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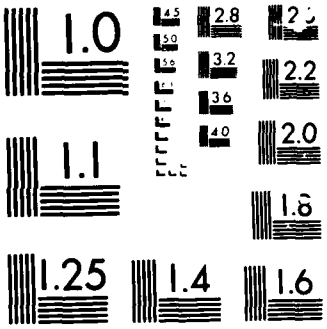
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Interpretation of Scientific or Mathematical Concepts: Cognitive Issues and Instructional Implications

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Scientific and mathematical concepts are significantly different from everyday concepts and are notoriously difficult to learn. It is shown that particular instances of such concepts can be identified or generated by different possible modes of concept interpretation. Some of these modes use formally explicit knowledge and thought processes, others rely on various kinds of compiled knowledge. Each mode has distinctive consequences in terms of attainable precision, likely errors, and ease of use. A combination of such modes can be exploited to formulate an "ideal" model for interpreting scientific concepts so as to achieve both reliable scientific effectiveness and cognitive processing efficiency. This model can be compared with the actual concept interpretations of expert scientists or novice students. All the preceding remarks are illustrated in the specific case of the physics concept "acceleration." The discussion elucidates both cognitive and metacognitive reasons why the learning of scientific or mathematical concepts is difficult. It also suggests instructional methods for teaching such concepts more effectively.

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Interpretation of Scientific or Mathematical Concepts: Cognitive Issues and Instructional Implications

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ABSTRACT

Scientific and mathematical concepts are significantly different from everyday concepts and are notoriously difficult to learn. It is shown that particular instances of such concepts can be identified or generated by different possible modes of concept interpretation. Some of these modes use formally explicit knowledge and thought processes, others rely on various kinds of compiled knowledge. Each mode has distinctive consequences in terms of attainable precision, likely errors, and ease of use. A combination of such modes can be exploited to formulate an "ideal" model for interpreting scientific concepts so as to achieve both reliable scientific effectiveness and cognitive processing efficiency. This model can be compared with the actual concept interpretations of expert scientists or novice students. All the preceding remarks are illustrated in the specific case of the physics concept "acceleration". The discussion elucidates both cognitive and metacognitive reasons why the learning of scientific or mathematical concepts is difficult. It also suggests instructional methods for teaching such concepts more effectively.

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1. COGNITIVE PROCESSING OF SCIENTIFIC CONCEPTS

1.1 Interest of Cognitive Studies of Scientific Concepts

There are several reasons why it is both intellectually interesting and practically important to achieve an improved understanding of the underlying thought processes involved in using and learning scientific concepts. (Unless stated otherwise, the following discussion uses the phrase "scientific concepts" to include also mathematical concepts.)

Scientific Importance. Numerous special concepts (such as "acceleration", "force", "electric field", "derivative", ...) are introduced in science or mathematics as the basic building blocks of conceptual structures designed to explain or predict a wide range of phenomena. Such basic concepts play, therefore, a crucially important role in all of science. In particular, the ability to interpret and use such concepts is an essential prerequisite for solving problems in scientific fields.

Indeed, many students' major problem-solving difficulties can be traced to deficiencies in their abilities to interpret the needed basic concepts. Unless adequately remedied, such conceptual deficiencies remain a bottleneck vitiating attempts to teach students useful general strategies for solving scientific or mathematical problems (Reif & Heller, 1982; Reif 1983; Schoenfeld, 1985).

Cognitive Interest. The thought processes required to interpret scientific concepts are significantly different from those needed to deal with everyday concepts. They also involve particular complexities and considerable amounts of special ancillary knowledge (Reif, 1985). Hence it is of interest to understand these thought processes in greater detail and to elucidate scientists' knowledge that is often largely tacit.

Educational Importance. Science and mathematics courses, at the pre-college or college level, devote a large fraction of their time teaching basic concepts. (Introductory courses, in particular, often introduce many new concepts in quick succession and spend relatively little time applying them extensively.) Such courses have well-deserved reputations of being "tough" since many students experience considerable difficulties learning the basic concepts of mathematics, physics, chemistry, and similar quantitative sciences. Instructors and textbook authors face thus formidable challenges.

There is considerable evidence that these challenges are not being adequately met and that common teaching methods are often far less effective and efficient than one might naively hope. Students' acquired knowledge of scientific concepts remains all too frequently diffuse and largely nominal, so that students are unable to interpret and apply these concepts flexibly in various contexts. Indeed, numerous recent investigations have revealed that many students, despite seemingly good performance in their prior science courses, often exhibit pre-scientific conceptions or gross scientific misconceptions, even

when dealing with quite elementary situations (Caramazza, McCloskey, & Green, 1981; Clement, 1982; Cohen, Eylon, & Ganiel, 1983; Halloun & Hestenes, 1985 a, 1985 b; McCloskey, Caramazza, & Green, 1980; McDermott, 1984; Trowbridge & McDermott, 1980, 1981; Viennot, 1979).

Limitations of past work. The last several years have seen many investigations of students' naive conceptions, preconceptions, and misconceptions in various scientific domains (such as mechanics, electricity, heat and temperature, ...). These investigations include those cited in the preceding paragraph, some books (Driver, Guesne, & Tiberghien, 1985; West & Pines, 1985; Lesh & Landau, 1984), and work reported at several international conferences (Helms & Novak, 1983; Centre National de la Recherche Scientifique, 1984). Hence there now exist rich data about students' interpretation and use of various scientific concepts.

However, with a few exceptions (e.g., diSessa, 1983; Reif, 1985), most of this work is descriptive rather than analytical. Thus it yields only relatively few theoretical insights into the underlying thought processes accounting for the observed results. Correspondingly, it provides also rather little specific guidance about how to teach scientific concepts more effectively.

1.2 Scope of Present Work

The work discussed in this paper is an attempt to transcend some of the limitations of past work by focusing less on what particular conceptions or misconceptions are exhibited by students or experts, and more on how scientific concepts are interpreted and used. In particular, the central interest is to understand better the underlying thought processes and forms of knowledge used to interpret and apply scientific concepts — and then to exploit the resulting insights to design principled instructional methods for teaching scientific concepts more effectively.

To limit the scope of this undertaking to manageable proportions, the present study focuses on the following kinds of concepts and uses of them:

Concepts of interest. The concepts of primary interest are basic scientific or mathematical concepts of the kind commonly taught in high-school or college courses. These concepts are either entities or properties.

Entity concepts denote either particular entities or, more commonly, generic entities (i.e., "variables"). For example, "the sun" is a particular entity, while "a triangle is a generic entity (i.e., any member of the class of three-sided polygons).

Property concepts are generic concepts which are used to describe other concepts (and are thus functions of these other concepts considered as independent variables). For example, "area" is a property concept describing the independent variable "surface" [i.e., a number, with units of $(\text{length})^2$, is

associated with every surface]. "Ratio" is a property describing two independent variables ("numerator" and "denominator") having numerical values. "Acceleration" is a property describing three independent variables, i.e., a "particle", a "reference frame" (needed for the specification of position), and the "time". Such property concepts are more complex than entity concepts since they describe functional relationships. But, for that very reason, they are also most important in all scientific work.

Concept uses of interest. The most fundamental use of a concept is the "basic interpretation" needed to identify or construct particular instances of the concept in various possible situations. The central interest in this paper is focused on the underlying knowledge and thought processes needed to perform such interpretation tasks.

Examples of concept identification tasks include the following: For an entity, like "triangle", identifying whether a particular diagram represents a triangle or not. For a property, like "acceleration", determining whether a specified value of the acceleration of an object has been properly identified or not.

Examples of concept construction tasks include the following: For an entity, like "triangle", drawing a particular triangle. For a property, like "acceleration", finding the value of the acceleration of a car moving in a specified way.

The discussion of this paper deals primarily with such basic interpretations of scientific concepts. It does not examine more sophisticated interpretation or problem-solving tasks involving scientific concepts, nor the principles or broader conceptual frameworks relating various concepts. But, as will become amply apparent, even basic concept-interpretation tasks are far from trivial, may require complex thought processes, and can lead to many kinds of errors and misinterpretations. Furthermore, such basic interpretation skills are essential prerequisites for using scientific concepts and for solving scientific problems.

