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**Abstract:**
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Sporadic Narrowband Radio Emissions From Uranus

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Among several different types of radio emissions discovered at Uranus during the Voyager 2 encounter in January 1986 is a very sporadic, bursty signal which consists of very narrow bands lying in the frequency range from about 3 to 10 kHz. The bursty emission was virtually undetectable from the dayside portion of the Voyager 2 trajectory but was observed out to beyond 300 R\textsubscript{U} during the outbound trajectory through the pre-dawn sector. While the narrowband tones making up this emission are reminiscent of escaping continuum radiation observed near Earth, Jupiter and Saturn, the Uranian signals show large amplitude variations on time scales of a second suggesting a very different type of generation mechanism.
I. INTRODUCTION

During the Voyager 2 flyby of Uranus in January 1986, several types of radio emissions were discovered by the plasma wave investigation [Gurnett et al., 1986] and the planetary radio astronomy investigation [Warwick et al., 1986]. The bulk of the radio spectrum of Uranus lies above about 30 kHz, however, there is evidence of nonthermal continuum radiation at frequencies down to about 1 kHz. As Voyager 2 left Uranus in the pre-dawn sector still another type of emission was detected in a band centered near 5 kHz [Gurnett et al., 1986].

The new band of radio emission found on the nightside of Uranus is remarkable in that while it displays a spectrum similar to escaping nonthermal continuum radiation detected at Earth [Kurth et al., 1981], Jupiter [Gurnett et al., 1983], and Saturn [Gurnett et al., 1981a], it is much more bursty and sporadic in nature than the continuum radiation. In fact, the time scale for amplitude variations of on the order of 30 dB is typically one second.

In this report we investigate the details of the sporadic narrowband emission from the magnetosphere of Uranus. We will emphasize the peculiar bursty nature of the emission and argue that a rather exotic source mechanism may be responsible for the emission. The observations presented herein are all taken from the Voyager 2
plasma wave receiver which has been described in detail by Scarf and Gurnett [1977].
II. OBSERVATIONS

An overview of the observations of the bursty radio emission in the frequency range of a few kHz is presented in Figure 1. Illustrated are the peak and average power fluxes detected by the spectrum analyzer channels centered at 3.11, 5.62, and 10.0 kHz as a function of time. The height of the solid black areas represent approximately 11-minute average values in each channel, while the peak power fluxes observed during the same 11-minute averaging intervals are plotted as a line above the averages.

It is clear in Figure 1 that the bulk of the emission is in the 5.62-kHz channel although considerable activity can be seen at 3.11 kHz and some emissions spread upward into the 10-kHz channel early in the plotted interval. It is important to point out, however, that during the interval from late on January 27 through January 29 the spacecraft traversed the bow shock several times [Bridge et al., 1986; Ness et al., 1986] and the response at 3 kHz is primarily due to electron plasma oscillation (Langmuir wave) activity in the upstream regions associated with these shock crossings. The activity at 3.11 kHz prior to late January 27 is the lower frequency extent of the waves of interest here. Very little activity is observed after January 31, however, there is evidence for the bursts on February 5 at about 570 R_j.
The bursty nature of the emission is apparent in Figure 1. In addition to the highly sporadic fluctuations in the peak amplitudes, the very large peak-to-average ratios which are in many cases much greater than 10 point to the highly variable amplitudes of the emission. In fact, that one cannot discern a difference between the temporal character of the radio emission and the electron plasma oscillations seen on days 28 and 29 of Figure 1 underscores the temporal variability since plasma oscillations are typically one of the most sporadic types of plasma waves.

Figure 2 shows several examples of high resolution frequency-time spectrograms which detail both the temporal and spectral characteristics of the radio emission. Each spectrogram shows the intensity of waves as a function of both frequency (ordinate) and time (abscissa). The most intense waves are darkest. Each panel represents a 10-second interval and the frequency range covered is 0 to 12 kHz. The Fourier transforms used to calculate the spectra provide about 28 Hz resolution and a new spectrum is calculated every 60 msec. Panels A through E in Figure 2 show examples of the spectrum of the radio emission responsible for the activity shown in Figure 1 at 5.62 kHz and sometimes the surrounding channels.

