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A BREAKDOWN SURFACE MODEL FOR THERMAL BACKSCATTERING FROM THE EXHAUST PLUME OF A SPACE-BASED HF LASER

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The purpose of this report is to present a breakdown surface model for evaluating thermal backscattering flow from the supersonic exhaust plume of a gaseous mixture of H, HF, H₂, DF and He. Fluxes of these species are considered separately. The model is carefully analyzed and is shown to overestimate the flux. Actual flux levels of the heavy corrosive molecules (HD, DF) have been found to be exceedingly low. It is concluded that the contribution of thermal backscattering to contaminating flux of HF and DF can be neglected. This work is an extension and modification of the recent thesis work done by S. E. McCarty at the Naval Postgraduate School.
ABSTRACT

The purpose of this report is to present a breakdown surface model for evaluating thermal backscattering flux from the supersonic exhaust plume of a space-based HF laser. The plume is of ring symmetry. It consists of a gaseous mixture of H, HF, H₂, DF and He. Fluxes of these species are considered separately. The model is carefully analyzed and is shown to overestimate the flux. Actual flux levels of the heavy corrosive molecules (HF, DF) have been found to be exceedingly low. It is concluded that the contribution of thermal backscattering to contaminating flux of HF and DF can be neglected. This work is an extension and modification of the recent thesis work done by S. E. McCarty at the Naval Postgraduate School.
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The ideas leading to this work crystalize through numerous discussions with LCDR Scott E. McCarty and Distinguished Professor Allen E. Fuhs. This Contractor report constitutes in fact an extension and generalization of LCDR McCarty's MSAE Thesis. Their help and cooperation are gratefully acknowledged.
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1. INTRODUCTION

This report is a presentation of one part of a study on the contaminating backflow from the exhaust plume of a large space-based HF laser (Figure 1). The exhaust plume is an underexpanded supersonic ring-jet, designed to stay clear of the spacecraft by maintaining a Prandtl-Meyer turning angle at the nozzle lips of well below 90°. However, it is well known from experience with rocket plumes in space[1,2] that cavity regions (where continuum gasdynamic theory predicts vacuum) are filled with a free-molecular flow. This backflow is largely due to viscous effects, which give rise to a "spill-over" of the boundary layer around the nozzle lip[3]. Assuming the boundary layer can be eliminated (e.g., an expanding step design of the nozzle lip), there are two more mechanisms which lead to backflow: thermal backscattering and scattering by ambient molecules traveling at orbital speeds. Since these effects are a small perturbation to the exhaust flow field, they can be considered independently (the total backflow will be a superposition of contributions due to these two effects). As a first phase of our broader study, we consider solely the contribution of thermal backscattering to backflow from a ring-plume of an HF laser, via a simple model of molecular effusion from a breakdown surface, fashioned after ideas suggested by Noller[4]. Our results indicate that the backflow of the heavier contaminants (HF, DF) due to thermal backscattering is negligible.

Naturally, our study pertains to presumably typical operating conditions of the HF laser. These operating conditions were largely determined from a report on some HF laser tests conducted at TRW in 1971[5] (in particular, Table 5, Test III, of this report). The typical parameters at the nozzle exit
are:

Composition (mole fractions):

\[ [\text{H}] = 0.091, [\text{HF}] = 0.091, \]
\[ [\text{H}_2] = 0.104, [\text{DF}] = 0.135, \]
\[ [\text{He}] = 0.579 \]

Specific Heats Ratio: \( \gamma = 1.54 \) (assuming ideal gas)

Mach Number \( M_1 = 4.0 \)

Average Molecular Weight \( \mu = 7.27 \text{ [kg/kg mole]} \) (1.1)

Stagnation Temperature \( T_o = 2300 \text{ [K]} \)

Stagnation Density \( \rho_o = 0.0075 \text{ [kg/m}^3\text{]} \)

Molecular Diameter, assuming it is uniform for all species (hard-sphere collisions)

\( D = 2.5 \times 10^{-10} \text{ [m]} \)

The exit Mach number can be chosen higher than \( M_1 = 4 \), but not considerably lower than this value, since \( M_1 = 4 \) results in a modest clearance angle of \( 41^\circ \) between the limiting (vacuum) characteristic of the lip-centered rarefaction fan and the spacecraft. We assume isentropic flow throughout the diffuser \( ^5 \), so that upon specifying the composition and flow variables at the diffuser inlet, along with \( M_1 \) at the diffuser exit, the exit flow is fully determined. One exception to this definition, however, is the stagnation temperature, which was estimated as \( T_o = 1400 \text{ [K]} \) at the diffuser inlet \( ^5 \). We set \( T_o = 2300 \text{ [K]} \), which corresponds to complete hydrogen recombination, even though the flow in the diffuser is of a nearly frozen composition due to the low rate of hydrogen recombination \( ^5 \). The reason for this choice is that given the uncertainty in determining \( T_o \), which results from an uncertainty in the degree of hydrogen recombination, it is the most pessimistic choice, resulting in higher thermally backscattered flux.

The model that we propose for evaluating the backscattered flux arriving
at the spacecraft (Figures 1, 2) is based on the effusive breakdown surface concept suggested by Noller[4]. The gradual transition from continuum to collisionless flow, which invariably takes place at the outer fringes of exhaust plumes having a near-vacuum background environment, is replaced by an abrupt change. We assume that the flow regime in each stream tube changes from continuum (with local thermodynamic equilibrium) to collisionless, upon crossing some breakdown surface.

An important simplification is introduced in the case of a large-radius spacecraft (about 2.5[m]), by observing that the temperature along the breakdown surface decreases so sharply with the distance from the nozzle lip, that the segment contributing significantly to thermal backscattering is only about 0.01 to 0.1[m] long. Consequently, the lip-centered rarefaction ring-fan may well be approximated by the standard (planar) Prandtl-Meyer flow field.

The structure of this report is as follows. The breakdown surface and the molecular effusion flux from it are obtained in closed-form expressions in Section 2. Section 3 is devoted to the spatial integration scheme, which is the evaluation of the flux arriving at a certain point on the spacecraft. Results of flux distribution along the spacecraft for the presumed laser operating range are presented and discussed in Section 4, followed by a critical examination of the breakdown surface model. Conclusions are given in Section 5, and Section 6 is a list of references. The code RINGBD, which computes the flux by numerical integration over the breakdown (effusing) surface, is given in Appendix A.
2. **BREAKDOWN SURFACE AND EFFUSION FLUX**

Our model for the thermally backscattered flux arriving at the surface of the spacecraft is essentially a modification of Noller's concept of a breakdown effusive surface[4]. We substitute his definition of a breakdown surface by a similar one introduced by Bird[6, Section 8.3]. We obtain the one-sided effusion flux from the breakdown surface by integrating over velocity space as suggested by Noller[4], except for the fact that we compute flux rather than density and we also consider the flux of species having molecular weight different from the average. In the following, each one of these steps is described in some detail, beginning with the breakdown surface.