1.3 Central Questions

An attempt to understand how scientific or mathematical concepts are interpreted leads to the following central questions about the underlying cognitive processes:

Possible modes of concept interpretation. A person's interpretation of a concept involves the retrieval of some pertinent "concept-specification knowledge", stored in the person's mind, and some "interpretation process" whereby this knowledge is applied in a particular situation of interest. However, a particular concept may be interpreted in various possible ways, depending on what specific kind of stored knowledge is retrieved and how it is processed. Hence there arise the following questions:

* What are some of the major possible ways (or "modes") of interpreting a scientific or mathematical concept?

* For each such mode, what is the nature of the stored concept-specification knowledge which is retrieved to interpret the concept? And what cognitive processes are used to apply this knowledge?
* What are the consequences of each such mode?
  * Processing characteristics. (How speedily can the concept be interpreted with this mode? What mental effort is required?)
  * Output characteristics. (What is the quality of the resulting interpretation, e.g., its degree of unambiguity, precision, consistency, generality, etc. ? What kinds of errors are likely to ensue?
  * Resulting capabilities. (How easy is it to detect and correct errors of interpretation? How readily can the interpretation mode be applied to deal with unfamiliar situations or to learn new concepts?)

(The preceding questions are not meant to deal with all pertinent issues. For example, concept interpretation may also be affected by a person's general knowledge, e.g., by his or her mental models about the world or metacognitive knowledge about the nature of scientific concepts.)

Interpretation modes of particular interest. Some modes of concept interpretation are of special interest and lead to the following particular questions:

* What modes of scientific concept interpretation are optimal in satisfying scientific requirements and facilitating the requisite human thought processes?
* What modes of scientific concept interpretation are commonly used by expert scientists?
* What modes of scientific concept interpretation are used by novice students who have relatively little experience with science or mathematics?
* What are the similarities and differences between these modes of scientific concept interpretation and the ways that lay concepts are interpreted in everyday life?

Implications for learning and teaching. Answers to the preceding questions provide a basis for addressing the more complex questions involved in the acquisition of scientific concepts:

* What are effective ways of learning scientific or mathematical concepts?
* How can such learning be promoted by effective and efficient instructional methods for teaching such concepts?

1.4 Issues Addressed in this Paper.

The preceding questions constitute an ambitious research agenda and the following pages attempt merely to begin answering some of these questions. In particular, Section 2 identifies some important modes of concept interpretation, illustrates them with specific examples, and points out some of their

consequences. Section 3 builds on this discussion to outline how scientific concepts may be interpreted in ways that are both scientifically effective and cognitively efficient. Section 4 illustrates all these general issues in greater detail for the particular physics concept "acceleration". Finally, Section 5 examines some of the particular difficulties of scientific concepts, outlines some broader issues for future investigation, and suggests instructional methods for teaching scientific concepts more effectively.

2. MODES OF CONCEPT INTERPRETATION

This section identifies and examines several possible modes of interpreting scientific concepts. As mentioned previously, each mode is characterized by what kind of concept-specific knowledge is stored and how it is processed to interpret the concept. Furthermore, the use of each mode entails some distinctive consequences.

The five concept-interpretation modes discussed in the following paragraphs rely on increasingly coherent and explicit knowledge. The distinctions between these modes are not absolutely sharp. There are also variations within each mode (e.g., depending on the extent to which the interpretation processes are deliberately performed or rely on more automatic perceptual processes.) Finally, it is possible, and sometimes advantageous, to use several interpretation modes in combination.

The primary purpose of this section is to identify and compare these various concept-interpretation modes, rather than to examine any one in great depth. Accordingly, the following paragraphs describe briefly each mode, illustrate it with a few examples, and discuss some of its major consequences. (The examples are taken from data gathered in interviews, from some written assessment questions, and from some detailed protocols of subjects tape-recorded while trying to answer questions about the concept "acceleration".)

2.1 Use of Fragmentary Knowledge

In this mode the scientific concept is interpreted by retrieving from memory some fragmentary knowledge associated with the concept. (This knowledge is ordinarily not a definition of the concept.) The retrieval may involve either an automatic recognition process, or sometimes more deliberate search, to match this fragmentary knowledge with some features of a specific situation.

The following examples illustrate this mode of concept interpretation.

> Example 1: Speed of ball. After being hit by a bat, a baseball travels along the trajectory illustrated in Figure 1. What is the speed of the ball at the highest point of its trajectory?

Insert Figure 1 about here

Some novice physics students (and some non-physics faculty members) respond to this question by saying that the speed there is zero. Some, when then merely asked "What is your definition of 'speed'?", say something about it being change of position with time — and then spontaneously realize that their previous answer was incorrect and that the speed is not zero.

In this instance, a question about the familiar concept "speed" is promptly answered by invoking some associated knowledge fragment about speed. (This may be bit of familiar knowledge about the speed of a vertically thrown ball at its highest point, or perhaps merely something cued by perceptual attention focused selectively on motion along the vertical direction.) This answer is given despite the fact that it violates common sense and the everyday meaning of "speed". Note that the word "speed" in the question triggers no attempt to invoke any explicit definition of "speed", until the person is urged by the experimenter to do so.

Example 2: Angle between vectors. The arrows in Figure 2a represent three vectors **A**, **B**, and **C**, of equal magnitudes, whose sum is zero. What then is the angle between the vectors **A** and **B** ?

Insert Figure 2 about here

Many students, including those quite familiar with vectors, answer promptly that the angle is 60°. They do this quickly, without ever invoking any explicit definition of "angle between two vectors". They merely retrieve the familiar knowledge fragment that any angle in an equilateral triangle is 60°. Their answer is wrong. (As shown later, the correct answer, 120°, is straightforwardly obtained by a more explicit mode of interpreting the concept "angle between two vectors".)

Discussion: The concept-interpretation mode illustrated in the preceding examples relies on various compiled knowledge elements which have become associated with a concept as a result of past experience — and which can often be retrieved nearly automatically in response to various cues provided by a given situation. The use of such compiled knowledge makes interpretation of a concept quick and effortless.

On the other hand, such stored knowledge about a concept is fragmentary and incoherent, like much of everyday knowledge (diSessa, in press). Furthermore, this knowledge may be invoked by responding selectively to some salient features of a given situation, while ignoring others of potential relevance. Thus there is considerable vagueness and variability about what stored knowledge elements get invoked and about what situational features are heeded. As a result, concept interpretations based on the use of such fragmentary knowledge can often be inconsistent, context-dependent, and not reliably generalizable beyond previously encountered cases. Ambiguity and

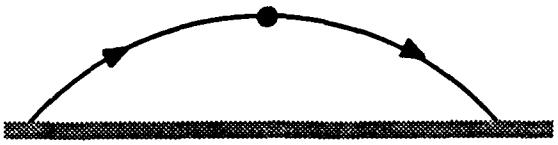

Figure 1

Trajectory of a baseball.

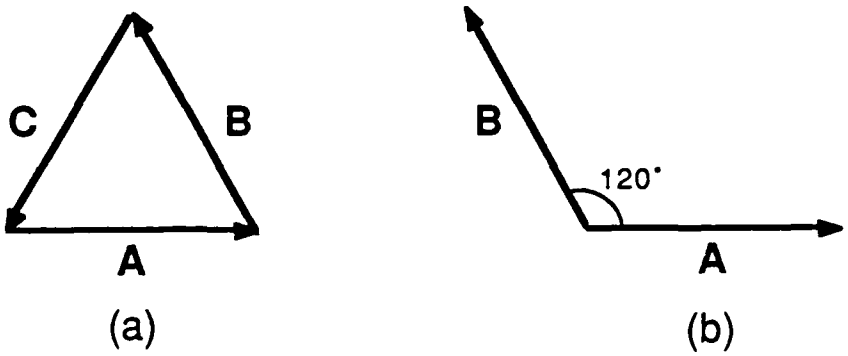

Figure 2

Three vectors, of equal magnitude, whose sum is zero.

vagueness are common since there are no good mechanisms ensuring fine discriminations. Interpretation errors occur thus quite frequently, particularly when the scientific concept is applied in somewhat unfamiliar situations.

2.2 Use of Standard Cases

This mode of concept interpretation uses knowledge stored in memory about certain important special cases of the concept. These cases are then used as "standard cases" that provide bases of comparison for interpreting other instances of the concept. [The utility of specially identified standard examples in mathematics has been discussed by Rissland (Michener, 1978; Rissland, 1985).]

The process used to interpret the concept in a particular situation consists then of (1) the retrieval of an appropriate standard case, and (2) a comparison used to match the given situation to this standard case. The comparison may involve either perceptual recognition processes or more explicit uses of analogy. (The matching to the standard case is thus usually partial rather than exact.) Sometimes the comparison may be a complex process involving several successive steps (e.g.,it may involve problem solving to transform a given situation to one closer to a standard case, matching this transformed situation to the standard, and then transforming back to the original situation.)

