A TWO-EQUATION TURBULENCE MODEL FOR A DISPERSED TWO-PHASED FLOW WITH VARIABLE DENSITY FLUID AND CONSTANT DENSITY PARTICLES

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A two-equation turbulence model has been developed for predicting a dispersed two-phase flow with variable density fluid and constant density particles. The two equations describe the conservation of turbulence kinetic energy and its dissipation rate for the fluid. They have been derived rigorously from the momentum equations of the carrier fluid in the two-phase flow. Closure of the time-mean equations is achieved by modeling the turbulent correlations up to third order. The new model eliminates the need to simulate in an ad hoc manner the effects of the dispersed phase on turbulent structure in situations where the compressibility of the fluid must be taken into account.
18. SUBJECT TERMS (continued)

Nonsteady Flow
Compressible Viscous Gas
Dilute Suspensions
Continuity Equation
Momentum Equation
Reynolds Time-averaging
Kinetic Energy
Dissipation Rate
PREFACE

This work was performed for the Defense Nuclear Agency under contract DNA 001-85-C-0022. Dr. George W. Ullrich was the DNA Contract Technical Manager. Dr. Allen L. Kuhl was the RDA program manager. The authors wish to thank Dr. Ullrich for his encouragement, Dr. Kuhl for suggestion of the research subject and helpful discussions, and Mrs. Vivian Fox for expert typing of the manuscript.
# CONVERSION TABLE

Conversion factors for U.S. Customary to metric (SI) units of measurement

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*The becquerel (Bq) is the SI unit of radioactivity; 1 Bq = 1 event/s.
**The Gray (Gy) is the SI unit of absorbed radiation.
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SECTION 1

INTRODUCTION

There are many problems of practical interest which require a mathematical model for a two-phase turbulent flow of suspended particles in a viscous fluid. Most one- and two-equation models for two-phase flows (e.g., Refs. 1 and 2) are based on ad hoc modifications of the single-phase turbulence-kinetic-energy and length-scale equations and fail to adequately predict the physical behavior of two-phase flows. Recently, Elghobashi et al. (Ref. 3) presented a rigorous derivation of the turbulence-energy and dissipation-rate equations from the momentum equations for an incompressible dispersed two-phase flow and successfully predicted the main features of a round gaseous jet laden with uniform-size solid particles (Refs. 4 and 5). In situations such as the dust transport by turbulence in nuclear burst flow fields (Ref. 6) and supersonic nozzle flows (Ref. 7) the compressibility of the fluid must be taken into account. The objective of the present work is to extend the two-equation turbulence model in Ref. 3 to become applicable to a compressible dispersed two-phase flow in which the fluid has variable density and the particles have constant density. The extended model can also be used to study such problems as compressible adiabatic mixing and low-speed isothermal mixing of two dissimilar two-phase flows (Ref. 8).
SECTION 2

EQUATIONS OF MOTION

We begin the formulation of the problem by stating the assumptions involved in deriving the equations. These are:

1. Both phases behave macroscopically as a continuum, but only the carrier fluid behaves microscopically as a continuum with variable density. This means that the volume-averaged equations are based on a control volume larger than the particle spacing but much smaller than the characteristic volume of the flow system. Mutual exclusion of the phases is also ensured.

2. The dispersed phase consists of rigid particles spherical in shape, uniform in size and constant in density. The uniformity of size reduces the magnitude of bookkeeping at this stage of the work, and thus concentrates the effort on understanding the mechanisms of interactions between the two phases. Extension to nonuniform size distribution is a straightforward matter (Ref 9).

3. The volume fraction of the dispersed phase is such that no collisions occur between the particles. This assumption renders the equations valid only for dilute suspensions.

4. Neither the suspended matter nor the carrier fluid undergoes any phase changes. Although this assumption rules out some situations of practical
interest, it is necessary to investigate complexities in a stepwise manner.

5. Additional assumptions on modeling of some of the turbulent correlations are stated in Sections 4, 5 and 6. The sparseness of experimental data for variable density flows allows us to assume that the forms and values of the coefficients of the turbulent correlations of constant-density flows apply to variable-density flows as well.

The instantaneous, volume-averaged momentum equations, in Cartesian tensor notations, of the carrier fluid are thus (Refs. 3 and 10)

\[
(Q_1 U_i)_t + (Q_1 U_j U_i)_j = -(1-\Phi_2)P,1 - K\Phi_1(U_i-V_i)
\]

\[+
\left[\mu_1\Phi_1(U_i, j+U_j, i)\right], j
\]

\[-\frac{2}{3}(\mu_1\Phi_1 U_{\delta, \delta}), i + Q_1 g_1 + F_{11},
\]

(1)

The corresponding equations for the particle phase are

\[
(Q_2 V_i)_t + (Q_2 V_j V_i)_j = -\Phi_2 P,1 + F\Phi_2(U_i-V_i)
\]

\[+
\left[\mu_2\Phi_2(V_j, j+V_i, i)\right], j - \frac{2}{3}(\mu_2\Phi_2 V_{\delta, \delta}), i
\]

\[+ Q_2 g_1 + F_{21},
\]

(2)
The continuity equations are

\[ \rho_1, t + (U_i), x_i = 0 \]

for the fluid and

\[ \rho_2, t + (U_i), x_i = 0 \]

for the particle phase. The global continuity equation is

\[ \rho_1 + \rho_2 = 1 = \frac{\rho_1}{\beta_1} + \frac{\rho_2}{\beta_2} \]  

(5)

In Eqs. (1) - (5) and throughout the report, the subscripts 1 and 2 denote the fluid and particle phase respectively. Partial derivatives are represented by a subscript consisting of a comma and an index (e.g., \( \rho, t \equiv \partial \rho / \partial t \); \( U_i, x_i \equiv \partial U_i / \partial x_i \); \( U_i, x_k x_l \equiv \partial^2 U_i / \partial x_k \partial x_l \)) where \( x_i \) (i = 1, 2, 3) are the rectangular spatial coordinates. \( U_i \) are the velocity components of the fluid. \( V_i \) are the velocity components of the particle phase. \( \rho \) and \( \rho \) are the material density and viscosity. \( \beta = \rho \phi \) is the apparent density. \( \phi \) is the volume fraction. \( F \) is the pressure. \( g_1 \) is the component of gravitational acceleration in the \( i \) direction. \( F_i \) is the component of body forces other than that due to gravity, and \( F \) is the interphase friction coefficient \( = \mu \omega \phi \) for Stokes' flow around a particle of diameter \( d \). \( K \) is the local effectiveness of momentum transfer from the particle phase to the fluid and is discussed in detail in Ref. 10. It suffices here to state that \( K \) equals unity for unaccelerated particle phase and assumes lower values when the phase is decelerated, the minimum value of \( K \) being zero. In general, \( K \) depends on the local properties of the fluid and
turbulence, the slip velocity, and the particulate size and concentration.

