Distributed and point-delay time-lag control systems. Feedback stabilization, state reconstruction, and tracking controllers for time-lag systems. Parameter identification of linear, bilinear and polynomial input-output differential systems.

Research is summarized for the state feedback stabilization of point delay and distributed delay time lag control systems via the development of a reducing transformation technique. The approach facilitates the controller design using well established delay-free methods once the unstable pole set is delineated for the time-lag system. Research is described on the dual problem of state reconstruction using input-output data and also on a tracking problem which employs integral action for the controller. Research is described in the use of a Fourier based modulating function technique for the least squares parameter identification of a class of polynomial input output differential systems.
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CONTROL AND IDENTIFICATION OF TIME VARYING SYSTEMS  

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Allan E. Pearson  
Division of Engineering  
Brown University  
Providence, RI 02912  

Report prepared by:  

Allan E. Pearson  
Professor of Engineering  
Principal Investigator  

Carl Cometta  
Executive Officer  
Division of Engineering  

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1. Introduction

This final technical report covers a one year period preceding August 31, 1986 during which support was provided under AFOSR Grant 85-0300. The research results described in Section 3 below were partially described in the AFOSR Proposal No. 86-NM-191 which is pending.

2. List of Scientific Collaborators

Y. A. Fiagbedzi* Post-doctoral Research Associate
F. C. Lee Former Graduate Student
A. E. Pearson* Professor and Principal Investigator

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3. Completed Research

The published research consists of three journal articles [1-3] and five conference proceedings papers [4-8]. There are three in-print or submitted-for-publication articles [9-11]. The results are described under the following headings.

3.1. Feedback Control and State Reconstruction of Time Lag Systems [3,6-11]

State feedback stabilization for the class of time lag systems described by the n-dimensional "point delayed" state and output equations

\[
\dot{x}(t) = A_0 x(t) + A_1 x(t-h_1) + B_0 u(t) + B_1 u(t-h_2)
\]

\[
y(t) = C_0 x(t) + C_1 x(t-h_2)
\]

has been developed in [3,6] via a "reducing transformation" method. This technique has its roots in the classical Smith regulator for stabilizing systems characterized by transfer functions with pure transport lag, but until the work in [3,6] the approach had been restricted to differential delay systems with delays in the control variables only.\(^1\) The distinguishing feature of this technique is that once the reducing transformation has been found, the design of the stabilizing feedback control can be carried out using well known delay-free methods. At the heart of finding the reducing transformation for a given system (1) is the "characteristic matrix equation" which is defined as the \(n \times n\) matrix equation:

\[
A - A_0 e^{-rA} A_1 = 0.
\]

Based on a partitioning of the unstable and poorly damped pole set for (1) into \(N\) disjoint sets of \(n\) eigenvalues each, it is shown in [3] how to construct \(N\) real matrix solutions to (3), each of which inherits \(n\) eigenvalues from the unstable pole set, in terms of the corresponding \(N\) sets of \(n \times n\) modal matrices. Each modal matrix is comprised of \(n\) linearly independent left eigenvectors corresponding to the \(n\) eigenvalues in each set. In this sense, (3) is referred to as the "left" characteristic matrix equation for (1). This terminology is suggested by our recent development of a corresponding observer theory for (1) using the output measurement equation (2) [11]. Here the pertinent matrix equation is

\[
F - A_0 e^{-rF} A_1 = 0.
\]

The algorithm devised for solving (4) is the same as the one for solving (3) only involving right eigenvectors instead of left; hence, the terminology in reference to (3) and (4) as the "left" and "right"

\(^1\) For example, the treatment in Kwon and Pearson: "Feedback Stabilization of Linear Systems with Delayed Control," *IEEE Trans. on Auto Contr.*, AC-25, no. 2, pp. 266-269, 1980.
characteristic matrix equations respectively. Together, the ability to construct solutions to these equations using well known eigenvector techniques constitutes the first step of the ensuing controller/observer design methodology outlined in \cite{3,11} using well established delay-free methods.\footnote{Actually, the first step is to check the spectral controllability/observability properties of (1) and (2) relative to the unstable pole set. However, this can be carried out using finite dimensional eigenvalue-eigenvector tests (the so called PBH tests) once the unstable pole set has been delineated.}

We believe that these equations will prove fundamental to many other system theoretic problems involving delay equations such as system approximation and filtering theory.

