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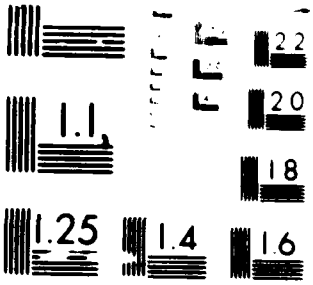
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APPROXIMATION FOR BAYESIAN ABILITY ESTIMATION

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APPROXIMATION FOR BAYESIAN ABILITY ESTIMATION

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Abstract

An approximation is proposed for the posterior mean and standard deviation of the ability parameter in an item response model. The procedure assumes that approximations to the posterior mean and covariance matrix of item parameters are available. It is based on the posterior mean of a Taylor series approximation to the posterior mean conditional on the item parameters. The method is illustrated for the two-parameter logistic model using data from an ACT math test with 39 items. A numerical comparison with the empirical Bayes method shows that the point estimates are very similar but the standard deviations under empirical Bayes are about two percent smaller than those under Bayes. The effect of sample size is demonstrated by illustrating the increase in the standard deviations for a smaller data set.

Ability Estimation

We consider estimating the ability of an individual based on the person's dichotomous responses to a set of test items whose characteristics are partially known through responses to the same set from other individuals belonging to the same population. Each item is characterized by an item response function which defines the probability of a correct response to the item by an individual with ability θ. We assume such a function to be known except for some parameter ξ and use the notation $P(y|\theta,\xi)$ to represent the probability of correct response (y=1) or incorrect response (y=0) by an individual with real valued ability θ to an item with parameter ξ. An example of such a function is the two-parameter logistic model defined by

$$P(y|\theta,\alpha,\beta) = \frac{\exp\{y\alpha(\theta-\beta)\}}{1+\exp\{\alpha(\theta-\beta)\}}, \qquad (1)$$

$y = 0,1$, where $-\infty < \theta < \infty$, $\xi = (\alpha,\beta)$ is two dimensional with $\alpha > 0$ and $-\infty < \beta < \infty$.

We assume the calibration of the test is based on responses to k items by a group of n individuals representative of some target population for which abilities are to be measured. We let $y_{ij} = 0$ or 1 according as the response by examinee i to item j is incorrect or correct. We assume conditional independence among the responses so that the joint probability of the nxk matrix $\underset{\sim}{y}$ of responses y_{ij} is given by

$$P(\underset{\sim}{y}|\underset{\sim}{\theta},\underset{\sim}{\xi}) = \prod_{ij}\prod P(y_{ij}|\theta_i,\xi_j) \qquad (2)$$

where $\underset{\sim}{\theta} = (\theta_1, \ldots, \theta_n)$ represents the abilities of the n individuals and $\underset{\sim}{\xi} = (\xi_1, \ldots, \xi_k)$ the parameters of the k items.

One standard method used in calibration is to obtain the joint maximum likelihood estimate $(\hat{\underset{\sim}{\theta}}_L, \hat{\underset{\sim}{\xi}}_L)$ of $(\underset{\sim}{\theta}, \underset{\sim}{\xi})$ using a procedure such as LOGIST (Wingersky, Barton, and Lord, 1982). The calibrated items are then used to measure the ability θ for a new individual with item responses $\underset{\sim}{x} = (x_1, \ldots, x_k)$ by finding the value of θ which maximizes the probability

$$P(\underset{\sim}{x} | \theta, \hat{\underset{\sim}{\xi}}_L) = \prod_j P(x_j | \theta, \hat{\xi}_{Lj}).$$

This procedure is straight forward and relatively easy to implement, however it is known to be biased outwards for extreme values of θ (Lord, 1983a), is subject to an occasional nonexistence of a solution, and fails to provide a good measure of uncertainity in the estimated θ.

A related empirical Bayes procedure is to first assume a prior distribution π of θ and estimate $\underset{\sim}{\xi}$ by marginal maximum likelihood. Then derive the posterior mean of θ conditionally on $\underset{\sim}{\xi}$ assumed to equal the marginal maximum likelihood estimate. The resulting estimate of θ is an empirical Bayes (Cox and Hinkley, 1974) estimate and has been demonstrated on different models by Bock and Aitkin (1981) and Rigdon and Tsutakawa (1983), among others.

The Bayesian approach, which we adopt here, assumes prior distributions on both $\underset{\sim}{\theta}$ and $\underset{\sim}{\xi}$ and uses an approximate posterior mean and variance of θ to make inferences regarding the unknown θ. The approximation for the posterior mean $E(\theta | \underset{\sim}{X})$ is based the posterior expectation of a Taylor series approximation of the

conditional posterior expectation $E(\theta|\underset{\sim}{x},\underset{\sim}{\xi})$. When the posterior distribution of $\underset{\sim}{\xi}$ is normal, it reduces to Lindley's (1980) approximation whose general form requires the computation of a large number of 3rd partial derivatives of the loglikelihood function. We will first discuss our method when the individual whose θ is being measured, is a member of the calibrating set, e.g. $\theta = \theta_1$. We then show how the same approximation can be modified and used on a new individual from the same target population.

