INCREASING THE MARGIN OF STABILITY OF ARBITRARILY
FINITE MODES OF FLEXIBLE (U) FLORIDA UNIV GAINESVILLE
DEPT OF MATHEMATICS I LASIECKA ET AL. 03 AUG 86
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This research project focuses on a canonical component of flexible large space structures, which is modeled by a hyperbolic second order equation (wave equation) with damping, in an arbitrary number of dimensions. The object is to provide a simple, implementable boundary feedback of a specific class which will (i) increase the margin of stability of finitely many modes while (ii) at least preserving the margin of stability of the remaining modes and, moreover, (iii) guarantee the exponential uniform decay of all feedback solutions with the same upper bound enjoyed by the free solutions (homogeneous boundary conditions). An analysis of the distributed parameter model is given by the co-principal investigators, which provides a theoretical solution of the above problem in an essentially constructive way. Numerical implementations of the theoretical proof have been carried out by a Ph.D. student, J. Bartolomeo. They show a behavior of the eigenvalues distribution of the feedback system as predicted, and in agreement with, the theoretical results.
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Annual Technical report
on research project entitled
"Increasing the margin of stability of large arbitrarily finite modes of flexible space structures with damping"
supported by Air Force Office of Scientific Research, Grant AFOSR - 84 - 0365.
Co-principal Investigators:
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1. **Summary**

This research project focuses on a canonical component of flexible large space structures, which is modeled by a hyperbolic second order equation (wave equation) with damping, in an arbitrary number of dimensions. The object is to provide a simple, implementable boundary feedback of a specified class which will (i) increase the margin of stability of finitely many modes while (ii) at least preserving the margin of stability of the remaining modes and, moreover, (iii) guarantee the exponential uniform decay of all feedback solutions with the same upper bound enjoyed by the free solutions (homogeneous boundary conditions). An analysis of the distributed parameter model is given by the co-principal investigators, which provides a theoretical solution of the above problem in an essentially constructive way. Numerical implementations of the theoretical proof have been carried out by a Ph.D. student, J. Bartolomeo. They show a behavior of the eigenvalues distribution of the feedback system as predicted by, and in agreement with, the theoretical results.

2. **Technical statement of research objectives and of results achieved.**

**Status of the research.**

This research project focuses on a canonical component of flexible large space structures, which is modeled by a hyperbolic equation with damping. The object is to provide a simple, implementable boundary feedback of a specified class, which will (i) increase at will the margin of stability of finitely many modes while (ii) at least preserving the margin of stability of remaining modes. Engineering approximations of the distributed system by lumped systems have failed. In contrast, the approach proposed by the authors applies directly to the distributed system. This has proved successful.

More precisely: consider the following boundary feedback for the damped wave equation.
(1) \( x_t(t, \xi) = \Delta x(t, \xi) - 2x_t(t, \xi) \quad \text{in} \ (0, \infty) \times \Omega \)

I.C. \( x_0 \) and \( x_1 \)

(2) \( x(t, \sigma) = \left[ \langle x(t, \cdot), w_1 \rangle + \langle x(t, \cdot), w_2 \rangle \right] g(\sigma) \quad \text{in} \ (0, \infty) \times \Gamma \)

Here \( k > 0 \) is the coefficient of viscous damping and the right hand side of (2) is the "interior observation" acting on the boundary.

For \( g = 0 \) and 'small' \( k > 0 \), the eigenvalues \( \lambda_{m}^{\pm} \) of the free system lie on the vertical line

\[ \text{Re} \lambda = -k \]

and it is a classical result that all free solutions decay with upperbound \( e^{-kt} \) in the energy norm for \( t > 0 \).

For \( g \neq 0 \), the following result is proved in [L-T.5], [T.4].

