In this paper, we study the existence results for systems of first order partial differential equations. If the coefficients of the gradient terms are different, proving existence results for the system by the method of characteristics seems to be difficult. However, if we employ monotone iterative technique, this difficulty can be eliminated, since in this case, we can reduce the study of the given system to the study of linear uncoupled systems. For this purpose, we first investigate comparison results and then develop monotone technique in the context of quasi-solutions and mixed monotone operators. One of comparison results proved provides bounds for solutions in terms of solutions of ordinary differential equations, which in turn contains as a very special case, the well known Haar's lemma.
FINAL REPORT

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6. AUTHOR(S) OF REPORT: A.S. Vatsala and G.S. Ladde

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8. SCIENTIFIC PERSONNEL SUPPORTED BY THIS PROJECT AND DEGREES AWARDED DURING THIS REPORTING PERIOD:

   1. Dr. A.S. Vatsala
   2. Dr. G.S. Ladde
   3. Dr. V. Lakshmikantham

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SUMMARY OF THE REPORT

The area of research pursued during the period of September 1, 1984 to August 31, 1986 are:

(a) Systems of first order partial differential equations,
(b) Periodic boundary value problem for first order partial differential equations,
(c) Multiple time-scale singularly perturbed stochastic systems,
(d) Singular perturbations of linear systems with n time scales,
(e) Nonlinear equations with singular linear part,
(f) Periodic boundary value problem in a Banach space,
(g) Ornstein-Uhlenbeck operators with wiener functionals of ITO and McShane calculus,
(h) McShane type of Stochastic differential systems.

In papers 1,2,3,6,7 using the lower and upper solutions and the well-known fruitful technique [\*], existence of extremal solutions of, systems of partial differential equations, PBVP of first order partial differential equation, Integro differential equation with periodic boundary conditions, nonlinear equations with singular linear part, periodic boundary value problem in a Banach space are proved.

In papers 4,5 multiple Time-scale singularly perturbed Linear Stochastic Systems, and Linear Systems with multi-parameters and Multiple Time-Scale are studied by non-singular linear transformation. Finally in papers 8,9,10 Ornstein-Uhlenbeck operator and Wiener functionals generated by ITO - and McShane calculus, existence of solutions of McShane type Stochastic differential systems and Random initial value and Nicolettic Boundary value problems are studied.

[\*] G.S. Ladde; V. Lakshmikantham and A.S. Vatsala Monotone Iterative Technique for Nonlinear Differential Equations, Pitman Publishing Co. 1985

AUTHORS: G.S. Ladde and A.S. Vatsala

ABSTRACT:

In this paper, we study the existence results for systems of first order partial differential equations. If the coefficients of the gradient terms are different, proving existence results for the system by the method of characteristics seems to be difficult. However, if we employ monotone iterative technique, this difficulty can be eliminated, since in this case, we can reduce the study of the given system to the study of linear uncoupled systems. For this purpose, we first investigate comparison results and then develop monotone technique in the context of quasi-solutions and mixed monotone operators. One of comparison results proved provides bounds for solutions in terms of solutions of ordinary differential equations, which in turn contains as a very special case, the well known Haar's lemma.


AUTHOR: A.S. Vatsala

ABSTRACT:

The following periodic boundary value problem for the first order partial differential equation

\[ u_t = f(t,x)u_x = g(t,x,u), \quad u(0,x) = u(2\pi,x) \]

is considered. Existence, uniqueness and comparison results related to (1) are developed. Finally by recalling a recent result of the existence of the periodic solution when the lower solution is bigger than the upper solution for the ordinary differential equation is utilized to develop the monotone iterative technique for (1).

AUTHORS: S.K. Kaul and A.S. Vatsala

ABSTRACT:

Consider the integrodifferential equation with periodic boundary conditions. By developing the comparison results relative to PBVP for integrodifferential equations and using a fixed point theorem of nonlinear operators whose domain and range are different, we develop the monotone method. Consequently, we prove the existence of extremal solutions of the PBVP of the integrodifferential equation.


AUTHORS: G.S. Ladde and O. Sirisaengtaksin

ABSTRACT:

By applying diagonalization transformation, generalized variation of constants formula and theory of differential inequalities, the mean square convergence of solution process of a singularly perturbed linear stochastic differential system of Itô-type is investigated. Moreover, slow and fast modes decomposition provides an auxiliary decoupled system whose solution processes are incorporated in approximating the solution processes of the original system.


AUTHORS: G.S. Ladde and S.G. Rajalakshmi

ABSTRACT:

In this paper, an alternate approach to the method of asymptotic expansions for the study of a singularly perturbed, linear system with multi-parameters and multiple time scales, is developed. The method consists of developing a linear, non-singular transformation that enables one to transform the original system into an upper triangular form. This process of upper triangularization will enable us to investigate (i) stability and (ii) approximation of solutions of the original system in terms of the overall reduced system and the corresponding boundary layer systems.

AUTHORS: V. Lakshmikantham and A.S. Vatsala

ABSTRACT:

Consider the equation

\[ Ax = F(x) \]

where \( F(x) \) is a nonlinear function in \( \mathbb{R}^n \) and \( A \) is an \( n \times n \) singular matrix. Monotone method is developed relative to (1) when \( F \) possesses mixed quasimonotone property. This gives the existence of extremal quasi solutions of (1) and also a unique solution under uniqueness assumptions. Similar results are obtained when \( F \) does not possess mixed monotone property by the technique of mixed monotony.


AUTHORS: S.W. Du and A.S. Vatsala

ABSTRACT:

In this paper we have considered the PBVP for first order differential equations in a general Banach space. Using the compactness assumption of the Kuratowski's type, monotone method has been developed which gives the existence of minimal and maximal solution of the PBVP of first order differential equations in a Banach space.

8. TITLE: Ornstein-Uhlenbeck Operator and Wiener Functionals Generated by Itô- and McShane - Calculus Stochastic Analysis and Applications (to appear)

AUTHORS: G.S. Ladde and Hu Bijin

ABSTRACT:

In this paper, we discuss the relationship between the vector Ornstein-Uhlbeck operator \( L \) and Itô- and McShane - Calculus, and use the variational calculus to prove an important property of the operator \( L \). The component of \( L_i, 1 \leq i \leq q \), on \( q = \text{dim} \). Standard Wiener space are discussed to replace the sum operator \( \sum L_i \) in earlier work.

AUTHORS: G.S. Ladde and S. Seikkala

ABSTRACT:

A system of stochastic differential equations of McShane type is studied. The Schauder fixed point theorem is applied to obtain existence of solutions and Osgood type existence and uniqueness results are derived using successive approximations. The noise processes are not required to be martingales or quasi-martingales. As a byproduct of our approach, upper estimates for solutions of McShane type stochastic differential systems are obtained.

10. TITLE: On Sample Solutions of Random Initial Value and Nicoletti Boundary Value Problems (to appear)

AUTHORS: G.S. Ladde and S. Seikkala

ABSTRACT:

By employing successive approximations existence and uniqueness results for a system of nonlinear differential equations with random coefficients are obtained. As a consequence results concerning Nicoletti boundary value problem are investigated.
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