The mean flow equations are now obtained from the instantaneous ones, for variable \( \rho_1 \), constant \( \rho_2 \) and \( \mu_1 \) and zero \( \mu_2 \) (consistent with the dilute suspension approximation stated in Ref. 10, p. 256), by performing the conventional Reynolds time-averaging of Eqs. (1) to (5). (The density-weighted averaging of Favre, Ref. 11, does not render significant simplification. We use Reynolds averaging mainly because most experimental data refer to time averaging correlations). The mean momentum equations of the fluid are

\[
(Q_1 u_i + q_{1u_i}),t + (Q_1 u_j u_i),j = -(1-K\phi_2)\rho_i + K\phi_2 \rho_i
\]

\[
\quad - KF[\phi_2(U_i - V_i) + \phi_2(u_i - v_i)]
\]

\[
\quad + \mu_1[\phi_1(U_i, J + U_j, i) + \phi_1(u_i, j + u_j, i)], j
\]

\[
\quad - \frac{2}{3} \mu_1[\phi_1 U_j, j + \phi_1 u_j, j], i
\]

\[
\quad - (Q_1 u_i u_j + U_1 q_{1u_i} + U_j q_{1u_j} + q_{1u_i u_j}), j
\]

\[
\quad + Q_1 g_i + F_{1i}.
\]
The mean momentum equations of the particle phase are

\[ (Q_2 v_i + q_2 v_i)_t + (Q_2 v_j v_i) = -[\dot{\phi}_2 p, i + \dot{\phi}_2 p, i] \]

\[ + F[\dot{\phi}_2 (U_i - v_i) + \dot{\phi}_2 (U_i - v_i)] \]

\[ - (Q_2 \overline{v_i v_j} + v_i q_2 \overline{v_j} + v_j q_2 \overline{v_i} + q_2 \overline{v_i v_j}), i \]

\[ + Q_2 q_1 + F_{2i} \]  

(7)

The mean continuity equation of the fluid is

\[ Q_1, t + (Q_1 U_1 + q_1 U_1), i = 0 \]  

(8)

The mean continuity equation of the particle phase is

\[ Q_2, t + (Q_2 v_1 + q_2 v_1), i = 0 \]  

(9)

which can be written as

\[ \dot{\phi}_2, t + (\dot{\phi}_2 v_1 + \dot{\phi}_2 v_1), i = 0 \]  

(10)

since \( \rho_2 \) is constant. The mean global continuity equation is

\[ \dot{\phi}_1 + \dot{\phi}_2 = 1 \]  

(11)

which, when subtracted from Eq. (5), gives

\[ \dot{\phi}_1 + \dot{\phi}_2 = 0 \]  

(12)

In Eqs. (6)-(12) capital letters (except \( K \) and \( F \)) denote time-mean quantities, lower-case letters (except \( \mu_1 \) and \( g_i \)) designate fluctuating components and overbars indicate
Reynolds-averaged correlations. For constant quantities \( (\mu_1, \rho_2 \text{ and } g_1) \) the mean and instantaneous values are equal, whereas for all other variables the instantaneous value consists of a mean component and a fluctuating component (e.g., \( Q + q, U + u, \rho_1 + \rho_1' \ldots \text{ etc.} \)).
SECTION 3

TURBULENCE-KINETIC-ENERGY AND DISSIPATION-RATE EQUATIONS

The first step in the derivation of the equations of the fluid's turbulence kinetic energy \( k \equiv u_i u_i / 2 \) and its dissipation rate \( \epsilon \equiv \nu_1 u_i, k u_i, k \) where \( \nu_1 = \mu_1 / \rho_1 \) is to obtain a transport equation for \( u_i \) by subtracting Eq. (6) from Eq. (1). The \( k \) equation is produced by multiplying the \( u_i \) equation throughout by \( u_i \) and then time-averaging. The \( \epsilon \) equation is obtained by differentiating the \( u_i \) equation with respect to \( x_k \), multiplying throughout by \( 2 \nu_1 u_i, k \) and finally time-averaging.

The resulting \( k \) and \( \epsilon \) equations are given in Appendix A. The closure of these equations is discussed in Sections 5 and 6, respectively.
SECTION 4

CLOSURE OF THE MOMENTUM EQUATIONS

The turbulent correlations appearing in Eqs. (6) and (7) are of five types:

1. Correlation of velocity fluctuations with those of the volume fraction or apparent density, e.g., \( \phi_1 u_i \) or \( q_1 u_i \);

2. The pressure interaction correlation \( \phi_{2P,i} \);

3. Multiple correlations among various components of velocity fluctuations with those of the apparent density, e.g., \( q_1 u_i u_j \);

4. Correlations of strain rate fluctuations with those of the volume fraction, e.g., \( \phi_{1i,j} \);

5. Multiple correlations of various components of velocity fluctuations, e.g., \( u_i u_j \).

The first four types occur only due to the presence of the second phase; their modeling is discussed below.

