted Results at 514.5 nm with Variations in the Boundary Conditions Considered for Case 5 for the (a.) Near-field and (b.) Far-field Solutions.
None of the $k$ values tried yielded an acceptable fit to the simulated data. A value of $k$ of 0.8 gave a reasonable fit to the far-field solution for altitudes below 10 km but an unacceptable fit above 10 km. This agrees with the results for the individual aerosols layers (see Sections 3.1.4 and 3.1.5). One can conclude from this, that one power law cannot represent an atmosphere with distinct aerosol layers.

3.1.6 Case 2 Plus Aged Volcanic Aerosols

Figures 25 (a.) and (b.) show the backscatter and extinction coefficients as a function of altitude for an atmosphere with aged volcanic aerosols added to an aerosol-free background. The backscatter and extinction coefficients, as shown in Figure 26, are those in LOWTRAN 6 [Shettle, private communication, 1986].

Figures 27 (a.) and (b.) give the results from the inversions with different values of $k$ and Figures 28 (a.) and (b.) give the results from the variation in the boundary conditions with $k=1$. As with the background stratospheric aerosol results (e.g. Figure 15), a power law with $k=1$ will reproduce the aerosol-free parts of the atmosphere but not those with aerosols present.

3.1.7 Case 2 Plus Fresh Volcanic Aerosols

Figures 29 (a.) and (b.) show the backscatter and extinction coefficients as a function of altitude for an atmosphere with fresh volcanic aerosols added to an aerosol-
Rayleigh + Aged Volcanic

Wavelength - 514.5nm

(a)

(b)

Figure 25. (a.) Backscatter and (b.) Extinction Coefficients As A Function of Altitude for a Midlatitude Summer Atmosphere at 514.5 nm With Aged Volcanic Aerosols
Figure 26. Backscatter Coefficients As A Function of Extinction Coefficients at 514.5 nm For A Midlatitude Summer Atmosphere With Aged Volcanic Aerosols
Figure 27. Comparison of Simulated and Inverted Data at 514.5 nm for Case 6 for Various Values of k for the (a.) Near-field and (b.) Far-field Solutions.
Figure 28. Inverted Results at 514.5 nm with Variations in the Boundary Conditions Considered for Case 6 for the (a.) Near-field and (b.) Far-field Solutions
Rayleigh + Fresh Volcanic
Wavelength - 514.5nm

Figure 29. (a.) Backscatter and (b.) Extinction Coefficients As A Function of Altitude for a Midlatitude Summer Atmosphere at 514.5 nm With Fresh Volcanic Aerosols.
free background. The backscatter and extinction coefficients, as shown in Figure 30, are those in LOWTRAN 6 [Shettle, private communication, 1986].

Figures 31 (a.) and (b.) give the results from the inversions with different values of $k$ and Figures 32 (a.) and (b.) give the results from the variation in the boundary conditions with $k=1$. As with the background stratospheric aerosol results (e.g. Figure 14), a power law with $k=1$ will reproduce the aerosol-free parts of the atmosphere but not those with aerosols present.

### 3.1.8 Case 2 Plus Combined Background Stratospheric and Fresh Volcanic Aerosols

Figures 33 (a.) and (b.) show the backscatter and extinction coefficients as a function of altitude for an atmosphere with 50% background stratospheric and 50% fresh volcanic aerosols added to an aerosol-free background. Figure 34 displays the backscatter and extinction coefficients.

Figures 35 (a.) and (b.) give the results from the inversions with different values of $k$ and Figures 36 (a.) and (b.) give the results from the variation in the boundary conditions with $k=1$. Under these conditions, a power law with $k=1$ does a reasonable job of reproducing the simulated data.
Figure 30. Backscatter Coefficients As A Function of Extinction Coefficients at 514.5 nm For A Midlatitude Summer Atmosphere With Fresh Volcanic Aerosols
Figure 31. Comparison of Simulated and Inverted Data at 514.5 nm for Case 7 For Various Values of k for the (a.) Near-field and (b.) Far-field Solutions
Figure 32. Inverted Results at 514.5 nm with Variations in the Boundary Conditions Considered for Case 7 for the (a.) Near-field and (b.) Far-field Solutions
Rayleigh + Background + Fresh Volcanic