The radio emission appears in the spectrograms in Figure 2 as numerous narrowband signals, many of which have durations of only a few seconds. The narrowband tone at 2.4 kHz is interference from the Voyager power supply and the band at 1.8 kHz is also thought to be interference since it also appeared in the Saturn data set; however, the source of this lower frequency interference is not known. The
radio emissions of interest seem to be composed of both the brief bursts as well as an underlying, more continuous system of narrow bands. In some spectrograms, the weaker, more stable bands tend to coalesce into weak, diffuse bands. The more intense bursts are occasionally seen superimposed upon the diffuse bands.

The observations presented in Figures 1 and 2 provide solid evidence that the bursty emissions are freely propagating electromagnetic waves. The radio emissions are at frequencies sufficiently high that the waves can freely propagate into the solar wind. The solar wind plasma frequency $f_p$ is typically in the range of 2 to 3 kHz at the orbit of Uranus based on the frequency of upstream plasma oscillations [Gurnett et al., 1986] and $f_p$ in the magnetotail is even less. The spectrum, while showing random variations in detail, remains constant in general even though the spacecraft has moved through the magnetotail, magnetosheath, and solar wind from January 25 through 31. While uncertainties in the receiver calibration caused by a failure in the spacecraft data system and the very bursty nature of the emissions make it difficult to detect a $1/R^2$ intensity variation, it is clear that both the amplitude and occurrence rate of the emissions drops dramatically over the 7 days illustrated.

The frequency range of the narrowband radio emission lies between about 3 and 10 kHz. The more diffuse portion of the spectrum is limited to the range from about 3 to 6 kHz. If there is a preferred frequency for the bursty emissions, it is 5 kHz; but the bursts can be found over the entire range from 3 to 10 kHz.
More details of the spectrum of the Uranian radio emission near 5 kHz can be seen in Figure 3 which shows power flux as a function of frequency for three selected time periods. Each spectrum is a 0.6-second average and the strong band of noise below 1 kHz is interference thought to be associated with the operation of the tape recorder on board the spacecraft. The interference tones at 1.8 and 2.4 kHz have been removed. In the top panel of Figure 3 there is a general rise in wave amplitudes from about 3 to 7 kHz corresponding to the more diffuse component of the emission. Superimposed upon this generally elevated spectrum are a few, very narrow lines with power fluxes at least a factor of 10 greater than the background levels.

The middle panel of Figure 3 again shows some evidence for a diffuse component between 3 and 5 kHz but also strong lines which extend from just above 3 kHz to almost 10 kHz. The bandwidths of the lines often do not exceed 100 Hz. For a line near 10 kHz, the bandwidth is only 1 percent of the center frequency.

The bottom panel of Figure 3 shows a single intense spike at about 5.5 kHz. The diffuse component is not present, however, Voyager 2 was greater than 300 R$_U$ from Uranus at this time and the amplitude of the diffuse emission may have simply dropped below the detection threshold because of the $1/R^2$ effect. We should point out, however, that the diffuse component does not necessarily vary only as $1/R^2$; there are spectrograms much closer to Uranus which show evidence for only the bursty component, hence, it is likely that there are temporal variations in the diffuse component with time scales perhaps much greater than a few seconds.
The narrowband nature of these Uranian radio emissions is similar to escaping continuum radiation at the Earth and similar emissions observed at Jupiter and Saturn. However, the very bursty nature of the Uranian emissions shown in Figures 1 and 2 set the Uranian emissions apart from those with which we are familiar. Figure 4 illustrates the very short time scales of intensity variations characteristic of the bursty emission. In Figure 4 we have plotted the intensity (power flux) of radio waves as a function of frequency and time in a three dimensional perspective plot to show the rapid variations in time for this narrowband burst. The frequency range is from 4 to 6 kHz and the entire time represented is 12 seconds. The original data have a temporal resolution of 60 msec and a spectral resolution of about 28 Hz, however, we have averaged each point over the five surrounding spectral and five surrounding temporal components (a two-dimensional boxcar average) to obtain the surface represented in Figure 4.