As mentioned in the introduction, the lip-centered rarefaction fan is approximated by a planar Prandtl-Meyer flow field (Figure 2). The standard expressions for this flow field have Mach number ($M$) as the independent parameter, thus $M$ varies between $M = M_1$ at the exit and $M = \infty$ at the limiting (vacuum) characteristic. (Index 1 always refers to exit conditions, i.e., to parameters evaluated at $M = M_1$).

\[
\psi(M) = -\zeta(M) + \xi_1 + u_1 + \frac{\gamma}{2}
\]

\[
\zeta(M) = \Gamma^{1/2} \text{ARCTAN}\left[\Gamma^{-1/2}(M^2 - 1)^{1/2}\right]
\]

\[
\Gamma = \frac{\gamma+1}{\gamma-1}
\]

\[
u(M) = \text{ARCSIN}(M^{-1})
\]

\[
\theta(M) = \psi(M) - \nu(M)
\]

where $\psi$ is the angle of characteristic lines, and $\theta$ is the angle of the velocity vector (or streamline).
Adopting Bird's definition of a breakdown parameter, which was first introduced in conjunction with a spherical source flow[6, Section 8.3] and later was shown to be meaningful also in a Prandtl-Meyer flow[7], we define the breakdown surface as having a constant value of \( B \), where \( B \) is given by:

\[
B = \frac{U}{\nu} \frac{1}{\rho} \left| \frac{d\theta}{dS} \right|
\]  

(2.2)

Here \( \rho, U, \nu, S \) are local flow density, speed, collision frequency, and coordinate along streamlines (thus restricting this definition of \( B \) to stationary flows). From the geometrical relationships in a Prandtl-Meyer fan (Figure 2) and from (2.1) we get:

\[
\frac{d\theta}{dS} = - \left( \frac{1}{R} \right) \left( \frac{d\theta}{d\psi} \right) \sin \mu = - \frac{2}{\gamma+1} \left[ M^{-1}(\gamma^2 - 1)^{1/2} \right] \left( \frac{\rho}{R} \right)
\]

(2.3)

\[
\rho(M) = \rho_0 \left( 1 + \frac{\gamma^{-1}}{2} \frac{N_t^2}{\gamma} \right) \frac{1}{\gamma-1}
\]

Using the expression for collision frequency[6]:

\[
\nu_0 = 4(\pi/\gamma)^{1/2} \left( N_0 D^2 C_0 \right)
\]

(2.4)

where \( N_0, C_0, D \) are stagnation number density, stagnation sound speed, molecular diameter, and using \( U = MC \) in conjunction with (2.2) and (2.3), we get:

\[
R_B(M) = (BN_0 D^2)^{-1} \left( \frac{\nu}{\pi} \right)^{1/2} \left( \frac{N_t^2-1}{2(\gamma+1)} \right)^{1/2} \left( 1 + \frac{\gamma-1}{2} N_t^2 \right)^{1/2} \frac{1}{\gamma-1}
\]

(2.5)

This expression is almost identical to that of Bird[7], the main difference being in assuming a constant collision diameter (hard spheres), which we
believe to be commensurate with the overall crudeness of the model. The breakdown surface as defined by equations 2.5 and 2.1, starts at point \([R_B(M_1), \psi(M_1)]\) on the exit characteristic \(M = M_1\), which we refer to as the initial point (see Figure 4). However, a breakdown in continuum flow also takes place on the segment of the exit characteristic between the corner and the initial point, since the value of the breakdown parameter there (Equation 2.2) is clearly larger than the value of \(B\) used in defining the breakdown surface (Equation 2.5). Hence, the breakdown surface defined by (2.5) should be supplemented by that segment. We refer to the combined surface as the augmented breakdown surface. The segment on the exit characteristic is referred to as the supplementary breakdown surface.

The one-sided directed effusion flux is defined as the number flux of molecules per unit area of an area element normal to the flux direction, per unit solid angle about the flux direction. It is obtained as a function of local Mach number and the angle \(\kappa\) between the flux direction and the local velocity vector, by repeating Noller's velocity integration scheme [4, Eq. (6)], with an added factor of molecular speed in order to obtain flux (rather than density as in Noller's work). The resulting expression for species \(i\) is readily obtained by using standard definite integrals:

\[
F_i(M) = h_i \frac{NoCo}{WA/W_i} (W_A/W_i)^{1/2} \left[ (1 + \frac{\gamma-1}{2} M^2) \right. \\
\left. \frac{\gamma+1}{2(\gamma-1)} \right] \\
\left[ (2\gamma\pi^3)^{-1/2} (1 + (1/2)\gamma\tilde{h}(2\cos^2\kappa)) \exp \left[ - \frac{\gamma M^2}{2} \right] + \right. \\
\left. (2\pi)^{-1} M \left( 3/2 + (1/2)\gamma\tilde{h}(2\cos^2\kappa) \cos\kappa \exp \left[ -(1/2)\gamma M^2 \sin^2\kappa \right] \right) \right] \\
\text{ERFC} \left[ - \frac{(\gamma/2)^{1/2} M \cos\kappa}{\sqrt{\gamma-1}} \right]
\]

\[
\text{ERFC}(v) = 2 \pi^{-1/2} \int_{-v}^{\infty} \exp \left[ -x^2 \right] dx
\]

(complementary error function)
\[ M = \left( \frac{W_i}{W_A} \right)^{1/2} M \]

\( h_i \) - Mole fraction of species \( i \).

\( W_i \) - Molecular weight of species \( i \).

(2.6 Continued)

The dependence of \( F_i(M) \) on the flux angle \( \phi \) is so sensitive that for some Mach number around \( M = 10 \), the backflow (in the typical operating range) is virtually negligible. In the following section we describe how the flux \( F_i(M) \) is integrated over the augmented breakdown surface, yielding the backscattered flux arriving at the surface of the spacecraft.
3. **FLUX INTEGRATION**

The effusion flux $F_i(M)$ given by (2.6) above, is defined in such a way that the number of molecules effusing from an area element $\Delta A_B$ of the breakdown surface and arriving at an area element $\Delta A_S$ on the spacecraft (per second), is given by:

$$F_i(M) (\Delta A_B \cos \alpha_B) (\Delta A_S \cos \alpha_S) L_{BS}^{-2} \text{ [molecules per second]} \quad (3.1)$$

where $\alpha_B$, $\alpha_S$ are the angles between the line-of-sight $L_{BS}$ (Figure 3) and the normals to the breakdown surface and the spacecraft surface respectively. $L_{BS}$ is the distance between the elements $\Delta A_B$ and $\Delta A_S$. Dividing equation (3.1) by $\Delta A_S$ and integrating over the breakdown surface, the flux per unit area of the spacecraft is given by:

$$Q_1 = \int F_i(M) \cos \alpha_B \cos \alpha_S L_{BS}^{-2} dA_B \quad \text{[Molecules per second per m$^2$]} \quad (3.2)$$

The integration scheme for $Q_1$ over the breakdown surface is expressed in terms of the set of polar coordinates $R, \psi, \phi$ (Figure 3). For a point $(R, \psi, \phi)$ on the breakdown surface, using Cartesian coordinates $(X, Y, Z)$ and the angle $\omega$ between X-axis and the line-of-sight $L_{BS}$ we obtain the following geometrical relationships:

$$X = R \cos \psi \quad ; \quad Y = (A_o + R \sin \psi) \cos \phi \quad ; \quad Z = (A_o + R \sin \psi) \sin \phi$$

$$\cos \phi_{max} = \left( \frac{A_o}{A_o + R(H) \sin \psi} \right)$$

$$\hat{u} = u(\cos \theta, \sin \theta \cos \phi, \sin \theta \sin \phi)$$

$$\frac{L_{BS}}{L_{BS}} = (\cos \omega, \sin \omega \cos \phi, \sin \omega \sin \phi)$$

$$\tan \beta = \frac{Z}{(Y-A_o)}$$
The cosines $\cos \kappa$, $\cos \alpha_B$, $\cos \alpha_S$ in (3.2) are expressed as scalar products of $\hat{l}_{BS}/|\hat{l}_{BS}|$ and unit vectors along the local velocity vector $\hat{u}$, along the local normal to the breakdown surface and along the local normal to the spacecraft surface, correspondingly.

