SKY SCREEN DATA REDUCTION TO OBTAIN MUZZLE VELOCITY AND LINEAR DRAG COEFFICIENT

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**SKY SCREEN DATA REDUCTION TO OBTAIN MUZZLE VELOCITY AND LINEAR DRAG COEFFICIENT**

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An analysis and associated computational technique were developed to extract muzzle velocity, velocity decay, and drag coefficient from the sky screen data. This technique utilizes the advantage of a closed form solution to the equation of motion for a flat fire antitank projectile. The only limitation of this technique is that the drag coefficient must be a linear function of velocity. However, the results show that this technique obtains excellent velocity decay even with slight nonlinearity in the drag coefficient.
INTRODUCTION

For most of the tank projectile test firings, sky screens are used to obtain the flight time as a function of range. These range data can be used to extract muzzle velocity, velocity decay, and drag coefficient. The most common technique to extract muzzle velocity is to use the difference method to obtain the secant velocities (i.e., \( v_i = \frac{x_{i+1} - x_i}{t_{i+1} - t_i} \)). The first few secant velocities are then fitted with a straight line or a parabola to obtain the muzzle velocity. There are two disadvantages with this type of data reduction technique: (1) the difference method magnifies the error in obtaining the secant velocities, (2) the velocity profile is neither a straight line nor a parabola. To remove these disadvantages, a closed form solution to the equations of motion is used to fit the flight time from the sky screens. A closed form solution is possible when the drag coefficient is assumed to be a linear function of velocity. Therefore, the drag coefficient can also be obtained by fitting the closed form solution to the sky screen data.

ANALYSIS

The equation of motion for a flat fire antitank projectile is

\[
M \ddot{v} = -\frac{1}{2} \rho V^2 SC_D
\]

where

- \( M \) = Projectile mass
- \( V, \dot{V} \) = Velocity and time rate of change of velocity
- \( \rho \) = Air density
- \( S \) = Reference area
- \( C_D \) = Drag coefficient

When the drag coefficient is a linear function of velocity, having a form of \( C_D = CD_a (1 + bV) \), equation 1 becomes

\[
M \ddot{v} = -\frac{1}{2} \rho V^2 CD_a (1 + bV)
\]

or

\[
\dot{V} = -AV^2 (1 + bV)
\]

where \( A \) is a constant and is defined as \( \frac{1}{2M} \rho SC_D a \), and \( b \) is a slope value for the linear drag expression. In most cases, \( b \) has a negative value.
The closed form solution for velocity as a function of range \((x)\) can be obtained by integrating equation 2 with \(V = \frac{dx}{dt}\). The solution is

\[
V = V_c(1 + bV)e^{-Ax}
\]

or

\[
V = \frac{V_c}{e^{Ax} - bV_c}
\]  

(3)

where \(V_c \equiv \frac{V_0}{1 + bV_0}\) with the initial condition of \(V = V_0\) at \(x = 0\).

Furthermore, the closed form solution of \(t(x)\) can be obtained from differential equation 3. It has the solution of

\[
t = t_o + \frac{1}{AV_c}(e^{Ax} - 1) - bx
\]  

(4)

with the initial condition of \(t = t_o\) at \(x = 0\).

Equation 4 expresses the projectile flight time as a function of range which is in the form to fit the sky screen data.

To obtain a best fit in the sense of minimizing the squares of the deviations, a residual is defined

\[
R_x = \sum_{i=1}^{N} (t_{ic} - t_{ie})^2
\]  

(5)

where \(t_{ic}\) is the time of flight calculated from equation 4 and \(t_{ie}\) is the time of flight obtained from the sky screen for the projectile to reach the \(i^{th}\) sky screen, and \(N\) is the total number of sky screens used for that test.

