This research was conducted to obtain better methods for image processing. It focused on several aspects of this problem, including parallel algorithms for image processing, knowledge-based techniques for image understanding, and modeling images using shape and texture. Eighteen technical reports were produced which will also appear as published papers in journals. In the paper *Holes and Genus of 3D Digital Images*, it was shown that certain geometric invariants of a digital image (number of components, number of holes, and number of cavities) do not determine the topology (in the sense of connectivity) of the image refuting the commonly believed assumption that they do. This research lays the groundwork for research on digital and computational geometry of 3D images. In the paper *Hough Transform Algorithms for Mesh-Connected for Mesh-Connected SIMD Parallel Processors*, several methods of Hough transform computation are studied in terms of suitability for implementation on a parallel processor, providing a valuable tool for straight-line detection.
FINAL REPORT
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Parallel Image Processing and Image Understanding

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This report summarizes the research conducted under Contract F49620-85-K-0009 during the period April 1985 – March 1986. Eighteen technical reports were issued under the contract during that period. Most of these reports have resulted in, or are expected to result in, published papers. Abstracts of the reports are given below in chronological order. (Numbers in brackets refer to these abstracts.) The reports deal with the following topics:

a) Parallel algorithms for processing images [18], geometric data structures [4,12], and graphs [1,5,7,9].

b) Knowledge-based image understanding [6,17].

c) Image modeling: digitization [10,11,15], texture [13,16], and shape [2,3,8,14].

Research will continue in all of these areas under a successor contract.
ABSTRACTS OF TECHNICAL REPORTS


ABSTRACT: Fast parallel algorithms are presented for updating connected components and bridges of an undirected graph when a minor change has been made to the graph, such as addition or deletion of vertices and edges. The machine model used is a parallel random access machine which allows simultaneous reads but prohibits simultaneous writes into the same memory location. The algorithms described in this paper require $O(\log n)$ time and use $O(n^2)$ processors. These algorithms are efficient when compared to previously known algorithms for finding connected components and bridges that require $O(\log^2 n)$ time and use $O(n^2)$ processors. The previous solution is maintained using an inverted tree (a rooted tree where a node points towards its parent) and after a minor change the new solution is rapidly computed from this tree.


ABSTRACT: Applications in robotics and autonomous navigation have motivated the study of motion planning and obstacle avoidance algorithms. The special case considered here is that of moving a point (the object) along the surface of a convex polyhedron (the obstacle) with $n$ vertices. Sharir and Schorr have developed an algorithm that, given a source point on the surface of a convex polyhedron, determines the shortest path from the source to any point on the polyhedron in linear time after $O(n^3\log n)$ preprocessing time. The preprocessed output requires $O(n^2)$ space. By using known algorithms for fast planar point location, the shortest path query time for Sharir and Schorr's algorithm is shown to be $O(k + \log n)$ where $k$ is the number of faces traversed by the path. We give an improved preprocessing algorithm that runs in $O(n^2\log n)$ time requiring the same query time and space. We also show how to store the output of the preprocessing algorithm in $O(n \log n)$ space while maintaining the same query time.


ABSTRACT: We present an algorithm that computes the Voronoi diagram of a set of points lying on the surface of a possibly nonconvex polyhedron. Distances are measured in the Euclidean metric along the surface of the polyhedron. The algorithm runs in $O(n^2\log n)$ time and requires $O(n^2)$ space to store the final data structure, where $n$ is the maximum of the number of edges and source points on the polyhedron. This algorithm generalizes or improves the running times of a number of shortest path problems both on polyhedra and in the plane.
amidst polygonal obstacles. By applying standard algorithms for point location, we can determine the distance from a query point to the nearest source in $O(\log n)$ time and can list the shortest path in $O(k + \log n)$ time, where $k$ is the number of faces traversed by the path. The algorithm achieves its efficiency by a novel method of searching the polyhedron’s surface.


ABSTRACT: The digital medial axis transform (MAT) represents an image subset $S$ as the union of maximal upright squares contained in $S$. Brute-force algorithms for computing geometric properties of $S$ from its MAT require time $O(n^2)$, where $n$ is the number of squares. Over the past few years, however, algorithms have been developed that compute properties for a union of upright rectangles in time $O(n \log n)$, which makes the use of the MAT much more attractive. We review these algorithms and also present efficient algorithms for computing union-of-rectangle representations of derived sets (union, intersection, complement) and for conversion between the union of rectangles and other representations of a subset.


