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Heliocentric Radius of the Cosmic Ray Modulation Boundary

by

B. A. RANDALL and J. A. VAN ALLEN

Department of Physics and Astronomy
The University of Iowa
Iowa City, Iowa 52242

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ABSTRACT

We have made a semi-empirical analysis of our extensive body of observed cosmic ray intensity data from Pioneers 10 and 11 and related spectral information from other authors in order to infer the radius R of the modulation region surrounding the sun. During the period 1972-1985 the inferred values of R vary with time systematically and in a manner generally similar to that of sunspot numbers. The range of values of R is from 42 AU at the time of minimum solar activity (∼1976) to 88 AU about 1.5 yr following the time of maximum solar activity (∼1980).

A specific, testable prediction is that Pioneer 10 will cross the modulation boundary in 1988 and will remain in its vicinity for several years thereafter.
INTRODUCTION

One of the classical problems of heliospheric physics is the observational determination of the boundary of the region around the sun within which the 11-year periodic modulation of galactic cosmic ray intensity is caused by the entrained magnetic field in the outflowing solar wind. This boundary has not yet been encountered by Pioneer 10 at its present heliocentric distance $r$ of 38 AU in the solar antipex direction nor by Pioneer 11 at 21 AU in the solar apex direction.

The scale size of the region is the reciprocal of the observed mean radial gradient of intensity, namely 50 AU. Crude physical arguments suggest that the radius $R$ of the modulation boundary lies between 50 and 100 AU [Van Allen and Randall, 1985]. Webber and Lockwood [1981] estimate that $R > 65$ AU; and McKibben et al. [1982] estimate that $R$ lies in the range 50 to 70 AU at both solar maximum and solar minimum. Both groups of authors use observed interplanetary data, modulation theory, and an assumed interstellar energy spectrum as the basis for their estimates. Kurth et al. [1984] have interpreted a distinctive episode (1983-84) of interplanetary radio emission in the frequency range 2 to 3 kHz as suggesting that the heliopause
(not accurately identical to the modulation boundary) lies at \( R \approx 46 \) AU.

The new aspect of the problem that is exploited herein is the recent availability of an extensive body of cosmic ray intensity data \( (E_p > 80 \text{ MeV/nucleon}) \) from Pioneers 10 and 11 over a complete cycle of solar activity and to great distances from the sun [Van Allen and Randall, 1985]. The intensity is a function of both radial distance \( r \) and phase of the solar activity cycle (as well as of sporadic solar activity) (Figure 1). Gleeson and Axford [1967, 1968] have shown that the convection-diffusion model of cosmic ray modulation [Parker, 1963, 1965] yields, under certain simplifying assumptions, a one-parameter characterization of the relationship of the differential intensity of a particular species of particle at infinity (i.e., in the interstellar medium) to that at an observing point interior to the modulation boundary [cf. Ehmert, 1960, and Freier and Waddington, 1965].

We use this approximate formulation together with our integral intensity data and spectral data from other authors to make speculative estimates of the heliocentric radius of the boundary of the modulation region as a function of phase of the solar cycle.
DEMODULATION MODEL

According to the formalism of the Gleeson-Axford theory [see also Goldstein et al., 1970], the effective change in energy of a charged particle that moves from a source point \( P \) on the boundary along a dynamical trajectory to an interior point \( Q \) is

\[-Z\Phi,\] irrespective of its path from \( P \) to \( Q \); that is, a particle whose observed kinetic energy at \( Q \) is \( T' \) must have had a kinetic energy \( (T' + Z\Phi) \) at \( P \). The unidirectional differential (in energy) intensity \( j'_Q(T') \) at \( Q \) along a dynamical trajectory leading from \( P \) to \( Q \) is related to that at \( P \) (in a quasi-steady state) by

\[
j'_Q(T') = \frac{j'_p(T' + Z\Phi)}{T'(T' + 2Amc^2)} \cdot \frac{T'(T' + 2Amc^2)}{(T' + Z\Phi)(T' + Z\Phi + 2Amc^2)} \tag{1}
\]