To minimize the residual, equation 5 is differentiated with respect to the four variables \((t_o, A, V_0, b)\) individually, and then set to zero. They are

\[
\frac{\partial R_x}{\partial t_o} = \sum_{i=1}^{N} (t_{ic} - t_{ie}) \frac{\partial t_{ic}}{\partial t_o} = 0
\]

(6)

\[
\frac{\partial R_x}{\partial A} = \sum_{i=1}^{N} (t_{ic} - t_{ie}) \frac{\partial t_{ic}}{\partial A} = 0
\]

\[
\frac{\partial R_x}{\partial V_0} = \sum_{i=1}^{N} (t_{ic} - t_{ie}) \frac{\partial t_{ic}}{\partial V_0} = 0
\]

\[
\frac{\partial R_x}{\partial b} = \sum_{i=1}^{N} (t_{ic} - t_{ie}) \frac{\partial t_{ic}}{\partial b} = 0
\]
The following partial derivatives of $t_{ic}$ with respect to the four variables can be obtained by differentiating equation 4:

\[
\begin{align*}
\frac{\partial t_{ic}}{\partial t_0} &= 1 \\
\frac{\partial t_{ic}}{\partial A} &= \frac{1}{\Lambda} [1 + e^{Ax_i}(Ax_i - 1)] \\
\frac{\partial t_{ic}}{\partial V_0} &= \frac{1}{\Lambda V_0} (e^{Ax_i} - 1) \\
\frac{\partial t_{ic}}{\partial b} &= (e^{Ax_i} - 1) - \lambda_i
\end{align*}
\]

With such a complex expression of the four variables in the set of equation 6, the easier approach in solving the four unknown variables is by using the iterative method. To obtain the corrections for the variables from the $j$th iteration to the $j + 1$st iteration, the $t_{ic}$ is expanded in a Taylor's Series and the series is truncated after the first order terms. Therefore

\[
t_{ic}(j + 1) = t_{ic}(j) + \frac{\partial t_{ic}}{\partial t_0} j (\Delta t_0) + \frac{\partial t_{ic}}{\partial A} j (\Delta A) + \frac{\partial t_{ic}}{\partial V_0} j (\Delta V_0) + \frac{\partial t_{ic}}{\partial b} j (\Delta b)
\]

Equation 8 is substituted into equation 6 for calculating the $j + 1$st corrections. For simplicity, the subscript $j$ and $j + 1$ will be omitted in the following expressions:

\[
\begin{align*}
\frac{\partial R_c}{\partial t_0} &= 0 = \sum_{i=1}^{N} \left[ (t_{ic} - t_{ic}) \frac{\partial t_{ic}}{\partial t_0} + \frac{\partial t_{ic}}{\partial A} \Delta t_0 + \frac{\partial t_{ic}}{\partial A} \Delta A + \frac{\partial t_{ic}}{\partial V_0} \Delta V_0 + \frac{\partial t_{ic}}{\partial b} \Delta b \right] \\
\frac{\partial R_c}{\partial A} &= 0 = \sum_{i=1}^{N} \left[ (t_{ic} - t_{ic}) \frac{\partial t_{ic}}{\partial A} + \frac{\partial t_{ic}}{\partial A} \Delta t_0 + \frac{\partial t_{ic}}{\partial A} \Delta A + \frac{\partial t_{ic}}{\partial V_0} \Delta V_0 + \frac{\partial t_{ic}}{\partial b} \Delta b \right] \\
\frac{\partial R_c}{\partial V_0} &= 0 = \sum_{i=1}^{N} \left[ (t_{ic} - t_{ic}) \frac{\partial t_{ic}}{\partial V_0} + \frac{\partial t_{ic}}{\partial A} \Delta t_0 + \frac{\partial t_{ic}}{\partial A} \Delta A + \frac{\partial t_{ic}}{\partial V_0} \Delta V_0 + \frac{\partial t_{ic}}{\partial b} \Delta b \right] \\
\frac{\partial R_c}{\partial b} &= 0 = \sum_{i=1}^{N} \left[ (t_{ic} - t_{ic}) \frac{\partial t_{ic}}{\partial b} + \frac{\partial t_{ic}}{\partial A} \Delta t_0 + \frac{\partial t_{ic}}{\partial A} \Delta A + \frac{\partial t_{ic}}{\partial V_0} \Delta V_0 + \frac{\partial t_{ic}}{\partial b} \Delta b \right]
\end{align*}
\]

