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COMPUTER PROGRAM FOR
ELECTROMAGNETIC PENETRATION INTO A CONDUCTING
CIRCULAR CYLINDER THROUGH A
NARROW SLOT, TM CASE

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by
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February 1987

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**COMPUTER PROGRAM FOR ELECTROMAGNETIC PENETRATION INTO A CONDUCTING CIRCULAR CYLINDER THROUGH A NARROW SLOT, TM CASE**

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**Aperture problems**  
**Circular cylinder**  
**Electromagnetic penetration**  
**Computer program**  
**Numerical solution**  
**Resonance**

A computer program is described and listed. This program calculates electromagnetic penetration of a TM plane wave into the cylindrical cavity for which \( \rho \leq a \) when the surface \( (\rho = a, 0 \leq \phi \leq 2\pi - \phi_0) \) is perfectly conducting. Here, \( \rho \) and \( \phi \) are cylindrical coordinates. The \( z \) directed electric field \( E_z \) in the slot aperture \( (\rho = a, 0 \leq \phi \leq \phi_0) \) is expressed as a linear combination of 4 even and 4 odd expansion functions. The method of moments is used to obtain the coefficients of these functions. The elements of the
20. ABSTRACT (continued)

moment matrix are obtained by expressing each expansion function as a Fourier series in \( \phi \) valid for \((0 \leq \phi \leq 2\pi)\).

Our moment solution remains accurate when the cavity becomes resonant because alternative expansion functions were chosen to prevent the moment matrix from becoming ill-behaved. As the aperture width \( 2\beta \) decreases, more and more terms in the Fourier series are needed. Thanks to Debye's asymptotic expansions for Bessel functions, we are able to handle 10,000 terms so as to obtain accurate results for \( \beta \) as small as 1.25.
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INTRODUCTION

In [1], a procedure is presented for calculating the electromagnetic field in the vicinity of an infinitesimally thin perfectly conducting circular cylindrical shell with an infinitely long slot illuminated by a TM wave. A computer program was written to implement this procedure. For the shell of [1, Fig. 1] illuminated by the incident electric field $E_i$ given by [1, eq. (1)]

$$E_i = u_z e^{jk\rho \cos(\phi - \alpha)}$$  \hspace{1cm} (1)

the program calculates the amplitude $|E_z|$ of the $z$ directed electric field at equally spaced points in the slot aperture and along the straight line that runs from the center of the cylinder to the center of the aperture. The radius of the shell is $a$, and the angular width of the slot aperture is $2\phi_0$. The incident electric field is the electric field that would exist if the shell were absent. In (1), $(\rho, \phi, z)$ are cylindrical coordinates, and $u_z$ is the unit vector in the $z$-direction. The incident wave (1) comes from the direction for which $\phi = \alpha$. In (1), $k = \omega\sqrt{\mu\varepsilon}$ where $\omega$ is the angular frequency, $\mu$ is the permeability of the homogeneous medium that surrounds the shell, and $\varepsilon$ is the permittivity of this medium.

The computer program consists of the subroutines BES, BESJ, BESJY, BESJJ, SFEO, DECOMP, and SOLVE, and a main program. The main program reads input data on the file MAUTZ3.DAT and writes output data on file MAUTZ6.DAT. The user who wants merely to run the program is advised to skip to Section VIII and to read only the descriptions of the input and output data there.
II. THE SUBROUTINE BES

The subroutine $\text{BES}(Y_0, N, X, BJ, BY)$ puts the Bessel function of the first kind $J_n(X)$ in $BJ(n+1)$ and the Bessel function of the second kind $Y_n(X)$ in $BY(n+1)$ for $n=0,1,...,N$ where $X$ is a positive real number. The subroutine $\text{BES}$ differs from that in [2, Sec. II] only in that common input variables $Y_0(1)$ to $Y_0(33)$, $P_12$, $P_14$, $P_17$, in [2, p. 6] have been changed to the subroutine arguments $Y_0(1)$ to $Y_0(33)$, $Y_0(34)$, $Y_0(35)$, and $Y_0(36)$, respectively, and the common output variables $BJ$ and $BY$ have been changed to the subroutine arguments $BJ$ and $BY$, respectively. The symbol $\emptyset$ in $Y_0$ denotes zero rather than the letter $0$. In other instances when it is obvious from the context that a zero is meant, the slash is not used.

Minimum allocations are given by

$\text{DIMENSION BJ}(N+1), BY(N+1), AJ(11+N+2*\lceil X \rceil)$ where $\lceil X \rceil$ is the largest integer that does not exceed $X$. 
LISTING OF THE SUBROUTINE BES

SUBROUTINE BES(YO, N, X, BJ, BY)

DIMENSION YO(36), BJ(100), BY(100), AJ(100)

M=10*N+2+1

IF(X.LT.1.E-3) M=4+N

AJ(M+1)=0.

AJ(M)=1.E-20

N1=M-1

X2=2./X

DO 16 K=1,N1

HX=M-K

AJ(HK)=X2*FLOAT(HK)*AJ(HK+1)-AJ(HK+2)

16 CONTINUE

ALP=.5*AJ(1)

DO 15 J=3,HZ.2

ALP=ALP+AJ(J)

15 CONTINUE

ALP=2.*ALP

NP=M+1

DO 17 K=1,NP

BJ(K)=AJ(K)/ALP

17 CONTINUE

IF(N.LE.0) RETURN

DO 18 K=1,NP

BY(K)=X2*FLOAT(K)*BY(K-1)-BY(K)

18 CONTINUE

RETURN

END
III. THE SUBROUTINE BESJ

The subroutine BESJ(X, BJZ, BJ1) puts \( J_0(X) \) in BJZ and \( J_1(X) \) in BJ1 where X is a positive real number.

If \( X \leq 3 \), then \( J_0(X) \) and \( J_1(X) \) are calculated in lines 5 to 20 in the listing of BESJ at the end of this section. As suggested in [3, Sec. 9.12., Example 1], \( J_{MZ}(X) \) and \( J_{MZ-1}(X) \) are set equal to the arbitrary values of zero and \( 10^{-20} \), respectively, where

\[
MZ = 11 + 2[X]
\]

(2)

where \([X]\) is the largest integer that does not exceed \( X \). This is done in lines 5 to 7. In DO loop 16, the recurrence relation [3, Eq. (9.1.27)]

\[
J_n(X) = \frac{2(n+1)}{X} J_{n+1}(X) - J_{n+2}(X)
\]

(3)

is used to calculate \( \{J_n(X), n = MZ-2, MZ-3, ..., 0\} \). Line 12 puts \( J_{MK-1}(X) \) in BJMK. According to [3, eq. (9.1.46)], the calculated values of \( J_0(X) \) and \( J_1(X) \) have to be normalized by dividing by \( \alpha \) where

\[
\alpha = J_0(X) + 2J_2(X) + 2J_4(X) + ...
\]

(4)

The division by \( \alpha \) is performed in lines 19 and 20.

If \( X > 3 \), then \( J_0(X) \) and \( J_1(X) \) are calculated in lines 22 to 38. The polynomial approximations [3, Eqs. 9.4.3 and 9.4.6] are used. With regard to [3, Eq. 9.4.3], lines 28 and 29 put \( f_0 \) in FZ, lines 30 and 31 put \( \theta_0 \) in TZ, and line 37 puts \( J_0(X) \) in BJZ. With regard to [3, Eq. 9.4.6], lines 32 and 33 put \( f_1 \) in FL, lines 34 and 35 put \( \theta_1 \) in TL, and line 38 puts \( J_1(X) \) in BJ1.
LISTING OF THE SUBROUTINE BESJ

SUBROUTINE BESJ(X,BJZ,BJI)
DIMENSION BJ(16)

IF(X-3.) 17,17,18

NZ=11+2*IFIX(X)
BJ(NZ+1)=0.
BJ(NZ)=1.E-20
M1=NZ-1
X2=2./X
DO 16 1=1,M1

MK=NZ-K
BJ(MK)=X2*FLOAT(MK)*BJ(MK+1)-BJ(MK+2)

16 CONTINUE
ALP=5.*BJ(1)
DO 19 J=3,NZ,2
ALP=ALP+BJ(J)
19 CONTINUE
ALP=2.*ALP
BJZ=BJ(1)/ALP
BJ1=BJ(2)/ALP
RETURN

18 X1=3./X
X2=X1*X1
X3=X2*X1
X4=X3*X1
X5=X4*X1
X6=X5*X1

19 FZ=.7978846-.77E-6*X1-.155274E-2*X2-.25512E-4*X3+
.137237E-2*X4-.12005E-3*X5+.14476E-3*X6
TZ=.7853982-.4166397E-1*X1-.3954E-4*X2+.262573E-2*X3- 
.54125E-3*X4-.29333E-3*X5+.1355E-3*X6
F1=.7978846+.156E-05*X1+.1659667E-01*X2+.17105E-03*X3- 
.24951E-02*X4+.113653E-02*X5-.20033E-03*X6
T1=X-2.356194+.1249961*X1+.565E-04*X2-.637879E-02*X3+ 
.74348E-03*X4+.79824E-03*X5-.2916E-03*X6
XX=SQRT(X)
BJZ=FZ/XX*COS(TZ)
BJ1=F1/XX*COS(T1)
RETURN
END
IV. THE SUBROUTINE BESJY

The subroutine BESJY(N, X, BJY) puts $J_N(X)$ $Y_N(X)$ in BJY where $J_N$ and $Y_N$ are, respectively, the Bessel functions of the first and second kinds of order $N$. Here, $X$ is a non-negative real number, and $N$ is large enough so that Debye's asymptotic expansions [3, eqs. (9.3.7) and (9.3.8)] apply to $J_N(X)$ and $Y_N(X)$. The subroutine BESJY is that in [2, Sec. VI] with DO loop 11 removed, the arguments $N_1$ and $N_2$ replaced by $N$, and the array BJY replaced by the single variable BJY.