According to Ref. 3 (Eq. (10)) and Ref. 12, we model the turbulent flux \( \phi_1 u_i \) by a gradient transport term and a convective transport term such that

\[
\phi_1 u_i = -\left( \frac{\nu_t}{\sigma_\phi} \right) \phi_{1,i} - \frac{1}{2} \phi_1 \left( \frac{\nu_t}{\sigma_\phi} \right)_{,i},
\]

(13)
where \( \nu_t \) is the kinematic eddy viscosity \( (= c_\mu k^2 \cdot \epsilon \text{ with } c_\mu = 0.09, \text{Ref. 13, under the provisional assumption that it has the same value as in the case of constant density}) \) and \( \sigma_\phi \) is the turbulent Schmidt number of \( \phi \). Similarly we model \( q_1 u_i \) by

\[
\overline{q_1 u_i} = -(\nu_t/\sigma_q)Q_{1,i} - \frac{1}{2} Q_1 (\nu_t/\sigma_q),i
\]

(14)

where \( \sigma_q \) is the turbulent Schmidt number of \( q \). As long as experimental data for \( \sigma_q \) are not available we make a provisional assumption that \( \sigma_q \) is equal to \( \sigma_\phi \) (= 1.0 for the sample calculation in Ref. 3).

According to Ref. 3 (Eqs. (11), (12), (14), (15) and (16)) and Refs. 14, 15 and 16 we model \( \phi_2 P,i \) by

\[
\phi_2 P,i = - \rho k \left[ \phi_1 \overline{u_i u_j} - \phi_2 \left( \frac{\overline{u_i u_j}}{k} - \frac{2}{3} \overline{u_k u_k} \right) \right]
\]

\[
+ \rho (0.8 \overline{u_k \phi_1} \overline{u_k u_k} - 0.2 \overline{u_k \phi_1} \overline{u_k u_k})
\]

\[
- \left[ -c_\phi \rho_1 k^{1/2} \overline{u_m \phi_1} \overline{e_m} \right.
\]

\[
+ c_\phi k^{3/2}/\epsilon \overline{u_n \phi_1} \overline{u_m \phi_1} \overline{e_n} \bigg]_i
\]

(15)

where \( e_m \) and \( e_n \) are unit vectors and the approximate values of the coefficients are

\[
c_\phi_1 = 4.3, \quad c_\phi_2 = 3.2, \quad c_\phi_3 = 1.0, \quad c_\phi_4 = 1.0
\]

(16)
According to Ref. 16 (Eq. (6.45)) we model $q_{1u_1u_j}$ by

$$q_{1u_1u_j} = -c_{45}(k/s)[ \overline{u_iu_j} (\overline{u_jq_1}),_i + \overline{u_ju_2} (\overline{u_iq_1}),_j ]$$

where the proportionality constant $c_{45}$ is approximately equal to 0.1.

The strain-rate volume-fraction correlations of the type $\phi_{1u_1,j}$ only appear multiplied by the molecular viscosity of the fluid and therefore will be neglected due to its relatively small magnitude.

The last correlation to be modeled in the momentum equations is that of the form $u_iu_j$. Again, to be consistent with the present level of closure, this quantity will be calculated from (Eq. (18) in Ref. 3)

$$\overline{u_iu_j} = -\nu_t(U_{i,j}+U_{j,i}) + \frac{1}{3} \delta_{ij} \overline{u_mu_m} - \frac{2}{3} \nu_t \delta_{ij} U_{j,j}$$

This completes the modeling of the momentum equations.
SECTION 5

CLOSURE OF THE TURBULENCE KINETIC-ENERGY EQUATION

The exact equation of the turbulence energy \( k \) for the carrier fluid appears in Appendix A and consists of 34 terms. They are classified into groups enclosed by square or curly brackets; each group is labeled according to its particular contribution to the conservation of \( k \).

The various correlations in these groups range from second to fourth order. We decide at the outset on neglecting all fourth-order correlations such as \( q_{ij} u_i u_j \) and \( u_i u_j \). Also, the contribution to the diffusion of turbulence energy due to the pressure interaction \( (u_i p) \) will be neglected as it is of relatively small magnitude (Ref. 17). Now the remaining terms will be modeled.

The five transient terms will be collectively approximated by \( (Q_1 k) , t \). The convection terms require no approximation. The production group contains the correlations \( q_{ij} u_j \) which have been evaluated earlier by Eqs. (17) and (18). The pressure velocity-divergence correlation \( p u_{i,i} \) in the turbulent-diffusion group cannot be neglected here since \( u_{i,i} \) does not vanish in two-phase flows. \( p u_{i,i} \) is evaluated following the approach of Ref. 18, thus (Eqs. (19) and (20) of Ref. 3)

\[
- p u_{i,i} = c_1 \frac{\beta_1}{2} \left( \frac{\varepsilon}{k} \right) (u_i u_i - \frac{2k}{3}) + \frac{(c_2 + 8) \beta_1}{22} (p_{ii} - \frac{2p}{3}) \\
+ \frac{(15c - 1) \beta_1}{55} (2k u_{i,i}) - \frac{(4c_2 - 1) \beta_1}{11} (D_{ii} - \frac{2p}{3}) \tag{19}
\]
where

\[
\begin{align*}
\text{P}_{ii} &= -2(\overline{u_i u_k} U_{i,k}) \\
\text{P} &= \frac{1}{2} \text{P}_{ii} \\
\text{D}_{ii} &= -2(\overline{u_i u_k} U_{k,i}) \\
c_1 &= 1.5 \\
c_2 &= 0.4
\end{align*}
\]

The last two terms in the turbulent-diffusion group can be modeled as (Eqs. (21) to (25) and Eq. (10) of Ref. 3)

\[
- \frac{u_i u_j}{Q_1} (Q_{ij}), - \frac{Q_1 u_i u_j}{Q_{ij}}, = \left\{ Q_1 \left[ (\nu_t / \sigma_k) k, j, \right. \right.
\]

\[
+ \frac{1}{2} k(\nu_t / \sigma_k), j \right\} \right\}
\]

(21)

where the turbulent Schmidt number of \( k \) is taken as \( \sigma_k = 1.0 \) (Ref. 13, under the provisional assumption that it has the same value as in the case of constant density).

There are eight terms in the extra production and transfer group, the last two of which are neglected for being of fourth order. The remaining six terms are modeled next.