Wavelength = 514.5 nm

50% Fresh Volcanic & 50% Background Stratospheric Aerosols Added

Atmospheric Background

Figure 33. (a.) Backscatter and (b.) Extinction Coefficients As A Function of Altitude for a Midlatitude Summer Atmosphere at 514.5 nm With 50% Background Stratospheric and 50% Fresh Volcanic Aerosols
Figure 34. Backscatter Coefficients As A Function of Extinction Coefficients at 514.5 nm For A Midlatitude Summer Atmosphere With Background and Fresh Volcanic Aerosols
Figure 35. Comparison of Simulated and Inverted Data at 514.5 nm for Case 8 for Various Values of k for the (a.) Near-field and (b.) Far-field Solutions.
Figure 36. Inverted Results at 514.5 nm with Variations in the Boundary Conditions Considered for Case 8 for the (a.) Near-field and (b.) Far-field Solutions
3.2 Evaluating the Boundary Conditions in a "Real World" Situation

In the preceding sections, it was assumed that the boundary conditions were known. Then, it was shown that inaccuracies in the boundary conditions could have a significant impact on the inverted results. It is fair to ask the question, "Can the boundary conditions be known accurately?"

If one were using the far-field solution approach, the extinction coefficients at or near the surface would be required. If the visibility is known, the extinction coefficient at visible wavelengths can be evaluated from the Koschmieder formula

\[ \sigma = \frac{3.912}{V} \quad (13) \]

where \( V \) is the meteorological range. If the observer visibility, \( \text{V}_{\text{obs}} \), is known, then \( V \) can be approximated as

\[ V = (1.3 \pm 0.3) \text{V}_{\text{obs}} \quad (14) \]

The observer visibility is not an exact measure of the transmission properties of the atmosphere and can vary greatly from one observer to another. Even with observer errors ignored, it is a quantity that can vary greatly during the day.

Over the ocean or uninhabited land areas, one would not have a way to calculate the extinction coefficient boundary conditions. Even if a surface extinction coefficient clima-
tology existed, it would still not be accurate enough for the inversion of lidar data.

For the near-field solution, the boundary condition represents the extinction at a high altitude, presumably in an altitude region well above any aerosol or cloud layers. In this case, the Rayleigh scattering values can be used for the boundary conditions. Assuming that the laser line was one in which little or no molecular absorption occurred, the assumption would be valid. Seeing that the Rayleigh scattering is primarily dependent upon the wavelength (there is a minor dependence of pressure and temperature upon the atmospheric index of refraction), the near-field boundary conditions could be evaluated utilizing Rayleigh scattering coefficients. The net effect is that the near-field boundary conditions can be evaluated with an accuracy that can not be achieved for the far-field solution.

3.3 Summary of Results Utilizing A Power Law Inversion Approach

The calculations just presented have demonstrated that a power law inversion approach can accurately invert an atmosphere that only consists of Rayleigh scatters. The real atmosphere is rarely free of aerosols and, as demonstrated, a power law approach does not accurately invert the optical properties of a Rayleigh and non-Rayleigh atmosphere. To complicate matters, the atmosphere contains mixtures of different kinds of aerosols. Therefore, even if a power law could invert the optical properties for a given
aerosol, one would have to know in advance what kind of aerosol was being probed so as to select the correct value of k. This requirement coupled with the lack of a physical justification for the power law assumption forces one to conclude that a power law inversion approach cannot be used for the inversion of data from a spaceborne lidar system. An alternate approach can be recommended that is rooted in the physics of radiative transfer. The alternate approach will now be presented in the next chapter.
4. AN ALTERNATE INVERSION APPROACH FOR A SPACEBORNE LIDAR SYSTEM

Assuming that one can measure the atmospheric density profile, an expected profile of signal returns can be calculated (e.g. Equation 1) for an atmosphere of pure Rayleigh scatterers, \( P_{\text{Ray}} \). From this profile for a pure Rayleigh atmosphere, one can calculate a profile of the ratio of the actual lidar returns to that expected from a Rayleigh atmosphere, \( R(r) \)

\[
R(r) = \frac{P_{\text{actual}}(r)}{P_{\text{Ray}}(r)}
\]  

(15)

The atmospheric extinction profile consists of molecular and aerosols components,

\[
\sigma(r) = \sigma_{\text{Ray}}(r) + \sigma_{\text{Aer}}(r)
\]  

(16)

Expressing Eq (15) in its component terms gives

\[
R(r) = \frac{\beta_{\text{bs},\text{Ray}}(r) + \beta_{\text{bs},\text{Aer}}(r)}{\beta_{\text{Ray}}(r)} \cdot \exp \left[ -2\sigma_{\text{Aer}}(r) \cdot dr \right]
\]  

(17)

where \( \beta_{\text{bs},\text{Aer}} \) and \( \beta_{\text{bs},\text{Ray}} \) are, respectively, the aerosol and Rayleigh backscatter coefficients.

