A Monte Carlo Fokker Planck Code is used to investigate the scattering loss of particles from a relativistic electron beam propagating in a low-pressure air channel. Two special cases are examined. The first corresponds to an experiment in which a one-cm radius, 90 KeV beam propagates in hydrogen channel of the same radius, 50 cm long with a density of 10 cm$^{-3}$ and a temperature of 0.5 eV assuming the interaction to take place only with the charged particles of the medium. It is shown that the beam will traverse such channel if the ionization drops below 20% noting that for such parameters the channel pressure corresponds to few tens of millitors. The second case corresponds to what is referred to as the Ion Focused Regime (IFR) where the pressure is about 0.1 torr. Scattering of beam particles by both neutral and charged particles is taken into account in this case as well as multiple scattering form neutral targets. It is shown that transmitted fraction is linearly proportional to the square of the channel radius, and the probability of multiple scattering increases with increasing channel radius.
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Interaction of Charged Particle Beams
with Pre-Ionized Channels

T. Kammash

J. Les

The University of Michigan
Abstract

A Monte Carlo Fokker Planck Code is used to investigate the scattering loss of particles from a relativistic electron beam propagating in a low-pressure air channel. Two special cases are examined. The first corresponds to an experiment in which a one-cm radius, 90 keV beam propagates in a hydrogen channel of the same radius, 50 cm long with a density of $10^{14} \text{cm}^{-3}$ and a temperature of 0.5 eV assuming the interaction to take place only with the charged particles of the medium. It is shown that the beam will not traverse such a channel if the ionization drops below 20% noting that for such parameters the channel pressure corresponds to few tens of militors. The second case corresponds to what is referred to as the Ion Focused Regime (IFR) where the pressure is about 0.1 torr. Scattering of beam particles by both neutral and charged particles is taken into account in this case as well as multiple scattering from neutral targets. It is shown that transmitted fraction is linearly proportional to the square of the channel radius, and the probability of multiple scattering increases with increasing channel radius.

Introduction and Analysis

In many applications involving the use of charged particle beams propagating in a gaseous medium the question often arises as to the efficiency of propagation and the processes that detract from efficient transmission. Of special interest in this connection is the propagation of intense relativistic electron beams in low-pressure, pre-ionized media. Although there are
several mechanisms that impact such a propagation the effect of scattering is especially important and requires extensive investigation. This effort is aimed at studying the role of scattering in electron beam propagation in low pressure channels. It is carried out with the aid of a Monte Carlo Fokker Planck (MCFP) computer Code which allows us to follow a given sample of non-interacting particles in full configuration and velocity space as the beam traverses a cylindrically symmetric gaseous channel. The four main parts of (MCFP) are: The Fokker Planck section, the large-angle scattering section, the neutral scattering section, and the particle loss section.

In the first section the code follows a sample of particles in time by advancing the position and velocity in accordance with the collisionless equations of motion. Particle deflections and energy deposition in the background plasma are statistically determined using the usual scale times derived from Fokker-Planck theory. For the deflection of beam particles MCFP assumes that each small angle scattering results in a deflection angle \( \Theta \) (usually set to 5° as input) and determines the number of deflections \( N_\theta \) encountered by the equation:

\[
N_\theta = \frac{\Delta t}{\tau_\theta}
\]

(1)

where \( \Delta t \) is a given time step, \( \tau_\theta = \tau_d \sin^2 \theta \) and \( \tau_d \) is the familiar deflection given in MKS units by

\[
\tau_d^{-1} = \frac{2g}{v^3} \sum_b \frac{Z_b^2}{\Lambda_b} \ln \Lambda_b \left\{ erf \left( \frac{v}{2q_b} \right) \left[ 1 - \frac{1}{2q_b} \right] + \frac{e^{-q_bv^2}}{\sqrt{\pi} q_b^{1/4} v} \right\}
\]