Approximate Posterior Mean and Variance of θ

Assume that $\theta_1, \ldots, \theta_n$ are independent and identically distributed (iid) according to some known prior $p(\theta)$. Also assume that $\xi_1, \ldots, \xi_k$ are independently distributed according to some known priors $p(\xi_1), \ldots, p(\xi_k)$ and independent of $\theta_1, \ldots, \theta_n$. Then the marginal posterior pdfs of $\underset{\sim}{\xi}$ and $\underset{\sim}{\theta}$ are given by

$$p(\underset{\sim}{\xi}|\underset{\sim}{y}) \propto p(\underset{\sim}{\xi}) \prod_i \int \prod_j P(y_{ij}|\theta_i, \xi_j) p(\theta_i) d\theta_i \qquad (4)$$

and

$$p(\underset{\sim}{\theta}|\underset{\sim}{y}) \propto p(\theta) \prod_j \int \prod_i P(y_{ij}|\theta_i, \xi_j) p(\xi_j) d\xi_j . \qquad (5)$$

As shown in Tsutakawa and Lin (1986) the EM algorithm can be used to compute the posterior mode $\hat{\underset{\sim}{\xi}}$ of $\underset{\sim}{\xi}$ and the posterior covariance of $\underset{\sim}{\xi}$ can be approximated by $\underset{\sim}{\Sigma}$, the inverse of the negative Hessian of the log posterior evaluated at $\hat{\underset{\sim}{\xi}}$. Due to the symmetry between (4) and (5), it may appear that similar methods may be adopted to compute the posterior mode of $\underset{\sim}{\theta}$. However, the analogous approach presents serious numerical problems since the integrals in (5) are multiple intergrals in contrast to the single integrals in (4).

Instead we shall try to derive the posterior expectation of $\underset{\sim}{\theta}$ by approximating the integral in the expression,

$$E(\underset{\sim}{\theta}|\underset{\sim}{y}) = \int E(\underset{\sim}{\theta}|\underset{\sim}{y}, \underset{\sim}{\xi}) p(\underset{\sim}{\xi}|\underset{\sim}{y}) d\underset{\sim}{\xi} , \qquad (6)$$

where $E(\underset{\sim}{\theta}|\underset{\sim}{y},\underset{\sim}{\xi})$ is the posterior expectation of $\underset{\sim}{\theta}$ conditionally on $\underset{\sim}{\xi}$, which is used by the empirical Bayes approach upon substituting $\underset{\sim}{\xi}$ with its marginal maximum likelihood estimate.

Let $w(\underset{\sim}{\xi}) = E(\theta_i|\underset{\sim}{y}_i, \underset{\sim}{\xi})$ and let w, w_r, w_{rs} and w_{rst} denote the values of $w(\underset{\sim}{\xi})$ and its first three derivatives all evaluated at $\hat{\underset{\sim}{\xi}}$. Then according to Lindley (1980), under regularity conditions (6) may be approximated to order $O(n^{-1})$ by

$$w + \tfrac{1}{2} \Sigma\, w_{rs}\, \tau_{rs} + \tfrac{1}{2} \Sigma\, \Lambda_{rst}\, w_u\, \tau_{rs}\tau_{tu} \quad , \qquad (7)$$

where Λ_{rst} is the third partial of the log posterior evaluated at $\underset{\sim}{\xi} = \hat{\underset{\sim}{\xi}}$ and τ_{rs} are the elements of $\underset{\sim}{\Sigma}$. The last term accounts for the skewness of the posterior distribution of $\underset{\sim}{\xi}$ and if, in particular, the posterior is normal, Λ_{rst} vanish(Anderson, 1958, p. 39) and (7) reduces to

$$m = w + \tfrac{1}{2} \Sigma w_{rs}\tau_{rs}. \qquad (8)$$

A heuristic justification for (8) can be given by considering the posterior expectation of the third order Taylor series approximation to $E(\theta_i|\underset{\sim}{y}_i,\underset{\sim}{\xi})$ about $\underset{\sim}{\xi} = \hat{\underset{\sim}{\xi}}$. To simplify the notation represent the components of $\underset{\sim}{\xi}$ and $\hat{\underset{\sim}{\xi}}$ by $(\mu_1,\ldots,\mu_q)$ and $(\hat{\mu}_1,\ldots,\hat{\mu}_q)$. Then the approximation is given by

$$\begin{aligned} w(\underset{\sim}{\xi}) \doteq\; & w + \sum_r w_r(\mu_r - \hat{\mu}_r) \\ & + 1/2 \sum_{rs}\sum w_{rs}(\mu_r - \hat{\mu}_r)(\mu_s - \hat{\mu}_s) \\ & + 1/6 \sum_{rst}\sum\sum w_{rst}(\mu_r - \hat{\mu})(\mu_s - \hat{\mu}_s)(\mu_t - \hat{\mu}_t). \qquad (9) \end{aligned}$$

If $p(\underset{\sim}{\xi}|\underset{\sim}{y})$ is normal with mean $\hat{\underset{\sim}{\xi}}$ and covariance matrix $((\sigma_{rs}))$, then the third moments of $\underset{\sim}{\xi}$ vanish and

$$\int w(\underset{\sim}{\xi})p(\underset{\sim}{\xi}|\underset{\sim}{y})d\underset{\sim}{\xi}$$

$$\doteq w + \tfrac{1}{2}\Sigma w_{rs}\sigma_{rs}. \qquad (10)$$

Upon replacing $((\sigma_{rs}))$ by its estimate $((\tau_{rs}))$ we have (8). Mosteller and Wallace (1964, p. 151) refer to (10) as the standard approximation for the case of known means and covariances and point out the need to consider the bias $\hat{\underset{\sim}{\xi}} - E(\underset{\sim}{\xi}|\underset{\sim}{y})$ and third derivatives when the distribution is nonsymmetric.

The posterior variances and covariances of $\underset{\sim}{\theta}$ may be similarly approximated by first obtaining an approximation to $E(\theta_i\theta_j|\underset{\sim}{y})$ by repeating (8) with $w(\underset{\sim}{\xi}) = E(\theta_i\theta_j|\underset{\sim}{y}_i,\underset{\sim}{y}_j,\underset{\sim}{\xi})$ and then making the appropriate substitutions in $E(\theta_i\theta_j|\underset{\sim}{y}) - E(\theta_i|\underset{\sim}{y})E(\theta_j|\underset{\sim}{y})$. The approximate variances, s^2, and covariances can be helpful in assessing the uncertainty of a particular individual's ability or in comparing the abilities of two individuals.