The second term, \(-Q_i U_i \overline{u_i u_j}, j\), is modeled following the proposal of Ref. 19 as \( Q_i U_i \nu_t U_{i,j}, j \). The correlation of the form \( \overline{u_i(q_i u_j)}, j \) which appears in the third and fourth terms is expanded as

\[
\overline{u_i(q_i u_j)}, j = \overline{(u_i q_i u_j)}, j - \overline{q_i u_j u_i}, j
\]

(22)
where the first term on the right is evaluated using Eq. (17), and the second term is modeled as

\[ \bar{q}_1 u_j u_i,j = -c_0 \frac{\varepsilon}{k} \left[ \frac{v_t}{\sigma_{q}} Q_{1,i} + \frac{1}{2} Q_{1} \left( \frac{v_t}{\sigma_{q}} \right)_i \right] \]  

(23)

where the provisional assumption on \( \sigma_q \) has already been mentioned in Section 4. The correlations \( \phi_1 u_i \) (in the first term) and \( \bar{q}_1 u_i \) (in the fifth term) have been discussed earlier (Eqs. (13) and (14)). The sixth term is approximated by

\[ U_i U_j \overline{u_i q_1},j \equiv U_i U_j [\overline{(u_i q_1)},j - \overline{q_i u_i},j] \]  

(24)

where \( \overline{q_i u_i},j \) is neglected for being relatively smaller than \( \overline{(u_i q_1)},j \).

The extra dissipation group contains three terms which exist only due to the slip between the two phases. According to Ref. 3 (Eq. (39)) and Refs. 20 and 21 we model the correlation \( u_i (v_i - u_i) \) by

\[ \overline{u_i (v_i - u_i)} = -\frac{1}{2} \overline{u_i^2} \left( 1 - \int_0^\infty \frac{\Omega_1 - \Omega_R}{\Omega_2} f(\omega) d\omega \right) \]  

(25)

where \( \omega \) is the frequency of turbulence and

\[ \overline{u_i^2} = \frac{1}{3} \overline{u_i u_i} = \frac{2}{3} k \]  

(26)

\[ \Omega_1 = (\omega/\alpha)^2 + \sqrt{6} (\omega/\alpha)^{3.2} + 3(\omega/\alpha) - \sqrt{6} (\omega/\alpha)^{1.2} + i \]  

(27)
\[ \Omega_2 = \beta^{-2}(\omega/\alpha)^2 + \sqrt{\beta} \beta^{-1}(\omega/\alpha)^{3/2} + 3(\omega/\alpha) \]
\[ + \sqrt{\beta} (\omega/\alpha)^{1/2} + 1 \]  
\[ (28) \]
\[ \Omega_R = \left[ (1-\beta)\omega/\alpha \beta \right]^2 \]  
\[ (29) \]
\[ f(\omega) = 4(\pi)^{3/2} \lambda \left[ (\frac{2}{3} \chi)_{1/2} \right] e^{-\frac{(\lambda \omega)^2}{(\frac{8}{3})\chi}} \]  
\[ (30) \]

in which
\[ \alpha = 12\mu_1/\rho_1 d^2 \]  
\[ (31) \]
\[ \beta = 3\rho_1/(2\rho_2 + \rho_1) \]  
\[ (32) \]

and \( \lambda \) is Taylor's microscale. The last term in the extra dissipation group contains the triple correlations \( \phi_2 \mu_{1i}u_i \) and \( \phi_2 \mu_{1i}v_i \) which can be modeled by Eqs. (33) and (34) of Ref. 3 as
\[ \phi_2 \mu_{1i}u_i = -2c_5 (k/\epsilon) \mu_{1i}u_j \mu_{1j} \]  
\[ (33) \]
and
\[ \phi_2 \mu_{1i}v_i = -c_5 (k/\epsilon) \left[ \mu_{1i}v_j \phi_5 \mu_{1j} \phi_5 \right] \]  
\[ (34) \]
where the double correlations on the right sides have been modeled earlier (Eqs. (13) and (18)).

The term \( \mu_1 \phi_{1i} u_j \phi_2 (u_i, j+u_j, i) \) constitutes the dissipation of \( k \) due to viscous action, and if \( \phi_2 \) is set to unity it is reduced
to the single-phase dissipation terms. This term is modeled as (Eq. (43) in Ref. 3)

\[ u_{i}^{1}(u_{i,j} + u_{j,i}) \]

The six terms in the viscous diffusion and dissipation group will be neglected due to their relatively small magnitudes as compared to the terms in the turbulent diffusion group.

The first term in the field forces effects group is modeled by Eq. (14). Similarly the second term in the group can be modeled as

\[ u_{i}f_{11} = -(\nu_{t}/\sigma_{f})F_{11,i} - \frac{1}{2}F_{11}(\nu_{t}/\sigma_{f}),i \]

where we make a provisional assumption that \( \sigma_{f} \) is equal to \( \sigma_{f} \).
CLOSURE OF THE TURBULENCE-ENERGY DISSIPATION-RATE EQUATION

The exact equation of the dissipation rate of turbulence energy $\epsilon$ for the carrier fluid appears in Appendix A and consists of 52 terms. They are classified into groups similar to those of the $k$ equation.

Again we neglect all fourth-order correlations as mentioned in the previous section.

All the terms in the first group, labeled Transient, are approximated collectively by $(Q_1\epsilon)_t$.

The convection group consists of eight terms of which only the first and the second are of higher magnitude than the other six at large Reynolds number. This is based on an order of magnitude analysis (Ref. 22) which shows that the first and second terms are greater than the others by at least a factor of $(\ell/\lambda)$, which is of order $(R_\eta)^{1/2}$. Here $\ell$ is the length scale of the energy containing eddies, $\lambda$ is a Taylor's microscale, and $R_\eta$ is the Reynolds number based on $\ell$.