(2)
In the above equation \( \text{erf}(z) \) denotes the error function and

\[
G = m e^{z^2/4} / 4\pi \varepsilon_0 m^3
\]

\[
A_b = m_b / 2 k T_b
\]

\[
\Lambda^{-1}_b = \begin{cases} 
\lambda_0 \frac{4\pi \varepsilon_0}{2 c^2} (3 k T_b) & ; b = e \\
\lambda_0 \frac{4\pi \varepsilon_0}{2 Z_e c^2} \left( \frac{m}{m_1 + m} \right) z E & ; b = e
\end{cases}
\]

\[ m = \text{mass of test particle} \]

\[ m_b = \text{mass of particle of species} \ "b" \]

\[ T_b = \text{temperature of particles} \ "b" \]

\[ Z_e = \text{charge of test particle} \]

\[ E = \text{energy of test particle} \]

\[ \Lambda_D = \text{Debye length} = \]

\[ \varepsilon_0 = \text{permittivity of free space} \]

The above small angle scattering information is then used to update the velocity vector of a given sample particle. The energy loss of a given test (beam) particle is obtained in a way similar to that of small angle scattering just described. In this case if we let \( E \) be the energy lost by a sample particle in time \( \Delta t \) then

\[
\Delta E = -E \frac{\Delta t}{\tau_E}
\]
where $E$ is the beginning energy, and $\tau_\text{E}$ is the familiar energy exchange time given by

$$\tau_\text{E} = -\frac{v^2}{g} \sum_b \ln \Lambda_b \sum_{b}^2 \left\{ (1 + \frac{m}{m_b}) \frac{4 a_{\lambda_b}^{-2} e^{-2 a_{\lambda_b} v_k}}{\sqrt{\pi}} \right\}$$

with the parameters defined as before. Thus the speed of the test particle can also be updated in each time step. It might be noted at this point that MCFP has an option that determines the background plasma temperature by employing the simple model in which the channel temperature difference $(T - T_0)$ is directly proportional to the total energy deposited, $\Delta E_t$, at time $t$. This clearly implies that the background plasma instantly thermalizes once energy is deposited. The newly determined temperature, $T$, is then used to calculate the new $d$ and $o$ temperature, $T$, is then used to calculate the new $d$ and $o$

assuming that the background plasma remains Maxwellian and thus a beam-channel feedback effect can be studied. If the changing background temperature model is not selected then MCFP assumes a constant temperature for an unperturbed Maxwellian background.

The code also takes into account the possibility that a beam particle suffers a large angle scattering with the background plasma. Using a random number the code determines such an event by sampling from the distribution.

$$P_{LA} = 1 - e^{-\frac{\Delta t}{\tau_{LA}}}$$

where

$$\tau_{LA} = \left( \frac{v}{\sum_{LA}} \right)^{-1}$$

with $\sum_{LA}$ being the large angle macroscopic scattering cross section and $v$ being the collision velocity. A scattering
event in the center of mass frame is then obtained by randomly sampling the equation

$$P_{LA}(\Theta_R) = \frac{2\pi}{\Theta_R} \int_0^{\Theta_R} J_R(E, \Theta') \sin \Theta' d\Theta'$$

(6)

where

$$J_R(E) = 2\pi \int_0^{\pi} J(E, \Theta') \sin \Theta' d\Theta'$$

(7)

and $J_R(E, \Theta')$ is the Rutherford cross section. The quantity $\Theta_R$ represents the angle that defines the onset of large angle scattering, and $E$ is the relative kinetic energy. In the interest of computation any deflection angle in the laboratory frame less than $10^\circ$ is ignored. When the kinetic energies of the interacting particles is above a certain threshold, $E_{th'}$, the code treats the background particle as a test (beam) particle meaning that its spatial and velocity coordinates are tracked in time. If the above requirement on the kinetic energies is not met then MCFP will relegate the energy of the less energetic particle to the background plasma. After that, only the higher energy particle from the interaction is followed allowing us a method by which "delta" rays can be identified and followed.

