INTRODUCTION

Many papers and reports have been written on the concepts, design and potential uses of lattice structures in outer space. Such structures include large antennae, solar power systems and habitable stations for support of space colonies.

Currently, both deployable and erectable concepts are being investigated for the implementation of lattice structures. Also, investigations of size considerations indicate that small antennae ranging from tens of meters in span to solar power collectors ranging up to several thousand meters have been proposed. Such structural sizes along with stringent operational requirements will require considerable information of dynamics, control, materials, nondestructive evaluation (NDE), environmental effects and wave propagation relating to their design and analysis.

Much has been written on the theoretical aspects of the control of such structures. Also, a large number of vibration analyses have been undertaken. However, despite a distinct recognition of the importance of wave propagation in many of the control, vibration and NDE investigations, virtually nothing can be found on wave propagation in large space structures (LSS). The goals of this program were to initiate and to pursue the development of several aspects of wave propagation analyses in LSS.
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WAVE PROPAGATION
LATTICE STRUCTURES
DYNAMIC FAILURE
LARGE SPACE STRUCTURES
ACCOMPLISHMENTS

A number of theoretical and experimental analyses have been undertaken. Some of these efforts have been documented in reports and some as yet remain undocumented in formal reports. First, the formally documented accomplishments will be described briefly; then the undocumented accomplishments will be listed.

Documented Accomplishments

As indicated above, the focus of our investigations has been the wave propagation of broadband and narrow-band signals in lattice structures for outer space applications. Theoretical and experimental analyses which have resulted in formal reports are listed here.

1. Theoretical formulations of the input-output relations for arbitrary pairs of locations on periodic structures have been initiated. In this formulation, we have used (and are continuing to use) a transfer matrix technique for analyzing periodic structures which can be modeled as one-dimensional continua, two-dimensional rectangular trusses and three-dimensional tetrahedral trusses. The elements in these structures are capable of transmitting longitudinal, shear and flexural waves. Our efforts thus far on this topic have resulted in a report as follows: J.H. Williams, Jr., F.C. Eng and S.S. Lee, "Wave Propagation and Dynamics of Lattice Structures", AFOSR/WEA Report, May 1984. This work is continuing to be actively developed.
2. The dynamic properties of two two-dimensional lattice structures have been investigated both analytically, using the NASTRAN finite element code, and experimentally, using an HP Fourier analyzer. The natural frequencies obtained via the two approaches were compared and shown to agree within seven percent of each other. These results are reported in a document as follows:


Further, in these efforts we have collaborated with colleagues at Stanford University and Virginia Polytechnic Institute and State University. In fact, both 5-bay and 22-bay lattice structures which we have fabricated and tested have been sent to colleagues at Stanford for testing.

3. In order to obtain the wave transmission and reflection coefficients in the joints of structures, we are pursuing both finite element and theoretical elasticity analyses of joints in two-dimensional rectangular lattices. The elements in these structures are capable of conducting longitudinal, shear and flexural waves. The results of these efforts have been documented as follows: J.H. Williams, Jr., R.H. Lailer and S.S. Lee, "Wave Propagation Through 'T' and 'L' Joints", AFOSR/WEA Report, October 1984.

4. In lattice structures where the signal propagation is dominated by longitudinal waves, we hope to develop general closed-form expressions for the input-output relations of arbitrary pairs of locations of the structure. This type of analysis requires
the reckoning of each wave front which leaves joint i and arrives at joint j, having undergone a series of reflections and transmissions at each intervening joint encountered along the way. Such an approach may be called a "wave summation analysis". Our efforts thus far on this topic have resulted in a report as follows: J.H. Williams, Jr. and H.K. Yeung, "Nondispersive Wave Propagation in Periodic Structures", AFOSR/WEA Report, January 1985. This work is continuing to be developed.

5. Most classical and modern control analyses utilize a modal frequency formulation. In very flexible large-scale structures where distinct wave packets can clearly propagate, reflect from boundaries and interact, we are convinced that the consideration of such wave packets in the formulation of a control strategy can be beneficial for some analyses. An introductory discussion and some preliminary analyses supporting this view are presented in the following report: J.H. Williams, Jr., G.A. Norris and S.S. Lee, "Feedforward Control of Waves in Lattice Elements", AFOSR/WEA Report, August 1985.