Extensions have been made in \cite{8,9} to more general systems such as the class of distributed delay systems described by

\[ \dot{x}(t) = \int_{-\infty}^{0} d\alpha(\theta)x(t+\theta) + \int_{0}^{\infty} d\beta(t)u(t+\tau) \]  

(5)

where $\alpha(\cdot)$ and $\beta(\cdot)$ are matrix valued functions of bounded variation and the integrals are of the Stieltjes type. This includes multiple point delayed systems akin to the system (1) as special cases. Some results on the state feedback stabilization problem for (5) are contained in \cite{8,9}, while others pertaining to the observer theory are still under development.

A tracking theory for the system (5) has been advanced in \cite{7,10} which facilitates integral action in the state feedback controller so that designated outputs will track step command inputs with zero steady state error while accomplishing stabilization with a prescribed degree of stability. It is intended to extend the theory by including an observer. When this extension is complete, it will represent a modern state space approach to the design of controllers for time lag systems which utilizes well-known delay-free methods, much in the same spirit as the design methodology for the classical Smith Predictor Controller. However, the work will by no means be complete at that point since, for example, we shall want to investigate issues of sensitivity and disturbance rejection, as well as perform a comparison with other approaches to the same class of control system problems such as those based on linear semigroup theory, the polynomial ring ideas, etc.

3.2. System Parameter Identification \cite{1,2,4,5}

A least squares parameter identification technique has been formulated in \cite{1} for the class of nonlinear systems modeled by the polynomial input-output differential equation:

\[ p^n y(t) + \sum_{i=1}^{n} \sum_{j=0}^{m} a_i(j,k)p^{n-i}[u(t)]^j[y(t)]^k = 0 \]  

(6)

\[ 0 \leq t \leq T, \quad a_i(0,0) = 0, \quad i=1...n. \]

Here $p$ is the differential operator $d/dt$ and the $a_i(j,k)$ represent parameters which are to be determined for a presumed given order $n$ based on the input-output data $[u(t),y(t)]$ observed over a single time interval $[0,T]$ for a one-shot estimate, or over a sequence of time intervals, each of duration $T$, for sequential least squares. The basis of the technique is Shinbrot's method of moment functionals using trigonometric modulating functions. It is shown in \cite{1} how the least squares identification can be formulated in a way that utilizes the computationally efficient FFT algorithm at each stage while avoiding the necessity to estimate unknown initial conditions for time limited data. In addition to the order of the system model and the number of parameters to be identified, the choice in modulating functions can be based to some extent on noise considerations, though much more remains to be done in this regard.
A special case of the model (6) is the bilinear input-output system

\[ p^n y(t) + \sum_{i=1}^{n} \alpha_i p^{n-i} y(t) = \sum_{i=1}^{n} p^{n-i} [\beta_i u(t) + \gamma_i u(t)y(t)] \]  

(7)

where the \((\alpha_i, \beta_i, \gamma_i)\) represent parameters to be identified. Within the framework of the modulating function approach and using sinusoidal probing signals on sequential time intervals, it was shown in [4] how the least squares identification of the system parameters can be accomplished using essentially the same underlying computation as would attend the identification of a linear differential system of the same order. The order determination problem for (7) was formulated in [5] with the order \(n\) to be determined in addition to the system parameters. However, in retrospect this initial formulation is considered to be a failure and a new formulation is planned for future investigation based on the singular value decomposition and "total" least squares theory.

Although the parameter identification problem for linear systems is ostensibly a special case of the problems discussed in [1,4], more substantive results are obtained in [2] for handling noise corrupted data. Specifically, it is shown how the frequency domain interpretation can be beneficial in enhancing the signal to noise ratio of the modulated data for the deterministic least squares estimate. Further, a maximum likelihood estimate is developed for the stochastic case of additive white gaussian noise in the data which effectively removes the bias when the parameter identification is considered in a recursive mode.

Future research in this area is planned on the problems of structure determination, e.g., determining the order \(n\) of the differential operator models in (6) and (7), and an extension of the theory to handle the deleterious effect caused by sensor dynamics.

4. Publications Under AFOSR 85-0300

4.1. Journal Articles


4.2. Conference Proceedings


4.3. Papers Submitted for Publication


