IMPACT RESPONSE OF STRUCTURES - GLOBAL LOCAL APPROACH

NORTHWESTERN UNIV EVANSTON IL  L M KEER 30 NOV 86
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## Impact Response of Structures—Global Local Approach

Leon M. Keer

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Static and dynamic solutions to indentation problems of large area contact with respect to the structural thickness were solved. Two classes of problems were solved during this time period. They involved the smooth indentation of transversely isotropic beams and the low velocity impact of transversely isotropic beams and plates. In addition to these problems, an investigation into the problem of three-dimensional scattering by a planar crack was begun.
RESEARCH OBJECTIVES

The objective of the research is to attempt to develop systematic techniques for determining the response of dynamic impact when the shape of the impacting object is known. The dynamic response can be obtained by the use of techniques that accurately model the stresses at the point of impact as well as the stresses away from the point of impact. This technique is one that involves a global-local technique, where a more exact description of the stress distribution is made near the contact (for example elastic theory) and a more approximate theory is used away from the point of load application (beam, plate or shell theory).

The research objectives include the study of both static and dynamic solutions to indentation problems. The static solutions are solved in order to understand elasticity results that will tend to form the bases for the local solution. This solution is then incorporated into a dynamic structural mechanics solution for a beam, plate or shell to obtain results of both the global and local character.

STATUS OF RESEARCH

During the tenure of the grant the following problems have been solved. For the static case:

"Smooth Indentation of Finite Layer" by L.M. Keer and G.R. Miller. The plane problem of a layer of finite length under the action of a frictionless indenter is studied. The solution obtained accounts for both the local contact behavior near the indenter and the overall beam behavior as well. The analysis superposes an infinite layer solution derived through the use of integral transforms with a pure bending beam
theory solution. The problem is reduced to a Fredholm integral equation of the second kind, which is solved numerically. Clamped and simply supported end conditions are studied; overall compliances and local indenter stresses are computed and plotted for various ratios of contact length to layer thickness. The results are compared to beam theory compliances and Hertz theory contact stresses.

"Smooth Contact Between a Rigid Indenter and an Initially Stressed Orthotropic Beam" by L.M. Keer and R. Ballarini. The elastostatics problem of a rigid punch in smooth contact with an initially stressed transversely isotropic layer is investigated. The problem is formulated by writing the field equations in terms of suitable displacement potentials. In addition, an elementary beam theory solution is superimposed on the elasticity solution to satisfy the support boundary conditions for a finite beam. The cases studied are those of a simply supported beam and a clamped beam.

"Smooth Indentation of an Isotropic Cantilever Beam" by L.M. Keer and W.P. Schonberg. The response of an isotropic cantilever beam of finite length under the action of frictionless cylindrical and flat indenters is studied. Solutions are obtained through a local-global technique, which accounts for both the local behavior near the indenter, as well as the global beam behavior. The method of analysis superposes an infinite-layer solution, derived through the use of integral transforms with a pure-bending beam-theory solution. Local indenter stresses, as well as displacements and rotations, are computed for each case and plotted for various ratios of contact width to beam thickness, and for
various positions of the indenter. Where possible, the results are compared to Hertz theory of contact stresses and to beam-theory displacement and rotation solutions.

"Smooth Indentation of a Transversely Isotropic Cantilever Beam" by L.M. Keer and W. Schonberg. The response of a transversely isotropic beam of finite length to a frictionless cylindrical and flat indenter is studied. Solutions are obtained through a global-local technique, which accounts for the local behavior near the indenter, as well as the global beam behavior. The method of analysis superposes an infinite elastic layer solution derived through the use of integral transforms with a beam theory solution. Local indenter stresses, as well as displacements and rotations are computed for each case and plotted for various ratios of contact width to beam length, and for various positions of the indenters. Each problem is solved using a nearly isotropic and a highly anisotropic material. Results are compared to those of the isotropic study, and to Hertz theory and beam theory solutions.
For the dynamic case the following studies were completed:

"Dynamic Impact of an Elastically Supported Beam -- Large Area Contact" by L.M. Keer and J.C. Lee. The dynamic contact problem of a rigid, smooth striker impacting an elastically supported beam is solved. Use is made of the superposition of an elastic layer solution together with an elementary beam theory solution that incorporates the dynamic effects. The problem is formulated in such a manner as to require the solution of a Volterra integral equation for each time increment. Numerical results are presented for various parameters.

"Low Velocity Impact of Transversely Isotropic Beams and Plates" by W.P. Schonberg, L.M. Keer and T.K. Woo. The dynamic structural contact problems of a rigid, smooth cylindrical striker impacting an elastically supported transversely isotropic beam and plate are solved. The solutions are obtained as a superposition of a static layer solution derived through the use of integral transforms with an elementary beam or plate theory solution that incorporates the dynamic effects. The problems are formulated in such a manner as to require the solution of a Volterra integral equation for each time step. Numerical results are presented for fixed and simply supported beams and plates. Three materials are used to study the effects of anisotropy: magnesium, which is nearly isotropic; cadmium, which is moderately anisotropic; and graphite/epoxy, which is highly anisotropic.
"Scattering by a Planar Three-Dimensional Crack" by W. Lin and L.M. Keer. The scattering of a three dimensional crack is formulated by the boundary integral equation method. A numerical scheme for solving the derived integral equations by using triangular boundary element is described in detail. The computed results include the stress intensity factors and the scattered patterns for cracks having an elliptical shape.

LIST OF WRITTEN PUBLICATIONS


LIST OF PAPERS PUBLISHED IN PROCEEDINGS


PERSONNEL ASSOCIATED WITH RESEARCH EFFORT

W.P. Schonberg, Research Assistant - Civil Engineering, PhD received June 1986.

T.K. Woo, Research Assistant - Applied Math. (partially supported) MS received June 1986

J.C. Lee, Post-Doctoral Fellow (partially supported)

W. Lin, Post-Doctoral Fellow (partially supported)
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