The third term in the production group is decomposed as

$$-2\nu_1Q_1(u_iu_j,k^i,k),j = -2\nu_1Q_1(u_i,ju_j,k^i,k + u_iu_j,k^i,k + u_iu_j,k^i,k)$$

The third term on the right side of Eq. (37) and the second term in the production group differ only in their signs and thus cancel each other. The first term on the right side of
Eq. (37), which represents the production of $\epsilon$ by self-stretching of the vortex tubes, is the dominant one at large Reynolds number. It is larger than the second term by a factor of $R_4$ and larger than the first term in the production group by a factor of $R_4^{1/2}$.

We, therefore, retain only $-2\nu_1 Q_1 u_{i,j}u_{j,k}u_{i,k}$ as the main generation of $\epsilon$. This term and the extra production terms are modeled collectively as

$$-2\nu_1 Q_1 u_{i,j}u_{j,k}u_{i,k} + \text{extra production terms} = c_{\epsilon_1} G_k \epsilon / k, \quad (38)$$

where $G_k$ is the total production of $k$ discussed in Section 5, and $c_{\epsilon_1}$ is a constant of value 1.43 (Ref. 13, under the provisional assumption that it has the same value as in the case of constant density). "Total" here means the production terms which are common to the single-phase and two-phase $k$ equations in addition to the extra production and transfer terms.

The turbulent diffusion group contains six terms. At high Reynolds number only the last two terms will be retained; they are larger by at least a factor of $R_4^{1/2}$ than the other terms. These two terms will be modeled collectively as

$$-2\nu_1 (Q_1 u_{j}u_{i,k} + Q_1 u_{j}u_{i,k}u_{i,k})$$

$$= [Q_1 (\nu t / \sigma_\epsilon) \epsilon, j + \frac{1}{2} Q_1 \epsilon (\nu t / \sigma_\epsilon), j], j \quad (39)$$

All the terms in the extra production group except the mean pressure gradient term are smaller than the main production term, modeled in Eq. (38), by at least a factor of $R_4^{-1/2}$ and
thus can be neglected. The mean pressure gradient term is included in the production term (Eq. (38)).

The first term in the viscous diffusion and dissipation group represents the main dissipation of $\varepsilon$; it reduces to the single-phase form when $\Phi_1$ equals unity. This term is larger than the other terms in the group by a factor of $R^{3/2}$ and thus it is the only one retained. Now the total dissipation of $\varepsilon$ includes this term in addition to the extra dissipation terms. They are modeled collectively as $Q_1(\varepsilon/k)(c_{\varepsilon 2} \varepsilon + c_{\varepsilon 3} \varepsilon_e)$ where $\varepsilon_e$ represents the extra dissipation terms appearing in the $k$ equation, $c_{\varepsilon 2}$ is a constant of a value about 1.92 (Ref. 13) and $c_{\varepsilon 3}$ is a constant of value 1.2 (Ref. 3) (both under the provisional assumption that they have the same values as in the case of constant density).

Both terms in the field forces effects group appear multiplied by the kinematic molecular viscosity of the fluid and therefore will be neglected due to their relatively small magnitudes.
As an example of the application of the modeled $k$ and $\epsilon$ equations, let us consider the motion of the dusty air during a nuclear explosion (Ref. 6). In order to understand the essential features of the complicated phenomena, some highly idealized and well-controlled laboratory tests have been performed or planned such as high speed wind above a sand bed in a wind tunnel (Ref. 23) and shock wave sweeping a sand bed in a shock tube (Ref. 24). To interpret and correlate the results from such tests, we may use the two-dimensional version of the present model in Cartesian coordinates $x$ and $y$. The final form of the modeled $k$ and $\epsilon$ equations is given in Appendix B (under the assumptions that the diffusional fluxes in $y$ direction are much larger than those in $x$ direction, that the effective coefficient $K = 1$ and that $g_y = g$ and $g_x = F_{1x} = F_{1y} = 0$). The remaining modeled equations contain two mean continuity equations (from Eqs. (8) and (9)), one mean global continuity equation (Eq. (11)), four mean momentum equations (from Eqs. (6) and (7) and Section 4) and one mean equation of state such as the perfect gas equation

$$P = R\rho_1 T_1$$

where $R$ is the specific gas constant and $T_1$ is the absolute temperature of the air. Thus, for isothermal problems ($T_1 = \text{constant}$) we have ten equations for ten unknowns $Q_1$, $Q_2$, $\rho_1$, $U_x$, $U_y$, $V_x$, $V_y$, $P$, $k$ and $\epsilon$ (from which $\phi_1 = Q_1/\rho_1$ and $\phi_2 = Q_2/\rho_2$ can be readily obtained). With proper initial and boundary conditions, these equations can be solved numerically by a marching finite-difference procedure described in Ref. 5.
The results of such calculation will be presented in a forthcoming report.

If $T_1$ is variable and the energy exchange between the air and dust can be neglected we need to include a mean energy equation for the air in the numerical solution procedure.

If the energy exchange between the air and dust (at temperature $T_2$) is not negligible, we need to include the mean energy equations for both air and dust in the numerical solution.
SECTION 8

CONCLUSION

The $k - \varepsilon$ turbulence model for an incompressible dilute suspension of Ref. 3 has been extended to a compressible dispersed two-phase flow by introducing the apparent densities $Q_1$ and $Q_2$ and the material density $\rho_1$ as new variables. The fluid has variable density and the particles have constant density. This allows the application of the model to a wider class of practical problems.

As in Ref. 3, the $k$ and $\varepsilon$ equations are first rigorously derived from the two-phase momentum equations and then their closure is provided. This is in contrast to the usual approach based on ad hoc modifications of the single-phase turbulence-kinetic-energy and length-scale equations.