Let a real \( g \in L^2(\Gamma) \) be given, with \( (g_{\text{ext}}, e_m)_{L^2(\Omega)} \neq 0, m=1,2,\ldots \) where \( g_{\text{ext}} \) = harmonic extension of \( g \) onto \( \Omega \) and \( \{e_m\} \) are the eigenvectors of the Laplacian \( \Delta \) with zero Dirichlet B.C. Let an arbitrary sequence \( \{\epsilon_m^{\pm}\} \) of distinct, non zero complex numbers, with \( \epsilon_m^{+} \) = complex conjugate of \( \epsilon_m^{-} \), be given. It is assumed that, moreover, the sequence \( \{\epsilon_m^{\pm}\} \) satisfies some technical assumptions i.e. that a few infinite sums involving \( \{\lambda_{m}^{\pm}\}, \{\epsilon_m^{\pm}\} \) and \( (g_{\text{ext}}, e_m) \) be finite), which allow finitely many \( \{\epsilon_m^{\pm}\} \) \( M \), \( M \) arbitrary, to be preassigned at will, while the sequence \( \{\epsilon_m^{\pm}\} \) approach zero 'sufficiently fast'.

Then, one can construct real vectors \( w_i \in L^2(\Omega) i = 1,2 \), such that the feedback system (1)-(2) has the following two desirable properties:

(i) its eigenvalues \( \{\alpha_m^{\pm}\} \) are given precisely by \( \alpha_m^{\pm} = \lambda_{m}^{\pm} + \epsilon_m^{\pm} \) (so that any finitely \( \{\alpha_m^{\pm}\} \) \{\epsilon_m^{\pm}\} \) can be preassigned at will, while the distance \( |\alpha_m^{\pm} - \lambda_{m}^{\pm}| \) between new and original eigenvalues goes to zero);

(ii) its corresponding eigenvectors form a Riesz basis on \( L^2(\Omega) \times H^{-1}(\Omega) \).

In particular, we may require \( \text{Re} \epsilon_m^{\pm} < 0 \), in which case all feedback closed loop solutions preserve the overall decay upper bound \( e^{-kt} \), \( t > 0 \), in view of properties (i) - (ii). This result, in particular, implies that one can increase at will the margin of stability of
Arbitrarily finite modes of the damped wave equation (those presumed 'dominant') by means of the boundary feedback in (2), while the remaining new modes approach asymptotically the original ones from the left of the vertical axis \( \text{Re } \lambda = -k \).

Numerical testing of the procedure, which is essentially constructive, is being implemented by a Ph.D. student, G. Bartolomeo. Thus, the short term goal of the Project summary of the Original Proposal has been accomplished.

The attached graphs show the numerical results performed on a one-dimensional problem. The crosses indicate the numerically computed eigenvalues of the suitable closed loop boundary feedback system provided by the theoretical investigation. As desired, such eigenvalues accumulate asymptotically at the left of the vertical axis \( \text{Re } \lambda = -k \).

3. **List of publication resulted from research study.**


   J. Bartolomeo, Numerical tests on the problem of eigenvalues assignment for the damped wave equation with boundary feedback, to be submitted.

4. **Professional personnel**

   Co-principal investigators:

   I. Lasiecka, Professor of Mathematics, University of Florida, Gainesville, Florida

   R. Triggiani, Professor of Mathematics, University of Florida, Gainesville, Florida
5. Interactions.
The main results of the research paper in List of Publications above were (or will be) presented by R. Triggiani also at the following conferences:

(i) Second International Workshop on "Distributed parameter systems" held in Vorau (Austria) on July 9-16, 1984, organized by University of Graz, Austria; published in Springer-Verlag Lecture Notes on Information and Control, # , 1985.


(iii) Annual Meeting, American Mathematical Society, New Orleans, Jan. 7-11, 1986, Special Session on "Operator Methods on Optimal Control Problems"

In addition, the main content of the present research was the object of seminar talks given by R. Triggiani at: Scuola Normale Superiore, Pisa, Italy, December 1983; Mathematics Department, University of Florida, February 1985.
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