Suppose we now have the n+1st or a new individual, with unknown θ, whose response to the same k items is $\underset{\sim}{x} = (x_1,\ldots,x_k)$. We can repeat the entire procedure including the updating of $(\hat{\underset{\sim}{\xi}},\underset{\sim}{\Sigma})$ after replacing $\underset{\sim}{y}$ with $(\underset{\sim}{y},\underset{\sim}{x})$. However this would be quite costly for routine evaluations. It would be much simpler retaining $(\hat{\underset{\sim}{\xi}},\underset{\sim}{\Sigma})$ from the first n individuals and using $\underset{\sim}{x}$ in $w(\underset{\sim}{\xi}) = E(\theta|\underset{\sim}{x},\underset{\sim}{\xi})$ only. When n is moderately large, the change in (m,s^2) through the updated $(\hat{\underset{\sim}{\xi}},\underset{\sim}{\Sigma})$ by the addition of a single individual would be quite negligible relative to the variability in $E(\theta|\underset{\sim}{x},\underset{\sim}{\xi})$ or $E(\theta^2|\underset{\sim}{x},\underset{\sim}{\xi})$ due to different values of $\underset{\sim}{x}$.

Computational Details for the Two-parameter Logistic Model

In this section we present computational expressions for the evaluation of w and w_{rs} in the case of the two-parameter logistic model (1) with a N(0,1) prior on θ. A derivation of $(\hat{\underset{\sim}{\xi}}, \hat{\underset{\sim}{\Sigma}})$ based on the EM algorithm can be found in Tsutakawa and Lin (1986).

For this model we have

$$w(\underset{\sim}{\xi}) = E(\theta_i | \underset{\sim}{Y}_i, \underset{\sim}{\xi})$$

$$= \int \theta_i p(\theta_i | \underset{\sim}{Y}_i, \underset{\sim}{\xi}) d\theta_i, \qquad (11)$$

where

$$p(\theta_i | y_i, \underset{\sim}{\xi}) = \frac{p(\theta_i) \prod_{j=1}^{k} p(y_{ij} | \theta_i, \alpha_j, \beta_j)}{\int p(\theta_i) \prod_{j=1}^{k} p(y_{ij} | \theta_i, \alpha_j, \beta_j)}$$

and

$$p(\theta) = \frac{1}{\sqrt{2\pi}} \exp(-\theta^2/2), \quad -\infty < \theta < \infty.$$

Following the notation in Tsutakawa (1984), for each i = 1,...,n and u = 1,...,k let

$$g_1(i,u,\theta) = \frac{\partial P(y_{iu} | \theta, \alpha_u, \beta_u)}{\partial \alpha_u} / P(y_{iu} | \theta, \alpha_u, \beta_u), \qquad (12)$$

$$g_2(i,u,\theta) = \frac{\partial P(y_{iu}|\theta,\alpha_u,\beta_u)}{\partial\beta_u}/P(y_{iu}|\theta,\alpha_u,\beta_u),$$

$$h_{11}(i,u,\theta) = \frac{\partial^2 P(y_{iu}|\theta,\alpha_u,\beta_u)}{\partial\alpha_u^2}/P(y_{iu}|\theta,\alpha_u,\beta_u),$$

$$h_{12}(i,u,\theta) = \frac{\partial^2 P(y_{iu}|\theta,\alpha_u,\beta_u)}{\partial\alpha_u\partial\beta_u}/P(y_{iu}|\theta,\alpha_u,\beta_u),$$

$$h_{22}(i,u,\theta) = \frac{\partial^2 P(y_{iu}|\theta,\alpha_u,\beta_u)}{\partial\beta_u^2}/P(y_{iu}|\theta,\alpha_u,\beta_u),$$

$$\phi_{\theta u} = \{1 + \exp[-\alpha_u(\theta - \beta_u)]\}^{-1}, \text{ and } \psi_{\theta u} = 1 - \phi_{\theta u}.$$

By taking derivatives it is readily shown that

$$\begin{aligned} g_1(i,u,\theta) &= (y_{iu} - \phi_{\theta u})(\theta - \beta_u). \\ g_2(i,u,\theta) &= -\alpha_u(y_{iu} - \phi_{\theta u}), \\ h_{11}(i,u,\theta) &= (y_{iu} - \phi_{\theta u})(\psi_{\theta u} - \phi_{\theta u})(\theta - \beta_u)^2, \\ h_{12}(i,u,\theta) &= -(y_{iu} - \phi_{\theta u})[1 + \alpha_u(\psi_{\theta u} - \phi_{\theta u})(\theta - \beta_u)], \end{aligned} \tag{13}$$

and

$$h_{22}(i,u,\theta) = (y_{iu} - \phi_{\theta u})\alpha_u^2(\psi_{\theta u} - \phi_{\theta u}).$$

(We note that there is a minus sign missing in the expression for h_{12} in Tsutakawa (1984)). Now define the following posterior expectations of the derivatives and their products by

$$\bar{g}_s(i,u) = \int_{-\infty}^{\infty} g_s(i,u,\theta)p(\theta|y_i,\underset{\sim}{\xi})d\theta,$$

$$\bar{h}_{st}(i,u) = \int_{-\infty}^{\infty} h_{st}(i,u,\theta)p(\theta|y_i,\underset{\sim}{\xi})d\theta, \tag{14}$$

$$\bar{d}_{st}(i,u,v) = \int_{-\infty}^{\infty} g_s(i,u,\theta)g_t(i,v,\theta)p(\theta|y_i,\underset{\sim}{\xi})d\theta,$$

$s,t = 1,2, u,v = 1,\ldots,k, u \neq v$, where $p(\theta|y_i,\underset{\sim}{\xi})$ is defined by (11).