The integration is performed numerically in two phases, the first being the integration along the supplementary breakdown surface (Figure 4). For this first phase, the straight line segment which constitutes the supplementary breakdown surface is divided into several intervals of length $\Delta R$ (typically 10 intervals). Each interval generates a half-strip by rotating it from $\phi = 0$ through $\phi = \phi_{\text{max}}(M_1)$. This strip is in turn subdivided into several sub-intervals of $\Delta \phi$ each (typically 10 intervals). The total flux arriving at $X_s$ is obtained by summing contributions from each sub-interval (two-dimensional integration). When the integration along the supplementary breakdown surface is concluded, it is continued into the breakdown surface, where $\Delta R$ intervals are replaced by breakdown surface intervals that correspond to a fixed Mach number increment $\Delta M$ (typically $\Delta M=0.1$). The integration proceeds along the breakdown surface (2.5) until the contribution of the last $\Delta M$ strip is negligibly small. The computation time is modest (about 1 second CPU per $X_s$ point, on IBM 3033 mainframe computer). The computations were carried out by a code RINGBD written specifically for this purpose. Further details of the scheme and programming can be obtained by reading this code which is given in Appendix A.
4. RESULTS AND DISCUSSION

4.1 Presentation of Results

The molecular flux backscattered to the spacecraft from the surface of continuum flow breakdown in the lip-centered rarefaction fan, has been computed for all five species H, HF, H₂, DF, He. The results are depicted in Figures 5 to 9 respectively. For each species two more cases were computed in addition to the nominal case (1.1), where the stagnation density \( \rho_0 \) was replaced by \( \rho_0/10 \) and by \( \rho_0 \times 10 \) (see Figures 5 to 9). This has been done in order to demonstrate the effect of variations in exit flow conditions on the flux. The particular choice of \( \rho_0 \) was motivated by the fact that the effects of changing \( \rho_0 \) are not obvious. The effects of changing the exit Mach number \( M_1 \) or the stagnation temperature \( T_0 \) are rather obvious (a higher flux would result from either a decrease in \( M_1 \) or an increase in \( T_0 \)). It turns out that for points lying not too near the nozzle lip (\( X_e > 0.1 \, \text{m} \)), the lower density flow generates a higher backscattered flux!

In addition to varying \( \rho_0 \), we also varied the breakdown parameter \( B \), obtaining a surprising result. The computation was performed for a particular species (HF), and the results obtained upon replacing \( B=0.05 \) (nominal value) by \( B/2 \) and by \( B \times 2 \) are brought in Figure 10.

It turns out that the \( B/2 \) case has the higher flux. This is somewhat surprising, since a lower value of \( B \) in a centered rarefaction fan (equation 2.5) means that the breakdown of continuum flow takes place in a region further out from the corner. In a source flow (e.g., a spherical source), that implies lower density and temperature, which would give rise to lower thermally backscattered flux.
An explanation to these seemingly counterintuitive results, along with some deeper insight into the breakdown surface model as it is applied to a centered rarefaction flow, can be obtained by taking a close look at the flow field and the breakdown surface in the vicinity of the corner. We take up this matter in the following sections.

We conclude the presentation of results, by comparing the flux (in the nominal case) of the five species with each other (Figure 11). This figure underlines the fact that the flux of light species (H, H₂, He) is many orders of magnitude (typically $10^{15}$) times that of heavy species (HF, DF). Indeed, these results demonstrate a well known effect: When an expanding gaseous mixture of light and heavy molecules experiences a breakdown of continuum flow, a separation of species takes place (see e.g., the work of Cattolica et. al. [8]).

### 4.2 The Breakdown Surface and Streamlines

Consider the parametric description $R_B(M)$ for the breakdown surface (Equation 2.5). Normalizing $R$ relative to the exit mean free path $\lambda_1$, we get:

$$R_B(M) = R_B(M_1) \left( \frac{M^2 - 1}{M_1^2 - 1} \right)^{1/2} \left( \frac{1 + \frac{\gamma-1}{2} M_1^2}{1 + \frac{\gamma-1}{2} M^2} \right)^{1}$$

$$R_B(M_1)/\lambda_1 = \left( \frac{\gamma\pi/2}{(\gamma+1)B} \right) \left( M_1^2 - 1 \right)^{1/2}$$

$$\lambda_1 = \left( 2 \frac{1/2}{\pi D^2 N_0} \right)^{-1} \left( 1 + \frac{\gamma-1}{2} M_1^2 \right)^{1/2}$$  \hspace{1cm} (4.1)

The normalized surface $R_B(M)/\lambda_1$ is thus independent of stagnation density, depending only on $\gamma$, $M_1$, $B$. 
Let us now derive a parametric equation $R_s(M)$ for a streamline that enters the fan at point $R_s(M_1)$ on the exit characteristic. The following geometrical relationship is readily obtained by considering two characteristic lines $\psi$ and $\psi + \Delta \psi$ and a streamline inclined at the Mach angle $\mu$ to them:

$$\frac{dR_s(\psi)}{d\psi} = -R_s(\psi) (\tan \mu)^{-1} \quad (4.2)$$

Using the standard Prandtl-Meyer functions (2.1), we get the following differential equation for $R_s(M)$:

$$\frac{1}{R_s(M)} \frac{dR_s(M)}{dM} = \left(\frac{\gamma+1}{2}\right) M(1 + \frac{\gamma-1}{2} M^2)^{-1} \quad (4.3)$$

This equation is readily integrated, giving:

$$R_s(M) = R_s(M_1) \left[\left(1 + \frac{\gamma-1}{2} M^2\right)/(1 + \frac{\gamma-1}{2} M_1^2)\right]^{-\frac{\gamma+1}{2(\gamma-1)}} \quad (4.4)$$

As pointed out by Bird[7], there is a particular streamline $R_{sa}(M)$ which asymptotically approaches the breakdown surface for large $M$, since the ratio $R_s(M)/R_B(M)$ tends to a constant (not zero) when $M \to \infty$. (Strictly speaking, this holds only for hard-sphere molecules, i.e., only when $\omega = 0.5$ in[7]). The limit is:

$$\lim_{M \to \infty} \frac{R_s(M)}{R_B(M)} = \frac{R_s(M_1)}{R_B(M_1)} \left[\frac{M_1^2 - 1}{M_1^2 + \frac{2}{\gamma-1}}\right]^{1/2} \quad (4.5)$$

For the limiting streamline $R_{sa}(M)$, the ratio $R_{sa}(M)/R_B(M)$ should tend to 1. This determines the point $R_{sa}(M_1)$ at which the limiting streamline enters the fan, as well as the entire line $R_{sa}(M)$:

$$R_{sa}(M_1) = R_B(M_1) \left[\frac{M_1^2 + \frac{2}{\gamma-1}}{M_1^2 - 1}\right]^{1/2} \quad (4.6)$$
Clearly, \( R_{sa}(M) \) is larger than \( R_{B}(M) \) for any \( M > M_1 \), so that no streamline beyond \( R_{sa}(M) \) can cross the breakdown surface. This pattern is shown in Figure 4, where \( R_{sa}(M) \) is denoted "streamline 2", and "streamline 1" is the streamline \( R_{B}(M_1) = R_{B}(M_1) \).