Example 3: Resistors in parallel. When students are asked whether the resistors R_1 and R_2 are connnected "in parallel" in the three situations illustrated in Figure 3, all say that they are in parallel in Figure 3a; some say that the are not in parallel in Figure 3b; and many say that they are in parallel in Figure 3c.

Insert Figure 3 about here

Figure 3a is the conventional standard diagram used to illustrate resistors connected in parallel. Students apparently answer the questions by comparing all situations with this standard case. It is then easy to understand the students' answers since Figure 3b looks perceptually different from Figure 3a, while Figure 3c looks quite similar.

Most of the students' answers are actually wrong. As shown later, a more explicit mode of interpreting the concept "parallel connection" leads to the conclusion that the resistors are connected in parallel in Figure 3b, but not in Figure 3c.

Example 4: Component vector. What is the component vector of the vector **A** along the direction **i** ? Most students answer this question easily when the question is asked about the situation in Figure 4a. But they have difficulties, or answer incorrectly, when the situation is that in Figure 4b.

Insert Figure 4 about here

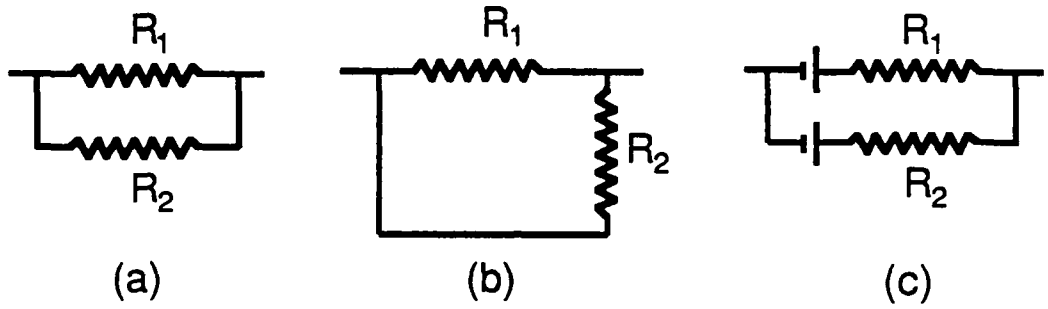

Figure 3

Different connections of two resistors
(and also of two batteries in diagram c).

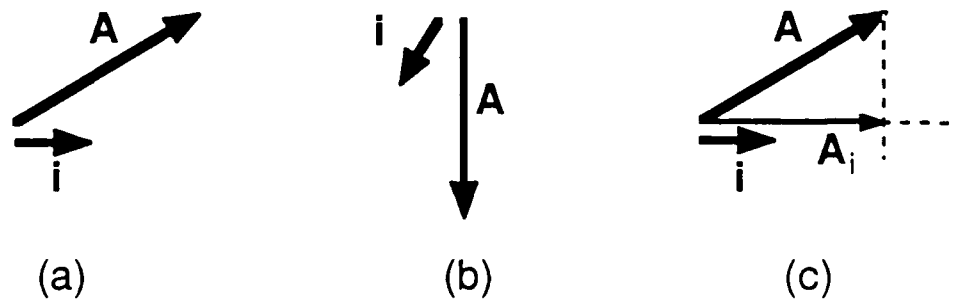

Figure 4

Component vector of a vector **A** along a direction **i**.

The reasons are quite clear. Students remember compiled knowledge about the special standard cases where the direction of a component vector is either horizontal (as in Figure 4c) or vertical. The situation of Figure 4b is then troublesome because the direction i, along which the component vector is to be found, is at an odd angle. Indeed, it is fairly common to see students who physically rotate the piece of paper, with the diagram of Figure 4b, until i is horizontal. These students interpret the concept "component vector" by using auxiliary rotations to obtain a transformed situation which can then be directly compared with the familiar standard case of Figure 4c.

Discussion: Comparisons with a few carefully identified standard cases may be sufficient to interpret a concept in many instances. Indeed, a single standard case may sometimes be prototypical of the entire domain of instances of a concept (as Figure 3a is for the concept "parallel connection"). Standard cases need not even be consciously explicit. The interpretation becomes then similar to the prototype comparisons used to specify entity concepts in everyday life (Rosch 1975, 1978) or to categorize some concepts in computer science (Adelson, 1985).

When instances of interest match known standard cases fairly closely, the concept interpretation process can be quick and effortless, often relying on automatic recognition or easy analogical comparison. Furthermore, the availability of well-identified standard cases helps to specify the particular knowledge to be retrieved for concept interpretation (i.e., it provides more explicit guidance for concept interpretation than interpretation based on less well specified fragmentary knowledge of the kind discussed in Section 2.1).

However, the criteria for determining the degree of match between such a standard case and a specific situation are often implicit or only vaguely specified. Hence concept interpretation based on standard cases can also be ambiguous and imprecise, nor can it deal reliably with instances far removed from the standard cases. In particular, inadequately specified comparison processes can easily lead to various discrimination errors. For example, important discriminations may not be made because significant features are not heeded (as illustrated by student responses to Figure 3c). Conversely, inappropriate discriminations may be made which are irrelevant to the definition of a concept (as illustrated by student responses to Figure 3b).

2.3 Use of Classified Types of Cases

This interpretation mode is an extension of the preceding one, but involves a partial or complete classification of the domain of concept instances into relatively few distinct types of cases. Sometimes this classification may be expressed in the form of explicit rules. Interpreting the concept in a particular instance involves then (1) retrieving the classification scheme, and (2) matching the given instance to fit one of the existing types.

Example 5: Acceleration. Most novice students know the characteristics of "acceleration" for some of the following possible types of motion: Straight-line motion with constant, increasing, or decreasing speed; and circular motion with constant speed. For instance, the last type of case can be described by the explicit rule "If a particle moves around a circle with constant speed, its acceleration is directed toward the center".

A more complete classification would comprise knowledge about the characteristics of the acceleration for all types of cases, i.e., for motion along a straight line with constant, increasing, or decreasing speed; and also for motion along any curved path with constant, increasing, or decreasing speed.

Discussion: A good classification into types of cases allows concept interpretation that is quick and requires little mental effort. If the distinct types are well-specified, the interpretation can also be fairly unambiguous and precise. Indeed, classification can provide a powerful basis for problem solving (Clancey, 1985).

Difficulties arise if the classification is incomplete. For then there are concept instances which cannot be interpreted, except by uncertain analogies to known types of cases. Another obvious source of difficulty occurs when the information in the typology is erroneous. For example, one of the subjects studied in our protocols answered many questions about "acceleration" by using his knowledge about various types of cases. But his knowledge included explicitly the incorrect rule that the acceleration is zero in the case where the speed of a particle is zero.

Even if a classification includes all types of cases, it lacks the degree of coherence of other concept-interpretation modes (discussed later) which encompass all cases in a more unified way. This lack of coherence reduces the ability to make general inferences and can result in special kinds of errors.

Example 6: Incorrect inference about acceleration. "If the speed of a particle at some instant is zero, can its acceleration be zero?" In one of our protocols the subject answered this question by successively examining the acceleration for various possible cases of motion along straight lines and curves. He made correct statements about several difficult cases, but forgot to consider the simplest case where a stationary particle remains at rest. This failure to examine exhaustively all possible cases led him to the incorrect conclusion that the acceleration cannot be zero. (By contrast, appeal to a general definition of acceleration would have been much less laborious and would have easily led to the correct answer.)

Example 7: Area. Science and engineering students in introductory college physics courses have dealt with the concept of "area" since elementary school. But their knowledge of the concept involves predominantly familiarity with the formulas for finding the areas of various types of geometrical figures (e.g., rectangles, parallellograms, triangles, circles, ...). It often does not include a more unifying conception of area (e.g., the number of little squares needed to cover a surface). As a result, such students often cannot make inferences about the areas of unfamiliar figures, about the areas of three-dimensional surfaces (e.g.,

the area of a person's skin), or about scaling properties (e.g., how would the area of a surface change if all its linear dimensions were three times as large?).

2.4 Use of Feature Specification

This mode of concept interpretation relies on stored knowledge defining a concept by an explicit specification of its characterizing features. The process of interpreting the concept in any particular instance then involves the following major steps: (1) Invoking the stored feature-specification knowledge. (2) Describing this knowledge in sufficient detail. (3) Devising a procedure for using this knowledge to identify or construct the concept. (4) Implementing this procedure in the particular instance of interest.

