NONLINEAR EFFECTS IN LONG RANGE UNDERWATER PROPAGATION
FINAL REPORT UNDER CONTRACT N00014-82-K-0805

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14 July 1985

Final Report

1 September 1982 - 30 November 1984

Approved for public release, distribution unlimited

Prepared for

OFFICE OF NAVAL RESEARCH
DEPARTMENT OF THE NAVY
ARLINGTON, VA 22219
# Nonlinear Effects in Long Range Underwater Propagation

**Final Report Under Contract N00014-82-K-0805**

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**Contracts or Grant Number:** N00014-82-K-0805

**Performing Organization Name and Address:**

Office of Naval Research  
Department of the Navy  
Arlington, Virginia 22219

**Abstract:**

Research was carried out to determine the extent to which nonlinearity affects long range underwater propagation. The research was divided into three tasks.

**Task I.** Shock pulse propagation in a homogeneous ocean. A numerical propagation program based on weak shock theory, with account taken of real-ocean absorption and dispersion, was developed. Refraction (due to inhomogeneity of the medium) was neglected.
20. (cont'd)

Task II. Nonlinear propagation in a depth dependent ocean. The effect of refraction on nonlinear propagation distortion was the subject of this task. The approach was analytical. Ordinary absorption and dispersion were neglected.

Task III. Nonlinear propagation in a caustic region. A time-domain treatment of propagation of a finite-amplitude pulse near a caustic was carried out.

After Tasks I and II were completed, a merger of the two tasks was begun, which was completed after the contract ended. In a separate effort, a brief study of scattering of sound by sound was carried out.

The research was reported in two technical reports and five papers presented at meetings including three that have written versions in proceedings.
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I. INTRODUCTION

Contract N00014-82-K-0805 between Applied Research Laboratories, The University of Texas at Austin (ARL:UT), and the Office of Naval Research (ONR) provided for research on nonlinear effects in long range underwater propagation. The period of the contract was 1 September 1982 - 30 November 1984.

The general purpose of the research was to determine the extent to which nonlinear effects are important in long range underwater propagation. Pressure pulses of the sort produced by explosives were considered. In order to work on several aspects of the problem at the same time, we divided the research into three tasks.

Task I. Shock pulse propagation in a homogenous ocean. In this task the effect of inhomogeneity of the ocean is suppressed, and the pulse is treated simply as a spherically spreading wave. Ordinary nonlinear effects--the decay of the shock at the head of the pulse and the waveform distortion of the tail behind--are included as are real-ocean attenuation and dispersion.

Task II. Nonlinear propagation in a depth dependent ocean. Here the inhomogeneity of the ocean, in particular, stratification, is taken into account. The primary concern is the effect of inhomogeneity on finite-amplitude distortion. Shock effects per se and ordinary attenuation and dispersion are ignored.

Task III. Nonlinear propagation in a caustic region. In this task the special problem of the passage of a finite-amplitude pulse through a caustic region is taken up.

In the end, of course, it would be desirable to combine the results of the three tasks. We wished to be able to calculate the propagation of a finite-amplitude pulse through a real, inhomogeneous ocean, even when the propagation path includes caustics.
For administrative purposes, a small additional project on nonlinear mixing of two beams of sound was later added to the contract.

Although D. T. Blackstock was the principal investigator for the contract, the primary senior person who planned and carried out the research was C. L. Morfey. A faculty member at Institute of Sound and Vibration Research (ISVR), University of Southampton, Morfey did most of his work while on leave at ARL:UT during the first half of 1983. At other times(15,264),(982,752) during the life of the contract, he pursued the research as a consultant. R. Buckley, also a consultant from ISVR, performed the work on the problem of caustics (Task III). ARL:UT staff members who assisted Morfey were J. M. Estes (development of numerical propagation programs for Task I), T. L. Foreman (advice on the ray tracing program for Task II), and J. Hardin and D. Gring (computation assistance). The separate project on nonlinear mixing of sound beams was the responsibility of consultant P. J. Westervelt (Brown University).