IC LISTING OF THE SUBROUTINE BESJY

SUBROUTINE BESJY(N, X, BJY)

F1 = N
F2 = F1 * F1
F3 = F2 * F1
F4 = F3 * F1
XN = X / F1
T1 = 1. / SQRT(1. - XN * XN)
T2 = T1 * T1
T3 = T2 * T1
T4 = T3 * T1
T5 = T4 * T1
T6 = T5 * T1
T7 = T6 * T1
T8 = T7 * T1
T9 = T8 * T1
T10 = T9 * T1
T12 = T10 * T2
U5 = T1 / (3.141593 * F1) * (U5 + U6) * (U5 - U6)
RETURN
END
V. THE SUBROUTINE BESJJ

The subroutine BESJJ(N, NR, X, BJJ) puts \( J_N(X(NR)) \) in BJJ(L) for \( L = 1, 2, \ldots, NR \). Here, \( X(L) \geq 0 \) and \( N \) is large enough so that Debye's asymptotic expansion applies to \( J_N \). Minimum allocations are given by

\[
\text{DIMENSION } X(NR), \text{ BJJ}(NR), \text{ U5}(NR), \text{ U6}(NR)
\]

Consider \( J_N(XX) \) where

\[
XX = X(L)
\]

Debye's asymptotic expansion for \( J_N(XX) \) is [3, eq. (9.3.7)]

\[
J_N(XX) = \frac{e^{N(tanh \alpha - \alpha)}}{\sqrt{2\pi N tanh \alpha}} \left( 1 + \sum_{k=1}^{4} \frac{u_k(coth \alpha)}{N^k} \right)
\]

where \( u_k \) is given by [3, formula 9.3.9] and

\[
tanh \alpha = \sqrt{1 - \left( \frac{XX}{N} \right)^2}
\]

From [4, formula 702.], we have

\[
\alpha = \frac{1}{2} \ln \left( \frac{1 + \tanh \alpha}{1 - \tanh \alpha} \right)
\]

Substituting (7) into (8) and rationalizing the denominator of the argument of the logarithm, we obtain

\[
\alpha = \ln \left[ \frac{N}{XX} (1 + \tanh \alpha) \right]
\]

Substitution of (9) into (6) gives

\[
J_N(XX) = \frac{1}{\sqrt{2\pi N tanh \alpha}} \left( \frac{XX e^{tanh \alpha}}{N(1+tanh \gamma)} \right)^N \left( 1 + \sum_{k=1}^{4} \frac{u_k(coth \alpha)}{N^k} \right)
\]

Thanks to (10), we obtain

\[
\frac{J_N(XX)}{J_N(X(NR))} = \left( \frac{U6(L)}{U6(NR)} \right)^N \left( \frac{U5(L)}{U5(NR)} \right)^N
\]
where

\[ U_5(L) = \frac{XX e^{\tanh \alpha}}{1 + \tanh \alpha} \]  \hspace{1cm} (12)

and

\[ U_6(L) = \frac{1 + \sum_{k=1}^{4} \frac{u_k (\coth \alpha)}{N^k}}{\sqrt{\tanh \alpha}} \] \hspace{1cm} (13)

Here, \( \tanh \alpha \) depends on \( L \) by means of (7) with \( XX \) given by (5).

In the listing of the subroutine BESJJ at the end of this section, line 5 inside DO loop 11 obtains \( XX \) of (5). Lines 23 to 26 put \( u_1, u_2, u_3, \) and \( u_4 \) of (13) in \( U_1, U_2, U_3, \) and \( U_4, \) respectively. Line 28 obtains \( U_5(L) \) of (12). Line 29 obtains \( U_6(L) \) of (13). Inside DO loop 12, line 34 puts \( J_N(XX)/J_N(X(NR)) \) of (11) in BJJ(L).
LISTING OF THE SUBROUTINE BESJJ

SUBROUTINE BESJJ(NR.XDBJJ)
DO 11 L=1,NR
XX=X(L)
F1=1
F2=F1*F1
F3=F2*F1
F4=F3*F1
XN=XX/F1
TT=SQRT(1.-XN*XN)
T1=1./TT
T2=T1*T1
T3=T2*T1
T4=T3*T1
T5=T4*T1
T6=T5*T1
T7=T6*T1
T8=T7*T1
T9=T8*T1
T10=T9*T1
T12=T10*T2
U1=(3.*T1-5.*T3)/(24.*F1)
U2=(81.*T2-482.*T4+369.*T6)/(1152.*F2)
U3=(30375.*T3-369603.*T5+765765.*T7-425425.*T9)/(414720.*F3)
U4=(4485125.*T4-9412168.E+01*T6+3499224.E+02*T8-4461857.E+02*T10)/(3981312.E*01*F4)
U5(L)=XX*EXP(TT)/(1.+TT)
U6(L)=(1.+U1+U2+U3+U4)/SQRT(TT)
CONTINUE
55=U5(NR)
56=U6(NR)
DO 12 L=1,NR
BJJ(L)=U6(L)/SS=(U5(L)/SS)**N
CONTINUE
BJJ(NR)=1.
RETURN
END
VI. THE SUBROUTINE SFEO

The subroutine SFEO(N, P, SE, SO, FE, FO) puts $S_{JN}^e$ of [1, eqs. (A-18) to (A-21)] or [1, eqs. (A-32) to (A-35)] in SE(J), $S_{JN}^o$ of [1, eqs. (A-22) to (A-25)] or [1, eqs. (A-37) to (A-40)] in SO(J), $F_{JN}^e$ of [1, eqs. (B-5) to (B-8)] or [1, eqs. (B-18) to (B-21)] in FE(J), and $F_{JN}^o$ of [1, eqs. (B-9) to (B-12)] or [1, eqs. (B-22) to (B-25)] in FO(J), all for $J=1,2,3,$ and 4. The variables N and P are input arguments defined by

$$N = n$$
$$P = \phi_o$$

(14)

where $n$ and $\phi_o$ appear in [1, eq. (B-4)] so that [1, eq. (B-4)] becomes

$$b = N*P$$

(15)

The subroutine SFEO calls the subroutine BESJ.

The subroutine SFEO is listed at the end of this section. If $b > 2$, line 11 of this listing puts $J_0(b)$ and $J_1(b)$ in BJZ and BJ1, respectively. Lines 15 to 19 put $S_{JN}^e$ of [1, eqs. (A-18) to (A-21)] in SE(J) for $J=1,2,3,$ and 4. Lines 22 to 26 put $S_{JN}^o$ of [1, eqs. (A-22) to (A-25)] in SO(J) for $J=1,2,3,$ and 4. Lines 30 to 33 put $F_{JN}^e$ of [1, eqs. (B-5) to (B-8)] in FE(J) for $J=1,2,3,$ and 4. Lines 36 to 40 put $F_{JN}^o$ of [1, eqs. (B-9) to (B-12)] in FO(J) for $J=1,2,3,$ and 4.

If $b \leq 2$, lines 42 and 43 put $e_n\phi_o/2$ in PP. Lines 46 to 53 put $S_{JN}^e$ of [1, eqs. (A-32) to (A-35)] in SE(J) for $J=1,2,3,$ and 4. Lines 55 to 62 put $S_{JN}^o$ of [1, eqs. (A-37) to (A-40)] in SO(J) for $J=1,2,3,$ and 4. Lines 63 to 70 put $F_{JN}^e$ of [1, eqs. (B-18) to (B-21)] in FE(J) for $J=1,2,3,$ and 4. Lines 71 to 78 put $F_{JN}^o$ of [1, eqs. (B-22) to (B-25)] in FO(J) for $J=1,2,3,$ and 4.
LISTING OF THE SUBROUTINE SFEO

THE SUBROUTINE SFEO CALLS THE SUBROUTINE BESJ.