The set of equation 9 can be written in matrix notation, which is in the form of: $M \Delta B = C$
where

\[
M = \begin{pmatrix}
\sum_{i=1}^{N} \left( \frac{\delta t_i}{\delta t_0} \right)^2 & \sum_{i=1}^{N} \frac{\delta t_i}{\delta t_0} \frac{\delta t_i}{\delta A} & \sum_{i=1}^{N} \frac{\delta t_i}{\delta t_0} \frac{\delta t_i}{\delta V_0} & \sum_{i=1}^{N} \frac{\delta t_i}{\delta t_0} \frac{\delta t_i}{\delta b} \\
\sum_{i=1}^{N} \frac{\delta t_i}{\delta A} & \sum_{i=1}^{N} \frac{\delta t_i}{\delta A} \frac{\delta t_i}{\delta A} & \sum_{i=1}^{N} \frac{\delta t_i}{\delta A} \frac{\delta t_i}{\delta V_0} & \sum_{i=1}^{N} \frac{\delta t_i}{\delta A} \frac{\delta t_i}{\delta b} \\
\sum_{i=1}^{N} \frac{\delta t_i}{\delta V_0} & \sum_{i=1}^{N} \frac{\delta t_i}{\delta V_0} \frac{\delta t_i}{\delta V_0} & \sum_{i=1}^{N} \frac{\delta t_i}{\delta V_0} \frac{\delta t_i}{\delta b} & \sum_{i=1}^{N} \frac{\delta t_i}{\delta V_0} \frac{\delta t_i}{\delta b} \\
\sum_{i=1}^{N} \frac{\delta t_i}{\delta b} & \sum_{i=1}^{N} \frac{\delta t_i}{\delta b} \frac{\delta t_i}{\delta b} & \sum_{i=1}^{N} \frac{\delta t_i}{\delta b} \frac{\delta t_i}{\delta b} & \sum_{i=1}^{N} \frac{\delta t_i}{\delta b} \frac{\delta t_i}{\delta b}
\end{pmatrix}
\]

\[
\Delta t_0 = \sum_{i=1}^{N} (t_{ie} - t_{ic}) \frac{\delta t_i}{\delta t_0}
\]
\[
\Delta A = \sum_{i=1}^{N} (t_{ie} - t_{ic}) \frac{\delta t_i}{\delta A}
\]
\[
\Delta V_0 = \sum_{i=1}^{N} (t_{ie} - t_{ic}) \frac{\delta t_i}{\delta V_0}
\]
\[
\Delta b = \sum_{i=1}^{N} (t_{ie} - t_{ic}) \frac{\delta t_i}{\delta b}
\]

(10)

The subscript c for the partial derivatives has been omitted. There is no ambiguity because the partial derivatives can only be computed from the closed form solution. The corrections are obtained by inverting matrix M, that is

\[
AB = M^{-1}C
\]

The iteration will be terminated when a prescribed degree of convergence or number of iterations is reached.