In the case of scattering from neutral particles the probability for such an interaction is selected by first defining an important mean free time, namely

$$\mathcal{L}_C = \frac{1}{J V n}$$

(8)

where $V$ is the beam electron velocity, $n$ is the neutral density and $J$ is an appropriate interaction cross section. Although other
inelastic events could be considered we limit our study to ionization and the interaction probability can then be described by $\frac{dt}{t_\Sigma}$ when $dt$ is taken as a very small code time step. Given an appropriate differential scattering cross section a random scattering angle $\theta$ can be selected from the distribution

$$P(\theta) = \frac{\int_{\theta_{\text{min}}}^{\theta_{\text{max}}} d\sigma \, d\Omega}{\int_{\theta_{\text{min}}}^{\theta_{\text{max}}} d\sigma \, d\Omega} = \frac{\xi}{\xi_t}; \quad \theta_{\text{max}} = \frac{2\pi v_c}{A}$$

(9)

where $\xi$ is a random number. For relativistic electrons propagating in a low temperature medium it is expected that the distribution be highly peaked in the forward direction, and using the results from hydrogen appropriately sealed for higher elements we can write

$$\int_{\theta_{\text{min}}}^{\theta_{\text{max}}} \frac{d\sigma}{d\Omega} \, d\Omega = \int_{\theta_{\text{min}}}^{\theta_{\text{max}}} \frac{128 \pi^5 m^2 Z^2 e^4}{h^* k^4} \frac{\cos \theta}{\sin^3 \theta} \left[ 1 + \frac{1}{(1 + \frac{1}{2} k^* \sin^2 \theta)^4} \right] d\theta$$

$$= \frac{\xi}{\xi_t} \left\{ \frac{1}{1 + \beta \Theta_{\text{mi}}} + \frac{1}{1 + \beta \Theta_{\text{mi}}^3} + 4 \log \left[ \frac{(1 + \beta \Theta_{\text{mi}}^3) \Theta_{\text{ni}}^3 \Theta_{\text{mi}}^3}{(1 + \beta \Theta_{\text{ni}}^3) \Theta_{\text{ni}}^3 \Theta_{\text{mi}}^3} \right] + \frac{1}{\Theta_{\text{mi}}^3} \right\}$$

(10)

where in CGS units the parameters $\xi$ and $\beta$ are given by

$$\xi = \frac{3.2885 \times 10^{20}}{V^4}; \quad \beta = (1.74 \times 10^{-18}) \frac{V}{A^2}$$

(11)

In the above expression $m$ is the electron mass, $k$ is the Debroglie wave number, $h$ is Planck's constant, $A = 1.4 \sigma_0^2$

$\sigma_0$ is the Bohr radium, and $\Theta_{\text{mi}}$ is the minimum scattering angle given by
\[ \theta_{min} = \frac{2^{1/2}}{192} \frac{c}{\gamma v} \]  

(12)

where \( \gamma \) is the relativistic parameter and \( c \) is the speed of light. As we noted earlier, small angle scattering is included in the code by an implicit Fokker-Planck method where the probability for such a scattering event in the code time \( t \) is simply \( dt/\tau_d \) where \( \tau_d \) is given by Eq. (2), for plasma densities on the order of \( 10^{11} - 10^{15} \text{ cm}^{-3} \) which might be appropriate for IFR and for \( U \approx c \) it is readily seen that \( \tau_d \approx 10^{-1} \text{ seconds} \) so that at pulse lengths on the order of nanoseconds it is clear that scattering from charged particles can be neglected. These considerations are consistent with some experimental observations. (3)

