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 DEVELOPMENT AND APPLICATION OF THE P-VERSION OF THE FINITE ELEMENT METHOD

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19 ABSTRACT (Continue on reverse if necessary and identify by block number)

The p-version of the finite element method is a new, important, computationally efficient, approach to finite element analysis. It is more robust than the conventional h-version and its rate of convergence, for domains with corners and for other singularity problems, is twice that of the h-version.

Hierarchic elements which implement the p-version efficiently have been formulated so as to enforce C^0 or C^1 continuity in the planar case, and so as to enforce C^0 continuity in three dimensions.

* Continued on the reverse side.
Recent research accomplishments include:

1. Development of an algorithm that finds all roots of an analytic function in a finite domain.

2. Preprocessing procedures to restrict unbounded domains which contain roots to bounded ones.

3. A reliable numerical argument principle algorithm to compute number of zeros within a closed contour.

4. Formulation of equations which determine the nature of stress singularity at a corner of a plate composed of n estropic materials.

All of the above are used in the extraction method for p-version finite element analysis of composite materials with corners.
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1. DISCUSSION

There are now three basic approaches to finite element analysis. In all approaches the domain $\Omega$ is divided into simple convex subdomains (usually triangles or rectangles in two dimensions, and tetrahedra or bricks in three dimensions) and over each subdomain the unknown is approximated by a (local) basis function (usually a polynomial of degree $\leq p$). Basis functions are required to meet continuously at boundaries of subdomains in the case of planar or 3 dimensional elasticity, or smoothly in the case of plate bending. The approaches are:

1. The \textit{h-version} of the finite element method. In this approach the degree $p$ of the approximating polynomial is kept fixed, usually at some low number such as 2 or 3. Convergence is achieved by allowing $h$, the maximum diameter of the convex subdomains, to go to zero. Estimates for the error in energy have long been known. In all of these estimates $p$ is assumed to be fixed and the error estimate is asymptotic in $h$, as $h$ goes to zero.

2. The \textit{p-version} of the finite element method. In this approach the subdivision of the domain $\Omega$ is kept fixed but $p$ is allowed to increase until a desired accuracy is attained. The p-version is reminiscent of the Ritz method for solving partial differential equations but with a crucial distinction between the two methods. In the Ritz method a single polynomial approximation is used over the \textit{entire} domain $\Omega$ ($\Omega$, in general, is not convex). In the p-version of the finite element method polynomials are used as approximations over \textit{convex subdomains}. This critical difference gives the p-version a more rapid rate of convergence than either the Ritz method or the h-version.
3. The h-p version of the finite element method. In this approach both the degree \( p \) of the approximating polynomial and the maximum diameter \( h \) of the convex subdomains are allowed to change.

The p-version of the finite element method requires families of polynomials of arbitrary degree \( p \) defined over different geometric shapes. Polynomials defined over neighboring elements join either continuously (are in \( C^0 \)) for planar or three dimensional elasticity, and smoothly (are in \( C^1 \)) for plate bending. In order to implement the p-version efficiently on the computer, these families should have the property that computations performed for an approximation of degree \( p \) are re-usable for computations performed for the next approximation of degree \( p+1 \). We call families possessing this property hierarchic families of finite elements.

The h-version of the finite element method has been the subject of intensive study since the early 1950's and perhaps even earlier. Study of the p-version of the finite element method, on the other hand, began at Washington University in St. Louis in the early 1970's and led to a more recent study of the h-p version. Research in the p-version (formerly called The Constraint Method) has been supported in part of the Air Force Office of Scientific Research since 1976.

Recent Research Accomplishments include:

1. Development of an algorithm that finds all roots of an analytic function in a bounded domain.

2. A preprocessing procedure which finds a bounded subdomain of a given unbounded domain in which the roots of an analytic function are to be found.

3. A numerical argument principle which computes the number of zeros of an analytic function inside a closed contour. The crucial part of
this algorithm is a test to determine whether the argument change between two points on the contour is less than $\pi$ in absolute value.

4. Explicit formulation of the equations which determine the nature of the stress singularity in a plate wedge composed of $n$ isotropic materials meeting at a corner. The boundary conditions are either clamped or free.

All of the above are needed to use the extraction techniques developed earlier for the finite element analysis by the p-version of a composite material with corners.
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PAPERS PUBLISHED AND PRESENTED SINCE THE START OF THE PROJECT (1977)

Published Papers:


Presented Papers:


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