All this leads to the following observation regarding the continuum breakdown of the flow in a centered rarefaction fan[7]. Referring to Figure 4, the fluid entering the fan through the supplementary breakdown surface (i.e., through the exit characteristic between the corner and the initial point of streamline 1), experiences breakdown immediately upon crossing this surface. Every streamline between streamline 1 and streamline 2 crosses the breakdown surface at some Mach number \( M > M_1 \), and at that point the continuum flow breaks down. All fluid entering the fan beyond streamline 2 will never pass through the breakdown surface, and hence will maintain a continuum flow regime all the way to infinity. Of course, that is only true for planar centered rarefaction fans. When the exhaust flow emerges from a nozzle of finite width, and especially when the exhaust jet has a ring symmetry (as in our case), the breakdown surface gradually curves in a balloonlike shape towards the opposite nozzle lip, forming the familiar plume pattern (Figure 1).

4.3 Analysis and Discussion of Results

The foregoing analysis is now used to explain the variation in back-scattered flux due to a change in exhaust flow conditions at the nozzle exit. Specifically, we consider a tenfold decrease in stagnation density (i.e., the case \( \rho_0/10 \)), and hence a tenfold increase in the exit mean free path \( \lambda_1 \).

The effusion flux from the breakdown surface is proportional to the local density, so one would expect to observe a decrease in flux, rather than
an increase (see Figures 5 to 9, for $X_s > 0.1$ m). Other factors causing increased flux, must then be larger than 10 so that they more than offset the 1/10 factor in density. It turns out that these effects are mainly geometrical, in that a tenfold increase in $\lambda_1$ causes the domain of integration on the breakdown surface to increase more than tenfold. In the $(X,Y)$ plane there is a tenfold "blowup" of the breakdown surface, due to the self-similar structure of the Prandtl-Meyer flow field. As a result of this "blowup" in $(X,Y)$, the angular integration range $\theta_{\text{max}}$ also increases, albeit not linearly (Equation 3.3). Another geometrical effect is an increase in the flux incidence cosine factor $\cos \alpha_s$ (see Equation 3.2), which for points $X_s$ sufficiently far from the nozzle lip, increases roughly tenfold (while the other cosine factor $\cos \alpha_B$ is almost constant). All this provides a qualitative explanation for the observed increase in flux at far points ($X_s > 0.1$ m).

As for the near range ($X_s < 0.1$ m), another effect becomes increasingly significant as $X_s$ approaches the nozzle lip. The turning angle $\kappa$, by which backscattered molecules have to be deflected relative to the flow velocity vector in order to reach point $X_s$ on the spacecraft (Figure 2), increases with the size of the breakdown surface (fixed $M$ and $X_s$). Since the local effusion flux (Equation 2.6) decreases rather sharply as $\kappa$ is increased, the net result is a tendency to get a reduced backscattered flux at near points such as $X_s = 0.01$ m (Figures 5 to 9).

We now turn to the effect of changing the value of the breakdown parameter $B$. From equation 4.1 it is clear that multiplying $B$ by some factor
will have the same "blowup" effect as dividing $\lambda_1$ by the same factor. A
tenfold decrease in $B$ is thus geometrically equivalent to a tenfold decrease
in $p_0$. However, since the local effusion flux at the breakdown surface is
proportional to $p_0$ while it is independent of $B$, the $B/10$ case will have ten
times as much backscattered flux as the $p_0/10$ case. In order to illustrate
the sensitivity of the flux estimates to an uncertainty in the appropriate
value of $B$, we computed the cases $B/2$ and $B*2$ for one species (HF), and they
are presented in Figure 10. The variation in flux relative to the nominal
case ($B = 0.05$), is by a factor no larger than about 5. Results for other
species were found to exhibit comparable variations.

Does this observation about the dependence of the breakdown surface
on $B$ agree with the breakdown surface appropriate to the far field of the
exhaust plume? In stationary source flow into vacuum, and when $M \gg 1$, the
breakdown parameter varies with radius as $B \sim R^{\delta-1}$ ($\delta = 1$ for cylindrical
source, $\delta = 2$ for spherical source). In a ringjet, the stream tubes of the
exhaust plume generally diverge at a rate higher than that of stream tubes in
a cylindrical source flow, so the effective value of $\delta$ in a ringjet is $\delta > 1$. Hence, in this case the far field breakdown surface moves downstream along
each stream tube as the value of $B$ increases. This is indeed geometrically
compatible with the fact that near the corner of the lip-centered rarefaction
fan $B \sim R^{-1}$, as shown schematically in Figure 12. The dependence of the break-
down surface on $B$ near the corner and in the far field, thus assures that
complete breakdown surfaces corresponding to different values of $B$, do not
intersect (Figure 12).

In the foregoing discussion it was pointed out that variations in
flux caused by changes in parameters such as $p_0$ and $B$, were directly related
to the self-similar structure of the Prandtl-Meyer flow field. It has been further shown that these variations are well-understood within the framework of the breakdown surface model and that they are not excessively large. Are we to conclude that the thermally backscattered flux estimates of the present model are also physically plausible and reliable? In the following section we take up this matter, arriving at some interesting conclusions about this model and its range of validity.
Critical Examination of the Model

Consider the centered rarefaction flow field of a compressible fluid negotiating an expansive corner at supersonic speed (Prandtl-Meyer flow). The streamlines of this flow field have an orderly "layered" structure, with each streamline curving around the corner, starting at its point of entrance into the fan (see Figure 4).

The present model is based on the stipulation that there is a point of continuum flow breakdown on each streamline, provided this streamline is not beyond a certain limiting streamline. Consider a sample molecule effusing from this breakdown point toward the spacecraft. It advances at constant speed along a straight line trajectory, traversing all inner streamlines. Since the flow velocity vector points away from the spacecraft, and since the flow is highly supersonic so that the velocity of most individual molecules does not differ much from the flow velocity (i.e., it is a "cold" flow), any collision of the sample molecule with a mainflow molecule will most probably divert the sample molecule away from the spacecraft. What is the probability that a sample molecule would traverse this cross flow collisionlessly? This probability is simply $\exp(-n)$, where $n$ is the expected number of collisions along the straight-line trajectory from the point of breakdown to the spacecraft. In the typical operating conditions assumed here, we estimated $n$ to be roughly about 10. Since this no-collision probability factor is ignored in the formulation of the present model, the backscattered flux may be exaggerated by a factor of $\exp(10)$ or about $10^4$. We conclude that in all likelihood, the prediction of the breakdown surface model for thermally backscattered flux from a centered rarefaction flow, is substantially overestimated.
Can anything be done to improve the present model? One may be inclined to suggest at this point that the obvious remedy is to incorporate the no-collision probability factor into the model. Rather, we prefer to retain the breakdown surface model in its present form as a simple means of obtaining an overestimate to the thermally backscattered flux from a centered rarefaction flow. An improved model can be constructed by considering thermal backscattering from the entire flow field (tempered by the probability of no-collision), without resorting to the physically untenable notion of an abrupt transition from continuum flow to free molecular flow.
5. CONCLUSIONS

Some surprising similarity laws of the breakdown surface model were observed. It has been shown that they were a direct result of the self similar structure of the Prandtl-Meyer flow field to which the model was applied. Specifically, it was found (and shown plausible) that reduced values of either the exhaust stagnation density $\varrho_0$, or the breakdown parameter $B$, caused higher backscattered flux.

