MEASUREMENT OF VERY SMALL DISPLACEMENTS WITH A MODIFIED MICHELSON INTERFEROMETER (U)
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by

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Radar ESM Section
Electronic Warfare Division

DEFENCE RESEARCH ESTABLISHMENT OTTAWA
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ABSTRACT

A modified Michelson interferometer was built and its performance evaluated for the measurement of very small displacements. The instrument was assembled from components already available in the laboratory and suggestions are made for an improved version.

RÉSUMÉ

Un interféromètre de Michelson modifié a été construit et on a évalué ses performances pour la mesure de très petits déplacements. L'instrument fut monté à partir de pièces disponibles dans le laboratoire et des suggestions sont présentées pour la réalisation d'une version améliorée.
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LIST OF ABBREVIATIONS

SMI: Standard Michelson Interferometer
MMI: Modified Michelson Interferometer
M1: Mirror
M2: Mirror of the wedge
M3: Mirror of the wedge
BS: beamsplitter
L4: laser
MO: microscope objective
P: plane of observation of the fringes
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INTRODUCTION

The design of an illumination system for a multichannel Bragg cell being fabricated by Marconi went through many iterations before reaching its final state. At one particular stage in the project, the implementation of the design required four vertical translations of a holographic plate (see Fig. 1) with less than \( \lambda/20 \) (\( \lambda = 633 \text{ nm} \)) horizontal spurious motion in order to preserve the phase relationship between the five illumination channels (one stationary image and four translations). Measuring displacements of the order of 30 nm is not trivial and the particular design idea required an investigation of some possible techniques to achieve such precise measurements. It was decided to use a very elegant concept recently proposed by Chandra and Rhode where a Michelson interferometer is modified in such a way that small displacements can be measured to a precision of \( \lambda/70 \). We designed and constructed our own version of a Modified Michelson Interferometer (MMI) and then we carried out measurements to evaluate the accuracy attainable and to identify factors which limited its performance.

2.0 MEASUREMENT OF SMALL DISPLACEMENTS WITH A STANDARD MICHELSON INTERFEROMETER (SMI)

A Michelson interferometer (see Fig. 2) is often used to monitor small displacements or vibrations. The interferometer consists of a beam splitter (BS) that divides the incoming coherent light into two perpendicular paths. Mirrors (M1, M2) are placed at the end of each arm of equal length of the interferometer to reflect the light. The light reflected from the two mirrors is mixed by the beam splitter and produces an interference pattern at the photodetector, but only if the difference in optical pathlength between the two arms of the interferometer is less than the coherence length of the illuminating light source. When a point source is used as an input to the interferometer, the two diverging beams produce at the output a set of concentric rings. The state of the center of the pattern depends on the relative phase difference between the two output beams mixed by the beam-splitter, assuming the mirrors M1, M2, and the beamsplitter have flatness better than \( \lambda/10 \) (see Fig. 3). If the beams are in phase, the center of the pattern is bright, if the beams have 180° phase difference, the center of the pattern will be dark. When a Michelson interferometer is used to measure small displacements, the length of one arm is kept constant and the light coming from that arm via M1 is used as a reference. The length of the second arm is allowed to change either by having the vibrating or moving surface attached to M2. The movement of the object under test will then produce a succession of dark and bright centers in the output fringe pattern. Each transition from a dark to a bright center corresponds to a movement of \( \lambda/4 \) of the object. Proper data processing of the brightness measurement of the central fringe may give a \( \lambda/20 \) precision on the displacement [ref. 2]. When no special signal processing is used, a precision
FIGURE 1  REQUIRED TRANSLATION OF A HOLOGRAPHIC PLATE
FIGURE 2  MICHELSON INTERFEROMETER
FIGURE 3  TYPICAL OUTPUT FROM A MICHELSON INTERFEROMETER

a) Output beams in phase;
b) Output beams with a phase difference of 180°
of \(\lambda/8\) is a maximum reasonable value that falls slightly short of our requirement. Even the \(\lambda/20\) precision obtained with signal processing is somewhat unsatisfactory because it is never good practice to operate at the very limit of a measuring instrument.

3.0 MEASUREMENT OF VERY SMALL DISPLACEMENTS WITH A MODIFIED MICHELSON INTERFEROMETER (MMI)

It is possible to improve on the precision of the displacement measured by a Michelson interferometer. S. Chandra and R.S. Rohde recently published a method to "amplify" the motion of the moving mirror, [ref. 1]. The technique consists of adding to the mirror M2 assembly of Fig. 2, a second mirror M3 (see Fig. 4). M3 is fixed and the other M2 moving, in such a way that the light will bounce \(N\) times (see Fig. 5) between the mirrors before being reflected back. The motion of the moving mirror M2 is then amplified by a factor \(N\).