DIMENSION SE(4), SO(4), FE(4), FO(4)

FN=N
B=FN*P
B2=B*B
B4=B2*B2
B6=B4*B2

IF(B.LE.2.) GO TO 11
CALL BESJ(B, BJZ, BJ1)

SZ=BJZ/FN
BZ=B/SZ
B1=BJ1/FN
SE(1)=B1

SE(2)=((B2-6.)*B1+3.*BZ)/B2
SE(3)=((B4-27.*B2+120.)*B1+6.*(B2-10.)*BZ)/B4
SE(4)=((B6-63.*B4+1200.*B2-5040.)*B1+3.*(3.*B4-95.*B2+840.)*B

IF(N.EQ.0) PP=.5*PP

B8=B6*B2
B10=B8*B2


RETURN
PB = PB
56 1576. E+02)
58 188. E+01 - B10*13. / 2477281. E+03)
60 1143. / 1548288. E+02 - B10*13. / 1857946. E+03)
64 1)
66 13024)
68 108864. E+02)
70 1/8812800.)
72 128. E+02)
74 197504. E+02)
76 1/969408. E+02)
78 184256. E+02)
79 RETURN
80 END
VII. THE SUBROUTINES DECOMP AND SOLVE

The subroutines DECOMP(N, IPS, UL) and SOLVE(N, IPS, UL, B, X) solve a system of N linear equations in N unknowns. The input to DECOMP consists of N and the N by N matrix of coefficients on the left-hand side of the matrix equation stored by columns in UL. The output from DECOMP is IPS and UL. This output is fed into solve. The rest of the input to solve consists of N and the column of coefficients on the right-hand side of the matrix equation stored in B. SOLVE puts the solution to the matrix equation in X.

Minimum allocations are given by

    COMPLEX UL(N*N)
    DIMENSION SCL(N), IPS(N)

in DECOMP and by

    COMPLEX UL(N*N), B(N), X(N)
    DIMENSION IPS(N)

in SOLVE

More detail concerning DECOMP and SOLVE is on pages 46-49 of [5].
1C LISTING OF THE SUBROUTINE DECOMP
2 SUBROUTINE DECOIIP(N, IPS, UL)
3 COMPLEX UL(16), PIVOT, EM
4 DIMENSION SCL(4), IPS(4)
5 DO 5 I=1, N
6 IPS(I)=1
7 RM=0.
8 J=1
9 DO 2 J=1, N
10 ULK=ABS(REAL(UL(J1)))+ABS(AIMAG(UL(J1)))
11 J1=J1+N
12 1 RM=ULM
13 2 CONTINUE
14 SCL(1)=1./RM
15 CONTINUE
16 NH1=N-1
17 K2=0
18 DO 17 K=1, NH1
19 BIG=0.
20 DO 11 I=K, N
21 1 IPS(I)=IPS(I+1)
22 IF=IPS(I)
23 IPK=IP+K2
24 SIZE=ABS(REAL(UL(IPK)))+ABS(AIMAG(UL(IPK))) * SCL(I)
25 IF(SIZE>BIG) 11.11,10
26 10 BIG=SIZE
27 PV=1
28 IF(I PV-K) 14.15.14
29 14 J1=IPS(K)
30 IPS(K)=IPS(J1)
31 K1=K1+1
32 IPS(IPV)=J
33 KPP=IPS(K)+K2
34 PIVOT=UL(KPP)
35 KP=KPP
36 IF(KP=KP1) 1.2
37 IF(KP=KP1) 1.2
38 IF(KP=KP1) 1.2
39 EM=UL(IP)/PIVOT
40 18 UL(IP)=EN
41 DO 18 J=KP1, N
42 IF=IF+I
43 KP=KP+N
44 UL(IP)=UL(IP)+EM*UL(KP)
45 CONTINUE
46 K2=K2+N
47 CONTINUE
48 RETURN
49 END
50C LISTING OF THE SUBROUTINE SOLVE
51 SUBROUTINE SOLVE(N, IPS, UL, X)
52 COMPLEX UL(16), B(4), X(4), SUN
53 DIMENSION IPS(4)
54 WP1=N+1
55 IP=IPS(1)
56 X(1)=B(IP)
57 DO 2 I=2, N
58 IF(IP=IPS(I)) 11 IP=IPS(I)
59 1 PB=IP
60 IM1=I-1
61 SUN=0.
62 DO 1 J=1, IM1
63 SUN=SUN+UL(IP)*X(J)
64 IF(IP=IP+N) 2 X(1)=B(IP)-SUN
65 2 X(1)=B(IP)-SUN
66 K2=N*(N-1)
67 IP=IPS(N)+K2
68 X(N)=X(N)/UL(IP)
69 DO 4 I BACK=2, N
70 IF=IF+1-1 I BACK
71 K2=K2-N
72 IP=IPS(1)+K2
73 CONTINUE
74 SUN=0.
75 IF(IP=IP1) 3 SUN=SUN+UL(IP)*X(J)
76 3 SUN=SUN+UL(IP)*X(J)
77 4 X(1)=(X(1)-SUN)/UL(IP1)
78 RETURN
79 END
VIII. THE MAIN PROGRAM

The main program accepts input data from the file MAUTZ3.DAT and writes output data on the file MAUTZ6.DAT. See the open statements on lines 10 and 11 of the listing of the main program at the end of this section. The main program calls the subroutines BES, BESJY, BESJJ, SFEO, DECOMP, and SOLVE. The subroutine SFEO calls the subroutine BESJ. In this section, first the input data are described, then the output data are described, relevant formulas are given, the main program is described verbally, and finally the main program and sample input and output data are listed.

The input data are read early in the main program from the file MAUTZ3.DAT according to

\[ \text{READ}(20,25)(Y_0(I), I = 1, 36) \]

25 FORMAT(5E14.7)

\[ \text{READ}(20,28) N_1, N_2, N_3, N_A, N_R \]

28 FORMAT(I3, I5, 3I3)

\[ \text{DO 31 JW} = 1, N_3 \]

\[ \text{READ}(20,32) N_N, I_P, I_B, X, P, A_LP \]

32 FORMAT(3I3, 3E14.7)

31 CONTINUE

The input array \( Y_0 \) should not deeply concern the user because he will never have to change it. The values of the elements of \( Y_0 \) appear in the sample data listed after the main program. The curious reader will find the values of \( Y_0(1) \) to \( Y_0(33) \) explained on pages 4 and 5 of [2]. The values of \( Y_0(34), Y_0(35), \) and \( Y_0(36) \) are \( 2/\pi, \pi/4, \) and \( 3\pi/4, \) respectively.
The Bessel functions in the summations in [1, eqs. (41), (42), (47), and (48)] are approximated by Debye's asymptotic expansions whenever \( N_1 < n < N_2 \), and these summations are truncated at \( n = N_2 \).

The summations in [1, eqs. (44) and (50)] are truncated at \( n = N_1 - 1 \).

For the \( z \) directed electric field \( E^b_z(-M) \) along the straight line that runs from the center of the cylinder to the center of the aperture, the Bessel functions in the first summation in (62) are approximated by Debye's asymptotic expansions whenever \( N_1 < n < N_2 \), and this summation is truncated at \( n = N_2 \).

The aperture field \( E_z \) is evaluated at \( NA \) equally spaced points in the aperture. Here, \( NA \geq 2 \). With reference to [1, Fig. 1], the first point is at the beginning of the aperture at \( \phi = -\phi_0 \), and the \( NA \)th point is at the end of the aperture at \( \phi = \phi_0 \). The interior field \( E^b_z(-M) \) is evaluated at \( NR \) equally spaced points along the line that runs from the center of the cylinder to the center of the aperture. Here, \( NR \geq 2 \).

The first point is at the center of the cylinder, and the \( NR \)th point is at the center of the aperture.