Figure 4 demonstrates that the power flux for this burst can increase or decrease by 30 dB within a second or so. Similar analyses of the temporal variations of narrowband electromagnetic emissions from Jupiter show variations in power flux of a few db on the same time scale. Figure 5 is an example of narrowband electromagnetic radiation at 11 kHz escaping from the Jovian magnetosphere as observed by Voyager 1. Note that the only apparent variations in amplitude during the 48-second interval occur about 6 and 30 seconds into the frame. These two decreases in signal level are actually due to decreases in the gain of the automatic gain controlled receiver in
response to interference from a stepper motor on board the spacecraft. The band shows significant amplitude variations only on time scale of several minutes. The Uranian radio burst in Figure 4 will not be fully understood unless the generation mechanism can explain both the narrowband nature and the rapid variations in intensity. We will attempt to address both issues in the next section.

In an attempt to understand the location of the source of the 5-kHz radio emission we have plotted the magnetic latitude of Voyager 2 from January 24 to February 1, 1986, as a function of time in Figure 6 using information about the orientation of the magnetic dipole from Ness et al. [1986]. There is no clear evidence for the existence of the radio emission in the spectrum analyzer data prior to January 25, however, there is some evidence of either the diffuse emission or the bursts in some of the wideband frames obtained on January 24. The locations of the spacecraft for those times when there is evidence in the wideband frames only on January 24 are marked with 'X'. In some cases, it is possible that the signature in the wideband frames is a different type of radio emission and not the bursty emission discussed in this paper; hence, we have placed question marks by these questionable events.

After January 25, two types of markings were used to identify times (magnetic latitudes) when the sporadic, narrowband emission was present. The thin solid line represents times when there is weak evidence for the emission. That is, if bursts were too sparse to significantly affect the averaged power flux as plotted in Figure 1
when the spacecraft was less than about 150 R\textsubscript{U} from Uranus, a thin line was used in Figure 6. The broad lines imply the radio emission was more prominent. While the distinction here is mainly qualitative, it is important to provide a means of de-emphasizing the weaker emissions observed close to Uranus which would not be detectable at larger distances since we have not taken a 1/R\textsuperscript{2} trend into account.

The impression left by the pattern of detections, especially the more 'prominent' detections is that the emission was observed with confidence only after encounter (when the spacecraft was in the predawn location) and then usually at the lowest magnetic latitudes observed by the spacecraft.

In Figure 7 we have illustrated wideband data which may have a bearing on a possible source of the bursty radio emissions. This spectrogram was observed near the time when Voyager traversed the Miranda L-shell. The intense narrowband line at 9 kHz is of interest since it is a possible source for the electromagnetic emissions observed later. The intensity and narrowband character make the connection to the radio emissions a reasonable one, even though the connection cannot be proven. The large dot in Figure 5 late on January 24 marks the location of Voyager 2 at the time the data in Figure 2 were taken. It is interesting and highly suggestive that the latitude of this intense narrowband emission is very close to the most favorable latitude for observing the radio emission. We will discuss the possible significance of the narrowband emission in Figure 7 in the next section.
IV. DISCUSSION AND CONCLUSIONS

The observations presented above show the characteristics of a new radio emission discovered in the region around Uranus. The emission exhibits extremely narrowband bursts with bandwidths of a few percent or less and time scales of seconds. The bursts are accompanied at times by a diffuse underlying structure of more temporally continuous bands. It is conceivable that the diffuse component is the result of the superposition of several weak bursts. The emission is only weakly present during the close-in dayside trajectory of Voyager 2 and is prominent in the radial distance range of about 20 to 300 R$_U$ as the spacecraft leaves Uranus in the pre-dawn direction.