Scientific Officers for ONR, Code 425 Underwater Acoustics, were, successively, P. H. Rogers, W. I. Roderick, and R. M. Fitzgerald.
II. TECHNICAL DISCUSSION

An enormous amount of literature exists on long range propagation of sound in the ocean. Much broader than it is deep, the ocean is a natural waveguide, albeit a very complicated one. The medium is inhomogeneous and the boundaries are, to say the least, nonideal. The three main techniques that have been used in analytical studies of long range propagation are ray theory, the parabolic approximation, and normal mode theory. All are based on the linear wave equation. Although nonlinear effects are rarely considered, it is interesting to note that two of the techniques, ray theory and the parabolic approximation, may be, and in fact have been, generalized to include nonlinear propagation effects. It would be exceedingly difficult in any general way to modify normal mode theory to include nonlinearity.

Except for studies of finite-amplitude distortion of the shock pulse produced by an underwater explosion,1-4 little interest has been shown in possible effects of nonlinearity on long range propagation. The prevailing opinion seems to be that, while perhaps important near the source, where the amplitude is high, nonlinear effects surely become insignificant for ranges beyond a few hundred meters or so. This belief persists despite the fact that, as Marsh, Mellen, and Konrad3 pointed out in 1965, Arons' amplitude scaling law2 is known to hold over five or six decades in range. This law is that the peak amplitude $p_s$ of a wave from an explosion decays with range $R$ as

$$p_s \propto R^{-1.13}$$

(not, as given by linear theory, as $p_s \propto R^{-1}$). A second and more telling argument against considering nonlinear effects has to do with frequency. Because attenuation increases rapidly with frequency, many long range measurements are limited to low frequencies, and low frequency signals are known to be highly resistant to finite-amplitude distortion. Two counter arguments are as follows: (1) nonlinear distortion is cumulative and, though small over short distances, the distortion effects may add up to produce appreciable effects over long ranges;
studies of both finite-amplitude tones and finite-amplitude noise have shown that for the high frequency end of the spectrum no range exists at which linear theory is adequate. As an example of (2), consider a spherically spreading sound field generated by a pure tone source. It has been found that the amplitude of the nth harmonic distortion component $p_n$ varies as

$$p_n \propto R^{-n} e^{-n\alpha_1 R},$$

not, as expected from linear theory,

$$p_n \propto R^{-1} e^{-\alpha_n R}.$$  

Here $\alpha_1$ and $\alpha_n$ are the attenuation coefficients at the frequencies of the fundamental and the nth harmonic, respectively.

About a year before our work on the problem of finite-amplitude propagation in the ocean started, B. E. McDonald and W. A. Kuperman at Naval Ocean Research and Development Activity (NORDA) began to attack the problem by using the (time-domain) nonlinear parabolic equation. This equation is well suited to numerical solution, and their work may be characterized as computationally intensive. Taken into account are nonlinear propagation distortion, refraction (including caustics), and diffraction (they considered a beam of sound, as is appropriate when the parabolic equation is used). Not taken into account is ordinary absorption by the medium; however, shock losses are included. Among their reported results are the evolution of waveforms and the behavior at caustics, in particular the effect of nonlinearity on phenomena at caustics.

We decided to use ray theory for our investigation. Ray theory is simple and versatile and is appealing for physical interpretation. The extension to nonlinear acoustics is relatively well known. Finally, the analytical work can be carried a great deal further with ray theory than with the parabolic equation. Although numerical computation must in the end be used to calculate waveforms, the computation is in principle very simple. Moreover, the use of ray theory allows one to easily investigate the individual effects of the various physical mechanisms,
such as nonlinearity, absorption, and refraction, on the propagation. The only significant disadvantage associated with ray theory is that caustics pose a difficult problem. As already noted, we planned to deal with caustics as a separate task.

The two approaches, ours and that of the NORDA group, were complementary and provided complementary results.

A. Task I. Shock Pulse Propagation in a Homogenous Ocean

In this task nonlinear propagation distortion (described by weak shock theory) and real-ocean attenuation and dispersion are considered (inhomogeneity of the medium is considered in Task II, not here). Shock pulse parameters of particular interest are the peak pressure $p_s$ and the decay time behind the shock $\theta_s$.