Now denote the second derivatives of $w(\underset{\sim}{\xi})$ evaluated at $\underset{\sim}{\xi} = \hat{\underset{\sim}{\xi}}$ by

$$w_{11}(u,v) = \partial w(\hat{\underset{\sim}{\xi}})/\partial\alpha_u\partial\alpha_v,$$

$$w_{12}(u,v) = \partial w(\hat{\underset{\sim}{\xi}})/\partial\alpha_u\partial\beta_v, \tag{15}$$

$$w_{21}(u,v) = \partial w(\hat{\underset{\sim}{\xi}})/\partial\beta_u\partial\alpha_v,$$

and

$$w_{22}(u,v) = \partial w(\hat{\underset{\sim}{\xi}})/\partial\beta_u\partial\beta_v.$$

These derivatives may be computed using the following expressions.

$$w_{st}(u,u) = E\{\theta h_{st}(i,u,\theta)\} + \{2\bar{g}_s(i,u)\bar{g}_t(i,u) - \bar{h}_{st}(i,u)\}E(\theta)$$
$$- \bar{g}_s(i,u)E\{\theta g_t(i,u,\theta)\} - \bar{g}_t(i,u)E\{\theta g_s(i,u,\theta)\} \tag{16}$$

and, for $u \neq v$,

$$w_{st}(u,v) = \{2\bar{g}_s(i,u)\bar{g}_t(i,v) - \bar{d}_{st}(i,u,v)\}E(\theta) \tag{17}$$

$$+ E(\theta g_s(1,u,\theta) g_t(1,v,\theta))$$

$$- \bar{g}_t(1,v) E(\theta g_s(1,u,\theta)) - \bar{g}_s(1,u) E(\theta g_t(1,v,\theta)),$$

where E denotes posterior expectation with respect to the density $p(\theta_1 | y_1, \hat{\xi})$ defined by (11) when $\xi = \hat{\xi}$.

The numerical evaluation of $w_{st}(u,v)$ requires numerically integrating a number of integrals of the type

$$\int h(\theta_1) p(\theta_1 | y_1, \hat{\xi}) P(y_{1j} | \theta_1, \xi_j, \hat{\xi}_j) d\theta_1, \quad (18)$$

where the function $h(\theta_1)$ varies from one integral to the next. The use of scaling for improved accuracy in the Gauss-Hermite approximation is discussed in Tsutakawa (1984).

Example

Our method will now be illustrated on a sample data set from an ACT math test previously used by Tsutakawa and Lin (1986). The data matrix of responses on k = 39 items by n = 400 respondents. Using their method, $(\hat{\xi}, \Sigma)$ were initially recomputed. The posterior means and standard deviations (m,s) were then computed for the first 100 respondents and are plotted against the empirical counterparts, $[E(\xi \mid y, \hat{\xi}_L)$ and $SD(\xi \mid y, \hat{\xi}_L)]$, where $\hat{\xi}_L$ is the marginal maximum likelihood estimate of ξ] in Figures 1 and 2. We note that the means under the two procedures are almost identical but the standard deviations could be quite different. In particular, posterior standard deviations are 1.5 to 2.5 percent smaller under empirical Bayes than approximate Bayes. One explanation for this is that the empirical Bayes approach does not account for the uncertainity in $\hat{\xi}_L$ (Deeley and Lindley 1981).

A plot of s vs. m in Figure 3 shows that the larger s is associated with the larger |m|, suggesting the difficulty of estimating extreme ability values based on the given test. The quadratic relationship seen in the plot suggests that, for routine use, s may be expressed as a function of m and need not be computed for all individuals.

Effect of Sample Size n

An interesting conjecture involves the effect of the sample size n on our Bayesian estimates. Although the posterior moments of ability are calculated separately for each individual they depend on the values of $(\hat{\underset{\sim}{\xi}}, \underset{\sim}{\Sigma})$ which are estimated from a response matrix which may contain the particular individual's response. It is natural to believe that as the size of the response matrix decreases, the posterior standard deviation of an individual's ability would increase, as there is more uncertainty in the estimated $(\hat{\underset{\sim}{\xi}}, \underset{\sim}{\Sigma})$.

We previously found ability estimates for the first 100 examinees in a 400 x 39 response matrix using $(\hat{\underset{\sim}{\xi}}, \underset{\sim}{\Sigma})$ which were derived from the responses of all 400 examinees. To investigate our conjecture we reestimated the posterior moments of ability for the same set of 100 examinees, using $(\hat{\underset{\sim}{\xi}}, \underset{\sim}{\Sigma})$ derived from the first 100 examinees only.

The initial comparison produced an interesting result. From individual to individual it was found that the posterior standard deviation of ability was usually smaller than the corresponding standard deviation based on n = 400. We do not feel however that our initial conjecture is untrue as the following discussion shows.

It is well known that the parameters of the two-parameter logistic model are not unique. The item parameters which were estimated from the two samples are quite different although the items are exactly the same. In particular, the estimates of α_j

were larger for n = 100. This is consistent with Lord (1983b) who shows that the MLE of the discrimination parameter is positively biased, particularly for small n. (Although we are not using MLE's here, our prior is relatively flat.) Intuitively, larger values of the discrimination parameter will tend to produce smaller posterior standard deviations of ability. We feel that this explains why the standard deviations did not increase as expected.