The single scattering albedo, \( \omega_0 \), can be given as the ratio of the total scattering to extinction coefficients

\[
\omega_0 = \frac{\beta_S}{\sigma}
\]  

(18)

The single scattering albedo can also be given in terms of the phase function, \( p(\Omega) \),

\[
\omega_0 = \frac{1}{4\pi} \int p(\Omega) \, d\Omega
\]  

(19)
Substituting Equation (18) into Equation (17) gives

\[ R(r) = \frac{\beta(r)_{\text{Ray}} + \beta(r)_{S, \text{AEF}}(180)}{\beta(r)_{\text{Ray}}} \exp\left[-2\int \beta_s \, dz'/\omega_0\right] \]  

(20)

Equation 20 can be solved numerically with an iterative method, such as the Newton-Raphson technique, to solve for the total scattering coefficient.

A sample inversion using this alternate technique is given in Figure 37. The sample calculations were made using the aged volcano parameters of Case 6. The calculations assumed a constant single scattering albedo of 0.9519. The inversion result is quite good, with a maximum difference between the actual profile of about 8%. Figures 38 (a) and (b) show, respectively, the impact of variations in the single scattering albedo and fraction of scattering into the backwards direction. In both cases, the values were perturbed by + and -10%. The results for this case indicated that the inversion was somewhat less sensitive to variations in the single scattering albedo. A full set of sensitivity calculations should be performed utilizing other assumed aerosol models before any general conclusions can be made concerning what are the most sensitive parameters in this alternate inversion technique. For example, one must have the profile of Rayleigh scattering extinction

Figure 37. Inverted Results at 514.5 nm with the Alternate Inversion Techniques for a Simulated Atmosphere with Aged Aerosols
Figure 38. Inverted Results at 514.5 nm with Variations in (a.) the Single Scattering Albedo and (b.) the Fraction of Backscattered Radiation
coefficients and to do this one must know the atmospheric density profile. Also, one must have some information about the single scattering albedo and the fraction of backscattered radiation. One can obtain these either from the literature or from detailed sets of Mie calculations. In order to utilize that information, one must be willing to limit the range of possible values by assuming some knowledge about the aerosol layer. For example, one could say that if the returns are from a specific altitude region they must be from a certain type of aerosol. Also, using the latitude and longitude of the satellite track one could say whether or not the returns are from an urban or rural type of aerosol. The point to stress is that the alternate technique still requires some knowledge of the aerosol, primarily a guess of what type of aerosol is causing the return.
5. SUMMARY OF INVERSION TECHNIQUES AND RECOMMENDATIONS FOR FUTURE WORK

5.1 Summary of Results

An examination of inversion techniques applicable for a spaceborne lidar system has been performed. The purpose of the examination has been to recommend a technique that could be used with the lidar system under development by AFGL.

In a purely Rayleigh atmosphere with no or minimal gaseous absorption, a power law relationship between backscattering and extinction exists in which \( k=1 \). In an atmosphere with aerosols and clouds, the assumption of a power law relationship is tenuous at best. Although popular with researchers, the examination of the literature leads us to conclude that the power law relationship has no physical basis to support its extensive use. For any application probing different regions of the atmosphere and different types of aerosols, one cannot assume a single set of power law parameters for the entire atmosphere.

An alternate inversion approach has been presented based on the fundamental radiative transfer characteristics of the atmosphere. The approach is rooted in fundamental physics but, still, does not eliminate the problem of more unknowns than equations. For the approach to be used, one must still make an assumption about the type of aerosol responsible for the lidar signal.
5.2 Recommendations for Future Research

The alternate inversion approach that has been presented should be researched further to investigate its sensitivity to variations in the input parameters. This sensitivity study would help to establish the accuracy requirements of the input data.

The alternate inversion approach requires information on the phase function and the single scattering albedo of the aerosols. A review of the literature should be performed to determine what measurement database exists for the type of aerosols that would most likely be encountered by a spaceborne lidar system. Where measurements are lacking, calculations should be performed or results from previous calculations assembled.

Finally, the alternate inversion technique should be tested against the lidar data obtained from the recent test flight. This would help to establish the usefulness of the technique in the "real world" rather than the highly unrealistic situation in which the actual profiles are known.