The objective of Task I was to construct a numerical model of long range shock-pulse propagation which would allow $p_s$ and $\theta_s$ to be calculated as a function of range $R$. The calculation would start with an assumed initial waveform. Our intention was to compare the numerical model, in which relaxation attenuation and dispersion are included along with nonlinear waveform steepening, with

(1) experimental data as summarized in the Arons scaling laws, and
(2) weak-shock theory, which does not include relaxation effects.

Particular questions of interest were the following.

- Why do the empirical scaling laws for $p_s$ and $\theta_s$ fit the available data better than an exact solution based on weak-shock theory?

- Out to what distance may weak-shock theory be used to describe the shock pulse?

- What influence does the first bubble pulse have on the shock pulse parameters at long range? (According to the weak-shock model, there is no interaction.)
Beyond what distance may nonlinear effects be neglected, for purposes of describing the evolution with range of certain waveform parameters?

The propagation algorithm that was developed is closely related to one devised by Pestorius\textsuperscript{11} for airborne propagation of finite-amplitude waves through a tube. Modifications were made to allow for spherical spreading and a switch of the acoustical medium to sea water. One starts with the time waveform of the wave at the source and computes the change in waveform as incremental steps in range are taken (when the wave spreads spherically the steps are equal increments in $\ell \ln R$). For each step the wave is first subjected to ordinary attenuation and dispersion (appropriate for ocean conditions) over that step. This operation is carried out in the frequency domain. After a transformation back to the time domain is made, the waveform is corrected for nonlinear propagation distortion by using the relations of weak shock theory. The sequence of absorption and dispersion followed by nonlinear distortion is repeated for each step until the desired propagation distance is reached.

The algorithm was used to perform numerical propagation "experiments" for equivalent TNT charge weights in the range 0.6 g to 20 kg. The following initial waveforms were used.

\begin{align}
(1) \quad p &= e^{-T} \quad \text{(simple exponential tail),} \\
(2) \quad p &= [1-(1-k)T]e^{-kT} \quad \text{(Brode\textsuperscript{12}),} \\
(3) \quad \text{A modified version of Wakeley's waveform\textsuperscript{13} with the first bubble pulse included.}
\end{align}

Here $p = p/R_0 p_m$, $T = (t-R/c)/\theta_m$, $p$ is acoustic pressure, $R_0$, $p_m$, and $\theta_m$ are the starting values of $R$, $p_s$, and $\theta_s$, respectively, $t$ is time, and $c$ is sound speed; Brode's constant $k$ is 3/9.
Rogers used weak-shock theory to obtain an analytical solution for waveform (1). Ordinary absorption and dispersion were not included. In a sense our effort under Task I is an extension of Rogers's work, since we considered a more realistic ocean and more realistic waveforms. Indeed we were able to explain some, but not all, of the differences Rogers noticed when he compared his analytical solution, Arons's scaling law, and measured data. For details, see References 84-2 and 85-1.

The abstract from 85-1 summarizes the results under Task I.

"The propagation of idealized model waveforms, representative of underwater explosions measured at moderate distances (particle Mach number = 0.06), has been simulated numerically out to large distances in an idealized uniform ocean. Besides chemical relaxation, the calculation procedure includes nonlinear waveform distortion and thermoviscous dissipation at shocks; these nonlinear phenomena are described using the weak-shock approximation. Results from the uncorrected weak-shock model, which omits chemical relaxation effects, are known to deviate appreciably from long-range measurements [P. H. Rogers, J. Acoust. Soc. Am. 62, 1412-1419 (1977)]; the discrepancy is explained by showing, with the aid of the present model, that initial waveforms having different time scales exhibit markedly different long-range behaviors. Specific waveform properties examined include the peak pressure, decay time constant, positive-phase impulse, and shock rise time. Reasonable agreement is demonstrated between model predictions and long-range measurements in the ocean, as represented by empirical scaling laws."