The \( NA \) evaluations of the aperture field and the \( NR \) evaluations of the interior field \( E^b_z(-M) \) are done inside DO loop 31, that is, these evaluations are done \( N3 \) times, once for each value of \( JW \). If \( NN = 4 \), these fields are obtained by solving [1, eqs. (40) and (46)] in which \( Y^e \) and \( Y^O \) are, as stated in [1], \( 4 \times 4 \) matrices. If \( 1 \leq NN \leq 3 \), the fields are obtained by solving [1, eqs. (40) and (46)] with \( Y^e \) replaced by the \( NN \times NN \) matrix in its upper left-hand corner, with \( Y^O \) replaced by the \( NN \times NN \) matrix in its upper left-hand corner, and with \( V^e, \overline{\tau}^e, \) and \( V^O, \overline{\tau}^O \) truncated accordingly. We assume that \( 1 \leq NN \leq 4 \).
In DO loop 31, IP = p where p is the non-negative integer upon which the alternative expansion functions [1, eqs. (30) and (34)] depend. The main program was written under the assumption that IP < Ni. If there is an integer p such that \(|J_p(ka)|\) is very small, then the user should take IP = p. Here, k is the wave number and a is the radius of the cylinder. However, if there is no value of p such that \(|J_p(ka)|\) is very small, a non-negative value of IP must still be chosen, and the alternative expansion functions will be given by [1, eqs. (30) and (34)] with p = IP. In this case the fields calculated by the procedure of [1] should not depend on p so that IP can be any non-negative integer less than Ni. If IB ≠ 0, then the main program uses the value of \(J_{IP}(ka)\) calculated by the subroutine BES. If IB = 0, then \(J_{IP}(ka)\) is set equal to zero. If the circular cylinder of radius a was resonant such that \(J_{IP}(ka) = 0\), then, without IB, it would be virtually impossible to obtain \(J_{IP}(ka) = 0\) because the exact resonant value of ka would be difficult to enter, and even if it could be entered, the subroutine BES would probably not give exactly zero for \(J_{IP}(ka)\).

In DO loop 31,

\[
\begin{align*}
    IP &= p \quad \text{(16)} \\
    X &= ka \\
    P &= \zeta_o \\
    ALP &= \alpha
\end{align*}
\]

where p, ka, \(\zeta_o\), and \(\alpha\) appear in [1]. Here, P and ALP are in radians.

The meaning of p was clarified in the previous paragraph, ka is the product of the wave number k and the radius a of the cylinder, \(\zeta_o\) is one half the angular width of the aperture, and \(\alpha\) is the value of \(\phi\) in the direc-
tion from which the incident wave comes. The angles $\phi_0$ and $\alpha$ are shown in [1, Fig. 1].

Minimum allocations are given by

```
DIMENSION BJ(MAX(NI*NR, NA)), BY(NI*NR), XR(NR), RP(NR)
DIMENSION RN(NN*NR), RE(NR), BJJ(NR)
```

where MAX denotes the maximum value.

The main program writes output data on the file MAUTZ6.DAT. The input data of the first two read statements are written out immediately after they are read in.

The following write statements are in DO loop 31:

```
WRITE(21, 27) NN, IP, IB, X, P, ALP
27 FORMAT(' NN=', I3, ', IP=', I3, ', IB=', I3/
WRITE(21, 33) (BJ(NP), NP=JJ, JN)
33 FORMAT(' BJ/(IX, 5E14.7))
WRITE(21, 19) (BY(NP), NP = JJ, JN)
19 FORMAT(' BY/(IX, 5E14.7))
WRITE(21, 24) (SE(J), J=1, NN)
WRITE(21, 24) (SO(J), J=1, NN)
24 FORMAT(1X, 4E14.7)
WRITE(21, 49) U2, S1
49 FORMAT(' E=', 2E14.7, ', ABS(E) = ', E14.7)
WRITE(21,64) (BJ(L), L = 1, NA)
64 FORMAT(' APERTURE FIELD AMPLITUDE/(IX, 4E14.7))
WRITE(21, 60)(RE(L), L = 1, NR)
60 FORMAT(' INTERIOR FIELD AMPLITUDE/(IX, 4E14.7))
```
The above write statements are labeled the first, second, third, ..., eighth write statements. The data of the first write statement are merely input data. The data of the second and third write statements are defined by

\[(BJ(NP), NP = JJ, Jjn) = (Jn(ka), n = 0, 1, \ldots, N1-1)\]  
(17)

\[(BY(NP), NP = JJ, Jjn) = (Yn(ka), n = 0, 1, \ldots, N1-1)\]  
(18)

where \(J_n(ka)\) and \(Y_n(ka)\) are, respectively, the Bessel functions of the first and second kinds of order \(n\) and argument \(ka\). The data of the fourth and fifth write statements are defined by

\[(SE(J), J=1, NN) = (|V_e^j|, j=1,2,\ldots,NN)\]  
(19)

\[(SO(J), J=1, NN) = (|V_0^j|, j=1,2,\ldots,NN)\]  
(20)

where \(V_e^j\) appears in [1, eq. (7)] and \(V_0^j\) appears in [1, eq. (24)]. If \(NN < 4\), then \(V_e^j\) and \(V_0^j\) are truncated at \(j = NN\) because they are given in terms of \(\hat{V}_e^j\) and \(\hat{V}_0^j\) by [1, eqs. (53) and (54)] and, as indicated earlier, \(\hat{V}_e^j\) and \(\hat{V}_0^j\) are truncated at \(j = NN\).

In the sixth write statement, \(U2\) is the complex number whose real and imaginary parts are those of \(E^b_z(-M)\) at the center of the cylinder, and \(SL\) is \(|E^b_z(-M)|\) at the center of the cylinder. This \(E^b_z(-M)\) is calculated from [1, eq. (68)]. In the seventh write statement, \(BJ(L)\) is the magnitude of the aperture field \(E_z\) at \(\phi = a\) and \(\phi = \phi_L\) where

\[\phi_L = \frac{2(L - 1)}{NA - 1} \phi_o - \phi_o\]  
(21)

In the eighth write statement, \(RE(L)\) is \(|E^b_z(-M)|\) at \(k_o = k_oL\) along the line that runs from the center of the cylinder to the center of the aperture. Here,
This completes our description of the input and output data. The formulas to be programmed are presented next.

Assuming that

$$0 \leq p \leq N_l - 1,$$

we approximate the matrix elements \([1, \text{eqs. (41), (42), and (44)}]\) by

$$\begin{align*}
Y_{i1}^e &= \frac{p^e S_{1p}^e}{H_{p}^{(2)}(ka)} + J(ka) \sum_{n=0}^{N_l-1} \frac{p^e S_{n}^e}{J(n)(ka)H_{n}^{(2)}(ka)} + J(ka) Y(n)(ka) \sum_{n=1}^{N_l} \frac{p^e S_{n}^e}{J(n)(ka)Y(n)(ka)}, \\
Y_{ij}^e &= \frac{N_l-1}{N_l} \frac{F_{in}^e(S_{jn} + C_{jn}^e S_{in}^e)}{J_{n}(ka)H_{n}^{(2)}(ka)} + J(ka) \sum_{n=0}^{N_l-1} \frac{F_{in}^e(S_{jn} + C_{jn}^e S_{in}^e)}{J(n)(ka)Y(n)(ka)} \left.I_n^{j} = \sum_{n=0}^{N_l-1} \frac{J_{n}(ka)}{H_{n}^{(2)}(ka)} \right|_i
\end{align*}$$

In (25), \(C_{jn}^e\) is given by \([1, \text{eq. (31)}]\)

$$C_{jn}^e = - \frac{S_{jn}^e}{S_{1p}^e}$$

Expressions (24) and (25) are recast as

$$\begin{align*}
Y_{i1}^e &= S_{1p}^e (EF)_{1p} + J(ka) \sum_{n=0}^{N_l-1} E_{in}^e (EF)_{in}, \\
Y_{ij}^e &= \frac{N_l-1}{N_l} \frac{F_{in}^e(S_{jn} + C_{jn}^e S_{in}^e)}{J_{n}(ka)H_{n}^{(2)}(ka)} + J(ka) \sum_{n=0}^{N_l-1} \frac{F_{in}^e(S_{jn} + C_{jn}^e S_{in}^e)}{J(n)(ka)Y(n)(ka)} \left.I_n^{j} = \sum_{n=0}^{N_l-1} \frac{J_{n}(ka)}{H_{n}^{(2)}(ka)} \right|_i
\end{align*}$$
\[ Y_{ij}^e = \sum_{n=0}^{N2} \frac{E_{jn}^e (EF)_{in}}{n+p} , \quad \begin{cases} i=1,2,3,4 \\ j=2,3,4 \end{cases} \] (29)

where

\[ (EF)_{in} = \begin{cases} \frac{F_{in}}{H^{(2)}_n(ka)} , & n=0,1,\ldots,N1-1 \\ j \frac{F_{in}}{J_{n}(ka)} , & n = N1, N1+1,\ldots,N2 \end{cases} \] (30)

\[ E_{jn}^e = \begin{cases} S_{jn}^e + C_{jn}^e \frac{S_{jn}^e}{J_{n}(ka) Y_{n}(ka)} , & n=0,1,\ldots,p-1,p+1,\ldots,N1-1 \\ S_{jn}^e , & j=1 \\ 0 , & j=2,3,4 \end{cases} \] (31)

\[ C_{jn}^e = \begin{cases} S_{jn}^e , & j=2,3,4 \end{cases} \] (32)