Some of these steps (particularly the first) are sufficiently simple that they may be implemented implicitly without conscious processing. However, the third step can be complex since it requires translating a feature-specification into a procedure for identifying or constructing the concept. Identifying an instance of the concept requires merely a procedure for checking that all specified features are present. But constructing an instance of the concept can be much more difficult, since one must then devise a procedure for generating an instance satisfying all constraints imposed by the feature specification. (Indeed, in mathematics one can sometimes prove the existence of concepts which one does not know how to construct.)

The following examples illustrate feature specifications used to define some of the concepts already exemplified previously.

Example 8: Resistors in parallel. Two resistors are said to be connected "in parallel" if one terminal of the first is directly connected to one terminal of the second, and if the other terminal of the first is directly connected to the other terminal of the second. (By checking these features it becomes apparent why the resistors in Figure 3b are connected in parallel, and why those in Figure 3c are not because of the presence of intervening batteries.)

Example 9: Angle between vectors. The "angle between two vectors" is the angle (between 0° and 180°) enclosed by the arrows that represent these vectors and emanate from the same point. (Accordingly, the angle between the vectors **A** and **B** in Figure 2a is the 120° angle shown in Figure 2b.)

Example 10: Component vector. The "component vector of a vector **A** along a direction i" is that particular vector, parallel to i, which yields the original vector when it is added to another vector perpendicular to i. (This definition is consistent with Figure 4c.)

Discussion: A feature specification can be used to define a concept unambiguously and precisely, as well as with the generality needed to specify it throughout its entire domain of applicability. (This is why mathematical and

scientific concepts are often defined by feature specifications.) Furthermore, a feature specification can define a concept in a very compact way. However, this compact specification can also be quite opaque (as may perhaps be apparent from Examples 9 and 10). Thus it may be quite unclear how to translate such a feature specification into a procedure for identifying or constructing the concept.

Interpretation errors can arise either because of defects in the feature specification itself, or because of defects in the process interpreting this specification.

Defects in the feature specification can occur if this specification is insufficiently complete to specify the concept unambiguously. They can also arise, in more subtle ways, if the feature specification is somehow inconsistent. Indeed, in mathematics one may need to prove that a particular concept can actually be defined in a manner consistent with the logical structure of the domain. In a science, one must ensure not only logical consistency, but also consistency with observable phenomena and the underlying laws of nature. For example, one of Einstein's major contributions was his insight that the concept "time between two events at different places" can not be defined consistently in an absolute way, but only relative to some specified observers. (Inconsistencies in a feature specification often become apparent only when one tries to use it to devise an actual procedure for identifying or constructing the concept.)

Even if the feature specification of a concept is flawless and well-known, errors may arise because the interpretation process is faulty. Thus a person may never invoke his or her available specification knowledge, may describe it improperly when trying to elaborate it, may not translate it properly into a procedure for identifying the concept, or may make mistakes in trying to implement such a procedure. All these kinds of errors are observed among novice students.

2.5 Use of Procedural Specification

In this mode the stored definitional knowledge about a concept is an explicit procedure specifying how to identify or construct the concept. (The specification knowledge, unlike in the case of feature specification, is thus procedural rather than declarative.) The process of interpreting the concept in any particular instance then involves the following major steps: (1) Invoking the stored knowledge about the specification procedure. (2) Describing this procedure in sufficient detail. (3) Implementing this procedure in the particular instance of interest.

A procedural specification of a concept may sometimes merely translate its feature specification into a procedure for identifying or constructing the concept. However, this procedure can then be stored as part of the remembered specification knowledge (rather than having to be generated by problem solving during the interpretation process). The advantages are that the concept

specification becomes thereby more explicit and its interpretation more straightforward.

The following are examples of procedural specifications of concepts previously defined by feature specifications in Examples 9 and 10. These procedural definitions are significantly easier to understand and interpret.

> **Example 11: Angle between vectors.** The major steps of the procedure specifying this concept are the following: (1) Draw the arrows, representing the two vectors, so that they start from the same point. (2) Identify the angle (between 0° and 180°) enclosed by these arrows. Call it "the angle between the vectors".
>
> **Example 12: Component vector.** The major steps of the procedure specifying this concept are the following and are illustrated in Figure 4c: (1) Consider the arrow representing the vector **A** of interest. (2) From the beginning of this arrow, draw a line parallel to the direction **i** of interest. (3) From the end of this arrow, draw a line perpendicular to **A**. (4) Draw the arrow from the beginning of **A** to the intersection point of these two lines. The vector represented by this arrow is called "the component vector of **A** along **i**".
>
> Because this procedural specification is explicit and general, it is as easily applied to find the component vector in the situation of Figure 4b as in that of Figure 4a. Interpretation of the concept "component vector" by means of this procedural specification leads thus to consequences distinctly different from Example 4 which relied on comparisons with standard cases.

Discussion: A procedural specification provides the most explicit and detailed specification of a concept. It can be made unambiguous and precise, and can ensure both generality and consistency. This is why procedural specifications ("operational definitions") are very useful in scientific work.

A procedural specification spells out the actual process required to identify or construct a concept. Hence a concept can be interpreted more straightforwardly, with appreciably less problem solving, than would be required by a feature specification which leaves this process unspecified. Significant time and mental effort may, neverthelesss, be required to implement a procedural specification in an explicit and systematic manner. Furthermore, the explicit details delineated by a procedural specification may obscure important general characteristics made apparent by a more compact feature specification. (More metaphorically, attention focused on all the individual trees may make it more difficult to see some features of the forest.)

Use of a procedural specification can lead to errors either because of defects in the procedural specification or in the interpretation process. Defects of the first kind may appear if the procedural specification itself is somehow ambiguous, imprecise, incomplete, or inconsistent. Defects of the second kind may be caused by failures to invoke the procedural specification, to describe its steps adequately, or to implement these steps properly. Errors due to all such

defects can be observed among novice students, and sometimes even among more expert individuals.

3. GOOD INTERPRETATION OF SCIENTIFIC CONCEPTS

The preceding discussion of several possible modes of interpreting scientific concepts provides the requisite background to address the following question: What mode, or combination of modes, is particularly useful (or optimal) for interpreting scientific concepts? The answer to this question constitutes an "ideal model" of scientific concept interpretation. The formulation of such an ideal model is of interest because it identifies important scientific thought processes, yields a useful comparative basis for examining the concept interpretations of expert scientists or novice students, and helps to design instruction for teaching scientific concepts.

An ideal model of scientific concept interpretation may be "prescriptive" rather than merely descriptive (Heller & Reif, 1984). In other words, it need not necessarily mimic what experts do nor merely assume that experts perform optimally, but may be based on a more general analysis of the thought processes needed to interpret scientific concepts. Accordingly, the following paragraphs examine some essential criteria that must be satisfied if scientific concept interpretation is to be reliably effective as well as efficient. An attempt to specify how these criteria can actually be met then leads to the formulation of an ideal model of good scientific concept interpretation.

3.1 Issues of Scientific Effectiveness

The central goal of science or mathematics is to invent conceptual structures allowing the most parsimonious prediction and explanation of the largest range of phenomena (directly observable phenomena in the case of a science, purely symbolic phenomena in the case of mathematics).

Accordingly, the individual concepts, used as building blocks of these conceptual structures, must be capable of satisfying the following essential requirements: (1) They must be specified explicitly and unambiguously to ensure that all predictions are definite and clearcut, and that well-specified meanings are assigned to the words used for communication between scientists. (2) They must be sufficiently precise to make fine discriminations and to achieve predictions of any desired degree of precision. (3) They must be consistent so as to avoid contradictory predictions or paradoxes. (4) They must be very generally applicable to ensure that predictions and explanations can be achieved in the most parsimonious way.

Scientific concepts need not always satisfy these criteria in all contexts. However, they must be capable of refinement to the point where they can satisfy these criteria to the maximum needed extent.

These stringent requirements can only be met by interpreting scientific concepts by methods which are sufficiently "formal", i.e., which are deliberately designed to be systematic, explicit, precise, and general. Such formal methods are those that rely on explicit feature specifications or procedural specifications. (By contrast, the other interpretation modes discussed in Section 2 are more informal, i.e., less explicit and less coherently general.)