In order to put both sets of estimated item paramters on a common scale and make them more comparable, we propose the following transformation. Define

$$\sigma = \left\{ \prod_{j=1}^{k} \alpha_j \right\}^{1/k},$$

$$\mu = -\sigma \frac{\sum_{j=1}^{k} \beta_j}{k}, \qquad (19)$$

$$\alpha_j^* = \frac{\alpha_j}{\sigma}, \text{ and}$$

$$\beta_j^* = \mu + \sigma\beta_j, \quad j = 1,\ldots,k.$$

The corresponding transformation for the ability parameter is

$$\theta_i^* = \mu + \sigma\theta_i. \qquad (20)$$

We refer to this process as rescaling and it is easily seen that $\prod_{j=1}^{k} \alpha_j^* = 1$ and $\sum_{j=1}^{k} \beta_j^* = 0$. Under this transformation we now have

$$E(\theta_i^* | \underset{\sim}{y}) \simeq \mu + \sigma E(\theta_i | \underset{\sim}{y})$$

and (21)

$$SD(\theta_i^* | \underset{\sim}{y}) \simeq \sigma SD(\theta_i | \underset{\sim}{y}).$$

One reason for putting the restriction on the item parameter space rather than on the abilities is because the item parameters are common to both sets while the ability parameters in the two sets are not identical, except for the first 100. Upon applying the rescaling to the two sets separately, the rescaled posterior standard deviations for n = 100 were .6 to 14 percent larger than those found using n = 400. This can be seen graphically in Figure 4.

Discussion

The approximation to the posterior mean and variance of the ability parameter proposed here is relatively simple to compute once the posterior mode and approximate covariance matrix of the item parameters are available. Other approximations, which do not appear as readily adaptable to ability estimation, yet deserve further study, include those by Leonard (1982) and Tierney and Kadane (1986). We now briefly examine the computational requirements of these approximations for evaluating the posterior mean θ of an individual with response $\underset{\sim}{x}$ who appears subsequent to the calibration based on n individuals with data $\underset{\sim}{Y}$.

For the Tierney and Kadane approximation define

$$\ell(\underset{\sim}{\xi}) = \prod_{i=1}^{m} \int \prod_{j=1}^{k} P(y_{ij}|\theta_i, \underset{\sim}{\xi}_j) p(\theta_i) d\theta_i, \tag{22}$$

$$L(\underset{\sim}{\xi}) = m^{-1} \log \pi(\underset{\sim}{\xi})\ \ell(\underset{\sim}{\xi}), \tag{23}$$

and

$$L^{*}(\underset{\sim}{\xi}) = m^{-1} \log E(\theta|\underset{\sim}{X}, \underset{\sim}{\xi})\ \pi(\underset{\sim}{\xi})\ \ell(\underset{\sim}{\xi}), \tag{24}$$

where $\pi(\underset{\sim}{\xi})$ is the prior for $\underset{\sim}{\xi}$, $m = n+1$, $\theta_{n+1} = \theta$ and $(y_{n+1,1} \cdots y_{n+1,k})^T = \underset{\sim}{x}$ so that the posterior expectation of θ can be expressed as

$$E(\theta|\underset{\sim}{y}, \underset{\sim}{x}) = \frac{\int e^{mL^{*}(\underset{\sim}{\xi})} d\underset{\sim}{\xi}}{\int e^{mL(\underset{\sim}{\xi})} d\underset{\sim}{\xi}} . \tag{25}$$

Suppose $\hat{\underset{\sim}{\xi}}'$ and $\hat{\underset{\sim}{\xi}}^*$ are the modes of L and L* and $\underset{\sim}{\Sigma}'$ and $\underset{\sim}{\Sigma}^*$ are minus the inverse Hessians of L and L* evaluated at $\hat{\underset{\sim}{\xi}}'$ and $\hat{\underset{\sim}{\xi}}^*$, respectively. Under regularity conditions, the posterior expectation may be approximated, according to Tierney and Kadane by

$$\tilde{\theta} = \frac{|\underset{\sim}{\Sigma}^*|^{\frac{1}{2}}}{|\underset{\sim}{\Sigma}'|^{\frac{1}{2}}} \exp\{m[L^*(\hat{\underset{\sim}{\xi}}^*) - L(\hat{\underset{\sim}{\xi}}')]\} . \qquad (26)$$

One of the requirements is that $E(\theta|\underset{\sim}{x},\underset{\sim}{\xi})$ be a positive valued function of $\underset{\sim}{\xi}$. But as pointed out by R.D. Bock (personal communication, April 29, 1986) since the origin of θ is arbitrary we may take it to be a large positive number, such as 5, and if necessary truncate the prior of θ to make $E(\theta|\underset{\sim}{x},\underset{\sim}{\xi})$ positive.

Although the computation of $(\hat{\underset{\sim}{\xi}}',\underset{\sim}{\Sigma}')$ can be performed using the EM algorithm as shown in Tsutakawa and Lin (1986), the computation of $\hat{\underset{\sim}{\xi}}^*$ can be a major problem, since, unlike the computation for $\hat{\xi}'$, the maximization of $L^*(\underset{\sim}{\xi})$ requries working with all item parameters simultaneously.

For the Leonard approximation, we begin with the joint posterior of $(\theta,\underset{\sim}{\xi},$,

$$p(\theta,\underset{\sim}{\xi}|\underset{\sim}{x},\underset{\sim}{y}) \propto p(\underset{\sim}{x}|\theta,\underset{\sim}{\xi})p(\theta)p(\underset{\sim}{\xi}|\underset{\sim}{y}) \qquad (27)$$

For fixed θ, let $\hat{\underset{\sim}{\xi}}_\theta$ be the mode of $\underset{\sim}{\xi}$ and $\underset{\sim}{R}_\theta$ the negative inverse Hessian of the log of (27) with respect to $\underset{\sim}{\xi}$ evaulated at $\hat{\underset{\sim}{\xi}}_\theta$. Then, under regularity conditions, the marginal posterior pdf of θ is approximated by

Ability Estimation

$$p(\theta|\underset{\sim}{x},\underset{\sim}{y}) = (2\pi)^{q/2}\, p(\theta,\hat{\underset{\sim}{\xi}}_\theta\, |\underset{\sim}{x},\underset{\sim}{y})/|R_\theta|^{\frac{1}{2}}\,, \qquad (28)$$

where q is the dimension of $\underset{\sim}{\xi}$. Following Leonard and Novick (1986), the posterior moments of θ may be computed from (28) using numerical integration.