The proposed closure of the equations accounts for the interaction between the two phases and its influence on the turbulence structure. Sparseness of experimental data for variable-density flows necessitates some provisional assumptions that forms and values of the coefficients in the turbulent correlations of constant-density flows apply to those of variable-density flows. Such assumptions indicate areas of needed experimental investigations which, when completed, can in turn modify the present work and enhance its validity.
SECTION 9

LIST OF REFERENCES


APPENDIX A

EXACT EQUATIONS OF KINETIC ENERGY OF TURBULENCE AND DISSIPATION RATE OF THAT ENERGY

The exact equation of the turbulence kinetic energy \( k \equiv \frac{1}{2} u_i u_i \) of the carrier fluid is

\[
\left( Q_1 \frac{u_i u_i}{2} \right)_t + Q_i, t \frac{u_i u_i}{2} + U_i, t \frac{u_i q_1}{2} + U_i u_i q_1, t
\]

\( \text{Transient} \)

\[
+ \frac{u_i (q_1 u_i), t}{2}
\]

\( \text{Convection} \)

\[
\left[ (Q_1 U_{jk}), j + k(Q_1 U_j), j \right] =
\]

\( \text{Production} \)

\[
- \left( Q_1 U_{ij}, j \frac{u_i u_j}{2} + U_{ij}, j q_1 u_i u_j + U_{ij}, j q_1 u_i u_j \right)
\]

\( \text{Turbulent Diffusion} \)

\[
- \left\{ (1 - K \phi_2) \left[ \left( \frac{u_i p}{i - pu_i}, i \right) + K \phi_1 \frac{u_i p}{i} + \frac{u_i u_i (Q_1 u_j)}{} \right] \right\}
\]

\( \text{Extra Production and Transfer} \)

\[
+ Q_1 u_i u_j \frac{u_i u_j}{2} + Q_1 U_{ij}, u_i u_j, j + U_{ij}, u_i (Q_1 u_j), j
\]

\( \text{Extra Dissipation} \)

\[
+ (V_{i - U_i}) \phi_2 u_i + \phi_2 u_i (V_{i - U_i}) + \mu_1 \left[ \phi_1 u_i (u_i, j u_j, i), j \right]
\]

\( \text{Dissipation} \)
The exact equation of the dissipation rate of $k$
\begin{align*}
\varepsilon & \equiv \nu_1 u_{i,k} \frac{u_{i,k}^2}{u_i} \\
\text{Viscous Diffusion and Dissipation} & \quad + \mu_1 \left\{ \phi_1, j \frac{u_i(u_i, j+u_j, i)}{u_i} + \left[ (U_i, j+U_j, i) \phi_1 \right] \right\} u_{i,j} \\
+ \left[ (u_i, j+u_j, i) \phi_1 \right] u_{i,j} - \kappa \left[ \phi_1 u_{i,j} - (\phi_1 u_{i,j}, u_{i,j} + (\phi_1 u_{i,j}, u_{i,j}) \right] \right\} \\
\text{Field Forces Effects} & \quad + \left[ q_i u_i q_i + \frac{u_i f_{i,j}}{u_i} \right] \\
\text{Transient} & \quad + \left\{ \left( Q_1 u_i, k u_{i,j} + (Q_1 u_i, k u_{i,j}, k) \right) + \left( Q_1 u_i, k u_{i,j} \right) \right\} \\
\text{Convection} & \quad \left\{ \left( Q_1 u_i, k u_{i,j} + (Q_1 u_i, k u_{i,j}, k) \right) + \left( Q_1 u_i, k u_{i,j} \right) \right\} = -2\nu_1 \left[ Q_1 u_i, k u_{i,j} \right] \\
\text{Production} & \quad - Q_1 u_{i,j} u_{i,j} + (Q_1 u_{i,j}, k u_{i,j}, k) \\
\text{Extra Dissipation} & \quad + 2\nu_1 K F \left[ \phi_2 (v_i - u_i), k u_{i,j} + \phi_2 (v_i - U_i), k u_{i,j} \right] \\
\text{Production} & \quad - 2\nu_1 \left[ Q_1, k u_{i,j} u_{i,j} \right] \\
\end{align*}
\[ + Q_{1,k} u_{1j}, j+u_{1i}, jui,k + Q_1, j u_{1j}, kui,k + P, kui.k \]

**Turbulent Diffusion**

\[ + Q_{1jui,k, k} + Q_{1jui,k, k} + 2\nu_1 \left\{ K\phi_2 p, i, k^2i, k \right\} \]

\[ - K P, k i \phi_{1i,k} - P, i u_{1i,k} \phi_{1i,k} - (K\phi_{1P, i}, k^2i,k) \]

\[ - u_{1i,k} \left[ (q_{1U_l}), ku, jui,k + (q_{1U_l}), ku, jui,k - u_{1i} q_{1u}, kuj,j \right] - u_{1i} q_{1u}, kuj,j \]

\[ - u_{1jui,k} (q_{1U_l}), jk - u_{1jui,k} (q_{1U_l}), jk - u_{1j} (q_{1U_l}), jk^2i,k \]

**Extra**

\[ - \left\{ (q_{1U_l}), jui, ku, j + (q_{1U_l}), ku, jui,k \right\} \]

**Production**

\[ - \left\{ U_1 U_1 q_{1l}, jk+q_{1l} (U_1 U_1, j+U_1 U_1, j) \right\} u_{1i,k} \]

\[ - \left\{ q_{1l} (U_1, jU_l), j+U_1 U_1, kq_1, j+q_{1l} (U_l, jU_l), k \right\} u_{1i,k} \]

\[ - u_{1jui,k} q_1, j + \left\{ u_{1jui,k} q_1, j+u_{1jui,k} \right\} u_{1i,k} \]

\[ - \left\{ q_{1l} (U_1, jU_l), j+U_1 U_1, kq_1, j+q_{1l} (U_l, jU_l), k \right\} u_{1i,k} \]

\[ + 2\rho_1 u_{1i,k} \left\{ \phi_1 (u_{1i,j+uj,i}), jk^2i,k + \phi_1 (U_1, j+U_1, i), jk^2i,k \right\} \]

\[ + \frac{1}{2} \left\{ (\phi_1 u_{1l}), i, k^2i,k \right\} \]

**Viscous Diffusion and Dissipation**

\[ + (\phi_1 U_1, i, k^2i,k + (\phi_1 u_{1l}), i, k^2i,k \right\} \]

\[ + 2\nu_1 \left\{ \phi_1 q_1, k^2i,k + \phi_{1i,i}, ku, ;i,k \right\} \]

**Field Forces Effects**
APPENDIX B
THE MODELED FORM OF $\kappa$ AND $\varepsilon$ EQUATIONS

The modeled conservation equations of the kinetic energy and the dissipation rate of that energy for the carrier fluid in the sample application are listed here.