where \( A \) is the atomic mass number of a particular species of energetic ion, \( Z \) is its charge, \( mc^2 \) is the rest energy of the proton, \( T' \) is the kinetic energy of the ion at the observing point \( Q \) and \( (T' + Z\Phi) \) is its kinetic energy at the boundary. (Note that \( \Phi \) is positive for ions and negative for electrons.)
Denote the energy per nucleon by $T$ such that $T' = AT$. Then (1) becomes

$$\frac{J_Q(T)}{T(T + 2mc^2)} = \frac{J_P(T + \frac{Ze\phi}{A})}{(T + \frac{Ze\phi}{A})(T + \frac{Ze\phi}{A} + 2mc^2)}$$

In the present study we use observed data only for protons ($Z/A = 1$) and alpha particles ($Z/A = 0.5$).
DIFFERENTIAL SPECTRA

The high energy (T > 30,000 MeV/nucleon) portions of the differential spectra of primary protons and alpha particles at 1 AU [Meyer, 1969] [Ryan et al., 1972] [Fulks, 1975] can be well represented by:

\[ J(T) = \frac{K}{(T + mc^2)^\gamma} \]  \hspace{1cm} (3)

with spectral index \( \gamma = 2.7 \).

We assume that the spectral form of (3) is valid down to \( T = 80 \) MeV/nucleon in the nearby interstellar medium. This assumption is plausible and there is no present evidence against it but it must be regarded as tentative and may later be found to be wrong.

Then combining (2) and (3) and denoting \( Ze\Phi/A \) by \( \varphi \), we have the following two working formulae for protons and alpha particles:

\[ J_{p,\alpha}(T) = \frac{T(T + 2mc^2)}{(T + \varphi_{p,\alpha})(T + \varphi_{p,\alpha} + 2mc^2)} \frac{K_{p,\alpha}}{(T + \varphi_{p,\alpha} + mc^2)^\gamma_{p,\alpha}} \]  \hspace{1cm} (4)
In (4), \( j(T) \) is the unidirectional spectral intensity at the observing point \( Q \), \( T \) denotes kinetic energy per nucleon, and \( \varphi_p = 2\varphi_\alpha \).

We first made least squares fits to available 1 AU spectral data [previous references] for protons over the energy range \( 10^2 < T < 10^6 \text{ MeV/nucleon} \) using a logarithmic version of equation (4) in order to find \( K_p, \gamma_p, \) and \( \varphi_p \). Only \( \varphi_p \) was allowed to be a function of epoch. Taking \( \varphi_\alpha = \varphi_p/2 \) at each epoch, we then made a corresponding treatment of the 1 AU spectral data for alpha particles to obtain \( K_\alpha \) and \( \gamma_\alpha \). The values of the four time-independent parameters in (4), as so determined, are:

\[
K_p = 1.99 \times 10^5, \quad \gamma_p = 2.71, \quad K_\alpha = 4.27 \times 10^4, \quad \gamma_\alpha = 2.84
\]

when \( j(T) \) is measured in \( \text{(cm}^2 \text{ sec sr MeV/n)}^{-1} \).

Then adopting these values of \( K_p, K_\alpha, \gamma_p, \) and \( \gamma_\alpha \), we found \( \varphi_p \) and \( \varphi_\alpha \) separately for 17 sets of proton and alpha particle spectra at various epochs and radial distances as observed by McDonald and collaborators with particle spectrometers on Pioneers 10 and 11, Helios 1 and 2, ISEE 3, and Voyager 1 and reported in unpublished memoranda [McDonald, 1982] [McDonald et al., 1984]. The portions of the spectra below our integration threshold, 80 MeV/nucleon, (i.e., the anomalous component) were ignored.
The most directly useful sets of the spectral data of McDonald et al. are the six sets obtained with Pioneer 10 because these observations were made on the same spacecraft as were ours. Numerical data for these six cases are given in Table 1. The fitted values of $\varphi_p$ and $\varphi_\alpha$ are given in columns 3 and 4 and the corresponding integral omnidirectional intensities $J_o (T > 80 \text{ MeV/nucleon})$ (sum of protons and alpha particles), in column 5. The mean counting rates of our detector C on Pioneer 10 for the same epochs and mean radial distances are given in column 6; and the apparent omnidirectional geometric factors $g$ of our detector (ratio of corresponding entries in columns 6 and 5), in column 7. The essential constancy of $g$ supports the empirical usefulness of formulae (4), irrespective of the physical validity of their derivation.