The breakdown surface model for thermally backscattered flux from a centered rarefaction fan, has been shown to overestimate the flux arriving at the spacecraft. It is suggested that an improved model be constructed by considering thermal backscattering from the entire flow field, along with the probability factor for a side-scattered molecule traversing the main flow collisionlessly.

The molecular flux of corrosive species (HF, DF) arriving at the spacecraft (Figures 6 and 8) is no larger than about $10^7$ (sec$^{-1}$ m$^{-2}$), which is negligible since it corresponds to about $10^{-5}$ molecular monolayers per year. This conclusion is reliable since even this flux level is an overestimate.

The maximum thermally backscattered flux of light species (H, H$_2$, He) is in the range of $10^{20}$ to $10^{22}$ (sec$^{-1}$ m$^{-2}$) (see Figures 5, 7, 9). Thus, we conclude that while thermal backscattering would contribute significantly to the flux of light molecules arriving at the spacecraft, it is utterly negligible as far as heavy molecules are concerned.
5. REFERENCES


APPENDIX A. The Computer Code RINGBD

We present a printout of the code RINGBD along with the results of the nominal case (printout of actual run). This is preceded by a brief description of the subroutines and a summary of major variables with their code and report notations.

A.1 Description of Subroutines

MAIN PROGRAM - Computes flux integration by summation of segment contributions (centered). Printing of results.

INIDAT - Definition of all data (no input file). Preparatory evaluation of parameters. Printing of data.

FLUX - Evaluates flux emitted from a single point on breakdown surface (mean segment values) to a point on spacecraft (XS).

BREAKR - Computes point on breakdown surface for given Mach number.

BREAKM - Computes point on breakdown surface for mean Mach number of a segment.
A.2 **Code Versus Report Notation**

XC(I) - [AB] - Mole fraction of species AB. (I=1,2,3,4,5 corresponds to H, HF, H₂, DF, He).

WC(I) - Wᵢ - Molecular weight of species i.

WAV - Wₐ - Average molecular weight

TO - T₀ - Stagnation temperature

RHOO - ρ₀ - Stagnation density

G - γ - Specific heat ratio

EMI - Mₑ - Exit Mach number

LAMDA₁ - λ₁ - Exit mean free path

AO - Aₒ - Spacecraft radius

R - R - Distance from corner (X=0, Y²+Z² = Aₒ²).

DIST - Lₐₕₜ - Distance between emitting point on breakdown surface and receiving point (XS) on spacecraft.

XS - Xₛ - Point on spacecraft (X=Xₛ, Y=Aₒ, Z=0).

PSI - ψ - Characteristic angle

AMU - u - Mach angle

TETA - θ - Velocity vector angle

PHI - φ - Rotation angle for flux integration

W - ω - Angle between x-axis and line-of-sight Lₐₕₜ

BETA - β - Angle between Y-axis and projection of Lₐₕₜ on (Y,Z) plane.

DMO - ΔM - Mach number increment for flux integration

EM - M - Mach number

PBIRD - B - Breakdown parameter
A.3 Code Listing (Run of Nominal Case):

```fortran
$JOB
RINGBD.NOXREF
1 IMPLICIT REAL*8(A-H,O-Z,$) 
2 COMMON /GAMA/G1,G2,G3,G4,G5,G6,G7,G8,G9,G10,G11,G12,G13,G14,G15,
3 G16,G17,G18,G19,G20 
4 COMMON /PAR/CO,ENO,EM1,D,TLIM,ETALIM,CLIM,ELO,QO,TO,
5 PBIRD,RBIRD,DMO,DEG,OMEGA,XSV(51) 
6 COMMON /NPAR/NETA,NC,NT,NEMO,NPHI,NXS,NRO,NSPEC 
7 COMMON /GEOM/APF,PAI,PAI2,SW,CW,BETA,SBETA,CBETA,PSI1,SPSI1,
8 CPSI1,PSIF,SPSIF,CPSIF,AK,SK,CK,AO,RF,XF,YF,ZF,
9 PHI,SPHI,CPHI,RMIN,RMAX,XS,DIST, 
10 AMU1,ZETA1,XN,YN,ZN,PSIM,SPSIM,CPSIM,R0 
11 COMMON /EPSIL/EPSQ,EPSETA,EPST,EPSC,EPSEM 
12 COMMON /EXTREM/TEXT,ETALIM,CLIM,ELO,QO,TO, RIN00150
13 DIMENSION DSUM(5) 
14 PRINT 101 
15 101 FORMAT('RINGBD - FLUX INTEGRATION FROM BREAKDOWN', 
16 'SURFACE') 
17 CALL INIDAT 
18 PRINT 110,XNAME 
19 110 FORMAT(///1X, 
20 NX',' NEM',' XS ',' PHIMAX',' QMAX ',' 
21 5(4X,A6,1X,'/ LOG',1X)) 
22 DO 200 NX=1,NXS 
23 EM=EM1 
24 CALL BREAKRCEM,RF) 
25 IF(NRO.3T.O) RF=RO 
26 XF=RF*CPSI1 
27 YF=RF*SPSI1+AO 
28 XS=XSV(NX) 
29 QMAX=0. 
30 DO 45 N=1,NSPEC 
31 FLUXC(N)=0. 
32 45 CONTINUE 
33 DO 1 NEM=1,NEM0 
34 RN=RF 
35 XN=XF 
36 YN=YF 
37 IF(NEM.GT.NRO) GO TO 41 
38 RF=RF+DFLOAT(NEM)*(RMIN-R0)/DFLOAT(NRO) 
39 XF=RF*CPSI1 
40 YF=RF*SPSI1+AO 
41 RMEAN=(RF+RF)/2.D0 
42 EMMEAN=EM1 
43 PSIM=PSI1 
44 CPSIM=SPSI1 
45 GO TO 42 
46 CONTINUE 
47 EM=EM+DM0 
48 CALL BREAKRAM(EM,RF) 
49 EMMEAN=EM+DM0/2.D0 
50 CALL BREAKRAM(EMMEAN,RMEAN) 
51 ALONG=DSQRT((XF-XN)**2+(YF-YN)**2) 
52 SALFA=(YF-YN)/ALONG 
53 CALFA=(XF-XN)/ALONG 
54 PHIMAX=DARCOS(AO/(AO+RMEAN*SPSIM)) 
55 BPHI=PHIMAX/NPHI 
```

DO 44 N=1,NSPEC
DSUM(N)=0.
CONTINUE
DO 2 NP=1,NPHI
PHI=(DFLOAT(NP)-0.5DO)*DPHI
CALL FLUX(EMMEAN,RMEAN)
CROSS1=SHWBET
CROSS2=(SALFA)*(-CW)*(-CALFA*CPSHI)*(-SW*CPSI)
1
GOREM=CROSS1*CROSS2*DPHI*(AO+RMEAN*SPSIM)*ALONG/DIST**2
DO 24 N=1,NSPEC
DSUM(N)=G.
CONTINUE
IF(QMAX.GE.QEXT) GO TO 25
QMAX=QEXT
CONTINUE
C PRINT 22,NEM,NP,EMMEAN,RMEAN,PHIMAX*DEG,DARCOS(CW)*DEG, W,BETA,PHI,
C 1
1X,'CROSS1,CROSS2,ALONG,DIST,GOREM=',5D15.5/
C 2
C 3
C 4
GO TO 10
CONTINUE
1 CONTINUE
7 CONTINUE
IF ((DSUM(N)/FLUXC(N)).GT.EPSEM) GO TO 28
CONTINUE
DO 27 N=1,NSPEC
IF((DSUM(N)/FLUXC(N)).GT.EPSEM) GO TO 28
CONTINUE
GO TO 10
CONTINUE
IF(CROSS1+CROSS2+GOREM).LT.0.5D15.5/ GO TO 28
CONTINUE
GO TO 10
STOP
END