It is tempting to associate the source location for the bursty radio emissions with the Miranda L-shell. During the brief passage through the Uranian magnetosphere, there is little chance to sample anything but the smallest region dictated by the flyby trajectory. Hence, it is quite unlikely that the spacecraft passed through an active source region for the bursty radio emissions detected during the outbound leg. There are many reasons, however, to suspect that the outbound Miranda L-shell crossing is a region which is either close to or similar to the radio emission source region. The narrowband emission at 9 kHz in Figure 7 is likely to be an electrostatic emission near the local plasma frequency or upper hybrid resonance
frequency \( f_{\text{UH}} \). While most of the radio bursts discussed herein are at lower frequencies, there are a few lines observed at 9 kHz. It is interesting to note that according to Ness et al. [1986] the range of magnetic field strengths of the L-shells traversed by Miranda correspond to electron gyrofrequencies ranging from about 5 to 9.5 kHz. Should the line at 9 kHz in Figure 7 be a Bernstein mode, lying between the electron gyrofrequency and its second harmonic, then one might expect similar lines at a wide range of frequencies extending down to the lower end of the radio emission frequency range. Plasma densities from the plasma experiment [J. W. Belcher, personal communication, 1986] are consistent with an upper hybrid resonance frequency \( f_{\text{UH}}^2 = \frac{f_g^2}{2} + \frac{f_p^2}{2} \) in the range of about 9 kHz at 1920 SCET on January 24; \( f_g \) is the electron gyrofrequency.

Another interesting observation from the region associated with the outbound Miranda L-shell crossing is the presence of intense, low frequency bursts of plasma wave activity. Figure 8 shows spectrum analyzer data from the one-hour interval around the Miranda L-shell crossing. The wideband data shown in Figure 7 were obtained at the time of the data gap shortly before 1920 SCET. The line crossing through the 5.62 kHz channel represents \( f_g \) according to information provided by the magnetometer team [N. F. Ness, private communication, 1986]. It is clear that several different types of wave modes are present during this interval. The bursty emissions above \( f_g \) correspond to the line at 9 kHz in Figure 7 and are likely plasma waves associated with \( f_{\text{UH}} \). The intense emissions with limited bandwidth in the region from perhaps 0.25 to 0.5 \( f_g \) are likely to be
whistler modes. Emissions in the range below a few hundred Hz could be Doppler-shifted ion-acoustic waves.

With the presence of the bursty plasma wave emissions at very low frequencies, it is possible to conceive of 3-wave interactions involving one of the low frequency emissions, the intense narrow band near 9 kHz, and an electromagnetic mode which we detect as a freely propagating radio wave as Voyager leaves Uranus.

The above-described 3-wave process is similar to one of the favored mechanisms for the generation of continuum radiation at the Earth, Jupiter, and Saturn. That is, the continuum radiation is thought to be a nonlinear process involving an intense band near \( f_{\text{up}} \) and some low frequency plasma wave which produces an electromagnetic wave (the continuum radiation) at nearly the same frequency as the upper hybrid band [see, for example, Melrose, 1981]. An alternative hypothesis is a linear conversion from the upper hybrid band to an electromagnetic mode [see, for example, Jones, 1976].

The theories for generation of continuum radiation, then, are perhaps applicable to the Uranian emission near 5 kHz, and if so, point to the region near Miranda's orbit because intense narrow bands near \( f_{\text{up}} \) are detected there. (There is little evidence to help us choose between the linear or nonlinear generation mechanisms, however.) But the major problem with this generation mechanism is that the continuum radiation usually has a very smooth temporal behavior, as shown in Figure 5, strikingly different from that demonstrated in Figures 1, 2, and 4. Very little has been mentioned in the literature about the temporal variability afforded by the theories for
continuum radiation so it is not possible to immediately assess
whether either the linear and/or nonlinear theories can account for
smooth temporal variations at Earth, Jupiter, and Saturn, and the very
bursty behavior at Uranus.