Assuming that (23) holds, we approximate the matrix elements [1, eqs. (47), (48), and (50)] by

\[ Y_{11}^o = \frac{F_{ip}^o S_{lp}^o}{H^{(2)}_p(ka)} + J_{p,(ka)}^o \left[ \sum_{n=1}^{N1-1} \frac{F_{in}^o S_{ln}^o}{J_{n}(ka) H^{(2)}_n(ka)} + j \sum_{n=N1}^{N2} \frac{F_{in}^o S_{ln}^o}{J_{n}(ka) Y_{n}(ka)} \right] , \quad i=1,2,3,4 \] (33)

\[ Y_{ij}^o = \sum_{n=1}^{N1-1} \frac{F_{in}^o (S_{jn}^o + C_{jn}^o S_{jn}^o)}{J_{n}(ka) H^{(2)}_n(ka)} + j \sum_{n=N1}^{N2} \frac{F_{in}^o (S_{jn}^o + C_{jn}^o S_{jn}^o)}{J_{n}(ka) Y_{n}(ka)} , \quad \begin{cases} i=1,2,3,4 \\ j=2,3,4 \end{cases} \] (34)
\[ I_1^o = 2 \sum_{n=1}^{N_l-1} \frac{j^n F_{in}^o \sin (n\alpha)}{H_n(2)(ka)} \text{, } i=1,2,3,4 \] (35)

where

\[ J_p^o(ka) = \begin{cases} 
1 & , \ p = 0 \\
J_p(ka) & , \ p > 0 
\end{cases} \] (36)

and [1, eq. (35)]

\[ C_j^o = \begin{cases} 
0 & , \ p = 0 \\
S_{ip}^o & , \ p > 0 \\
-S_{ip}^o & , \ p > 0 
\end{cases} \] (37)

Expressions (33) and (34) are recast as

\[ Y_{11}^o = S_{1p}^o (OF)_{ip} + j_p^o(ka) \sum_{n=0}^{N_2} E_{1n}^o (OF)_{in} \text{, } i=1,2,3,4 \] (38)

\[ Y_{ij}^o = \sum_{n=0}^{N_2} E_{jn}^o (OF)_{in} \text{ } \begin{cases} 
i=1,2,3,4 \\
j=2,3,4 
\end{cases} \] (39)

where

\[ (OF)_{in} = \begin{cases} 
\frac{F_{in}^o}{H_n(2)(ka)} & , \ n=0,1,\ldots,N_l-1 \\
j F_{in}^o & , \ n=N_l,N_l+1,\ldots,N_2 
\end{cases} \] (40)

\[ E_{jn}^o = \begin{cases} 
S_{jn}^o + C_j^o S_{jn}^o & , \ n=0,1,\ldots,p-1,p+1,\ldots,N_l-1 \\
S_{jn}^o & , \ n=N_l,N_l+1,\ldots,N_2 
\end{cases} \] (41)
The terms for which $n=0$ do not contribute to (38) and (39) because
$S^o_{j0} = P^o_{10} = 0$ [1, eqs. (27) and (49)]. The terms for which $n=0$ have been
added to make (38) and (39) similar to (28) and (29) and thus easier to
program.

For convenience, [1, eqs. (40), (46), and (53)] are repeated
here:

\[ v^o_v^e = \frac{r^e}{r} \]  
\[ v^{o+}_v^o = \frac{r^o}{r} \]  
\[ v^e = J_p(ka) \hat{v}^o_1 + \sum_{i=2}^{4} C^e_v^e \]  
\[ v^e_j = \hat{v}^e_j, \quad j=2,3,4 \]  

With $J^o_p(ka)$ given by (36) and $C^o_j$ by (42), [1, eqs. (54) and (55)] combine:

\[ v^o_1 = J^o_p(ka) \hat{v}^o_1 + \sum_{i=2}^{4} C^o_v^o \]  
\[ v^o_j = \hat{v}^o_j, \quad j=2,3,4 \]  

Expression [1, eq. (68)] for the field $E^b_z(-M)$ at the center of the
cylinder is

\[ [E^b_z(-M)]_{n=0} = \begin{cases} 
S^e_{10} \hat{v}^e_1, & p = 0 \\
\frac{1}{J_o(ka)} \sum_{j=1}^{4} S^o_{j0} v^e_j, & p \neq 0 
\end{cases} \]
The aperture field $E_z$ is the $\phi$ component of $M$ of [1, eq. (4)]:

$$E_z = M^e + M^o$$  \hspace{1cm} (48)

Substituting [1, eqs. (7) and (24)] into (48), using [1, eqs. (8) and (25)], and evaluating $E_z$ at $\phi = \{\phi_L, L = 1,2,\ldots,NA\}$ where $\phi_L$ is given by (21), we obtain

$$[E_z]_L = [\sum_{j=1}^{4} (\phi_L \nu_j^o + \nu_j^e) (\phi_L)^2] \sqrt{1 - (\phi_L)^2}, \quad L=1,2,\ldots,NA$$  \hspace{1cm} (49)

Along the line from the center of the cylinder to the center of the aperture, $\phi = 0$ so that expression [1, eq. (62)] for the interior field $E^b_z(-M)$ reduces to

$$E^b_z(-M) = \sum_{n=0}^{\infty} \frac{B^e_n J_n(k\rho)}{J_0(k\rho)}$$  \hspace{1cm} (50)

where $B^e_n$ is given by [1, eqs. (63) or (65)].

$$B^e_n = \begin{cases} \sum_{j=1}^{4} \nu_j^e S_j^e, & n \neq p \\ J_p(k\rho) \tilde{\nu}_p^e, & n = p \end{cases}$$  \hspace{1cm} (51)

Truncating the summation in (50) at $n = N2$, substituting (51) into (50), and then letting $k\rho = \{k\rho_L, L=1,2,\ldots, NR\}$ where $k\rho_L$ is given by (22), we have

$$[E^b_z(-M)]_L = S^e_{1p} J_0(k\rho_L) \tilde{\nu}_1^e + \sum_{j=1}^{4} \nu_j^e (\sum_{n=0}^{N2} \frac{S^e_{jn} J_n(k\rho_L)}{J_0(k\rho)}), \quad L=1,2,\ldots, NR$$  \hspace{1cm} (52)

Expression (52) is recast as

$$[E^b_z(-M)]_L = (RP)_{L} \tilde{\nu}_1^e + \sum_{j=1}^{4} (RN)_j L \nu_j^e, \quad L = 1,2,\ldots, NR$$  \hspace{1cm} (53)

where
Expression (55) was obtained by assuming that (23) holds. We plan to use Debye's asymptotic expansions for the Bessel functions in the second summation in (55).

If \( L = 1 \), then \( k_0 L = 0 \) and (53) reduces to

\[
[E^b_{z(-M)}]_1 = \begin{cases} 
S^e_1 \hat{V}_1^e, & p = 0 \\
\frac{4}{\pi} \sum_{j=1}^{N_2} \left( \frac{1}{J_0(ka)} \right) V_j^e, & p \neq 0
\end{cases}
\]  

(56)

Therefore, any difference between the computed values of the field (47) at the center of the cylinder and the field (53) at \( L = 1 \) is due to roundoff error.

If \( L = NR \), then \( k_0 L = ka \) and (52) reduces to

\[
[E^b_{z(-M)}]_{NR} = S^e_{1p} J_p(ka) \hat{V}_1^e + \sum_{j=1}^{N_2} V_j^e \left( \sum_{n=0}^{N_2} S^e_j n \right)
\]  

(57)

Using (45) to express \( J_p(ka) \hat{V}_1^e \) in terms of \( V_j^e, j=1,2,3,4 \), we obtain

\[
J_p(ka) \hat{V}_1^e = V_1^e - \sum_{i=2}^{4} C_i^e V_i^e
\]  

(58)

In view of (32), substitution of (58) into (57) gives

\[
[E^b_{z(-M)}]_{NR} = \sum_{j=1}^{N_2} V_j^e \left( \sum_{n=0}^{N_2} S^e_j n \right)
\]  

(59)

Equating [1, eqs. (8) and (12)] at \( \phi = 0 \), we obtain

\[
\sum_{n=0}^{\infty} S^e_{jn} = \begin{cases} 
1, & j=1 \\
0, & j=2,3,4
\end{cases}
\]  

(60)
If

$$\sum_{n=0}^{N_2} S_{jn}^e = \sum_{n=0}^{\infty} S_{jn}^e$$  \hspace{1cm} (61)

then (60) would reduce (59) to

$$\begin{pmatrix} E_z^b(-M) \end{pmatrix}_{NR} = V_1^e$$  \hspace{1cm} (62)

If NA is odd so that $\phi_L = 0$ when $L = (NA+1)/2$, then (49) gives

$$[E_z]_L = V_1^e, \quad L = (NA+1)/2$$  \hspace{1cm} (63)

Noting that (62) and (63) have the same right-hand side and recalling that (61) was used to obtain (62), we deduce that any difference between the computed values of the aperture field (49) at the center of the aperture and the interior field (53) at $k\phi_L = ka$ is due to the error in truncating the summation with respect to $n$ in (55), the error in calculating $\{S_{jn}^e\}$, and, of course, roundoff error.