In practice the major difficulty would be in computing $\hat{\xi}_\theta$. The EM algorithm can again be used, with some modification of the procedure in Tsutakawa and Lin (1986). However since the computation of each $\hat{\xi}_\theta$ will require as much effort as computing $\hat{\xi}$ and a fair number of $(\hat{\underset{\sim}{\xi}}_\theta, |\underset{\sim}{R}_\theta|)$ pairs will be needed for each individual, without some modification, Leonard's approximation cannot be recommended for routine use.

In practice the simplicity of our approximaiton is obtained by separating the item parameter and ability estimation. Although there is some loss of information from not using all current data successively update the posterior distribution of the item parameters, (i) the extensive computation is completed at the calibration stage and (ii) ability measurement is done uniformly and without excessive computation for subsequent examinees.

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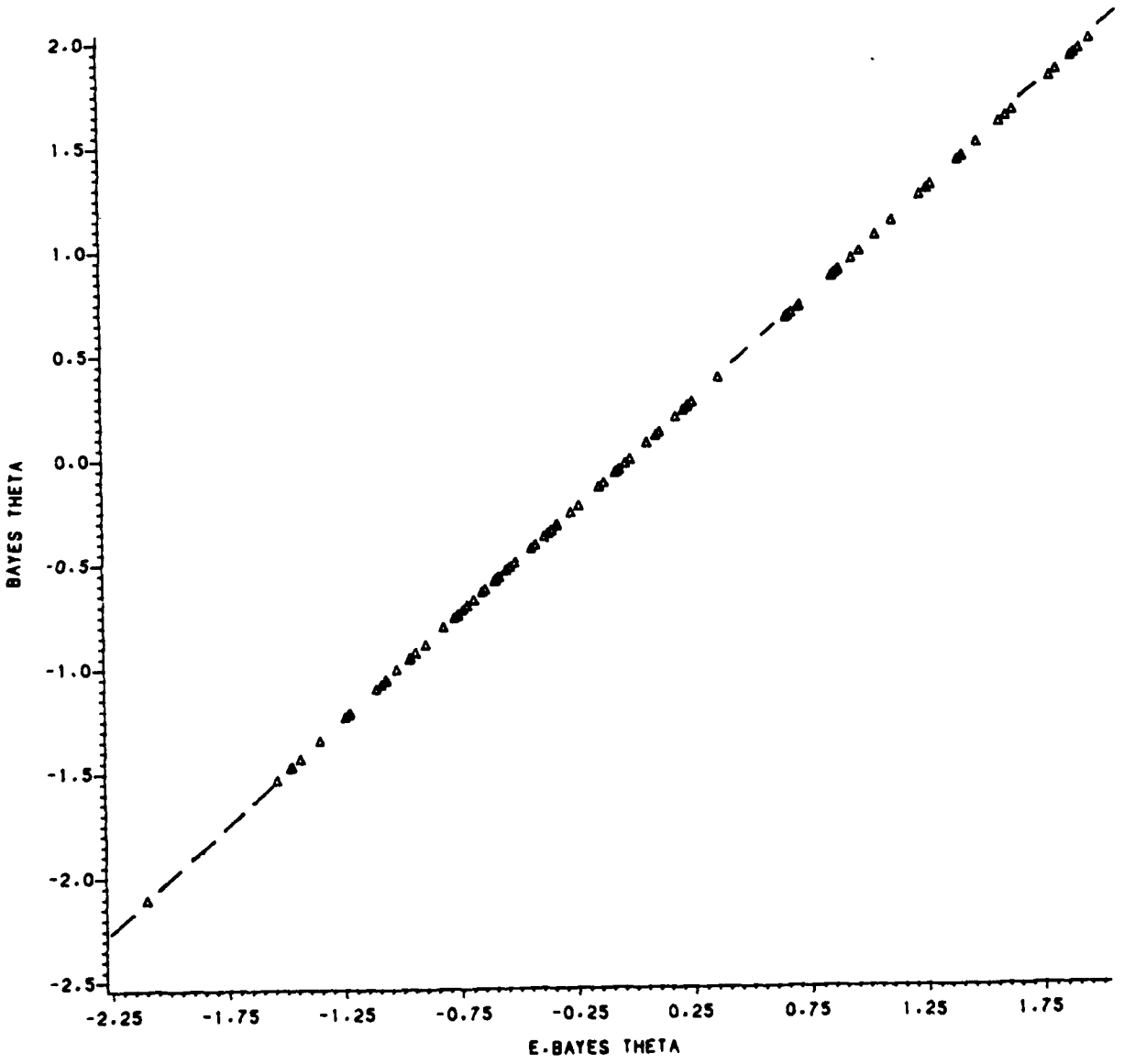

Figure 1. Bayes vs. Empirical Bayes $\hat{\theta}$.