6. The wave propagation characteristics of a tetrahedral truss model consisting of fiberglass reinforced composite rods and aluminum joints were studied experimentally. A significant observation obtained in this study was the experimental qualitative verification of wave propagation reciprocity between pairs of joints. The details of the fabrication processes, the testing and the qualitative observations are documented in the

7. The wave propagation characteristics of a 5-bay aluminum planar lattice were studied experimentally. Wave propagation speeds and frequency spectra were obtained, and wave propagation reciprocity was observed. Wave propagation attenuation was quantitatively measured and an attenuation parameter expressed on a per-bay basis was defined. These results are reported as follows: J.H. Williams, Jr., J.J. Zhang and S.S. Lee, "Wave Propagation Measurements on Two-Dimensional Lattice", AFOSR/WEA Report, September 1985.

Undocumented Accomplishments

During the past two years, several theoretical efforts have been initiated, but have not yet been formally documented. Most of these investigations will be pursued during a subsequent contract, with several of these undertakings resulting in reports during the coming year. Such investigations include but are not limited to the following:

1. Wave propagation through joints via joint coupling matrices;
2. Wave propagation in LSS via wave-mode coordinates and scattering matrices;
3. Failure propagation and arrest studies of lattice structures; and
4. Failure propagation and arrest studies of orthotropic continuum models of LSS.
SUMMARY

As an overview of both the documented and the undocumented accomplishments of this program, an Appendix containing copies of transparencies which summarize our efforts is provided. (These transparencies were presented during the Third Forum on Large Space Structures sponsored by the AFOSR, which was held at Texas A&M University in July 1985.) These transparencies summarize the major topics we've considered and present some typical results. As accentuated on pages 9 and 27, these transparencies emphasize the fact that Wave Propagation in LSS is at the center of our research program as we attempt (1) to rationalize some of our work with vibration analyses and (2) to illustrate the broad applicability of our work in failure propagation and arrest, NDE, and control of LSS.
APPENDIX *

WAVE PROPAGATION AND DYNAMICS IN LARGE SPACE STRUCTURES (LSS)

James H. Williams, Jr.

(AFOSR - A. K. Amos)

* Because the original transparencies were multicolored, the quality of these copies is relatively poor.
Wave Propagation and Dynamics Studies

Wave Propagation in LSS

- Vibration
- Wave Propagation
- Failure Propagation
- NDE
- Control
Vibration Studies of 2-Dimensional Lattice Structures

Fabrication of 5-Bay and 22-Bay Lattices

Numerical: Mode Shapes and Natural Frequencies

Experimental: Mode Shapes and Natural Frequencies

Experimental: Damping (Stanford)
Typical Mode Shapes for 5-Bay Planar Lattice

5-Bay Planar Lattice Structure

Mode 2
69.2 Hz

Mode 22
945.0 Hz

Mode 36
1300.1 Hz
Frequency Versus Mode Number for 5-Bay Planar Lattice

Measured Results

Finite Element Results
Wave Propagation Studies of 1-Dimensional, 2-Dimensional and 3-Dimensional Lattice Structures

Experimental:
"Reciprocity" in 3-D Trusses

Numerical:
Transmission and Reflection Coeffs. of Joints in Lattice via Finite Element Calculations

Analytical:
Pulse Propagation in 1-D Structure Having Arbitrary Sections via Pulse Summation and Method of Characteristics

Analytical:
Wave Propagation in 1-D, 2-D and 3-D Structures via Transfer Matrices

Analytical:
Wave Propagation Through Joints via Joint Coupling Matrices

Analytical:
Wave Propagation in LSS via Wave-Mode Coordinates & Scattering Matrices
WAVE PROPAGATION IN LSS

Transfer Matrix Method

Consider Harmonic Excitation Frequency \( \omega \).
State Vector \( \mathbf{z} \) at Sections A and B.
Transfer Matrix \( \mathbf{T} \)
\[
\mathbf{z}_B = \mathbf{T} \mathbf{z}_A
\]

1-D Structure
2-D 3-Bay Planar Lattice
3-D Tetrahedral Truss

Frequency Response \( H(\omega) \)
\[
H(\omega) = \frac{\text{Response}}{\text{Excitation}}
\]