Having presented relevant formulas, we begin to verbally describe the main program. In the equations that were presented in this section, $Y^e$ and $Y^o$ are $4 \times 4$ matrices, $\tilde{V}^e$, $\tilde{V}^o$, $\tilde{I}^e$, and $\tilde{I}^o$ are $4 \times 1$ column vectors, and $\{V_j^e, V_j^o, j=1,2,3,4\}$ are defined by (45) and (46). The equations that were programmed are those that were presented in this section with the order 4 replaced by NN, that is, $Y^e$ and $Y^o$ are replaced by the $NN \times NN$ submatrices in their upper left-hand corners, and $\tilde{V}_j^e$, $\tilde{V}_j^o$, $\tilde{I}_j^e$, $\tilde{I}_j^o$, $V_j^e$, and $V_j^o$ are retained only for $j \leq NN$. We describe the main program by relating variables therein to quantities that appear in the previous text of this section. What follows is clarified in Table 1.

Line 27 and DO loop 61 put...
\[ J_n(0) = \begin{cases} 1 & , n = 0 \\ 0 & , n > 0 \end{cases} \] (64)

in BJ(n+1) for \(n=0,1,...,N_1-1\). Equation (64) can be obtained from [3, formula 9.1.10]. In DO loop 18, line 38 puts \(J_n(k_0L)\) in BJ(n+1+(L-1)*N1) for \(n=0,1,...,N_1-1\) where \(k_0L\) is given by (22). Line 38 also puts \(Y_n(k_0L)\) in BY(n+1+(L-1)*N1) for \(n=0,1,...,N_1-1\) although only \(\{y_n(k_0L) = y_n(ka)\}, n=0,1,...,N_1-1\) is needed.

Table 1. Key lines in the main program, the variables read in, defined, or incremented therein, the corresponding quantities in the text, and the numbers of the equations where these quantities appear.

<table>
<thead>
<tr>
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<td>p and ka</td>
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<td>P and ALP</td>
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<td>CE(J)</td>
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<tr>
<td>Line Number</td>
<td>Variable(s) in the main program</td>
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<tr>
<td>85</td>
<td>{SE(J), $J=1,2,3,4$}</td>
<td>${s^e_{Jn}, J=1,2,3,4}$</td>
<td>31</td>
</tr>
<tr>
<td>85</td>
<td>{SO(J), $J=1,2,3,4$}</td>
<td>${s^o_{Jn}, J=1,2,3,4}$</td>
<td>41</td>
</tr>
<tr>
<td>85</td>
<td>{FE(I), $I=1,2,3,4$}</td>
<td>${f^e_{In}, I=1,2,3,4}$</td>
<td>26, 30</td>
</tr>
<tr>
<td>85</td>
<td>{FO(I), $I=1,2,3,4$}</td>
<td>${f^o_{In}, I=1,2,3,4}$</td>
<td>35, 40</td>
</tr>
<tr>
<td>88</td>
<td>BJN</td>
<td>$J_n(\text{ka})$</td>
<td>26, 30</td>
</tr>
<tr>
<td>91</td>
<td>U2</td>
<td>$1/H_n(2)(\text{ka})$</td>
<td>26, 30</td>
</tr>
<tr>
<td>93, 94</td>
<td>CSA</td>
<td>$\epsilon \cos(n\alpha)_n$</td>
<td>26</td>
</tr>
<tr>
<td>95</td>
<td>SNA</td>
<td>$2 \sin(n\alpha)$</td>
<td>35</td>
</tr>
<tr>
<td>97</td>
<td>EF(I)</td>
<td>$F^e_{In}/H_n(2)(\text{ka})$</td>
<td>30</td>
</tr>
<tr>
<td>98</td>
<td>OF(I)</td>
<td>$F^o_{In}/H_n(2)(\text{ka})$</td>
<td>40</td>
</tr>
<tr>
<td>99</td>
<td>CUE(I)</td>
<td>nth term</td>
<td>26</td>
</tr>
<tr>
<td>100</td>
<td>CUO(I)</td>
<td>nth term</td>
<td>35</td>
</tr>
<tr>
<td>105</td>
<td>YE(I)</td>
<td>$S^e_{1p}(\text{EF})_{Ip}$</td>
<td>28</td>
</tr>
<tr>
<td>106</td>
<td>YO(I)</td>
<td>$S^o_{1p}(\text{OF})_{Ip}$</td>
<td>38</td>
</tr>
<tr>
<td>110</td>
<td>RP(L)</td>
<td>$(\text{RP})_L$</td>
<td>54</td>
</tr>
<tr>
<td>116</td>
<td>BJJ(L)</td>
<td>$J_n(\text{ka})/J_n(\text{ka})$</td>
<td>55</td>
</tr>
<tr>
<td>120</td>
<td>BJN</td>
<td>$J_n(\text{ka})Y_n(\text{ka})$</td>
<td>31, 41</td>
</tr>
<tr>
<td>122</td>
<td>EF(I)</td>
<td>$j^e_{In}$</td>
<td>30</td>
</tr>
<tr>
<td>123</td>
<td>OF(I)</td>
<td>$j^o_{In}$</td>
<td>40</td>
</tr>
<tr>
<td>Line Number</td>
<td>Variable(s) in the main program</td>
<td>Quantity in the Text</td>
<td>Equation number(s)</td>
</tr>
<tr>
<td>------------</td>
<td>--------------------------------</td>
<td>---------------------</td>
<td>-------------------</td>
</tr>
<tr>
<td>125</td>
<td>{BJJ(L), L=1,2,...,NR}</td>
<td>{J_n (k_L)/J_n (k_a)}</td>
<td>55</td>
</tr>
<tr>
<td>128,131</td>
<td>E</td>
<td>E_J</td>
<td>67</td>
</tr>
<tr>
<td>129,132</td>
<td>0</td>
<td>O_J</td>
<td>68</td>
</tr>
<tr>
<td>135</td>
<td>YE(IY)</td>
<td>nth term</td>
<td>28 or 29</td>
</tr>
<tr>
<td>136</td>
<td>YO(IY)</td>
<td>nth term</td>
<td>38 or 39</td>
</tr>
<tr>
<td>143</td>
<td>RN(JR)</td>
<td>nth term</td>
<td>55</td>
</tr>
<tr>
<td>148,152</td>
<td>{VE(J), I=1,2,...,NN}</td>
<td>{\hat{V}_I^e, I=1,2,...,NN}</td>
<td>43</td>
</tr>
<tr>
<td>149,154</td>
<td>{VO(I), I=1,2,...,NN}</td>
<td>{\hat{V}_I^o, I=1,2,...,NN}</td>
<td>44</td>
</tr>
<tr>
<td>163</td>
<td>VO(1)</td>
<td>\hat{V}_I^o</td>
<td>46</td>
</tr>
<tr>
<td>164</td>
<td>U3</td>
<td>\hat{V}_I^e</td>
<td>43</td>
</tr>
<tr>
<td>165</td>
<td>VE(1)</td>
<td>\hat{V}_I^e</td>
<td>45</td>
</tr>
<tr>
<td>167</td>
<td>SE(J)</td>
<td></td>
<td>\hat{V}_J^e</td>
</tr>
<tr>
<td>168</td>
<td>SO(J)</td>
<td></td>
<td>\hat{V}_J^o</td>
</tr>
<tr>
<td>173</td>
<td>SE(J)</td>
<td>{S_{J0}^e, J=1,2,3,4}</td>
<td>47</td>
</tr>
<tr>
<td>175</td>
<td>U2</td>
<td>S_{10}^e \hat{V}_1</td>
<td>47</td>
</tr>
<tr>
<td>181</td>
<td>U2</td>
<td>{E_z^{b(-M)}}_o=0</td>
<td>47</td>
</tr>
<tr>
<td>188</td>
<td>XX</td>
<td>\phi_L/\phi_o</td>
<td>49</td>
</tr>
<tr>
<td>192</td>
<td>U2</td>
<td>[E_z]_L</td>
<td>49</td>
</tr>
<tr>
<td>195</td>
<td>BJ(L)</td>
<td></td>
<td>[E_z]_L</td>
</tr>
<tr>
<td>201</td>
<td>U2</td>
<td>(RP)_L \hat{V}_1</td>
<td>53</td>
</tr>
<tr>
<td>204</td>
<td>U2</td>
<td>[E_z^{b(-M)}]_L</td>
<td>53</td>
</tr>
<tr>
<td>206</td>
<td>RE(L)</td>
<td></td>
<td>[E_z^{b(-M)}]_L</td>
</tr>
</tbody>
</table>
Line 61 puts $S_{Jp}^e$ of (32) in $SE(J)$ for $J=1,2,3,$ and 4. Line 61 also puts $S_{Jp}^o$ of (42) in $SO(J)$ for $J=1,2,3,$ and 4. Lines 62 to 67 put $J (ka)$ of (28) in $BJPE$ and $\{C_J^e, J=1,2,\ldots,NN\}$ of (32) in $\{CE(J), J=1,2,\ldots,NN\}$. Lines 68 to 79 put $J^o_p (ka)$ of (33) in $BJPO$ and $\{C_J^o, J=1,2,\ldots,NN\}$ of (42) in $\{CO(J), J=1,2,\ldots,NN\}$.