**DISCUSSION**

The computer program listing for this sky screen data reduction technique is shown in appendix A. The sky screen data for round number 63 for the 105-mm APFSDS projectile M833 fired on 20 April 1984 at Aberdeen Proving Ground was chosen as a test case for this data reduction technique. The computer printout for this test case is given in appendix B. The results show that it only took four iterations to achieve the desired convergence even though the initial guessed muzzle velocity and drag coefficient were different by as much as 35% from their best-fit values. Therefore, for reasonably well-behaved sky screen data, this iterative method converges very rapidly.
To investigate the deviation between the path the projectile travelled and the ground distance due to the trajectory curvature, the deviations were calculated for ground ranges of 2,500 meters and 3,000 meters for the 105-mm HEAT projectile M456 and the 105-mm APFSDS projectile M833. The super-elevation needed to reach 2,500 meters for the M456 projectile is 0.84 degree and to reach 3,000 meters for the M833 projectile is 0.41 degree. This deviation is approximately 0.025 meters for the M833 projectile and 0.07 meters for the M456 projectile at their respective ranges. These deviations are within the accuracy of the sky screen data, therefore, the trajectory curvature can be neglected in the data reduction.

**CONCLUSIONS**

The only limitation of this sky screen data reduction technique is that the drag coefficient has to be a linear function of velocity. From results of some test cases, it indicates that this technique still rendered excellent results, even with the drag coefficient deviating slightly from the linearity requirement.
SYMBOLS

A  Constant, defined as \( \frac{1}{2M} S C_D a \), per meter

b  Slope value in drag coefficient expression, per meter per second

C_D  Drag coefficient

C_D_a  Constant in drag coefficient expression

M  Projectile mass, kg

\( \rho \)  Air density, kg/meter\(^3\)

S  Projectile reference area, meter\(^2\)

t  Time of flight, seconds

t_o  Time at the gun muzzle, seconds

V  Projectile velocity, meters per second

V_o  Muzzle velocity, meters per second

V_c  Defined as \( \frac{V_o}{1 + bV_o} \), meters per second

x  Range, meters

\( t' \)  Time rate of change, \( \frac{d}{dt} \)