There is another mechanism which comes to mind which might
explain the very bursty temporal variations of the Uranian 5-kHz radio
emission. Soliton collapse has long been considered a viable mecha-
nism for solar Type III radio bursts generated in the interplanetary
medium [see, for example, Goldman, 1983]. Type III bursts do not show
a rapidly varying component, but the model assumes the collective
action of many solitons and electromagnetic radiation from the
'condensate' radiation left by soliton collapse and burn-out [Goldman,
1984].

There is some speculation that electromagnetic radiation could be
generated from the collapse of a single soliton if ion-acoustic waves
were present to take up momentum and serve as the third wave in
wave-vector and frequency matching conditions. The electron plasma
oscillations responsible for the ponderomotive force which collapses
the soliton is the pump wave in this scenario. This mechanism would
certainly explain the time scales of the bursts reported herein.

We have examined the energetics implied by generation of individ-
ual bursts via soliton collapse. If we assume the burst illustrated
in the bottom panel of Figure 3 was beamed into a solid angle of only
1 steradian (prompted by the restricted observability of the emission
illustrated in Figure 6) and the source is a plasma consisting of 1
keV electrons with a density of 1 cm$^{-3}$ (very optimistic in view of the
plasma observations in the region near Miranda's L-shell [R. L. McNutt, personal communication, 1986]), then 100 percent of the thermal plasma energy from a volume of \((1000 \lambda_{\text{Debye}})^3\) would have to be converted into electromagnetic waves in the process of soliton collapse to account for the observed amplitude of the radio burst at greater than 300 Ry. Since soliton collapse generally results in plasma heating, this efficiency for electromagnetic wave generation is high by several orders of magnitude. Further, the frequency of the pump (Langmuir) wave would vary throughout the collapse process and the resulting electromagnetic wave would almost certainly not be monochromatic, as observed.

The energy problem could be solved if many solitons were radiating, however, the effect of superimposing radiation from several collapsing solitons would be to spread both the temporal and spectral profile of the radio emission. Type III solar bursts are long lived and have relatively broad bandwidths because of these collective effects. We must conclude, then, that soliton collapse is not a viable mechanism for the observed waves and we are led to consider other possibilities.

It is possible to produce a short, monochromatic pulse response in a detector simply by sweeping a narrow beam of monochromatic waves over the detector. The pulse length is determined by the beam width and angular velocity of the beam. The very short pulse durations reported herein imply a very narrow beam or very large angular velocities. The monochromatic spectrum is inherent in the source.
One excellent method of producing a monochromatic, highly collimated beam is via a laser mechanism since these are two basic features of a laser. We suggest that the 5-kHz Uranian bursts could be the result of some lazing process operating in the magnetosphere of Uranus. Calvert [1982] has suggested that several properties of auroral kilometric radiation at the Earth can be explained by coupling the wave amplification mechanism of Wu and Lee [1979] with a positive feedback mechanism. High resolution spectrograms of auroral kilometric radiation do not show the type of temporal and spectral behavior seen in Figure 2, however. Instead, the terrestrial emission is composed of narrowband tones which are nearly continuous in amplitude, and which vary in frequency very rapidly.

If we use current ideas on the generation of continuum radiation instead of the Wu and Lee mechanism for auroral kilometric radiation, we might begin to fashion a viable mechanism for the Uranian bursty emission. This choice is suggested by the similarity in bandwidth and slowly varying spectral behavior of the continuum radiation and the Uranian emission coupled with the apparent low magnetic latitude of the source at Uranus. The upper hybrid bands at the Earth associated with continuum radiation source regions are usually located near the magnetic equator.

The model is complicated by the reliance of both the linear and nonlinear theories for the generation of continuum radiation on coupling from the electrostatic Bernstein mode into the electromagnetic mode. For a simple laser model, one has an amplification process which operates directly on the electromagnetic mode which
executes multiple reflections through the amplification region before being emitted. Perhaps it would be possible for the electrostatic waves to make multiple reflections through a region of growth and have the resulting, highly amplified electrostatic mode couple into the electromagnetic mode at the reflecting boundary, presumably the wall of a duct.