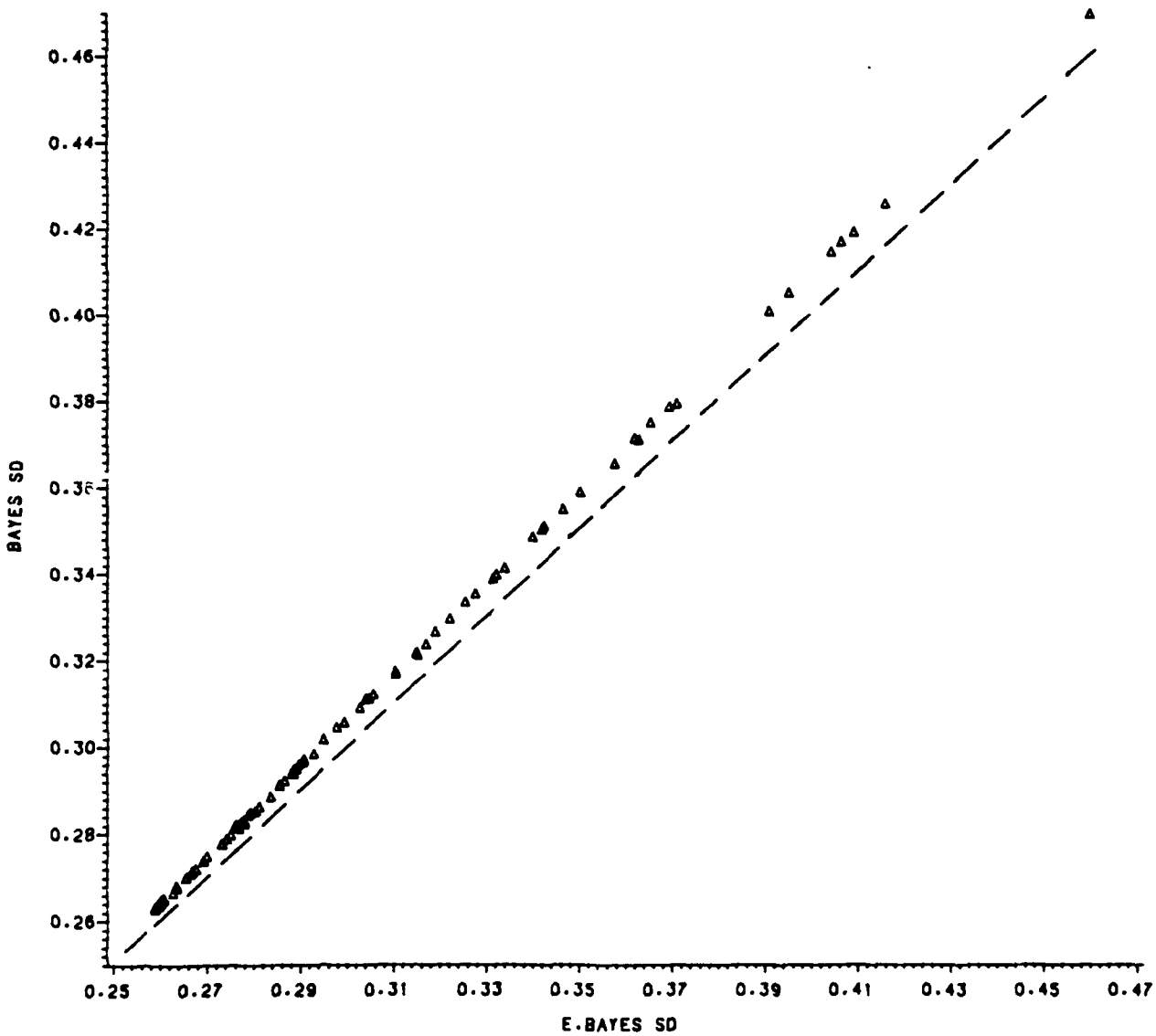

Figure 2. Bayes vs. Empirical Bayes Standard Deviation.