Subscript

c  Values computed from the closed form solution

e  Values from range data

i  Values pertaining to the ith sky screen

j  Values pertaining to the jth iteration
APPENDIX A

COMPUTER PROGRAM LISTING
100 PROGRAM MUFIT(INPUT,TAPES=INPUT),OUTPUT,TAPES=OUTPUT)
110 DIMENSION X(16),TE(16),TC(16),TD(16),WXX(5),C(5,6),D(5)
120 DIMENSION AA(4),M(16),XX(16),TITLE(8),FF(4)
130 DIMENSION R(7),V(7)
140 DATA X/56.31,74.81,92.81,110.8,128.81,239.76,453.22,779.93,
150 /982.1,1251.18,1479.0,1741.45,1984.35,2475.85,2733.21,2983.21/
160 DATA AA/3H10=,3H20=,3H30=,3H40=,/
170 DATA FF/.01000005,3000000/
180 DATA R/0.0,0.500.0,1000.0,1500.0,2000.0,2500.0,3000.0/
190 700 FORMAT(BAi)
200 701 FORMAT(1611)
210 702 FORMAT(Bfl0.2)
220 10 READ(5,700) TITLE
230 IF(EOF(5) .NE. 0,0) STOP
240 READ(5,702) (TE(I),I=1,16)
250 READ(5,702) (D(I),I=1,4)
260 READ(5,701) M
270 L=0
280 DO 50 I=1,16
290 IF(M(I) .EQ. 0) GO TO 50
300 L=L+1
310 X(L)=XX(I)
320 TE(L)=TE(I)
330 50 CONTINUE
340 NT=L
350 800 FORMAT(1H1)
360 WRITE(6,800)
370 600 FORMAT(16/H0/10X,8A10)
380 WRITE(6,600) TITLE
390 601 FORMAT(//10X,*NUMBER OF SKY SCREENS = *,12)
400 WRITE(6,601) NT
410 WRITE(6,601)
420 801 FORMAT(//10X,*RANGE DATA-METERS*/)
430 WRITE(6,602) (X(I),I=1,NT)
440 602 FORMAT(10X,10F10.3)
450 WRITE(6,602)
460 802 FORMAT(//10X,*TIME DATA-SECONDS*/)
470 WRITE(6,603) (TE(I),I=1,NT)
480 603 FORMAT(10X,10F10.6)
490 WRITE(6,603)
500 803 FORMAT(//10X,*INITIAL GUESSES---*)
510 WRITE(6,604) (D(I),I=1,4)
520 604 FORMAT(10X,F12.6,F12.7,F12.2,F12.6)
530 NR=0
540 E=0.0
550 805 FORMAT(//4X,13HITERATION NO.,6X,9HRESIDUALS,7X,14HPROBABLE ERROR
560 +//10X,43HCORRECTIONS OF CONSTANTS FOR EACH ITERATION)
570 WRITE(6,805)
580 100 SR=0.0
590  DO 150 J=1,5
600  WXX(J)=0.0
610  DO 150 I=1,4
620  C(I,J)=0.0
630 150 CONTINUE
640  T0=D(1)
650  A=D(2)
660  VO=D(3)
670  B=D(4)
680  VC=VO/(1.0+B*VO)
690  AVC=A*VC
700  AAVC=A*AVC
710  VC2=VC**2
720  VO2=VO**2
730  DO 150 I=1,NT
740  AX=AVC(I)
750  EAX=EXP(AX)
760  EAX1=EAX-1.0
770  PTA=(-EAX1+EAX*AX)/AAVC
780  PTV=-EAX1/(A*VO2)
790  PTB=(EAX1-AX)/A
800  TC(I)=T0+EAX1/AVC-B*X(I)
810  TD(I)=TE(I)-TC(I)
820  SR=SR+TD(I)**2
830  C(1,1)=C(1,1)+1.0
840  C(2,1)=C(2,1)+PTA
850  C(3,1)=C(3,1)+PTV
860  C(4,1)=C(4,1)+PTB
870  C(1,2)=C(2,1)
880  C(2,2)=C(2,2)+PTA**2
890  C(3,2)=C(3,2)+PTA*PTV
900  C(4,2)=C(4,2)+PTA*PTB
910  C(1,3)=C(3,1)
920  C(2,3)=C(3,2)
930  C(3,3)=C(3,3)+PTV**2
940  C(4,3)=C(4,3)+PTV*PTB
950  C(1,4)=C(4,1)
960  C(2,4)=C(4,2)
970  C(3,4)=C(4,3)
980  C(4,4)=C(4,4)+PTB**2
990  C(1,5)=C(1,5)+TD(I)
1000 C(2,5)=C(2,5)+TD(I)*PTA
1010 C(3,5)=C(3,5)+TD(I)*PTV
1020 C(4,5)=C(4,5)+TD(I)*PTB
1030 150 CONTINUE
1040  NR=NR+1
1050  E1=E
1060 $t = 0.6745 \times \text{SORT}(SR/(NT-4))$
1070 WRITE(6,605) NR,SR,E
1080 605 FORMAT(//10X,12.2F20.8)
1090 IF (ABS(E1-E)-1.0E-05) 200,200,110
1100 110 IF (NR-15) 120,120,200
1110 120 CALL INV(C,4,5,WXX)
1120 DO 180 I=1,4
1130 F=C(I,5)
1140 IF (ABS(F) .GT. FF(I)) F=SIGN(FF(I),F)
1150 D(I)=D(I)+F
1160 180 CONTINUE
1170 WRITE(6,606) (C(I,5),I=1,4)
1180 606 FORMAT(/10X,5F18.6)
1190 GO TO 100
1200 200 CONTINUE
1210 CALL INV(C,4,4,WXX)
1220 DO 210 I=1,4
1230 210 WXX(I)=SORT(ABS(C(I,I)))*E
1240 806 FORMAT(//22X,I5REST-FIT VALUES,4X,14HPRORABLE ERROR)
1250 WRITE,(6,806)
1260 806 FORMAT(/10X,15HAS,4X,14HF)
1270 230 WRITE(6,607) AA(I),D(I),WXX(I)
1280 607 FORMAT(10X,F13.2,F13.6,F13.6)
1290 120 CONTINUE
1300 608 FORMAT(10X,F13.2,F13.6,F13.6,F13.6)
1320 WRITE(6,804)
1330 DO 240 I=1,NT
1340 WRITE(6,608) X(I),IE(I),TC(I),TD(I)
1350 240 CONTINUE
1360 VI=D(3)+D(1)*D(2)*D(3)**2*(1.0+D(3)*D(4))
1370 WRITE(6,609) VI
1380 609 FORMAT(//10X,*MUZZLE VELOCITY AT TIME EQUAL TO ZERO IS *,F10.2)
1390 V(1)=D(3)
1400 VC=D(3)/(1.0+D(4)*D(3))
1410 DO 250 I=2,7
1420 VAX=VC*EXP(-D(2)*(I-1)*500.0)
1430 DO 250 I=2,7
1440 250 CONTINUE
1450 WRITE(6,610) R
1460 WRITE(6,611) V
1470 610 FORMAT(/5X,1SHRANGE(METERS) =,7F12.1)
1480 611 FORMAT(5X,1HVELOCITY(MPS) =,7F12.1)
1490 GO TO 10
1500 END
SUBROUTINE INV(C,NC,NCS1,WXX)