(i) The $k$ equation:

\[
\frac{Q_1k_t}{t} + Q_1 \left[ \frac{U x_k + U y k_y}{x} \right] = \left\{ \begin{array}{l}
\left[ U_t x, y \right] \left( Q_1 U_x \right), y - \frac{2}{3} k \left[ Q_1 U_y \right], y \\
\left[ c_5 \left( \frac{k}{\varepsilon} \right) \left( \frac{U_t}{\sigma_\delta} \right) Q_1, y \right), y \left( \frac{U x}{\sigma_\delta} \right)^2 - \frac{4}{3} k U_y, y \\
- Q_1 \left[ \frac{6.4}{11} \left( U_t x, y \right) - \frac{2}{3} k U_y, y \right] - \frac{43}{35} k U_y, y \\
+ \left[ Q_1 \left( \frac{U_t}{\sigma_\delta} \right) k, y + \frac{1}{2} k Q_1 \left( \frac{U_t}{\sigma_\delta} \right), y \right], y \\
- \left( \frac{U_t}{\sigma_\delta} \right) Q_2, y + U_t Q_1 U_y, y, y \\
+ c_5 U_x \left[ \left( \frac{k}{\varepsilon} \right) U_t x, y \left( \frac{U_t}{\sigma_\delta} Q_1, y \right), y \right], y \\
- 2c_5 U_y \left[ \frac{4}{3} \left( \frac{k^2}{\varepsilon} \right) \left( \frac{U_t}{\sigma_\delta} Q_1, y \right), y \right], y + \left( \frac{6}{X} \right) \left( \frac{U_t}{\sigma_\delta} Q_1, y \right) + 0.5 Q_1 \left( \frac{U_t}{\sigma_\delta} \right), y \\
+ \left[ \left( \frac{U^2}{y} \right), y \left( \frac{U_t}{\sigma_\delta} Q_1, y \right) + U^2 \left( \frac{U_t}{\sigma_\delta} Q_1, y \right), y \right] \right\}
\]

\]

31
Extra Dissipation \((\varepsilon_e)\)

\[
- (\frac{F}{\rho_2}) \left[ k Q_2 \left( 1 - \int_0^{\frac{\Omega_1 - \Omega R}{\Omega_2}} f(\omega) d\omega \right) \right. \\
\left. - \left( \nabla_y \cdot U_y \left( \frac{\nu t}{\sigma_\phi} \right) Q_2, y \right) \right] \\
+ c_\phi Q_2 \left( \frac{F}{3\rho_2} \right) \left( \frac{k^2}{\varepsilon} \right) \left[ 1 - \int_0^{\frac{\Omega_1 - \Omega R}{\Omega_2}} f(\omega) d\omega \right] \left( \frac{\nu t}{\sigma_\phi} Q_2, y \right) \]
\]

\[- Q_1 \left[ g \left( \frac{\nu t}{\sigma_\phi} \right) Q_1, y + \frac{1}{2} Q_1 \left( \frac{\nu t}{\sigma_\phi} \right), y \right] \]

Dissipation Field Forces Effects

(ii) The \(\varepsilon\) equation:

\[
\left[ Q_1 \varepsilon, t \right] + Q_1 \left[ U_x \varepsilon, x + U_y \varepsilon, y \right] \\
\left[ \frac{\partial}{\partial t} \varepsilon \right] + \left[ U_x \frac{\partial \varepsilon}{\partial x} + U_y \frac{\partial \varepsilon}{\partial y} \right] \\
\] Transient Convection

= \[
\left[ c_{\varepsilon 1} \left( \frac{\varepsilon}{k} \right) (P + P_e) \right] + \left[ Q_1 \left( \frac{\nu t}{\sigma_\phi} \right) \varepsilon, y \right] \left[ \frac{1}{2} Q_1 \varepsilon \left( \frac{\nu t}{\sigma_\phi} \right), y \right] \\
\right. \\
\left. \text{Total Production} \quad \text{Diffusion} \right]
\]

- \left[ Q_1 \left( \frac{\varepsilon}{k} \right) \left( c_{\varepsilon 2} \varepsilon + c_{\varepsilon 3} \varepsilon \varepsilon \right) \right] \\
\left. \text{Total Dissipation} \right]
\]

The notations used in the partial derivatives have been explained in Section 2 of the text, thus \((\cdot), t\) means \(\partial(\cdot)/\partial t\), \((\cdot), x\) means \(\partial(\cdot)/\partial x\) and \((\cdot), y\) means \(\partial(\cdot)/\partial y\), where \(x\) and \(y\) are the distances along the horizontal and vertical directions, respectively. The values of the constants appearing in the two equations are:

\[
c_{\varepsilon 1} = 1.43, \quad c_{\varepsilon 2} = 1.92, \quad c_{\varepsilon 3} = 1.2, \quad c_\phi = 0.1, \quad \sigma_k = 1.0, \quad \sigma_\varepsilon = 1.3, \quad \sigma_\phi = 1.0
\]
APPENDIX C

NOMENCLATURE

$C_1$, $C_2$, $C_\mu$, $C_{e1}$, $C_{e2}$, $C_{e3}$, $C_{\phi1}$, $C_{\phi2}$, $C_{\phi3}$, $C_{\phi4}$, $C_{\phi5}$

constants in the turbulence model

d
particle diameter

$D_{ii}$
an expression defined in Eqs. (20)

$e_M$, $e_n$
unit vectors

$f(\omega)$
an expression defined in Eq. (30)

$f_i$
fluctuating component of body forces other than that due to gravity

$F_i$
instantaneous (in Eqs. (1) and (2)) or time-mean component of body forces other than that due to gravity