ABSTRACT: Fast parallel algorithms are presented for updating the distance matrix, shortest paths for all pairs and biconnected components for an undirected graph and the topological ordering of vertices of a directed acyclic graph when an incremental change has been made to the graph. The kinds of changes that are considered here include insertion of a vertex or insertion and deletion of an edge or a change in the weight of an edge. The machine model used is a parallel random access machine which allows simultaneous reads but prohibits simultaneous writes into the same memory location. The algorithms described in this paper require $O(\log n)$ time and use $O(n^3)$ processors. These algorithms are efficient when compared to previously known $O(\log^2 n)$ time start-over algorithms for initial computation of the above mentioned properties of graphs. The previous solution is maintained in multiple inverted trees (a rooted tree where a child node points towards its parent) and after a minor change the new solution is rapidly recomputed from these trees.
ABSTRACT: The goal of this research is to develop a robust control strategy for constructing image understanding systems (IUS). This paper proposes a general framework based on the integration of “related” hypotheses. Hypotheses are regarded as predictions of the occurrences of objects in the image. Related hypotheses are clustered together. A “composite hypothesis” is computed for each cluster. The goal of the IUS is to verify the hypotheses. We constructed an image understanding system, SIGMA, based on this framework and demonstrated its performance on an aerial image of a suburban housing development.

ABSTRACT: We define an algebraic structure on the paths in a graph based on a coloring of the arcs. Using this structure, basic classes of graphs (trees, hypercubes, arrays, cliques, etc.) are characterized by simple algebraic properties. The structure provides a framework for defining parallel contraction operations on a graph, in which many pairs of nodes are simultaneously collapsed into single nodes, but the degree of the graph does not increase. Such operations are useful in defining systematic strategies for simulating large networks of processors by smaller ones, or in building “pyramids” of networks.

ABSTRACT: This paper solves the problem of subdividing a unit square into rectangles of area $1/p$ in such a way that the maximal perimeter of a rectangle is as small as possible. The correctness of the solution is proved using the well-known theorems of Menger and Dilworth.

ABSTRACT: Fast parallel algorithms are presented for updating the transitive closure, the dominator tree, and a topological ordering of a directed acyclic graph (DAG) when an incremental change has been made to it. The kinds of changes that are considered here include insertion of a vertex or insertion and deletion of an edge. The machine model used is a parallel random access machine which allows simultaneous reads but prohibits simultaneous writes into the same
memory location. The algorithms described in this paper require \( O(\log n) \) time and use \( O(n^3) \) processors. These algorithms are efficient when compared to previously known \( O(\log^2 n) \) time algorithms for initial computation of the above mentioned properties of DAGs. We also describe a new algorithm for initial computation of the dominator tree of a DAG. Our algorithm improves the processor complexity of a previously known algorithm [14] by a factor of \( n \), but does not affect the time complexity, which remains \( O(\log^2 n) \).


ABSTRACT: We consider the graphs of functions representable in the form
\[
h(x) = \sum_{j=1}^{n} a_j f_j(x) \]
where the \( f_j \) constitute a linearly independent set of functions over \( R \). These graphs are digitized by the set of lattice points \((i, [h(i)])\). An algorithm is presented to determine if a given set of lattice points is part of such a digitization. By use of this algorithm we are able to supply simple proofs of previously known theorems for straight lines.

We also study the digitization of polynomials. An important tool used is the set of differences of the y-coordinates (digital derivatives). For example, if \( h(x) \) is a polynomial of degree \( n \), then its \( n \)th digital derivative is cyclic and its \( n+1 \)-st digital derivative has a bound which depends only on \( n \).