Both of these formal methods are useful in complementary ways. A procedural specification (or "operational definition") provides the most explicit and detailed specification of a concept. By contrast, a feature specification is declarative rather than procedural, and hence less explicitly detailed. Correspondingly, it is usually also more compact, more easily remembered, and sometimes more transparent in revealing general characteristics of a concept (characteristics that may be obscured by the very details of a procedural specification). However, considerable elaboration and problem solving may be required to use a feature specification to interpret a concept in particular instances. (Nor is it possible to avoid the problem of translating a declarative description of a concept into interpretive procedures. For the validity of scientific statements can only be assessed by specifying what one must ultimately do to determine whether statements involving the concept are true or false.)

3.2 Issues of Efficiency

When properly elaborated and implemented, the preceding formal methods can be sufficient to ensure the reliably effective interpretation of scientific or mathematical concepts. However, the systematic implementation of these methods is fairly slow and requires appreciable mental effort. Hence these methods alone are inefficient or impractical for carrying out many of the human thought processes needed for scientific work.

Such considerations of efficiency are important for the following reasons: (1) Scientific concepts are used predominantly for solving scientific problems and making numerous inferences. If the thought processes required to interpret individual scientific concepts are too slow and laborious, there is not enough mental capacity left to deal with the more complex reasoning processes needed for problem solving. (As an analogy, it would be impossible to read and comprehend a scientific article if slow and deliberate processing would be required to decode and interpret individual words.) (2) Great unambiguity and precision are not always required in scientific work. If they are not, it is wasteful to resort to unnecessarily formal methods of concept interpretation. (3) Even if unambiguous precision is ultimately required, it is often strategically best to start with vague descriptions and then to refine these by successive approximations.

To achieve such cognitive efficiency, it is useful to have available much more extensive knowledge than that provided by mere formal definitions. Such knowledge should be entailed and derivable from more formal knowledge, or should at least be consistent with it. However, as a result of past familiarity and

use, such knowledge should have become compiled (Anderson, 1982) so as to stored in memory in a form where it can be quickly retrieved without the need for much conscious processing. The elements of such compiled knowledge provide special cognitive building blocks whose retrieval can short-circuit the need to resort to more formal concept-interpretation processes. Concept interpretation can thereby become fast and effortless.

Such useful compiled knowledge includes all that used to interpret concepts by the more informal methods discussed in Sections 2.1 through 2.3. For example, a well-classified knowledge of various types of cases can make concept interpretation far quicker and easier than resort to a general definition which needs to be laboriously instantiated A knowledge of some well-selected standard cases helps to deal with important situations and with other instances that can be easily compared with them. More fragmentary knowledge about various special cases allows one to recognize immediately familiar instances of a concept Such fragmentary knowledge can also usefully include specific warnings about likely errors that should be avoided during concept interpretation.

Such compiled knowledge constitutes "intuitive scientific knowledge" which facilitates concept interpretation by recognition or analogical processes which are quick and effortless (although lacking the precision and coherence of more explicit interpretations.). Of course, the utility of this knowledge depends crucially on the extent to which it is consistent with formal scientific concept specifications and is adequately discriminated from other intuitive knowledge (e.g., everyday knowledge) which may be inconsistent with it.

The cognitive demands of concept interpretation can be further reduced by exploiting the flexibility provided by exploiting various descriptions. For example, concept interpretation can often be appreciably facilitated by expressing the relevant knowledge in terms of various symbolic representations (e.g., verbal, algebraic, pictorial,...).

3.3 "Ideal" Model of Concept Interpretation

The preceding discussion suggests that the interpretation of scientific or mathematical concepts is best achieved by a combination of formal and informal interpretation modes. Formal modes, based on explicit procedural or feature specifications of the kind described in Section 3.1, ensure that scientific concepts are interpreted in a fashion that is reliably unambigous, precise, and consistent. But more informal interpretations, based on compiled knowledge and more implicit proceses of the kind described in Section 3.2, facilitate more efficient human thought processes on scientific tasks.

According to the proposed ideal model of good scientific concept interpretation, these kinds of formal and informal knowledge are to be used jointly in the following way:

If one encounters a familiar situation, it is most efficient to interpret the concept by quick informal processes relying on compiled knowledge. If there are any reasons to doubt the result or to guarantee its correctness, one can then check this result by using more formal interpretation methods.

If one encounters a situation which is unfamiliar, or if one needs to correct errors or to resolve inconsistencies, or if one wants to make general inferences, then it is usually best to interpret the concept systematically by a formal interpretation method. As a check of consistency, the result may then be compared with available compiled knowledge.

In addition, concept instances may be indirectly identified by inferences from principles relating the concept to other concepts in a larger knowledge structure. For example, instead of reverting directly to any definitional knowledge about "acceleration", values of the acceleration may also be inferred from motion principles that relate acceleration to the concept of "force".

These formal and informal modes of concept interpretation will be specifically illustrated in Section 4 for the physics concept "acceleration".

3.4 Comparisons with Expert and Novice Interpretations

The preceding ideal model of good scientific concept interpretation approximates the behavior of expert scientists. The main difference is that experts cannot always fully articulate their formal concept interpretation knowledge, although their concept interpretations are usually consistent with it. As a result, experts occasionally make wrong inferences, sometimes run into paradoxes or inconsistencies, and often may not be able to explain some concepts clearly to students.

Novice students interpret scientific concepts in ways that depart appreciably from ideal behavior. They often base their concept interpretations on highly fragmented knowledge used in intuitive ways. This knowledge is often vague and partially inconsistent with scientific conceptions, reflecting instead various lay notions acquired in daily life or misleading conceptions aquired in prior schooling. Formal specifications of scientific concepts are rarely used and, even if they can be stated, they can often not be adequately interpreted.

The preceding comments will also be illustrated in the next section for the specific case of the concept "acceleration".

4. EXAMPLE: THE CONCEPT "ACCELERATION"

The general issues discussed in the preceding sections can be better understood by being illustrated in the case of a particular concept. This section discusses, therefore, the physics concept "acceleration". This concept is typical of the many property concepts encountered in the physical sciences or mathematics. It is a good candidate for investigation because it is a fairly elementary concept, yet involves complexities representative of more sophisticated scientific concepts. "Acceleration" is also a concept of fundamental importance in Newtonian mechanics. Furthermore, cognitive studies of this concept have practical importance for instruction since many students experience great difficulties in learning this concept and often keep misinterpreting it for a long time.

4.1 Good Concept Interpretation

As discussed in Section 3, good interpretation of a scientific concept involves the combined use of formal interpretation knowledge and less formal compiled knowledge. The following paragraphs illustrate these kinds of knowledge in the case of the concept "acceleration".

Formal interpretation knowledge. A declarative feature specification of the concept is provided by the qualitative statement that "acceleration is the rate of change of velocity with time". This specification can be expressed in precise quantitative form by the formula $\mathbf{a} = d\mathbf{v}/dt$ (where $\mathbf{a}$ is the acceleration of the particle of interest, $\mathbf{v}$ is its velocity, and t is the time — and where the bold-faced letters denote quantities which are vectors, i.e., which are characterized jointly by a magnitude and a direction).

Such a declarative specification becomes clear and explicit if it is elaborated by a procedure specifying how to identify or find the acceleration. Such a procedural specification consists of the five following major steps illustrated in Figure 5: (1) Identify the velocity $\mathbf{v}$ of the particle at the time t of interest. (2) Identify the velocity $\mathbf{v}'$ of the particle at a slightly later time t'. (3) Find the velocity change $\Delta\mathbf{v} = \mathbf{v}'-\mathbf{v}$ of the particle during the short time interval $\Delta t = t'-t$. (4) Divide $\Delta\mathbf{v}$ by Δt to find the ratio $\Delta\mathbf{v}/\Delta t$. (5) Imagine that the time t' is chosen sufficiently close to the time t so that the time interval Δt becomes infinitesimally small. (Denote the corresponding infinitesimal changes Δt and $\Delta\mathbf{v}$ by dt and $d\mathbf{v}$.) Find the limiting value $d\mathbf{v}/dt$ of the ratio $\Delta\mathbf{v}/\Delta t$, and call it the "acceleration of the particle at the time t".