SUBROUTINE INIDAT
IMPLICIT REAL*8(A-H,O-Z,S)
REAL*8 LAMDAO,LAMDA1
COMMON /GAMA/G,G1,G2,G3,G4,G5,G6,G7,G8,G9,G10,G11,G12,G13,G14,G15,G16,G17,G18,G19,G20
COMMON /PAR/CO,ENO,EM1,D,TLIM,ETALIM,CLIM,ELO,QO,TO, PBIRD,BIRD,DMO,DEG,OMEGA,XSV(51)
COMMON /GEOM/APF,PAI,P12,SN,SW,CW,BETA,SBETA,CBETA,PSI1,SPSI1,
1 CPSI1,PSIF,PSIFS,CPF,CK,AD,RF,XF,YF,ZF,
2 PH1,SPH1,CPH1,CRM1,CMAX,XS,DIST,
3 AMU1,ZETA1,XN,YN,ZN,PS11,SPS11,PSIM,CPSIM,R0
COMMON /EPSIL/EPSQ, EPSETA, EPSI, EPSM, EPSQ, EPSM, EPSI, EPSM
COMMON /EXTREM/TEXT, ETAEXT, CEXT, REXT, PSIEXT, EMEXT, BEXT, QEXT
DATA XC/0.091DO, 0.091DO, 0.104DO, 0.135DO, 0.579DO/
DATA MC/1.00D0, 2.00D0, 2.00D0, 2.10D0, 4.00D0/
DATA XNAME/"H", "HF", "H2", "DF", "HE"/
PAI=4.*DATAN(.1D 1)
AR=8.3143D3
AV=6.022D 26
C OMEGA=0.5 IS FOR HARD SPHERE COLLISIONS,
C AN AVERAGE RECOMMENDED VALUE IS ABOUT OMEGA=0.75
OMEGA=0.5DO
NSPEC=5
WAV=0.
DO 51 N=1,NSPEC
WAV=WAV+XC(N)*WC(N)
51 CONTINUE
DO 52 N=1,NSPEC
WCR(N)=DSQRT(WC(N)/WAV)
52 CONTINUE
AO=2.5DO
EM1=4.00D0
RHO0=0.0075DO
TO=2.50D0
D=2.50D-10
G=1.54D0
ENO=RHO0*WAV
CO=DSQRT(G*AR*TO/WAV)
PBRD=0.05D0*2.DO
C RO IS THE RADIUS FOR BEGINNING THE INTEGRATION ALONG THE M=M1
C CHARACTERISTIC(THE AUGMENTED BREAKDOWN SURFACE).
C NRO IS THE NUMBER OF INTEGRATION INTERVALS ON THIS SEGMENT.
C FOR NO INTEGRATION ALONG M=M1 CHARACTERISTIC, SET NRO=0.
RO=0.
NRO=10
DMO=0.1DO
NEMO=20.DO/DMO+NRO
C TO GET FLUX DUE TO AUGMENTED BREAKDOWN SURFACE SOLELY, ACTIVATE:
C NEMO=NRO
NPHI=10
NXS=13
XSI=1.D-2
XSF=1.D1
XSV(1)=XSI
IF(NXS.EQ.1) GO TO 111
DXL=(DLOG(XSF)-DLOG(XSI))/(NXS-1.DO)
XLI=DLOG(XSI)
DO 11 NX=2,NXS
XSV(NX)=DEXP(XLI+(NX-1.DO)*DXL)
11 CONTINUE
111 CONTINUE
EPSEM=1.D-5
DEG=180.DO/PAI
PAI2=PAI/2.DO
GAMMA=G
G1=G/2.DO
G2=(G+1.DO)/(2.DO*(G-1.DO))
G3=G/2.DO
G4=(G+1.DO)/(G-1.DO)
G5=DSQRT((G+1.DO)/(G-1.DO))
G6 = 1.0D0/(G-1.0D0)  
G7 = 2.0D0/(G+1.0D0)  
G8 = (G+1.0D0)/(2.0D0*(G-1.0D0))  
G9 = (G+3.0D0)/(2.0D0*(G-1.0D0))  
G10 = (G+5.0D0)/(2.0D0*(G-1.0D0))  
G11 = D5ORT(G/PAI)/2.0D0  
G12 = D5ORT(G/2.0D0)  
G13 = 1.0D0/D5ORT((G+1.0D0)/(G-1.0D0))**((G-1.0D0)/(G+1.0D0))  
G14 = (G+S.0D0)/(2.0D3*C3-1.0D0)  
G15 = (7.0D0-3.0D0*G)/(2.0D0*(G-1.0D0))  
G16 = D5RT(G/PAI)/(2.0D0*(G+1.0D0))  
G17 = D5RT(G/2.0D0)  
G18 = 1.0D0/D5RT((G+1.0D0)/(G-1.0D0))**((G-1.0D0)/(G+1.0D0))  
G19 = D5RT(G/PAI)/(2.0D0*(G+1.0D0))  
G20 = D5RT(G/2.0D0)  
G21 = (G+1.0D0)/(G-1.0D0)**((G-1.0D0)/(G+1.0D0))  
G22 = (G+S.0D0)/(2.0D3*C3-1.0D0)  
G23 = (7.0D0-3.0D0*G)/(2.0D0*(G-1.0D0))  
G24 = D5RT(G/PAI)/(2.0D0*(G+1.0D0))  
G25 = D5RT(G/2.0D0)  
G26 = 1.0D0/D5RT((G+1.0D0)/(G-1.0D0))**((G-1.0D0)/(G+1.0D0))  
G27 = D5RT(G/PAI)/(2.0D0*(G+1.0D0))  
G28 = D5RT(G/2.0D0)  
G29 = (G+1.0D0)/(G-1.0D0)**((G-1.0D0)/(G+1.0D0))  
G30 = D5RT(G/PAI)/(2.0D0*(G+1.0D0))  
G31 = D5RT(G/2.0D0)  
G32 = 1.0D0/D5RT((G+1.0D0)/(G-1.0D0))**((G-1.0D0)/(G+1.0D0))  
G33 = D5RT(G/PAI)/(2.0D0*(G+1.0D0))  
G34 = D5RT(G/2.0D0)  
G35 = (G+1.0D0)/(G-1.0D0)**((G-1.0D0)/(G+1.0D0))  
G36 = D5RT(G/PAI)/(2.0D0*(G+1.0D0))  
G37 = D5RT(G/2.0D0)  
G38 = 1.0D0/D5RT((G+1.0D0)/(G-1.0D0))**((G-1.0D0)/(G+1.0D0))  
G39 = D5RT(G/PAI)/(2.0D0*(G+1.0D0))  
G40 = D5RT(G/2.0D0)  
G41 = (G+1.0D0)/(G-1.0D0)**((G-1.0D0)/(G+1.0D0))  
G42 = D5RT(G/PAI)/(2.0D0*(G+1.0D0))  
G43 = D5RT(G/2.0D0)  
G44 = 1.0D0/D5RT((G+1.0D0)/(G-1.0D0))**((G-1.0D0)/(G+1.0D0))  
G45 = D5RT(G/PAI)/(2.0D0*(G+1.0D0))  
G46 = D5RT(G/2.0D0)  
G47 = (G+1.0D0)/(G-1.0D0)**((G-1.0D0)/(G+1.0D0))  
G48 = D5RT(G/PAI)/(2.0D0*(G+1.0D0))  
G49 = D5RT(G/2.0D0)  
G50 = 1.0D0/D5RT((G+1.0D0)/(G-1.0D0))**((G-1.0D0)/(G+1.0D0))  
G51 = D5RT(G/PAI)/(2.0D0*(G+1.0D0))  
G52 = D5RT(G/2.0D0)  
G53 = (G+1.0D0)/(G-1.0D0)**((G-1.0D0)/(G+1.0D0))  
G54 = D5RT(G/PAI)/(2.0D0*(G+1.0D0))  
G55 = D5RT(G/2.0D0)  
G56 = 1.0D0/D5RT((G+1.0D0)/(G-1.0D0))**((G-1.0D0)/(G+1.0D0))  
G57 = D5RT(G/PAI)/(2.0D0*(G+1.0D0))  
G58 = D5RT(G/2.0D0)  
G59 = (G+1.0D0)/(G-1.0D0)**((G-1.0D0)/(G+1.0D0))  
G60 = D5RT(G/PAI)/(2.0D0*(G+1.0D0))  
G61 = D5RT(G/2.0D0)  
G62 = 1.0D0/D5RT((G+1.0D0)/(G-1.0D0))**((G-1.0D0)/(G+1.0D0))  
G63 = D5RT(G/PAI)/(2.0D0*(G+1.0D0))  
G64 = D5RT(G/2.0D0)  
G65 = (G+1.0D0)/(G-1.0D0)**((G-1.0D0)/(G+1.0D0))  
G66 = D5RT(G/PAI)/(2.0D0*(G+1.0D0))  
G67 = D5RT(G/2.0D0)  
G68 = 1.0D0/D5RT((G+1.0D0)/(G-1.0D0))**((G-1.0D0)/(G+1.0D0))  
G69 = D5RT(G/PAI)/(2.0D0*(G+1.0D0))  
G70 = D5RT(G/2.0D0)  
G71 = (G+1.0D0)/(G-1.0D0)**((G-1.0D0)/(G+1.0D0))  
G72 = D5RT(G/PAI)/(2.0D0*(G+1.0D0))  
G73 = D5RT(G/2.0D0)  
G74 = 1.0D0/D5RT((G+1.0D0)/(G-1.0D0))**((G-1.0D0)/(G+1.0D0))  
G75 = D5RT(G/PAI)/(2.0D0*(G+1.0D0))  
G76 = D5RT(G/2.0D0)  
G77 = (G+1.0D0)/(G-1.0D0)**((G-1.0D0)/(G+1.0D0))  
G78 = D5RT(G/PAI)/(2.0D0*(G+1.0D0))  
G79 = D5RT(G/2.0D0)  
G80 = 1.0D0/D5RT((G+1.0D0)/(G-1.0D0))**((G-1.0D0)/(G+1.0D0))  
G81 = D5RT(G/PAI)/(2.0D0*(G+1.0D0))  
G82 = D5RT(G/2.0D0)  
G83 = (G+1.0D0)/(G-1.0D0)**((G-1.0D0)/(G+1.0D0))  
G84 = D5RT(G/PAI)/(2.0D0*(G+1.0D0))  
G85 = D5RT(G/2.0D0)  
G86 = 1.0D0/D5RT((G+1.0D0)/(G-1.0D0))**((G-1.0D0)/(G+1.0D0))  
G87 = D5RT(G/PAI)/(2.0D0*(G+1.0D0))  
G88 = D5RT(G/2.0D0)  