Impulse Response \( h(t) \)
\[
h(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} H(\omega) e^{i\omega t} \, d\omega
\]
WAVE PROPAGATION IN LSS

Wave-Mode Coordinates

Joint Coupling Matrix $B$

$$
\begin{bmatrix}
\omega \\
\mathbf{q}
\end{bmatrix}
= B \
\begin{bmatrix}
\mathbf{q}^T \\
\mathbf{v}
\end{bmatrix}
$$

Wave-Mode Coordinates $w$

$$
w = Y^{-1} \mathbf{y}
$$

where

$$
Y^{-1} I Y = W
$$

where $W$ is diagonal.
Then,
\[ w = \begin{bmatrix} w_{in} \\ w_{out} \end{bmatrix} \]

From \( S \),
\[ \begin{bmatrix} c_1 \\ \vdots \\ c_n \end{bmatrix} = S \begin{bmatrix} w_{in}^1 \\ \vdots \\ w_{in}^n \end{bmatrix} + \begin{bmatrix} c_{out}^1 \\ \vdots \\ c_{out}^n \end{bmatrix} \]

\begin{itemize}
  \item \textbf{Scattering Matrix} (Input Generates Output via \( S \))
  \item \textbf{Generation Matrix} (Forces Generate Output via \( S \))
\end{itemize}
Control Studies of Wave Propagation in 1-Dimensional Structures

Analytical:
Control of Wave Propagation in 1-D Structures

Feedback Control

Identification of System Instability

Feedforward Control

Identification of System Instability

Controller Designs to Avoid System Instability

System Transfer Function

System Sensitivity to Controller Error
CONTROL OF WAVE PROPAGATION

Wave Superposition

\[
\sigma(x, t_0) \\
\sigma(x, t_0 + \frac{1}{2} T) \\
\sigma(x, t_0 + T) \\
\sigma(x, t_0 + \frac{3}{2} T) \\
\sigma(x, t_0 + 2T)
\]
Actuator Action
Response of Controlled Substructure to Single Square Wave Input (Feedforward Control)
Schematic of Feedforward Control of Wave Propagation in Structures

Note: Leftwardly-Propagating Waves Generated by Actuator are Neglected by Control System.
Square of Magnitude of Transfer Function
for Feedforward Control of Wave Propagation
in Structures Versus Parameter \( \omega \)
Failure Propagation and Arrest Studies of 2-Dimensional Lattice Structures and Continuum Models of LSS

- Analytical: 2-D Lattice of Beams & Struts
- Analytical: Orthotropic Continuum Models of LSS

Definition of Failure Propagation and Failure Arrest

Failure Propagation and Failure Arrest Conditions

- Failure Arrest Using Deflector Concept
- Failure Arrest Using Arrester Concept

Post-Failure Residual Motion
FAILURE PROPAGATION IN LSS

2-D Planar Lattice Model

Failure Propagation and Failure Arrest Criteria

\[ \dot{G}_d = \frac{dW}{dA} - \frac{dV}{dA} - \frac{dT}{dA} \]

\[ \dot{G}_d = R \rightarrow \text{Failure Propagation} \]

\[ \dot{G}_a < R \rightarrow \text{Failure Arrest} \]

where \( \dot{G}_d = \text{Dynamic Energy Release Rate} \)
\( R = \text{Dynamic Failure Toughness} \)
Criteria for Failure Propagation and Failure Arrest

Dynamic Failure Toughness \( R \)

No Failure Propagation

Failure Propagation with Failure Arrest when First Reflection Occurs

Failure Tip

Supersonic Failure Propagation with No Arrest

No Failure Propagation

Initial Static Energy Release Rate \( G_{Hq}^2 \)
Location of Failure Arrest

Nondimensional Failure Speed $\frac{\dot{L}}{C_s}$

Failure Arrest Location $n^*$

- $\frac{L_c}{d} = 100$
- $\frac{L_c}{d} = 50$
- $\frac{L_c}{d} = 20$
- $\frac{L_c}{d} = 5$
- $\frac{L_c}{d} = 1$
Wave Propagation and Dynamics Studies

Wave Propagation in LSS

- Vibration
- Wave Propagation
- Failure Propagation
- NDE
- Control
END

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