Figure 2 displays the spectral data for the 2nd, 4th, and 6th cases in Table 1 and exhibits the quality of the fits.
USE OF INTEGRAL DATA FROM PIONEERS 10 AND 11 TO INFERR

A comprehensive body of omnidirectional integral intensities of cosmic rays for $T > 80$ MeV/nucleon is provided by the University of Iowa detectors on Pioneers 10 and 11 for the period 1972-85 [Van Allen and Randall, 1985]. A condensed example of these data for a matched pair of detectors (detectors C) on these two spacecraft is given in Figure 1. The two principal plots in this figure show, on a semilogarithmic scale of ordinates, the counting rates of these two detectors as a function of $r$. A straight line connecting two corresponding points (i.e., after allowance for solar wind propagation delay) on the Pioneer 10 and Pioneer 11 curves establishes the apparent radial gradient of intensity. Previously published results [Van Allen and Randall, 1985] of this procedure are reproduced as Figure 3. The insert in the lower right of Figure 1 shows the time dependence of the cosmic ray intensity as measured by IMP-8 at 1 AU (courtesy S. M. Krimigis and R. B. Decker).

The counting rates $C$ of our detectors are proportional to the sum of the omnidirectional intensities of protons and alpha particles. A consistent normalization of our counting rates to the integrals of the six available Pioneer 10 spectra of McDonald
et al. (at various epochs) is obtained [Table 1] by taking our
detectors C on Pioneers 10 and 11 to have an omnidirectional
geometric factor g of 0.157 cm², about 29% greater than our
previous best estimate of 0.122 cm² [Van Allen et al., 1980].
This discrepancy is not understood but it is not unreasonable
because it encompasses the uncertainties of absolute calibrations
of both the GSFC and Iowa detectors. Adopting the normalized
value of g, we integrated formulae (4) numerically from 80
MeV/nucleon to infinity to find "predicted" values of our
counting rate C as a function of φp. An abridged summary is
given in Table 2.

In particular, the value of C for φp = φα = 0, namely 1.83
count/sec, is taken to be C(R), i.e., the interstellar value.
This value is represented by the horizontal line near the top of
Figure 1.

Observed values of C as a function of time and radial
distance r were taken from our previous work [Van Allen and
Randall, 1985] and updates thereof. Further in accordance with
that work, we next assumed that the radius R of the modulation
region is given by

\[ R = r - \frac{1}{G} \ln \frac{C(r)}{C(R)} \]  

(5)
where $G$ is the observed mean integral radial gradient, having the value 0.0207 (AU)$^{-1}$. The graphical significance of (5) is evident in Figure 1. We adopt $C(R) = 1.83$ count/sec per Table 2. Values of $R$ as thus computed are plotted in Figure 4 for the recent solar cycle.

The inferred values of $R$ vary with time systematically and range from 42 AU at the time of minimum solar activity (~1976) to 88 AU about 1.5 yr following the time of maximum solar activity (~1980). Also shown in Figure 4 is a plot of monthly mean sunspot numbers [Solar-Geophysical Data, 1972-1986]. The general correlation between $R$ and the sunspot number is striking, though at the greater radial distances $R$ lags the sunspot number by about 1.5 year [cf. Van Allen and Randall, 1985].

Figure 4 presents the principal results of this paper.
DISCUSSION

We consider that a loose relationship to convection-diffusion theory, as sketched in the introduction, is a desirable aspect of our exposition. But such a relationship is not essential to our analysis.

Indeed, the analysis can be framed in purely empirical terms as follows:

(a) Each of formulae (4) can be regarded as an empirical one with one (positive) parameter $\varphi$, whose value is a function of radial distance and phase of the solar activity cycle and is zero at the modulation boundary.