*References cited in this style indicate items in the Chronological Bibliography, e.g., 84-2 means the second entry in the list for 1984.
B. Task II. Nonlinear Propagation in a Depth Dependent Ocean

The effect of stratification of the medium on finite-amplitude distortion is considered in this task. The approach is analytical. Nonlinear distortion is basically determined by the particle velocity amplitude relative to the local sound speed, the initial slope of the wave, the coefficient of nonlinearity $B$, and the distance traveled. Stratification greatly complicates the process. First, the amplitude depends on the area of the ray tube along which the wave is propagating, and the area in turn depends on the acoustical environment. The changing environment also affects distortion directly through the change in sound speed, impedance, and coefficient of nonlinearity. Despite the complicated relations involved, the problem can be dealt with analytically. The trick is to scale the dependent variable (pressure) and the travel distance in such a way that the nonlinear wave equation reduces to plane wave form. The known analytical solution for the plane wave problem may then be used.

The primary assumption that makes the (relatively simple) analytical approach feasible is that the ray paths for finite-amplitude waves are the same as for small-signal waves. The ray program used in our work is MEDUSA, a very versatile routine developed by Foreman.\(^{14}\)

The requisite scalings are as follows (84-1)

$$\tilde{p} = p\left(\frac{S}{S_0}\right)^{1/2}\left(\frac{\rho c}{\rho_0 c_0}\right)^{-1/2} \quad \text{(reduced pressure)} \quad (6)$$

$$\tilde{x} = \int_0^S \frac{\alpha(S)}{\alpha_0(S_0)}^{-1/2} \, ds' \quad \text{(reduced distance)}, \quad (7)$$

where $S$ is ray tube area, $\rho$ is density, $s$ is arc length along the ray path, and $\alpha$ is the thermodynamic quantity

$$\alpha = B(\rho c^5)^{-1/2} \quad . \quad (8)$$

Subscript zero means a reference value, normally the value at the source (or effective source). Another important quantity is the delay time along the path.
\[ t' = t - \int ds/c \] \hspace{1cm} (9)

An important special case is that of a homogenous medium, for which, since \( S = 4 R^2 \), Eq. (7) reduces to

\[ \tilde{x} = R_0 \ln(R/R_0) \] \hspace{1cm} (10)

This is the classical expression for the reduced distance for ordinary spherical waves of finite-amplitude (Task I). Because Morfey expected that Eq. (10) would give a reasonably good first estimate of Eq. (7), he introduced the quantity \( G \) by means of the definition

\[ \tilde{x} = s_0 \ln(Gs/s_0) \] \hspace{1cm} (11)

(\( G \) may be calculated by equating the right hand sides of Eqs. (7) and (11)). The departure of \( G \) from unity provides a measure of the extent to which stratification affects distortion.

Morfey's technical report (84-1) provides all details of the results for Task II. Topics include conversion of Eq. (7) to an integral in terms of ray coordinates, exact analytical expressions in certain cases (for example, for a linear sound speed profile), asymptotic approximations for \( G \), examples of the calculation of \( G \) for typical ocean profiles, and calculation of \( G \) when a caustic is approached. Appendices give formulas for acoustical properties (including \( \beta \)) for the ocean, useful approximate relations for \( \alpha, \beta, \) and \( \rho \), some asymptotic ray relations for small angles, an analysis of vertical propagation, and evaluation of \( G \) by means of a ray path integration.

C. Merger of Tasks I and II. Applications

Near the end of the contract, work was begun to merge the results of Tasks I and II, in particular, to generalize the Task I computer program to provide for propagation through an inhomogeneous ocean. Ray paths involving reflections and caustics were avoided. Although not completed under the present contract, the merger was continued and finished under Contracts N00014-75-C-0967 and N00014-84-K-0574. The
computer program was used to run a number of examples. The importance of the various physical mechanisms at work -- finite-amplitude distortion by itself (both with and without stratification), absorption and dispersion alone, finite-amplitude distortion combined with absorption and dispersion, and so on -- was investigated. The general question that gave rise to the research in the first place, "Are nonlinear effects important in long range propagation?" was also explored. A preliminary, oral report of this work (84-4) was followed by a thesis and technical report (85-3) and subsequently by a paper at an international symposium (86-1). The following excerpt from the abstract of 85-3 summarizes the results.