DIMENSION C(5,6),WXX(5),PIVOT(2),CC(5,10)

NCT=NC*2
NCP1=NC+1
DO 10 I=1,NC
DO 10 J=1,NC
10 CC(I,J)=C(I,J)
DO 20 I=1,NC
DO 20 J=NCP1,NCT
20 CC(I,J)=0.0
DO 30 I=1,NC
30 CC(I,NC+I)=1.0
DO 40 I=1,NC
40 PIVOT(1)=CC(I,I)
DO 50 K=1,NC
50 PIVOT(2)=CC(K,K)
IF(K-I) 135,130,140
130 DO 150 J=1,NCT
150 CONTINUE
GO TO 200
135 DO 160 J=1,NCT
160 IF(PIVOT(1)) 134,210,134
134 CC(K,J)=CC(I,J)/PIVOT(1)
136 DO 160 J=1,NCT
160 CONTINUE
GO TO 200
140 DO 170 J=1,NCT
170 IF(PIVOT(2)) 145,170,145
145 CC(K,J)=CC(K,J)/PIVOT(2)-CC(I,J)
146 DO 170 J=1,NCT
170 CONTINUE
GO TO 200
150 CONTINUE
GO TO 250
160 CONTINUE
GO TO 250
170 CONTINUE
GO TO 250
180 CONTINUE
GO TO 250
190 CONTINUE
GO TO 250
200 CONTINUE
GO TO 250
210 WRITE(6,600)
600 FORMAT(//10X,*DET IS EQUAL ZERO*)
DO 250 I=1,NC
DO 250 J=1,NC
300 C(I,J)=CC(I,J+NC)
NCS=NCS1
IF(NCS-NC) 500,500,400
400 DO 420 I=1,NC
420 WXX(I)=C(I,NCS)
410 DO 420 I=1,NC
420 C(I,NCS)=0.0
450 DO 450 I=1,NC
450 C(I,NCS)=C(I,NCS)+C(I,J)*WXX(J)
460 NCS=NCS-1
470 DO TO 350
500 CONTINUE
RETURN
END
APPENDIX B

COMPUTER OUTPUT EXAMPLE
**MH53 HD-65 20 APRIL 1984**

**NUMBER OF SKY SCREENS = 16**

**RANGE DATA-METERS**

<table>
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<tr>
<th>Value</th>
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**TIME DATA-SECONDS**

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**INITIAL GUESSES**

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**ITERATION NO.**

**RESIDUALS**

**PROBABLE ERROR**

**CORRECTIONS OF CONSTANTS FOR EACH ITERATION**

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Muzzle velocity at time equal to zero is 1468.42

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