(b) By fitting formulae (4) to observed differential spectra of protons and alpha particles at 1 AU and at various epochs, with the assumption that $\varphi_p = 2\varphi_\alpha$, values of $K_p$, $K_\alpha$, $\gamma_p$, and $\gamma_\alpha$ are found. (The overall results are insensitive to the assumption that $\varphi_p = 2\varphi_\alpha$ and are only slightly changed if the alpha particle component is ignored.)

(c) With the time-independent values of $K_p$, $K_\alpha$, $\gamma_p$, and $\gamma_\alpha$ from step (b), values of $\varphi_p$ and $\varphi_\alpha$ are found for a variety of independently observed spectra at various epochs and radial distances.
(d) As \( \varphi_p \) and \( \varphi_\alpha \) tend toward zero, formulae (4) reproduce the high energy portions of observed differential spectra as represented by formula (3).

(e) Using values of the spectral parameters for six Pioneer 10 cases, we find that the ratios of the integrals of formulae (4) to our observed integral omnidirectional intensities \( C \) are essentially constant [Table 1] over a range of a factor of two in \( C \).

(f) We then use the corresponding value of \( C(R) \) [Table 2] to infer values of \( R \) by way of equation (5), adopting the previously determined mean value of the radial gradient \( G \).

(g) An essential ad hoc assumption is that formula (3) is a valid representation of the interstellar spectra of protons and alpha particles down to \( T = 80 \) MeV/nucleon. If this assumption is wrong then the horizontal line in the upper portion of Figure 1 must be either raised or lowered as appropriate and our inferred values of \( R \) will be increased or decreased in a corresponding way.

Our use of a single mean value for the radial gradient is, of course, subject to criticism. It is clear from Figure 3 that piecewise fits to short portions of the body of data shown there yield a substantial spread in values of \( G \), including negative values. Also, a preliminary analysis of Pioneer 10/11 data for 1984-1986 yields \( G \approx 0.013 \) AU\(^{-1} \). (As with our previous
determinations of $G$, this analysis assumes cylindrical symmetry around the polar axis of the ecliptic plane.) The physical reason or reasons for short-term fluctuation in the apparent radial gradient are not known and we have chosen to use the mean value in equation (5) as appropriate to this paper, whose intent is heuristic and not definitive. The consequences of using different values of $G$ can be visualized with the help of time-labeled plots such as those in Figure 1.

The smooth curve in the lower portion of Figure 4 shows the radial distance of Pioneer 10 as a function of time. A specific, testable prediction of our work is that Pioneer 10 will reach the modulation boundary in 1988 and will remain in its vicinity for several years. A secondary prediction is that $C$ will increase from its early 1986 value of 1.0 to 1.83 count/sec in 1988.
ACKNOWLEDGMENTS

This work has been supported by Ames Research Center/NASA contract NAS2-11125, Office of Naval Research contract N00014-85-K-0404, and National Aeronautics and Space Administration grant NGL 16-001-002, all of which are gratefully acknowledged. We are indebted to F. B. McDonald and collaborators for the spectral data used in our analysis. The IMP-8 data in Figure 1 were kindly supplied by S. M. Krimigis and R. B. Decker of the Applied Physics Laboratory of Johns Hopkins University. We also thank the referees of a preliminary version of this paper for scathing, though quite valuable, criticism.
Table 1

Comparison of Integrals of Pioneer 10 Differential Spectra of McDonald et al. to Our Observed Counting Rates