In any case, angular motion of the beam seems to be assured by the dramatic tumbling motion of the magnetic field of Uranus which is due to the 60-degree tilt with respect to the angular momentum axis of the planet [Ness et al., 1986]. In fact, it could be that the large dipole tilt is responsible for the dramatic difference between the smoothly varying narrowband emissions from Jupiter, Saturn and Earth compared to the very bursty nature of the Uranian emission. While both Jupiter and Saturn rotate more rapidly than Uranus, the apparent motion of a spacecraft in magnetic latitude at Uranus, is much greater than at the other planets. This distinction would imply a beam which is broad in azimuth, but narrow in latitude. Jones [1980] has predicted strong latitudinal beaming as a consequence of the linear conversion of the electrostatic mode into the electromagnetic mode at Earth and Jupiter, however, Jones' model would predict rapidly drifting frequencies as the observer changed magnetic latitude. For the short time intervals for which we have observations, this drift is not apparent.

It is not within the scope of this paper to develop in detail a generation mechanism for the new radio emission described herein. The main purpose has been to fully describe the observations and draw some
general conclusions about the requirements on the source mechanism. These requirements lead quite naturally to a number of possibilities such as soliton collapse and lasing mechanisms. We suggest that the lasing mechanism holds the greatest potential for success based on the rather simple-minded approach taken above. It is obvious that considerable theoretical effort will be required to firmly establish any mechanism.
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FIGURE CAPTIONS

Figure 1  Power flux as a function of time for three of the Voyager 2 spectrum analyzer channels showing the low frequency radio emission as the spacecraft left Uranus in the pre-dawn direction. Notice that the bulk of the activity occurs in the 5.6-kHz channel, but occasionally spreads into the adjoining channels. Much of the activity at 3.1 kHz after the middle of day 27 are electron plasma oscillations associated with numerous bow shock crossings.

Figure 2  A series of frequency-time spectrograms which show the detailed spectral and temporal behavior of the bursty Uranian radio emission. The emission is characterized by brief, narrowband bursts superimposed on a weak diffuse component. The lines at 2.4 and 1.8 kHz are spacecraft interference.

Figure 3  High resolution spectra from three selected time intervals which show the rich spectral structure of the Uranian radio emission from 3 to 10 kHz. Some evidence for underlying diffuse bands is present in the upper two panels while only the intense, narrow band at 5.5 kHz is present in the bottom panel.
Figure 4  A perspective plot showing intensity as a function of frequency and time illustrating the extremely narrow-band and bursty nature of this radio emission at about 5 kHz.

Figure 5  A spectrogram taken by Voyager 1 at Jupiter showing the very steady nature of the Jovian narrowband emission at 11 kHz.

Figure 6  A plot of the magnetic latitude of Voyager 2 as a function of time based on information of the Uranian magnetic field provided by the Voyager magnetometer. The heavy dark lines show intervals when the 5-kHz emission was prominent. The emission was only very weakly detectable in waveform samples on the inbound (dayside) leg of the encounter (indicated by 'X's).

Figure 7  This wideband spectrogram may be representative of the source region of the bursty Uranian radio emission. The narrowband emission at 9 kHz is likely to be an electrostatic band near $f_{\text{UH}}$ which could couple into the escaping electromagnetic mode. These data were obtained in the vicinity of the Miranda L-shell.

Figure 8  Spectrum analyzer data from the region around the outbound Miranda L-shell crossing which is a possible source region for the 5-kHz Uranian radio emission.
Figure 1
Figure 4

VOYAGER 2
JANUARY 31, 1986
0004:11 SCET

POWER FLUX (W m⁻² Hz⁻¹)

10⁻¹⁴

10⁻¹₅

10⁻¹₆

10⁻¹₇

FREQUENCY (kHz)

0 4 5

TIME (SEC)

6 7 8 9 10

Figure 4
Figure 5
Figure 7
Figure 8