It is interesting to note from the analysis made by Chandra and Rohde that the incidence angle \(\alpha\) is related to the wedge angle \(\theta\) between the two mirrors by the equation (see Fig. 5):

\[
\alpha = (N-1)\theta
\]  

(1)

where \(N\) is the number of reflections on M2. The displacement sensitivity, \(S\), is then defined as the actual displacement of the mirror M2 required to produce one complete cycle at the detector and is given for small angles of incidence by:

\[
S = \frac{\lambda}{2nN}
\]  

(2)

where \(n\) is the index of refraction of the medium in which the light is propagating. For \(N = 1\), equation 2 becomes the expression for the sensitivity for a Standard Michelson Interferometer. Curves of the ratio of the displacement sensitivity \(S\) for a Modified Michelson Interferometer to the displacement sensitivity for a SMI as a function of the angle of incidence \(\alpha\) are given in Fig. 6. These curves indicate a possible improvement of the sensitivity by a factor of 100 of a MMI over a SMI. Chandra and Rohde have experimentally achieved a sensitivity of 1 cycle/(\(\lambda/72\)). For illumination at the wavelength of a He-Ne laser (6328\(\AA\)), one cycle of fringe is generated by a displacement of 88\(\AA\).

4.0 IMPLEMENTATION OF A MODIFIED MICHELSON INTERFEROMETER AT DREO

The (MMI) described in ref. 1 (see Fig. 4) was built using a point source produced by focusing a He-Ne laser beam using a low power microscope objective. Figure 7 shows the experimental arrangement. The mirrors were polished flat to \(\lambda/10\) in order to avoid unnecessary distortion of the wavefront of the
FIGURE 4  MODIFIED MICHELSON INTERFEROMETER
(from ref.2, p.1533)
FIGURE 5  MULTIPLE REFLECTIONS WITHIN THE MIRROR WEDGE IN A
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FIGURE 7  PHOTOGRAPH OF THE MODIFIED MICHELSON
INTERFEROMETER BUILT AT DREO
observed fringe pattern. The moving mirror M2 was rigidly fixed to a piezo-electric displacement unit. The set-up was extremely sensitive to vibrations and was isolated with an air supported table. The measurement of the angle and $\theta$ was the major limitation of the experiment. The setting of these angles was difficult because of the close proximity and relative sizes of M2 and M3. The interferometer was successively set for $N = 2, 4$ and 7 with $\theta$ around $5^\circ$ and $\alpha$ varying from $20^\circ$ to $30^\circ$. The displacement sensitivities $S$ thus achieved were $1 \text{ cycle} / (\lambda/4)$, $1 \text{ cycle} / (\lambda/8)$ and $1 \text{ cycle} / (\lambda/14)$ respectively. It is easy to determine when a change of half a cycle has occurred from the interference pattern at the output of the interferometer (see Fig.3), so, a sensitivity of $1 \text{ cycle} / (\lambda/14)$ is more than good enough for our present purpose that is to measure displacements of the order of $\lambda/20$ (316Å). However, a sensitivity of $1 \text{ cycle} / (\lambda/72)$ was demonstrated by Chandra and Rohde [ref.1] but getting a better than $1 \text{ cycle} / (\lambda/14)$ sensitivity in our system would necessitate substantial investment in custom made equipment. So we decided to stop the development of our MMI at that point.

5.0 CONCLUSIONS: KEY FACTORS IN THE CONSTRUCTION OF AN IMPROVED VERSION

A MMI with a sensitivity $1 \text{ cycle} / (\lambda/14)$ sufficient to meet our requirements was built. The experience acquired in the implementation of a MMI allowed us to identify two areas for improvement if better sensitivity is to be achieved. First, higher quality mirrors should be used for M2 and M3 (at least polished to $\lambda/20$) in order to avoid noticeable wavefront distortion of the output fringes since any irregularities of the surface of M2 and M3 are magnified by a factor $N$. Second, a special holder should be designed for M2 and M3 which would permit precise setting of $\alpha$ and $\theta$ and the alignment of M2 and M3.

6.0 REFERENCES


A modified Michelson interferometer was built and its performance evaluated for the measurement of very small displacements. The instrument was assembled from components already available in the laboratory and suggestions are made for an improved version.
MICHELSON INTERFEROMETER
MEASUREMENT OF SMALL DISPLACEMENTS
INTERFEROMETER

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