Being summations with respect to $n$, the matrix elements $I^e_I$ of (26), $Y^e_{IJ}$ of (28) and (29), $I^o_I$ of (35), $Y^o_{IJ}$ of (38) and (39), and $(RN)_J^L$ of (55) are accumulated in $CUE(I)$, $YE(IY)$, $CUO(I)$, $YO(IY)$, and $RN(JR)$, respectively, where

\[
\begin{align*}
IY &= I + (J-1)*NN \\
JR &= J + (L-1)*NN
\end{align*}
\]  

In DO loop 15,

\[
N = NP-1
\]

where $NP$ is the index of DO loop 15. The terms for which $n=N$ are added to $CUE(I)$ and $CUO(I)$ in DO loop 15. If $N \neq p$, then the terms for which $n=N$ are added to $YE(IY)$, $YO(IY)$, and $RN(JR)$ in DO loop 15. If $N=p$, then $S_{Jn}^e (EF)_I^p$ is added to $YE(I)$, $S_{Jn}^o (OF)_I^p$ is added to $YO(I)$, and $(RP)_L$ of (54) is put in $RP(L)$ in DO loop 15. More detail on the workings of DO loop 15 is presented in the following four paragraphs.

Line 85 puts $S_{Jn}^e$ of (31) in $SE(J)$, and $S_{Jn}^o$ of (41) in $SO(J)$ for $J=1,2,3,$ and 4. Line 85 also puts $F_{In}^e$ of (26) and (30) in $FE(I)$, and $F_{In}^o$ of (35) and (40) in $FO(I)$ for $I=1,2,3,$ and 4.

If $N < N_l$, then lines 87 to 118 are executed. Line 88 puts $J_n (ka)$ in $BJN$, line 91 puts $1/H_n^{(2)} (ka)$ in $U2$, lines 93 and 94 put $\zeta_n \cos (n\alpha)$ in $CSA$, and line 95 puts $2\sin(n\alpha)$ in $SNA$. In DO loop 40, $(EF)_I^{e}n$ of (30) is
put in EF(I), (OP)\_In of (40) is put in OF(I), the nth term of (26) is added to CUE(I), and the nth term of (35) is added to CUO(I). If N=p, then DO loops 41 and 53 are executed. In DO loop 41, S^e\_lp (EF)\_Ip of (28) is added to YE(I), and S^O\_lp (OF)\_Ip of (38) is added to YO(I). In DO loop 53, (RP)_L of (54) is put in RP(L). If N \neq p, then DO loop 55 is executed. In DO loop 55, the factor J^n (k0_L)/J^n (ka) in the first summation in (55) is put in BJJ(L).

If N \geq N_1, then lines 120 to 125 are executed. Line 120 puts J^n (ka) Y^n (ka) of (31) and (41) in BJN. DO loop 51 puts J^e\_In of (30) in EF(I) and J^O\_In of (40) in OF(I). In line 125, the factor J^n (k0_L)/J^n (ka) in the second summation in (55) is put in BJJ(L) for L=1,2,...,NR.

If N \neq p, then lines 126 to 145 are executed. Immediately before these lines are executed (EF)\_In of (30) will reside in EF(I), and (OP)\_In of (40) will reside in OF(I). Upon entry into DO loop 44, the quantity E\_j defined by

$$ E_j = \begin{cases} J^e_p (ka) E^e_{jn} & j = 1 \\ E^e_{jn} & j \neq 1 \end{cases} $$  \hspace{1cm} (67)

resides in E and the quantity O\_j defined by

$$ O_j = \begin{cases} J^O_p (ka) E^O_{jn} & j = 1 \\ E^O_{jn} & j \neq 1 \end{cases} $$  \hspace{1cm} (68)

resides in O. Here, E^e_{jn} is given by (31) and E^O_{jn} by (41). In DO loop 44, the nth term of the summation in (28) or (29) is added to YE(IY), and the nth term of the summation in (38) or (39) is added to YO(IY). In nested DO loops 56 and 57, the nth term of the summation in (RN)_JL of (55) is added to RN(JR).
Lines 147 to 154 put the elements of the solution $\vec{v}^e$ of (43) in $VE$ and the elements of the solution $\vec{v}^o$ of (44) in $VO$. In DO loop 45, $v^e_1$ of (45) is accumulated in $U2$ and $v^o_1$ of (46) is accumulated in $U3$. The series in (45) and (46) were, as indicated in the paragraph preceding (64), summed for $i$ up to $NN$ rather than 4. After lines 163-165 have been executed, the elements of $\vec{v}^e$ will reside in $VE$, the elements of $\vec{v}^o$ will reside in $VO$, and $v^e_1$ will be in $U3$. In DO loop 47, $|v^e_j|$ is put in $SE(J)$, and $|v^o_j|$ is put in $SO(J)$.

The rest of the main program, lines 173 to 210, serves to calculate the field (47) at the center of the cylinder, the aperture field (49), and the field (53) along the line from the center of the cylinder to the center of the aperture. The series in (47), (49), and (53) are summed with $i$ running from 1 to $NN$ rather than from 1 to 4.

Line 173 puts $S^e_{j0}$ in $SE(J)$ for $J=1,2,3,4$. If $p=0$, then line 175 puts $S^e_{10}$ of (47) in $U2$. If $p \neq 0$, then DO loop 48 is executed and line 181 puts $[E^b_z(-\pi)]_{\phi=0}$ of (47) in $U2$.

The index $L$ of DO loop 62 obtains the subscript $L$ in (49). Line 188 puts $(\phi_{1L}/\phi_0)$ in $XX$. DO loop 63 accumulates $[E^z_L]$ of (49) in $U2$. Line 195 puts $|E^z_L|$ of (49) in $BJ(L)$.

The index $L$ of DO loop 58 obtains the subscript $L$ in (53). Line 201 puts $(RP)_L\vec{v}^e$ of (53) in $U2$. DO loop 59 adds to $U2$ the summation with respect to $j$ in (53). Line 206 puts $|E^b_z(-\pi)|_L$ of (53) in $RE(L)$.

Sample input and output data appear after the listing of the main program. The sample input data are for a perfectly conducting circular cylindrical shell of electrical radius $ka = 2$ with a slot of half angular width $\phi_o = 1.25^\circ = 0.02181662$ radians centered at $\phi = 0^\circ$ illuminated by a TM plane wave that comes from the direction in which $\phi = 0^\circ$. The last
number in the sixth from the last row of the sample output data is exactly the same as the first number in the last row. This is expected because expression (56) is identical to expression (47) when p=0.

We ran the main program with IP of the input data changed from 0 to 1. The first four significant figures of each number in the last six lines of the resulting output data were the same as those that were obtained with IP=0. According to (56) and (47), the last number in the sixth from the last line of the output data may, due to roundoff error, be slightly different from the first number in the last line if IP≠0. These two numbers were identical to each other in the output obtained with IP=1.

Note that the third number in the fourth from the last line of the sample output data is slightly different from the last number in the last row. Reasons for this are given in the paragraph that contains (63).
LISTING OF THE MAIN PROGRAM

INPUT DATA ARE READ FROM THE FILE MAUTZ3.DAT ON UNIT 20.
OUTPUT DATA ARE WRITTEN ON THE FILE MAUTZ6.DAT ON UNIT 21.
THE SUBROUTINES BES, BESJ, BESJY, BESJJ, SFE0,
DECOMP, AND SOLVE ARE USED.