<table>
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<tr>
<th>Epoch</th>
<th>Mean Radial Distance</th>
<th>( \Phi_p ) (MeV/n)</th>
<th>( \Phi_{\alpha} ) (MeV/n)</th>
<th>( J_0 = 4\pi \int_0^\infty j , dT ) ( (\text{cm}^2 , \text{sec})^{-1} )</th>
<th>( C ) ( (\text{sec})^{-1} )</th>
<th>( C/J_0 = g ) ( \text{cm}^2 )</th>
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<td>3/1/77-6/15/77</td>
<td>AU 12.8</td>
<td>181 ± 10</td>
<td>80 ± 6</td>
<td>6.34 ± 0.15</td>
<td>0.97 ± 0.01</td>
<td>0.153</td>
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<tr>
<td>11/12/77-2/4/78</td>
<td>14.7</td>
<td>162 ± 9</td>
<td>78 ± 10</td>
<td>6.66 ± 0.18</td>
<td>0.99 ± 0.02</td>
<td>0.149</td>
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<tr>
<td>11/15/78-3/2/79</td>
<td>17.7</td>
<td>209 ± 8</td>
<td>104 ± 12</td>
<td>5.89 ± 0.14</td>
<td>0.88 ± 0.02</td>
<td>0.149</td>
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<tr>
<td>1/1/80-6/2/80</td>
<td>21.1</td>
<td>327 ± 19</td>
<td>164 ± 4</td>
<td>4.48 ± 0.17</td>
<td>0.70 ± 0.01</td>
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<tr>
<td>2/10/81-5/2/81</td>
<td>24.0</td>
<td>544 ± 15</td>
<td>318 ± 26</td>
<td>2.96 ± 0.09</td>
<td>0.50 ± 0.01</td>
<td>0.169</td>
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<tr>
<td>11/1/82-1/31/83</td>
<td>28.9</td>
<td>497 ± 13</td>
<td>281 ± 17</td>
<td>3.21 ± 0.08</td>
<td>0.53 ± 0.01</td>
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Table 2

"Predicted" Counting Rates
(count/sec)

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<tr>
<th>$\phi_p$ (MeV/n)</th>
<th>$C_p$</th>
<th>$C_\alpha$</th>
<th>$C = (C_p + C_\alpha)$</th>
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<td>0</td>
<td>1.69</td>
<td>0.13</td>
<td>1.83</td>
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<td>50</td>
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<td>1000</td>
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REFERENCES

Ehmert, A., Electric field modulation, pp. 1000-1008 of

Freier, P. S., and C. J. Waddington, The helium nuclei of
the primary cosmic radiation as studied over a solar
cycle of activity, interpreted in terms of electric
field modulation, Space Science Reviews, 4,

Fulks, G. J., Solar modulation of galactic cosmic ray electrons,

Gleeson, L. J., and W. I. Axford, Cosmic rays in the inter-

Gleeson, L. J., and W. I. Axford, Solar modulation of

Goldstein, M. L., L. A. Fisk, and R. Ramaty, Energy loss of
cosmic rays in the interplanetary medium, Phys. Rev.

Kurth, W. S., D. A. Gurnett, F. L. Scarf, and R. L. Poynter,
Detection of a radio emission at 3 kHz in the outer helio-


FIGURE CAPTIONS

Figure 1. The two principal curves give the counting rates of detectors C on Pioneers 10 and 11 as a function of radial distance, with the phase of the solar activity cycle as the implicit variable. The two slanting straight lines that bound these curves are drawn for a radial gradient of 0.0207 (AU)^{-1}. The horizontal line near the top of the figure at 1.83 count/sec represents our inferred value of the interstellar counting rate. In the lower right of the figure is a plot of the time-dependence of the cosmic ray intensity as measured by IMP-8 at 1 AU [courtesy of S. M. Krimigis and R. B. Decker].

Figure 2. Spectral data for protons and alpha particles for the second, fourth, and sixth cases of Table 1, all from Pioneer 10 [McDonald et al.]. The smooth curves are our least squares fits of formulae (4) as described in the text. The two dashed curves represent asymptotic spectra for \( \varphi_p = 0 \) and \( \varphi_\alpha = 0 \), respectively, i.e., in interstellar space.
Figure 3. A sample plot from Van Allen and Randall [1985] illustrating our determination of the mean radial gradient of cosmic ray intensity using data from Pioneers 10 and 11.

Figure 4. The central portion of the figure shows inferred values of the heliocentric radius of the cosmic ray modulation boundary in astronomical units (AU) over the recent solar activity cycle. The upper portion gives a plot of monthly mean sunspot numbers for the same period. The smooth curve in the lower portion gives the heliocentric radial distance of Pioneer 10.
Figure 2
END

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