Insert Figure 5 about here

This procedural specification makes apparent the great complexity of the concept "acceleration" (a complexity almost totally hidden by the compact formula $\mathbf{a} = d\mathbf{v}/dt$ or the equivalent verbal statement that "acceleration is the rate

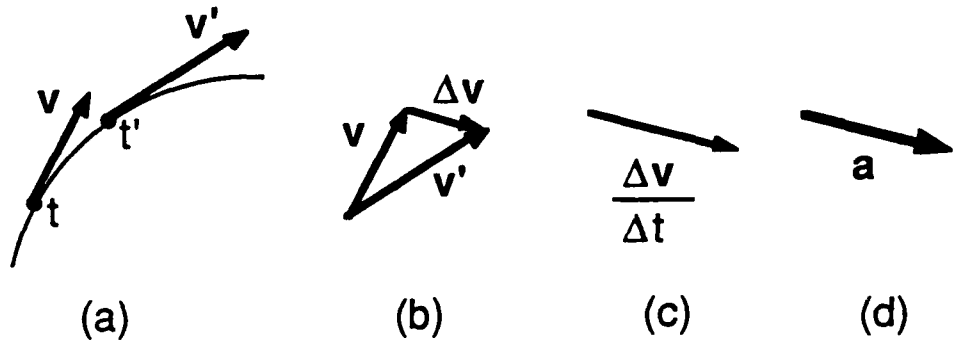

Figure 5

Diagram illustrating the procedural definition of "acceleration".

of change of velocity"). In particular, the procedure presupposes considerable prerequisite knowledge about vectors, about the concept "velocity", and about the notion of a limit. It makes explicit a complex process for comparing one velocity with another (both in magnitude and direction) and for finding a limiting case. Furthermore, the implementation of some steps in the procedure requires several subsidiary steps (such as those needed to subtract two vectors to find the change of velocity, or those needed to carry out the limiting process specified in the last step). An appreciation of these complexities makes it easy to understand why students find it so difficult to interpret the scientific concept "acceleration".

Useful compiled knowledge. As discussed in Section 3, the interpretation of scientific concepts can be greatly facilitated by various kinds of compiled knowledge. Although this knowledge can be derived from the formal definition of the concept, it is useful if it is directly stored so as to be retrievable without much processing. In the case of "acceleration", such compiled knowledge includes the following:

Knowledge about various types of cases corresponding to different ways that a particle can move (i.e., different ways that its velocity can change). These cases can be classified into the following types: (1) If a particle moves along a straight path with constant, increasing, or decreasing "speed" (i.e., magnitude of velocity), its acceleration is zero in the first case; non-zero and directed along the velocity in the second case; and non-zero and directed opposite to the velocity in the third case. (2) If a particle moves along a curved path, its acceleration is non-zero and directed toward the concave side of the path. Furthermore, if the particle's speed is constant, increasing, or decreasing, the direction of its acceleration is perpendicular to its velocity in the first case; at an angle less than 90° in the second case; and at an angle greater than 90° in the third case.

The preceding compiled knowledge allows one immediately to determine the qualitative properties of the acceleration in any particular instance, without needing to revert to the formal definition of the concept. Note that these important properties of the acceleration are not apparent from the definition of the concept without engaging in substantial reasoning processes. (Indeed, these properties seem counter-intuitive to most novice students.)

Compiled knowledge about certain standard cases is also very useful. For example, one important standard case is that of circular motion with constant speed. The corresponding knowledge is that the acceleration in this case is directed toward the center of the circle and that its magnitude if v^2/r (where v is the speed of the particle and r the radius of the circle). This compiled knowledge can then be used, without going back to the formal definition of the acceleration, as a standard of comparison to deal also with various related cases (e.g., motion with constant speed along any curved path, since any such path can be locally approximated by a circular arc).

Other more fragmentary knowledge also facilitates the interpretation and use of the acceleration concept. Such knowledge includes information about miscellaneous special cases (e.g., the fact that the acceleration is non-zero at the top of the path of a vertically projected particle). More importantly, it includes special caveats about likely errors (e.g., warnings not to confuse the scientific concept "acceleration" with the lay concept denoted by the same name, or warnings that the acceleration can be non-zero even if the speed of a particle is zero or remains constant).

Finally, Section 3.3 mentioned that alternate descriptions can often facilitate concept interpretation. Thus the acceleration, like any vector, can also be described in terms of numbers, i.e., by its components along convenient directions. Such a redescription leads to the following compiled knowledge: A change in the magnitude v of the velocity leads to a component of the acceleration along the velocity (equal to dv/dt); a change in the direction of the velocity leads to a component of the acceleration perpendicular to the velocity (toward the concave side of the path and equal to v^2/r, where r is the radius of curvature). This redescribed form of the knowledge is very useful since it allows quick identification of the qualitative features or quantitative values of the acceleration, without any need to go back to the formal definition of the concept.

The preceding comments illustrate some of the kinds of knowledge and thought processes useful for interpreting the concept "acceleration". Without repeating the general comments of Section 3.3, it should also be clear how joint use of formal and compiled knowledge can make the interpretation of this concept both reliably effective and efficient.

4.2 Concept Interpretation of Experts and Novices

Section 3.4 made some comments comparing theoretical ideas about good concept interpretation with the actual behavior of experts or novices. These comments can be well illustrated by data obtained from interviews or protocols of subjects asked to interpret the concept "acceleration".

Expert behavior. The following problem, presented to experts or novices, is very revealing.

> **Example 13: Pendulum problem.** A pendulum bob, suspended by a string from the ceiling, swings back and forth as illustrated in Figure 6a. At the extreme point A of its swing, the speed of the bob is momentarily zero. As it descends with increasing speed along a circular arc, the bob passes the point B and attains its maximum speed when it is a the lowest point C where the string is vertical. Then the bob continues moving with decreasing speed, going through the point D and finally reaching the extreme point E where the speed of the bob is again momentarily zero.
>
> At each of the points A, B, C, D, and E, is the acceleration of the bob zero or not? If not, draw an arrow indicating its direction.

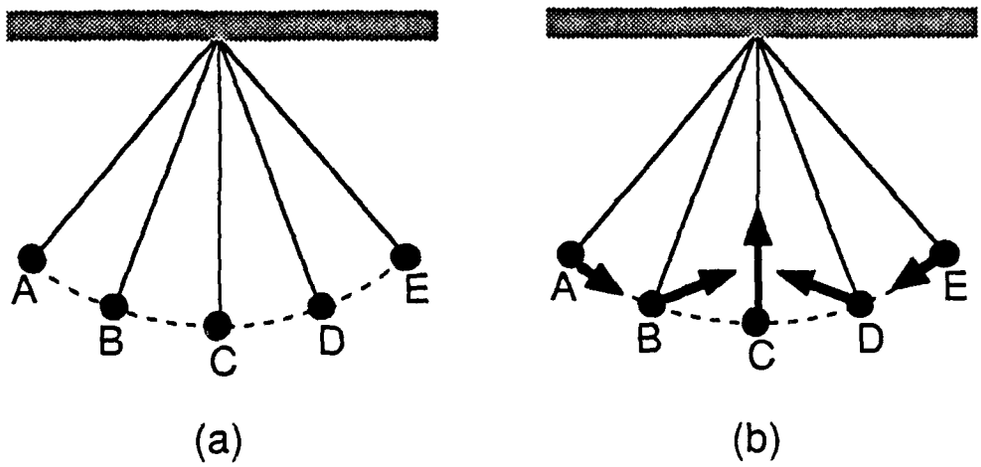

Figure 6

Acceleration of the bob of a swinging pendulum.

Insert Figure 6 about here

This is seemingly a very simple problem. It deals with the very basic concept "acceleration" which is ordinarily taught in the first couple of weeks of any introductory physics course; it involves the very familiar situation of a pendulum; and it only asks qualitative questions about the acceleration. Nevertheless, it is very rare to find any expert (physics professor or graduate student) who can quickly answer this problem intuitively without engaging in some explicit reasoning. This indicates that compiled knowledge is quite situation-specific. All experts have, through years of experience, acquired much compiled knowledge about acceleration and about pendula (which are favorite textbook examples). However, they have never, or rarely, encountered the specific questions asked in Example 13.

The following is a typical expert's solution: He first considers the point C and states that the acceleration there must be upward toward the point of suspension (as indicated in Figure 6b) because the situation there is like circular motion with constant speed. Then he considers the point A and finds the acceleration there by explicitly comparing the bob's velocity at A and at a slightly later time. Then he immediately uses symmetry to find the acceleration at E. Finally, he says that the acceleration at B must have a value somewhere between that at A and at C (i.e., like that shown in Figure 6b); and that, similarly, the acceleration at D must have a correspondingly symmetric value somewhere between that at E and C.