ABSTRACT: A 2D digital image \( S \) is represented conventionally by the union of grid squares containing pixels of \( S \) which we denote by \( F(S) \). This gives the correct topology for \( S \) with 8-adjacency, and with a little imagination, 4-adjacency can also be properly handled. However, one encounters difficulty in extending basic 2D results to 3D digital images. The last few years have seen the need for better methods which give a closer link with well developed continuous topology, especially with the advent of digital surface theory [11]. We define a new continuous model \( \hat{F}(S) \) by refining \( F(S) \). We show that this gives a better bridge between the two subjects, digital and continuous topologies. We also show how this space \( \hat{F}(S) \) is related to two other continuous models [4] [7]. Although we concentrate only on 2D images in this paper, the concepts and general ideas extend to 3D images. A 3D version of this paper is in preparation.

ABSTRACT: A region may be represented by specifying the curves that bound it. When \( p \) processors are available, typical operations on regions so represented can be performed much faster than using a single processor. This paper presents parallel algorithms to determine properties of regions, such as area; to perform operations on regions, such as union and intersection; to determine if a point lies inside a region; and to determine whether a given digital curve could be the boundary of a region. Some of the algorithms involve sorting, the time complexity of which depends on the particular model of parallel computation used.


ABSTRACT: Given a random field belonging to some specific class, and given a data sample generated by the random field, we consider the problem of finding a field of the given class that approximates the field that generated the sample. This paper derives a solution to this problem for the simple case of a field consisting of independent random variables. Subsequent papers will treat other types of fields, e.g., having Markov dependencies. Numerical examples are given, showing that good approximations can be obtained based on relatively small sample sizes. In particular, this approach can be used to find random field models that generate given samples of image texture, and so can be applied to texture classification or segmentation.


ABSTRACT: This paper solves the problem of decomposing a rectangle into \( p \) rectangles of equal area so that the maximum rectangle perimeter is as small as possible. The problem is considered both as a continuous geometric problem and also as a discrete problem on an integer grid. This problem arises in a number of flexible object packing and allocation applications. We give a solution to the continuous case that is optimal and requires only a constant number of arithmetic operations \( +, - , \times , /, \leq, \lfloor \rfloor \) and integer square root to characterize the structure of the decomposition and linear time to print the decomposition. We give an approximation to the corresponding discrete problem on the integer grid by approximating the continuous solution. We provide three algorithms that compute this approximation on a grid, providing tradeoffs between the quality of the approximation and the running time of the algorithm.

ABSTRACT: For a 3D digital image $S$ with direct or indirect adjacency, denote by $o(S)$ the number of components, by $c(S)$ the number of cavities, and by $h(S)$ the number of holes of $S$. We define Betti numbers $b_i(S) (i = 0, 1, 2)$. We prove that $o(S) = b_0(S)$, $c(S) = b_2(S)$, and $h(S) = b_1(S)$, and as a corollary that $h(S) = 0$ if and only if $S$ is simply connected. Using examples from ordinary (continuous) 3D topology, we show that the geometric invariants $o(S), c(S), h(S)$ do not determine the topology (in the sense of connectivity) of $S$, thereby refuting the generally accepted assumption that they do. Besides these concrete results, much of our work is devoted to explanations of relevant concepts, and careful definitions of terminology which will be necessary for further research on the digital (as well as computational) geometry of 3D images.


ABSTRACT: Given a random field belonging to some specific class, and given a data sample generated by the random field, we consider the problem of finding a field of the given class that approximates the field that generated the sample. This paper derives a solution to this problem for the case of a first-order Markov process (i.e., a one-dimensional random field, or stochastic process, in which each random variable is dependent on the preceding one). A numerical example is given showing that good approximations can be obtained based on reasonable sample sizes. In particular, this approach can be used to find random field models that generate given samples of image texture, and so can be applied to texture classification or segmentation.


ABSTRACT: This paper describes an image understanding program that combines image processing techniques with artificial intelligence methodology. It is concerned with road network detection on aerial photographs. The implementation makes use of production system techniques to generate symbolic descriptions of road pieces.

ABSTRACT: Hough transform techniques for straight line detection play a key role in the road following algorithms developed by the University of Maryland for the DARPA Autonomous Land Vehicle Project. This report compares several methods of Hough transform computation suitable for implementation on a mesh-connected SIMD parallel processor, such as Goddard Space Flight Center's Massively Parallel Processor (MPP) or Martin Marietta Corp.'s Geometric Arithmetic Parallel Processor (GAPP).
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