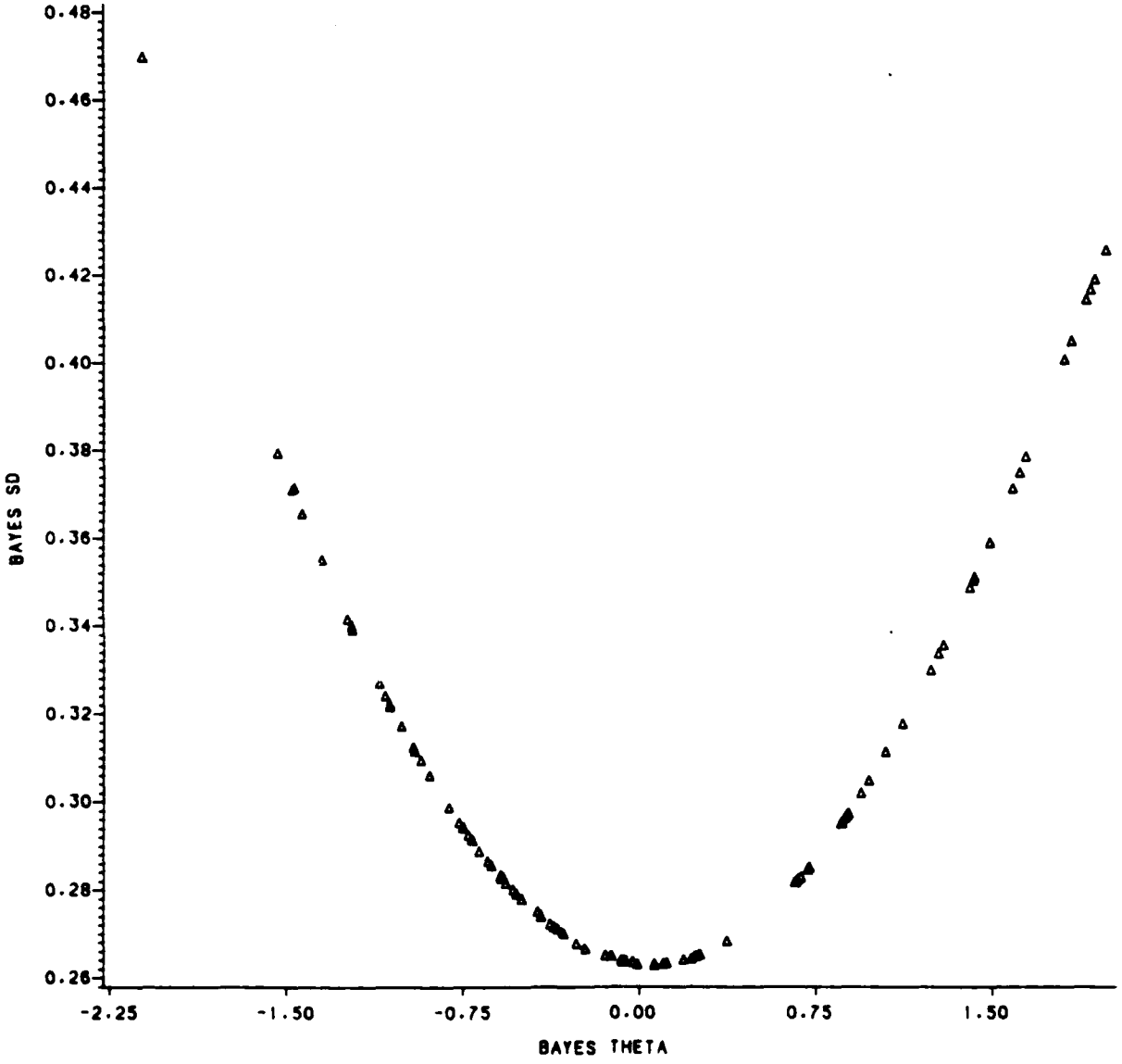

Figure 3. Bayes Standard Deviation vs. Bayes θ.

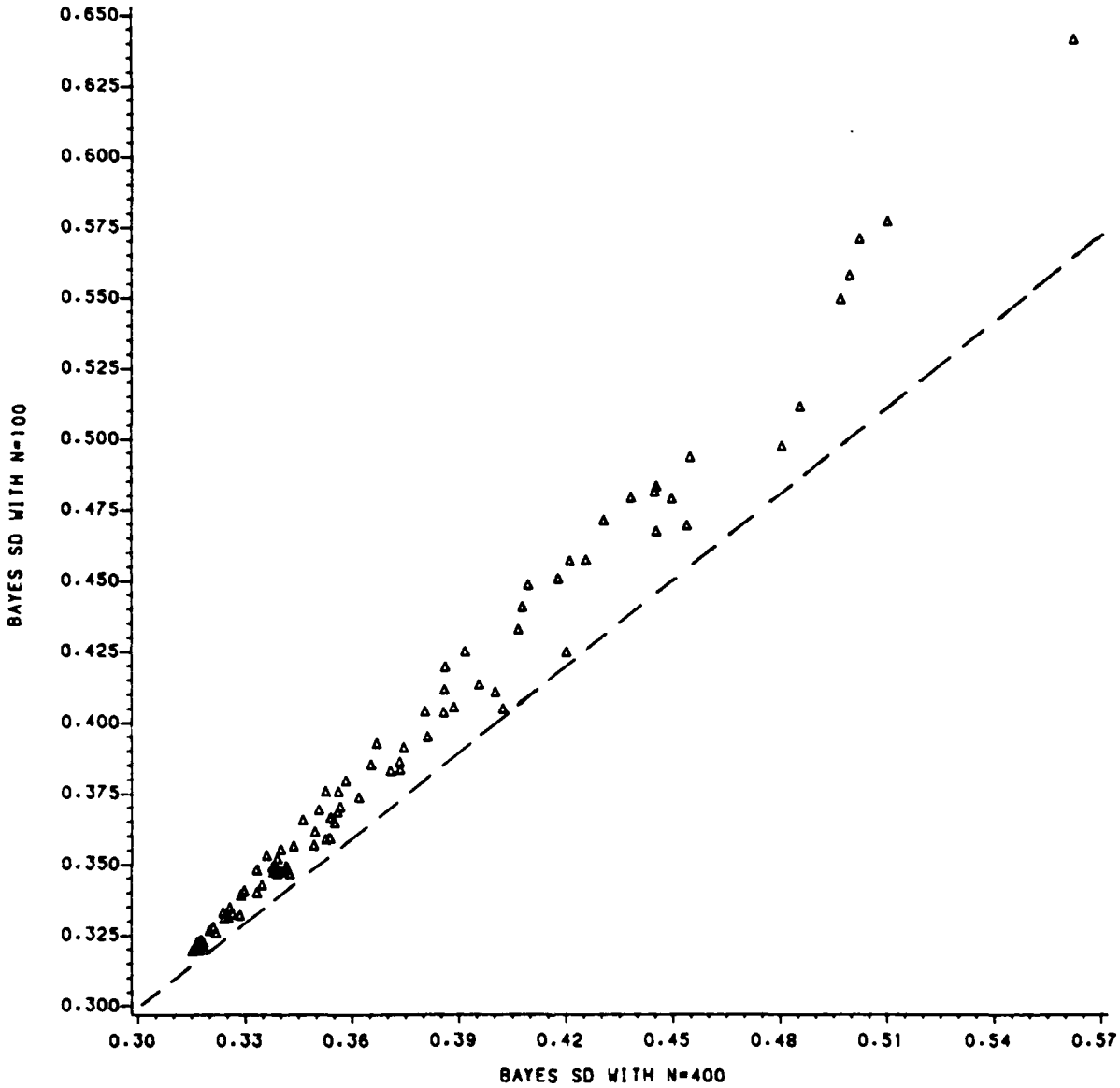

Figure 4. Rescaled Bayes Standard Deviations for N = 100 vs. 400

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