Let us examine more closely this solution which illustrates the combined use of compiled and formal definitional knowledge. Deliberately deviating from the order in which the questions are asked, the expert first considers the middle point C because he can there use his compiled knowledge about the standard case of circular motion with constant speed. Indeed, he nonchalantly applies this knowledge to the point C although the speed there is maximum and thus merely instantaneously unchanging. (The similarity between these two cases seems obvious to the expert, although it is sometimes difficult to understand for students.) Finding no readily compiled knowledge to answer the question about point A, the expert there applies systematically the comparison procedure formally used to define "acceleration". After that, he immediately finds the accelerations at all other points by means of quick symmetry or interpolation arguments.

Other experts' solutions differ somewhat from the one just described. For example, some do not rely on interpolation, but use the definitional procedure explicitly at both points A and B. Some use the definitional procedure also at point C, and then check their result by referring to the known case of circular motion with constant speed. Some first try to find the acceleration at the point A by identifying the forces acting on the pendulum bob; but, realizing that they lack requisite information about the magnitude of the force exerted by the string, they abandon this approach and revert to the procedural definition of

acceleration. Finally, some solve the problem efficiently by applying their compiled knowledge about the components of acceleration parallel and perpendicular to the velocity.

Quite a few physics graduate students, and even some physics professors, make mistakes and arrive at wrong answers. Indeed, some experts' performance resembles that of novices. Such observations indicate that nominal experts (i.e., persons designated as "expert" by virtue of their degrees, titles, or positions) can differ very widely in their actual competence. (To paraphrase George Orwell, some experts are much more equal than others). This should be a warning about the interpretation of many cognitive studies where "experts" are selected by purely nominal criteria, without specifying adequately the nature of their actual expertise.

Novice behavior. Novices' interpretation of scientific concepts reflects knowlege which is fragmented, inconsistent, and imprecise. Important discriminations are often ignored or forgotten. Formal definitions are rarely invoked and, if they are, they are often inadequately implemented. The pendulum problem of Example 13 can again be used to illustrate these remarks.

When this problem is presented to students (even those having completed mechanics courses where acceleration was extensively studied and used), it evokes a great variety of responses. The following are some examples: (1) Many students claim, with considerable assurance, that the acceleration of the pendulum bob at the point A is zero because the speed of the bob there is zero. (2) Some students say that the acceleration of the bob at A and at B is tangent to the path of the bob (directed along its velocity at these points), and that the acceleration of the bob at points E and D is similarly tangent to the path (directed opposite to its velocity at these points. (3) These students often also assert that the acceleration at the point C is zero. Some of the following reasons are used to justify this answer: The speed of the bob there does not change; the bob there does not move along the string because the string does not stretch; the force on the bob there is zero because the upward force exerted by the string balances the downward force by gravity. (4) A few students say that the acceleration at all the points is directed radially toward the point of suspension because the bob moves around a circular arc. (5) Other students simply claim that the acceleration at all points is directed downwards because of gravity.

All these responses are incorrect. A detailed analysis of the particular errors and misconceptions reflected by these responses is of interest, but is peripheral to the main focus of this paper. Here it is only worth pointing out how the students try to answer these questions about acceleration. Note that these questions do not evoke appeal to a definition of this concept or to other systematic knowledge. Students invoke, instead, various knowledge fragments associated with the notion of acceleration. Because this knowledge is fragmentary, and thus not constrained by the consistency requirements of a

more coherent knowledge structure, it is often applied inappropriately. In particular, important discriminations are often not made and crucial validity conditions are ignored. Furthermore, confusions and misconceptions are caused by various knowledge fragments acquired in everyday life or in prior science instruction.

Similar comments seem to describe the students, observed by Trowbridge and McDermott (1981), who were asked to interpret the acceleration concept in the simple case of linear motion. They are also consistent with our protocols of students asked to determine the acceleration in more complex cases. For example, students commonly confuse the properties of acceleration with those of velocity. They often fail to pay attention to changes in the direction of the velocity, e.g., they attribute no acceleration to a particle traveling with constant speed around a curved path. (This error is, of course, encouraged by confusion with the lay notion of acceleration which describes merely increases in speed.) They commonly fail to discriminate between velocity and a change of velocity. They often claim that the acceleration of a particle moving around a circle is always directed toward the center (thus ignoring the validity condition specifying that this result is only true if the speed of the particle is constant). Many other examples could me mentioned.

Yet, most students "know" the scientific definition of acceleration, i.e., they can readily state that "acceleration is the rate of change of velocity" or quote the precise definitional formula **a** = dv/dt. However, they rarely appeal to this definition, nor do they know how to interpret it adequately. The pendulum problem of Figure 6 provides again an example. When students claim that the acceleration of the bob at the point A is zero, they are asked to reexamine their answer by explicitly applying the definition of acceleration **a** = dv/dt; indeed, this formula is even written out for them in the elaborated form **a** = (**v'**–**v**)/(t'–t), [limit as t' --> t]. Some of these students persist, nevertheless, in saying that the acceleration at A is zero. It is only when they are led step-by-step through a procedure for interpreting the formula (i.e., when they are first asked to identify the velocity **v**, then to identify the velocity **v'** at a slightly later time, then to find the difference **v'**–**v**, ...) that they realize that the acceleration at A is not zero.

The preceding observations indicate an important point. Most experts readily translate a declarative definition of a concept (such as **a** = dv/dt) into the procedure necessary for its interpretation. However, many students do not or cannot do this, or don't do it systematically enough to interpret a concept reliably.

5. DISCUSSION AND INSTRUCTIONAL IMPLICATIONS

The preceding pages identified some centrally important questions about the interpretation of scientific or mathematical concepts. Then they attempted to answer these questions by examining various ways of interpreting such

concepts, by specifying ways that are both reliably effective and efficient, and by comparing these with the behavior of expert scientists or novice students.

The discussion restricted itself merely to basic interpretation processes whereby instances of scientific concepts can be identified or constructed. Nevertheless, the discussion was fairly lengthy, revealed more complexities than one might naively have expected, and illuminated many reasons why students experience difficulties and make frequent mistakes. This concluding section comments on the relevant concerns from a broader perspective. Thus it begins by examining more fully the complexities involved in using and learning scientific or mathematical concepts. Then it identifies broader issues involved in the interpretation of scientific concepts and outlines some useful directions for future investigations. Finally, it points out implications of the present work to the design of improved methods for teaching scientific concepts.

5.1 Complexities of Scientific Concepts

Scientific concepts are, in many respects, similar to the "lay concepts" used in everyday life. In both cases, concepts are used as basic building blocks of conceptual structures permitting people to explain or predict diverse phenomena (whether in scientific work or in daily life). In the words of Einstein, "the whole of science is nothing more than a refinement of everyday thinking". However, this refinement is often very substantial and thus gives rise to some major differences between lay concepts and scientific ones. As the following remarks indicate, these differences lead to some particular complexities of scientific concepts — and to corresponding difficulties in using or learning such concepts.

Lay concepts are used in the naturalistic context of everyday life with the implicit goal of ensuring adequate human functioning in daily activities. This goal can be attained with concepts which are specified somewhat ambiguously and vaguely, which may lead to occasional inconsistencies, or which may be used in context-dependent and partially incoherent ways. Such requirements can be adequately satisfied by specifying the meanings of everyday concepts implicitly by the contexts in which they are used, rather than by formally explicit definitions. Often such concepts can also be efficiently identified by comparisons with implicit prototypes, with heavy reliance on recognition processes and analogies (Rosch, 1975, 1978).

By contrast, scientific or mathematical concepts are used in scientific domains which are deliberately designed by humans, i.e., which are "artificial" (in the sense used by Simon, 1981). These scientific pursuits are undertaken with the explicit goal of inventing conceptual structures capable of optimum inferencing power, i.e., of achieving the most parsimonious explanation and prediction of the largest range of phenomena.

Explicit pursuit of optimum inferencing imposes some exacting requirements different from those needed to attain satisfactorily the more implicit goals of

everyday life. In particular, to make the most precise and far-ranging predictions, it is essential to use concepts which can be specified with minimum ambiguity, maximum precision, maximum mutual consistency, and highest generality. To satisfy such requirements, it is necessary to use special, and sometimes quite sophisticated, cognitive means different from those used in everyday life. As a result, particular complexities and difficulties arise at several levels.