$F$
interface friction coefficient

$g_i$
component of gravitational acceleration

$G_k$
total production of $k$

$k$
turbulence kinetic energy of the fluid

$K$
local effectiveness of momentum transfer from the particle phase to the fluid

$l$
length scale

$p$
pressure fluctuation

$P$
instantaneous (in Eqs. (1) and (2)) or time-mean pressure

$P$
an expression defined in Eqs. (20)

$P$
production (in Appendix B)

$P_e$
extra production (in Appendix B)

$q$
fluctuation of the apparent density

$Q$
instantaneous (in Eqs. (1)-(5)) or time-mean apparent density

$R$
specific gas constant

$R_g$
Reynolds number based on $l$
time
T  absolute temperature
ui  fluctuating velocity component of the fluid
U_i instantaneous (in Eqs. (1)-(3)) or time-mean velocity component of the fluid
v_i  fluctuating velocity component of the particle phase
V_i instantaneous (in Eqs. (1)-(4)) or time-mean velocity component of the particle phase
x_i  rectangular spatial coordinate
x  horizontal coordinate
y  vertical coordinate
Greek symbols
\alpha  an expression defined in Eq. (31)
\beta  an expression defined in Eq. (32)
\delta_{ij}  Kronecker symbol
\epsilon  dissipation rate of k
\epsilon_e  extra dissipation terms in the k equation
\lambda  Taylor's microscale
\mu  viscosity
\nu  kinematic viscosity
\nu_t  kinematic eddy viscosity
\rho  material density
\sigma\phi, \sigma_q, \sigma_k, \sigma_f, \sigma_e  turbulent Schmidt numbers
\phi  fluctuation of the volume fraction
\Phi  instantaneous (in Eqs. (1)-(5)) or time-mean volume fraction
\omega  frequency of turbulence
\Omega_1  an expression defined in Eq. (27)
\Omega_2  an expression defined in Eq. (28)
\Omega_R  an expression defined in Eq. (29)
Subscripts

1  fluid phase  
2  particle phase  
\( t \)  partial derivative with respect to \( t \)  
\( j \)  partial derivative with respect to \( x_j \)

Superscript

\( \bar{\cdot} \)  time-averaged value  
\( \cdot \)  fluctuating component
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<td>ASST TO THE SECY OF DEFENSE ATOMIC ENERGY</td>
<td>U S ARMY MATERIAL COMMAND</td>
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<td>ATTN: EXECUTIVE ASSISTANT</td>
<td>ATTN: DRCDE-D</td>
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<td>COMMANDER IN CHIEF, ATLANTIC</td>
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<td>ATTN: DRXMR-HH</td>
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<td>DEPARTMENT OF THE NAVY</td>
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<td>OFC OF THE DEPUTY CHIEF OF NAVAL OPS</td>
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<td>STRATEGIC SYSTEMS PROGRAMS (PM-1)</td>
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DEPARTMENT OF THE AIR FORCE (CONTINUED)

AIR FORCE WEAPONS LABORATORY, AFSC
ATTN: NTED J RENICK
ATTN: NTED LT KITCH
ATTN: NTED R HENNY
ATTN: NTEDA
ATTN: NTES
ATTN: SUL

AIR FORCE WRIGHT AERONAUTICAL LAB
ATTN: FIBC
ATTN: FIMG

AIR FORCE WRIGHT AERONAUTICAL LAB
ATTN: AFWAL/MLP
ATTN: AFWAL/MLTM

AIR UNIVERSITY LIBRARY
ATTN: AUL-LSE

BALLISTIC MISSILE OFFICE/DAA
ATTN: CAPT T KING MGEN
ATTN: CC MAJ GEN CASEY
ATTN: ENSN
ATTN: ENSR

DEPUTY CHIEF OF STAFF
ATTN: AF/RDQI

DEPUTY CHIEF OF STAFF
ATTN: AFRDS SPACE SYS & C3 DIR

FOREIGN TECHNOLOGY DIVISION, AFSC
ATTN: SDBG

STRATEGIC AIR COMMAND
ATTN: NRI/STINFO

STRATEGIC AIR COMMAND
ATTN: XPF

STRATEGIC AIR COMMAND
ATTN: XPQ

161 ARG ARIZONA ANG
ATTN: LTCOL SHERER

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ATTN: L-10 H KRUGER
ATTN: L-10 J CAROTHERS
ATTN: L-122 G GOUDREAU

ATTN: L-122 S SACKETT
ATTN: L-203 T BUTKOVICH
ATTN: L-22 D CLARK
ATTN: L-8 P CHRZANOWSKI

LOS ALAMOS NATIONAL LABORATORY
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ATTN: M T SANDFORD
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SANDIA NATIONAL LABORATORIES
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ATTN: ORG 7112 A CHABAI
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DEPARTMENT OF THE INTERIOR
ATTN: D RODDY

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ATTN: C WOLF

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ATTN: H MIRELS

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ATTN: N HIGGINS

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INFORMATION SCIENCE, INC
ATTN: W DUDZIAK

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ATTN: R RUETENIK
ATTN: W LEE

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ATTN: J MURPHY

MERRITT CASES, INC
ATTN: J MERRITT

NEW MEXICO ENGRG RESEARCH INSTITUTE
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ATTN: C K B LEE
2 CYS ATTN: G YEH
ATTN: J LEWIS
ATTN: P RAUSCH
2 CYS ATTN: S ELGHOBASHI

R & D ASSOCIATES
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S-CUBED
ATTN: A WILSON

S-CUBED
ATTN: C NEEDHAM

SCIENCE APPLICATIONS INTL CORP
ATTN: H WILSON

SCIENCE APPLICATIONS INTL CORP
ATTN: J COCKAYNE
ATTN: W LAYSON

SCIENCE APPLICATIONS INTL CORP
ATTN: A MARTELLUCCI

SCIENCE APPLICATIONS INTL CORP
ATTN: G BINNINGER

TRW ELECTRONICS & DEFENSE SECTOR
ATTN: G HULCHER
ATTN: P DAI

WEIDLINGER ASSOC, CONSULTING ENGRG
ATTN: P WEIDLINGER
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