\[
\begin{align*}
\text{SPI} &= \sin(\phi) \quad \text{RIN02390} \\
\text{CPHI} &= \cos(\phi) \quad \text{RIN02400} \\
\text{XMEAN} &= R \times \text{CPSIM} \quad \text{RIN02410} \\
\text{YMEAN} &= (A_0 + R \times \text{SPSIM}) \times \text{CPHI} \quad \text{RIN02420} \\
\text{ZMEAN} &= (A_0 + R \times \text{SPSIM}) \times \text{SPHI} \quad \text{RIN02430} \\
\text{TBETA} &= \text{ZMEAN} / (\text{YMEAN} - A_0) \quad \text{RIN02440} \\
\text{BETA} &= \text{PAI2} - \text{DATAN}(1.0 \times \text{TBETA}) \quad \text{RIN02450} \\
\text{SBETA} &= \sin(\text{BETA}) \quad \text{RIN02460} \\
\text{CBETA} &= \cos(\text{BETA}) \quad \text{RIN02470} \\
\text{DIST} &= \sqrt{(X - \text{XMEAN})^2 + (\text{YMEAN} - A_0)^2 + (Z - \text{ZMEAN})^2} \quad \text{RIN02480} \\
\text{CN4} &= -(X - \text{XMEAN}) / \text{DIST} \quad \text{RIN02490} \\
\text{SW} &= \sqrt{1 - \text{CN4}^2} \quad \text{RIN02500} \\
\text{GM} &= (A_0 + G_1 \times \text{EM} \times 2) \times (-G_2) \quad \text{RIN02510} \\
\text{AMU1} &= \text{DARSIN}(1.0 \times \text{EM}) \quad \text{RIN02520} \\
\text{TETA} &= \text{PSIM} - \text{AMU1} \quad \text{RIN02530} \\
\text{STETA} &= \sin(\text{TETA}) \quad \text{RIN02540} \\
\text{CTETA} &= \cos(\text{TETA}) \quad \text{RIN02550} \\
\text{CKAPA} &= (\text{CTETA}) \times (-\text{CN4}) + (\text{TETA} \times \text{CPHI}) \times (-\text{SBETA}) \quad \text{RIN02560} \\
\text{SKAPA} &= \sqrt{1 - \text{CKAPA}^2} \quad \text{RIN02570} \\
\text{QEXT} &= 0. \quad \text{RIN02580} \\
\text{DO} &= 1 \text{ to } \text{NSPEC} \quad \text{RIN02590} \\
\text{EMT} &= \text{EM} \times \text{SKAPA} \times G_1 \times \text{WCR}(N) \quad \text{RIN02600} \\
\text{POW} &= \text{EM} \times \text{EMT} \times 2 \quad \text{RIN02610} \\
\text{POWT} &= \text{POW} \times \text{SKAPA} \times 2 \quad \text{RIN02620} \\
\text{EXP1} &= \text{DEXP}(-\text{POW}) \quad \text{RIN02630} \\
\text{EXP2} &= \text{DEXP}(-\text{POWT}) \quad \text{RIN02640} \\
\text{ERFC1} &= \text{DERFC}(-\text{EMT}) \quad \text{RIN02650} \\
\text{ERFC1} &= 1.0 - \text{ERFC1} \quad \text{RIN02660} \\
\text{ERFC1} &= 1.0 \quad \text{RIN02670} \\
\text{QEXT} &= \text{EXP1} + \text{EXP2} \quad \text{RIN02680} \\
\text{QEXT} &= \text{QEXT} \times \text{CN4} \quad \text{RIN02690} \\
\text{QEXT} &= \text{QEXT} \times \text{CN4} \quad \text{RIN02700} \\
\text{QEC} &= \text{QEXT} - \text{QEXT} \times \text{CN4} \quad \text{RIN02710} \\
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\text{QEC} &= \text{QEC} \times \text{CN4} \quad \text{RIN02730} \\
\text{QEC} &= \text{QEC} \times \text{CN4} \quad \text{RIN02740} \\
\text{QEC} &= \text{QEC} \times \text{CN4} \quad \text{RIN02750} \\
\text{EVER1} &= \text{ENO} \times (\text{CN4} \times \text{WCR}(N)) \times \text{G1} \times \text{G3} \times \text{G4} \times \text{G5} \times \text{G6} \times \text{G7} \times \text{G8} \times \text{G9} \times \text{G10} \times \text{G11} \times \text{G12} \times \text{G13} \times \text{G14} \times \text{G15} \times \text{G16} \times \text{G17} \times \text{G18} \times \text{G19} \times \text{G20} \quad \text{RIN02760} \\
\text{EVER2} &= \text{ENO} \times (\text{CN4} \times \text{WCR}(N)) \times 0.5 \times \text{EN} \times \text{CN4} \times \text{WCR}(N) \times \text{CN4} \times \text{G1} \times \text{G3} \times \text{G4} \times \text{G5} \times \text{G6} \times \text{G7} \times \text{G8} \times \text{G9} \times \text{G10} \times \text{G11} \times \text{G12} \times \text{G13} \times \text{G14} \times \text{G15} \times \text{G16} \times \text{G17} \times \text{G18} \times \text{G19} \times \text{G20} \quad \text{RIN02770} \\
\text{EVER1} &= \text{EVER1} \times \text{EVER2} \quad \text{RIN02780} \\
\text{QEC} &= \text{QEC} \times \text{CN4} \quad \text{RIN02790} \\
\text{QEC} &= \text{QEC} \times \text{CN4} \quad \text{RIN02800} \\
\text{QEXT} &= \text{QEC} \times \text{CN4} \quad \text{RIN02810} \\
\text{CONTINUE} \quad \text{RIN02820} \\
\text{CONTINUE} \quad \text{RIN02830} \\
\text{END} \quad \text{RIN02840} \\
\text{SUBROUTINE BREAKR(EM, R)} \quad \text{RIN02850} \\
\text{IMPLICIT REAL*8(A-H, O-Z,$)} \quad \text{RIN02860} \\
\text{COMMON /GEOM/ APF, PAI, PAI2, SW, CH, BETA, SBETA, CBETA, PS1I, SPSI1,} \quad \text{RIN02870} \\
\text{CPSI1, PSIF, SPSI1, CPSI1, AK, CK, A0, RF, XM, YF, ZF,} \quad \text{RIN02880} \\
\text{PHI, SPSI, CPSI1, CPSI1, A0, RF, XM, YF, ZF,} \quad \text{RIN02890} \\
\text{AMUI, ZETAI, XN, YN, ZN, PSIM, SPSIM, CPSIM, R0} \quad \text{RIN02900} \\
\text{COMMON /PAR/ CS0, EN0, EM1, D, TLIM, ETA, CLIM, EL0, Q1, TO,} \quad \text{RIN02910} \\
\text{PBIRD, RBIRD, DOMO, DEG, OMEGA, XSV(51)} \quad \text{RIN02920} \\
\text{COMMON /GAMA/ G1, G2, G3, G4, G5, G6, G7, G8, G9, G10, G11, G12, G13, G14, G15, G16, G17, G18, G19, G20} \quad \text{RIN02930} \\
\text{R = RBIRD * DSQRT(EM * 2 - 1.0 * D) * (1.0 * G1 * EM * 2 + G6 - OMEGA + 0.5 * D)} \quad \text{RIN02940} \\
\text{ZETA = G5 * DATAN(DSQRT(EM * 2 - 1.0) / G5)} \quad \text{RIN02950} \\
\text{PSI = PAI2 + AMUI + ZETAI - ZETA} \quad \text{RIN02960} \
\end{align*}
\]
237 \( XF = RF \times DCOS(\psi) \)