In order to crudely model the propagation of a relativistic electron beam in either an IFR (4) or a force-free propagation mode we have incorporated the self magnetic field in a phenomenological way using Ampere's law for an infinite, uniform beam. We have neglected the radial electric field by taking the charge neutralization factor \( f \) to be unity. Since the code calculates the number of particles that traverse the channel, the fraction of the beam that is transmitted would then represent an upper limit for pulse lengths \( \tau_p \) that are shorter than \( \tau_{CN} \), the charge neutralization time, and a good estimate when \( \tau_p \approx \tau_{CN} \) since the instabilities which might occur in such time scale are ignored. Under these conditions the description of the scattering process might be deemed appropriate to that taking
place in a force neutralized, charge neutralized or force-free
regimes. With propagation having been observed in pressure
ranges of 0.1 - 5 torr it should be noted that at lower
pressures there are usually insufficient neutrals to provide an
adequate ion density for force neutralization and at higher
pressures multiple scattering could cause dispersal of the beam.
The latter effect has been incorporated in the code.

Finally, it should be noted that in assessing the
usefulness of MCFP code in beam propagation studies it should be
kept in mind that particle scattering can be accurately described
and that no assumptions need be made concerning the plasma
conductivity since ionization can be calculated self-
consistently. Particle and energy loss mechanisms can also be
readily included so long as the appropriate mean free paths for
these processes can be selected. Because of its three
dimensional simulation capability, the often used "paraxial"
approximation and others need not be made. Because of
limitations on the number of particles that can be simulated it
is clear that the treatment of self fields and the modelling of
instabilities, though possible, might be quite difficult.

Some Applications

When used in connection with the study of an electron beam
propagation in a low pressure, preformed channel given by the
following parameters: (6)

beam current = 1 kA
beam radius = 1 cm
beam energy = 90 keV
plasma species = hydrogen
channel radius = 1 cm
channel length = 50 cm
channel proton density = $10^{14}$ cm$^{-2}$
channel neutral density = $4 \times 10^{14}$ cm$^{-3}$
channel temperature = 0.5 eV
pulse length = $0.8 \times 10^{-8}$ sec
The code revealed that no propagation occurs below 20% ionization. At the above parameters the results show that about 33% of the beam particles were last in a distance of about 10 cm down stream from the injection point. When the neutral density was reduced by a factor of 10 most of the beam particles were able to traverse the channel.

In assessing the rate of multiple scattering by neutral particles the following case was examined:
channel radius = $R_c$
beam radius $R_b = 1.5$ cm
neutral density $n_n = 2 \times 10^{15}$ cm$^{-3}$
electrons used to simulate the beam $n_s = 5000$
Transmitted fraction of beam = $T$
Number of scattering off of a neutral particle =
The ratio $n_m/n_s = \delta$
The results are shown in Table 1.
We note that the transmitted fraction is almost linearly proportional to \( \lambda_c \) which is a direct consequence of the geometric effect. Another geometric consequence is the linear scaling of \( T \) and \( S \) with \( \lambda_c \). Since the probability of neutral-interaction is proportional to the path length traversed, and this length increases with \( \lambda_c \), it is expected that multiple scattering should increase with \( \lambda_c \). The variation of these parameters with the neutral density is shown in Table 2 for \( \lambda_c = 21.2 \) cm and \( \sigma_{\text{tot}} = 3 \times 10^{-18} \text{cm}^2 \).

### Table 1

<table>
<thead>
<tr>
<th>( \lambda_c ) (cm)</th>
<th>( T )</th>
<th>( m )</th>
<th>( S )</th>
</tr>
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<tr>
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<td>7338</td>
<td>1.47</td>
</tr>
<tr>
<td>5</td>
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<td>9258</td>
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<td>10</td>
<td>0.03</td>
<td>12183</td>
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<tr>
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<td>15003</td>
<td>3.0</td>
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<tr>
<td>21.2</td>
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<td>1780r</td>
<td>3.56</td>
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### Table 2

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<tr>
<th>( n ) (cm(^{-3}))</th>
<th>( m )</th>
<th>( S )</th>
<th>( T )</th>
</tr>
</thead>
<tbody>
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<td>0.95</td>
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<td>2x1016</td>
<td>68263</td>
<td>13.65</td>
<td>0.002</td>
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References


5. T. Fessenden et al., UCID - 17840, June 1978.

END

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