"In this report the propagation of finite amplitude acoustic signals through an inhomogeneous ocean is investigated both analytically and numerically. The effects of reflections and focusing are not considered....In the numerical study the ocean is assumed to be stratified. The effects of inhomogeneity, ordinary attenuation and dispersion, and nonlinear propagation are investigated using a numerical implementation of nonlinear geometrical acoustics. Two explosion waveforms are considered: a weak shock with an exponentially decaying tail and a more realistic waveform that includes the first bubble pulse. Numerical propagation of the simpler wave along a 58.1 km path starting at a depth of 300 m leads to the following conclusions: (1) The effect of inhomogeneity on nonlinear distortion is small. (2) Dispersion plays an important role in determining the arrival time of the pulse. (3) Neither nonlinearity nor ordinary attenuation (and dispersion) are paramount; both need to be included. For the more realistic wave the propagation is along a 23 km ray path starting from a depth of 4300 m. Two charge weights, 0.818 kg and 22.7 kg TNT, are assumed. In each case the energy spectrum of the signal obtained by considering finite amplitude effects for the entire 23 km path is compared with spectra obtained by neglecting finite amplitude effects (1) entirely, (2) after the first 150 m, and (3) after the first 100 m. Finite amplitude effects are found to be of small consequence in the case of the 0.818 kg TNT explosion.
for frequencies below 6 kHz at distances beyond 1100 m. For the 22.7 kg explosion the corresponding quantities are 4 kHz and 1100 m."

D. Task III. Nonlinear Propagation in a Caustic Region

The approach of a ray to a caustic indicates the impending breakdown of the ray description of the sound field. At the caustic itself the ray tube area vanishes and the pressure is predicted to become infinite. What actually happens is that diffraction, which is neglected in ray theory, limits the growth in amplitude. Also not included in ray theory is the phase shift that occurs when the wave passes through the caustic. For small signals the phase shift is $\pi$ for a three-dimensional focus, and $\pi/2$ for a two-dimensional focus (the usual caustic). These results, if applied without change to the finite-amplitude problem, imply that one need only add a $\pi/2$ phase shift to all frequency components of the finite-amplitude wave when it reaches the caustic (more accurately, the caustic produces a Hilbert transform of the incident wave). Indeed, just such a procedure has been used by Ostrovskii, Pelinovskii, and Fridman, and by Rogers and Gardner. Buckley undertook the development of a more accurate procedure for handling propagation through the caustic region. His work is summarized by the following abstract.

"Nonlinear acoustic ray tracing in the ocean fails whenever a wave front converges to a caustic or focus and neighboring rays intersect. It is the purpose of this paper to construct a numerical algorithm for dealing with propagation of large amplitude pulses in the neighborhood of a caustic. Introduction of a time-straining transformation of the nonlinear wave equation allows one to attempt a perturbation expansion of the pressure in powers of signal strength. An essential ingredient of the procedure is a knowledge of the solution of the corresponding linear problem. An efficient procedure for calculating the linear solution for
incident N wave pulses having an arbitrary two-dimensional wave front is described. The corresponding calculation of the nonlinear time straining is also presented. The procedure may readily be adapted to pulses of more general shape."

This abstract introduced an early version of a manuscript intended for publication. Current plans call for dividing the manuscript in two. The linear theory will be the subject of one manuscript (86-2), and nonlinear theory the subject of the other (86-3).

E. **Nonlinear Mixing of Two Sound Beams**

Westervelt carried out an analysis of the scattering of sound by sound. He has long maintained\(^ {17} \) that interaction of two primary sound beams produces no scattered difference frequency (or sum frequency) sound outside the interaction region unless the primary beams are collinear, that is, unless the angle of interaction is zero. A defense of this position with new calculations was presented in a paper given at an international symposium (84-3).
CHRONOLOGICAL BIBLIOGRAPHY

Code
JS = submitted for publication in a journal
O = oral presentation at a scientific meeting
P = paper in a proceedings
T = thesis
TR = technical report

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Code 1985


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*Represents work begun under the present contract and continued under Contract N00014-84-K-0574.

**Work primarily supported by Contracts N00014-75-C-0967 and N00014-84-K-0574 but contains material developed under the present contract.
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<td>Washington, DC 20375</td>
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