COMPLEX YE(16), Y0(16), CUE(4), CU0(4), U3, U2, EF(4), OF(4)
DIMENSION Y0(36), DJ(220), BY(220), XR(11), RP(11), RN(44), RE(11)
DIMENSION BJ(11), SE(4), SO(4), FE(4), FO(4), CE(4), CO(4), IPS(4)

OPEN(UNIT=20, FILE='MAUTZ3.DAT', STATUS='OLD')
OPEN(UNIT=21, FILE='MAUTZ6.DAT', STATUS='OLD')

READ(20, 25)(Y0(I), IMI, 35)
WRITE(21, 26)(IOM, 1X, 3E14.7)
READ(20, 26) M1, N2, N3, NA, NR.
WRITE(21, 27) M1, N2, N3, NA, NR

FOR I= 1, N3
READ(20, 32) M1, IP, IB, X, P, ALP
WRITE(21, 28) M1, IP, IB, X, P, ALP

DO 35 NP= 2, N1
BJ(NP)= 0.
CONTINUE

XR(1)= 0.
JJ= 1
FNR= NR-1
DO 35 L= 2, NR
XR(L)=(L-1)/FNR*X
IF(L.EQ.NR) XR(L)= X
JJ= JJ+ 1
CALL BES(Y0, M1, XR(L), BJ(JJ), BY(JJ))
CONTINUE

JJN= JJ+ M1-1
WRITE(21, 33) BJ(NP), NP= JJ, JJN
WRITE(21, 19) BY(NP), NP= JJ, JJN

WRITE(21, 33) BJ(NP), NP= JJ, JJN
WRITE(21, 19) BY(NP), NP= JJ, JJN

MN2=MN= MN
DO 35 I= 1, MN2
YE(I)= 0.
Y0(I)= 0.
CONTINUE

DO 35 I= 1, NN
CUE(I)= 0.
CU0(I)= 0.
CONTINUE
NRW=NR*NN
DO 52 J=1, NRW
RN(J)=0.
52 CONTINUE
M1R=MI*(NR-1)
IPP=IP+1+M1R
IF(IB.EQ.0) BJ(IPP)=0.
CALL SFEO(IP,P,SE,SO,FE,FO)
BJPE=BJ(IPP)
CE(I)=0.
IF(NN.EQ.1) GO TO 34
DO 37 J=2, NN
CE(J)=-SE(J)/SE(1)
37 CONTINUE
34 IF(IP.NE.O) GO TO 21
BJPO=1.
DO 38 J=1, NN
CO(J)=0.
38 CONTINUE
GO TO 23
21 BJPO=BJPE
CO(1)=0.
IF(NN.EQ.1) GO TO 23
DO 22 J=2, NN
CO(J)=-SO(J)/SO(1)
22 CONTINUE
U3=(1.,0.)
U=(0.,1.)
N2P=N2+1
DO 30 NP=1, N2P
N=NP-1
CALL SFEO(N,P,SE,SO,FE,FO)
IF(N.GE.NI) GO TO 50
MPN=NP+M1R
DJ(NP1)=BJ(NP)
S1=BJN
IF(ABS(S1).LE.1.E-10) S1=0.
U2=1./CMPLX(S1,-BY(NPN))
ALN=ALP*N
CSA=2.*COS(ALN)
IF(N.EQ.0) CSA=1.
SNA=2.*SIN(ALN)
DO 40 I=1, NN
EF(I)=FE(I)*U2
OF(I)=FO(I)*U2
CUE(I)=CSA*EF(I)*U3*CUE(I)
CUO(I)=SNA*OF(I)*U3*CUO(I)
40 CONTINUE
U3=U3*U
IF(N.NE.IP) GO TO 20
DO 41 I=1, NN
YE(I)=SE(I)*EF(I)+YE(I)
YO(I)=SO(I)*OF(I)+YO(I)
41 CONTINUE

107 41 CONTINUE
108      IJ=NP
109    DO 53 L=1, NR
110   RP(L)=SE(I)*BJ(IJ)
111      IJ=IJ+1
112 53 CONTINUE
113    GO TO 15
114 20 JJ=NP
115    DO 55 L=1, NR
116   BJJ(L)=BJ(JJ)/BJN
117      JJ=JJ+1
118 55 CONTINUE
119    GO TO 54
120 50 CALL BESJY(N*X,BJN)
121   DO 51 I=1, NN
122   EF(I)=EF(I)*U
123   O(F(I))=FO(I)*U
124 51 CONTINUE
125    CALL BESJJ(N*NR,XR,BJJ)
126 54 IT=0
127    DO 42 J=1, NN
128   E=(SE(J)+CE(J)*SE(I))/BJN
129   O=(SO(J)+CO(J)*SO(I))/BJN
130    IF(J.NE.1) GO TO 43
131   E=BJPE*E
132   O=BJPO*O
133   DO 44 I=1, NN
134      IT=IT+1
135   YE(IY)=EF(I)+TF(I)*Y1(T)
136   YO(IY)=FO(I)+TO(I)*YO(T)
137 44 CONTINUE
138 42 CONTINUE
139    JR=0
140    DO 56 L=1, NR
141   DO 57 J=1, NN
142      JR=JR+1
143   RN(JR)=BJJ(L)*SE(J)+RN(JR)
144 57 CONTINUE
145 56 CONTINUE
146 15 CONTINUE
147    IF(NN.NE.1) GO TO 17
148   VE(1)=CUE(I)/YE(I)
149   VO(1)=CUO(I)/YO(I)
150    GO TO 16
151 17 CALL DECOMP(NN,IPS,YE)
152    CALL SOLVE(NN,IPS,YE,CUE,VE)
153    CALL DECOMP(NN,IPS,YO)
154    CALL SOLVE(NN,IPS,YO,CUO,VO)
155 16 CE(I)=BJPE
156   CO(I)=BJPO
157     U2=0.
158     U3=0.
159   DO 45 I=1, NN
U2=CE(1)*VE(1)+U2
U3=CD(1)*VO(1)+U3

45 CONTINUE

V0(1)=U3
U3=VE(1)
VE(1)=U2

DO 47 J=1,NN
SE(J)=CABS(VE(J))
SO(J)=CABS(V0(J))

47 CONTINUE

WRITE(21,24)(SE(J),J=1,NN)
WRITE(21,24)(SO(J),J=1,NN)

24 FORMAT(1X,4E14.7)
CALL SFEO(0,P,SE,SO,FE,FO)
IF(18,N.E.0) GO TO 45
U2=SE(1)*U3

GO TO 39

48 U2=0.

DO 48 J=1,NN
U2=SE(J)*VE(J)+U2

48 CONTINUE

U2=U2/BJ(N1R+1)
S1=CABS(U2)
WRITE(21,49) U2,S1

49 FORMAT(1X,2E14.7) 'ABS(E)='E14.7)
FNA=2./(NA-1)
DO 62 L=1,NA
U2=0.
XX=1.+(L-1)*FNA
X2=XX*XX
S1=SQRT(1.-X2)

62 CONTINUE

DO 63 J=1,NN
U2=S1*(XX*VO(J)+VE(J))+U2

63 CONTINUE

DJ(L)=CABS(U2)

63 CONTINUE

WRITE(21,64)(BJ(L),L=1,NA)

64 FORMAT(' APERTURE FIELD AMPLITUDE'/1X,4E14.7))
JR=0

DO 58 L=1,HR
U2=RP(L)*U3

DO 59 J=1,NN
JR=JR+1
U2=RM(JR)*VE(J)+U2

59 CONTINUE

RE(L)=CABS(U2)

58 CONTINUE

WRITE(21,66)(RE(L),L=1,HR)

66 FORMAT(' INTERIOR FIELD AMPLITUDE'/1X,4E14.7))

31 CONTINUE

STOP

END
### Listing of the Input Data File MAUTZ3.DAT

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<th>0.1710234E+02</th>
<th>-0.2271001E+00</th>
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<td>0.2181862E-01</td>
<td>0.0000000E+00</td>
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### Listing of the Output Data File MAUTZ6.DAT

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<th>-0.608510E+03</th>
<th>0.1710234E+02</th>
<th>-0.2271001E+00</th>
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</tbody>
</table>

| M1= 20, M2=10000, N3= 1, NA= 5, NR= 3 |
| MN= 4, IP= 0, IB= 1 |
| X= 0.2000000E+01, P= 0.2181862E-01, ALP= 0.0000000E+00 |

### BJ

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### BY

| E=-0.1121177E-02 | 0.2728021E-03, ABS(E)= 0.1153889E-02 |

**Aperture Field Amplitude**

| 0.0000000E+00 | 0.4136272E-01 | 0.4655501E-01 | 0.4136272E-01 |
| 0.0000000E+00 |                |                |                |

**Interior Field Amplitude**

| 0.1153889E-02 | 0.1597396E-02 | 0.4656652E-01 |
REFERENCES


END
4-87
DTIC