238 \( YF = RF \times DSIN(\psi) + A0 \)

239 \( ZF = 0 \)

240 \( 1 \)

241 CONTINUE

242 \( C \)

243 ENTRY BREAKM(EM, R)

244 \( R = RR \times RD SQRT((EM - 2.9) \times (1.9 + G1 \times EM)) \times (G6 - OMEGA + 0.5D) \)

245 \( ZETA = G5 \times DAT4(DSQRT((EM - 2.9)/G5)) \)

246 \( PSIM = PA12 + AMU1 + ZETA1 - ZETA \)

247 \( SP SIM = DS IN(P SIM) \)

248 RETURN

249 END

$ENTRY

**RINO300 - FLUX INTEGRATION FROM BREAKDOWN SURFACE**

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END RIN000 RUN

STATEMENTS EXECUTED = 1281233
Figure 1. Thermal Backscattering from Laser Exhaust Plume
(Schematic)

Figure 2, Prandtl-Heyer Centered Karman 3D and Breakdown Surface

- BACKSCATTERED FLUX
- LIMITING CHARACTERISTIC
- BREAKDOWN SURFACE
- STREAMLINE
Figure 3. Flux Integration Scheme

- $B$ - Point on Breakdown Surface
- $(X, Y, Z)$ - Point on Breakdown Surface (Same point as $B(R, \psi)$)
- $\mathbf{LBS}$ - Line of Sight
Figure 5. Flux of Species H at Various Stagnation Densities
Figure 6. Flux of Species HF at Various Stagnation Densities
FLUX OF SPECIES $H_2$  \([H_2] = .104\)

\(M_1 = 4\)  \(\gamma = 1.54\)

\(T_0 = 2300\ (K)\)  \(W_A = 7.27\)  \(B = .05\)

---

**Figure 7.** Flux of Species $H_2$ at Various Stagnation Densities
FLUX OF SPECIES DF \[ [DF] = 0.135 \]

\[ M_1 = 4 \quad \gamma = 1.54 \]

\[ T_0 = 2300 \text{ (K)} \quad W_A = 7.27 \quad B = 0.05 \]

Figure 8. Flux of Species DF at Various Stagnation Densities
FLUX OF SPECIES He $[\text{He}] = .579$

$M_1 = 4 \quad \gamma = 1.54$

$T_0 = 2300 \text{ (K)} \quad W_A = 7.27 \quad B = .05$

Figure 9. Flux of Species He at Various Stagnation Densities
Figure 10. Flux of Species HF at Various Values of Breakdown Parameter
Figure 11. Flux of All Species at Typical Operating Conditions
Figure 12. Schematic Display of Complete Breakdown Surface in a Ringjet Exhaust Plume
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Monterey, California 93943                                                      | 2      |
| 3.  | Department Chairman, Code 67  
Department of Aeronautics  
Naval Postgraduate School  
Monterey, California 93943                                                      | 1      |
| 4.  | Distinguished Professor Allen E. Fuhs  
Code 72  
Naval Postgraduate School  
Monterey, California 93943                                                      | 4      |
| 5.  | Mr. Neil Griff  
Pentagon  
SDIO/DEO  
Washington, DC 20301-7100                                                        | 3      |
| 6.  | Mr. Bruce Pierce  
Pentagon  
SDIO/DEO  
Washington, DC 20301-7100                                                        | 1      |
| 7.  | Dr. Joseph Falcovitz  
Code 72  
Naval Postgraduate School  
Monterey, CA 93943-5100                                                          | 8      |
| 8.  | Associate Professor Oscar Biblarz  
Department of Aeronautics  
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END
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