For example, fine discriminations must often be made to ensure that concepts are defined unambiguously and precisely (and distinguished from related concepts in daily life). Scientific concepts are often specified quite abstractly to attain great generality; but this abstractness must be achieved without vagueness since a concept needs to be unambigously interpretable in any specific instance. Concepts must also be used consistently and coherently throughout very broad domains. Furthermore, a very careful use of language and of other symbolic representations is needed to ensure that all symbols are unambigously related to their referents and to each other. All the preceding requirements impose special cognitive demands different from those of daily life and sometimes appreciably more difficult.

In addition, the differences between science and daily life lead to complexities at the metacognitive level, i.,e. at the level of understanding adequately the nature of science and the thought processes required in this domain. For example, familiar everyday words often turn out to be scientifically meaningless because they are too ambiguously defined, not clearly relatable to any observable phenomena, or not useful for explanatory or predictive purposes. On the other hand, new scientific concepts and corresponding words may be freely invented, provided only that this can be done consistently and is found to be scientifically useful. Furthermore, deliberate active effort is often required to learn a new scientific concept so that it can be interpreted reliably, precisely, and with intuitive ease. By contrast, many concepts in everyday life are spontaneously acquired over longer times with less conscious deliberateness and less concern for precision; and many concepts learned in school are all too often merely memorized without becoming flexibly interpretable.

These general characteristics of scientific concepts are ordinarily not recognized by students, nor are they usually explicitly taught. Indeed, students approach their study of scientific concepts with implicit mental models about the nature of science and about the requisite thought and learning processes. These mental models, derived from everyday experience where they are reasonably adequate, are often quite naive and inappropriate in scientific domains. They are also probably even more resistant to change than students' mental models about the physical world. They can crucially determine how students learn and how they direct their attention — and they can thus cause subtle and far-reaching difficulties for students' acquisition of scientific concepts.

In summary, there are many substantial reasons why scientific and mathematical concepts are difficult to learn or teach. Efforts to understand these

reasons in greater detail offer, therefore, also the prospects of devising better ways of reducing these difficulties.

5.2 Further Issues of Concept Interpretation

Extensions of present work. It would clearly be useful to undertake more detailed analyses and to gather more extensive data on the thought processes involved in various ways of interpreting scientific concepts. For example, my collaborators and I have been gathering extensive protocol data on subjects asked to interpret the concept "acceleration" in various situations. A partial analysis of these data has yielded some valuable insights, but has also suggested modified approaches and some further questions for investigation. Work dealing with the interpretation of other scientific or mathematical concepts (e.g., "force", "area", ...) would also have obvious utility, both in its own right and in establishing the generality of any conclusions. In all such work, the emphasis should not just be on what conceptions or misconceptions are exhibited by various subjects, but on how they think and use their knowledge to arrive at their interpretations.

The present paper has focused only on the most basic concept-interpretation knowledge. But, as pointed out elsewhere (Reif, 1985), other ancillary knowledge is also important in facilitating the interpretation of scientific concepts and such knowledge merits more extensive study. For example, such ancillary knowledge includes that needed to identify all the independent variables required for the unambiguous specification of a property concept. It also includes that needed to describe the same scientific concept in various symbolic representations, to translate between such representations, and to choose the representation most likely to be useful for particular purposes.

Broader issues of concept interpretation. In the attempt to limit the scope of the present investigation to manageable proportions, the discussion in the previous pages has examined the interpretation of individual scientific concepts without considering adequately their interaction with more extensive knowledge. But the effective interpretation of a scientific concept depends ultimately on the larger scientific knowledge structure within which the concept is embedded. The following remarks point out some of the resulting broader issues which, although more complex, are ultimately quite important and deserve investigation.

A new scientific concept is formally defined in terms of other previously defined concepts and/or in terms of certain primitives (primitive undefined concepts in mathematics, or primitive concepts directly related to some specific observables in a science). In this way the concept is given a deliberately "assigned meaning" which ensures its unambiguous specification and permits clear communication among scientists. (Indeed, some scientific concepts, such as "meter" or "second", are assigned specific meanings ratified by formal international conventions.) However, the initial introduction of the concept is motivated by particular characteristics of a larger scientific knowledge structure.

Furthermore, once the concept has been introduced, it becomes complexly related to most of the other concepts in this structure.

It is this web of interrelationships which provides the concept with a rich "contextual meaning" extending far beyond its formal definition. Indeed, the validity and utility of a scientific concept are ultimately determined by the extent to which the concept contributes to an overarching knowledge structure which is internally consistent — and capable of parsimonious and correct inferences about all the phenomena which it is intended to describe. Furthermore, the efficient interpretation of a concept requires that significant portions of this larger knowledge structure have become compiled so that the concept can often be interpreted quickly with intuitive ease.

A concept is usually related to the following kinds of other knowledge in a larger knowledge structure: (1) The primary concepts in terms of which the concept is formally defined — and thus indirectly also all the other concepts related to these. (For example, in the case of "acceleration", these primary concepts include "velocity" and "time", and indirectly concepts such as "position", "vector", ...). (2) The implications of the concept's definition for special cases and for the properties of the concept. (For example, the definition of acceleration entails special implications for the cases of motion along straight lines or curves.) (3) Implications and principles which relate the concept to other concepts. (For example, Newton's motion principle $m\mathbf{a} = \mathbf{F}$ relates the acceleration $\mathbf{a}$ to the concept "force" $\mathbf{F}$.) (4) Specifications of the kinds of measurements which (in a science) relate the concept unambiguously to observable phenomena.

These remarks can best be illustrated for a fairly complex scientific concept, such as "absolute temperature". (1) This concept is formally defined in terms of the rate of change of the "entropy" of a system with respect to its "energy". (Each of these latter two concepts is quite complex and dependent on a knowledge of very many other more basic concepts.) (2) Implications of this general definition include the special cases of low and high absolute temperatures, of zero temperature (i.e., "absolute zero"), and of negative absolute temperature (a possible case which seems naively strange). They also include conclusions about how the absolute temperatures of systems are related to the direction of energy flow between them. (3) More far-reaching implications relate the absolute temperature to other concepts (like "pressure") which describe gases and liquids, to concepts (like "magnetic susceptibility") which describe magnetic materials, and to many others. Indeed, it is only because the "absolute temperature" concept can consistently relate the properties of fluids, of magnetic materials, and of many other macroscopic systems, that this concept is valid, useful, and also intuitively meaningful. (4) Specifications which relate the absolute temperature to observable measurements are quite complex. Furthermore, completely different measurement methods are required to determine the absolute temperature near room temperature, near absolute zero, or at very high temperatures.

This paper's discussion of concept interpretation has dealt with the importance of relating a scientific concept carefully to the primary concepts involved in its formal specification; it has also pointed out the importance of compiled knowledge about special cases (standard cases, classified types of cases, and others) But it has not considered the role of other related concepts in a larger knowledge structure, nor some of the subtle questions which may arise in relating a concept to directly observable phenomena. Such issues certainly deserve attention in a more complete study of concept interpretation.

Section 5.1 mentioned how metacognitive knowledge about the nature of science, and about the thought processes in this domain, can affect how students interpret and learn scientific concepts. Anecdotal evidence suggests that the resultant effects can be subtle and important. Hence the role of such metacognitive knowledge also merits future investigation.

5.3 Implications for Teaching

General guidelines. The proposed model of good concept interpretation suggests the following general guidelines for teaching scientific or mathematical concepts:

(1) Teach explicitly both formal interpretation knowledge and useful compiled knowledge about a concept. This guideline is frequently violated. Mathematically oriented courses often define concepts formally without developing students' intuitive knowledge about them. Conversely, basic science courses often introduce concepts by vague verbal statements or analogies, without providing clear definitions. (Such lack of clarity contributes to students' confusions and misconceptions.)

(2) To ensure that a scientific concept is reliably interpretable, teach students explicit procedures for identifying or constructing the concept. Make sure that students can invoke such procedural specifications, can describe them adequately, and can implement them in particular instances.

(3) Make sure that students' compiled knowledge about a scientific concept is both correct and intuitive. This requires that their compiled knowledge is consistent with formal scientific knowledge and can be checked against it by the students themselves. It also requires that students have adequate familiarity and practice with particular instances of the concept, and have compared them sufficiently with their preexisting notions, so that compiled scientific knowledge is automatically retrieved without interference from everyday intuitive knowledge.

(4) Allow adequate time for students' learning of scientific concepts, so that they have a chance to cope with the inherent cognitive complexities. In particular, give them opportunities to interpret a new scientific concept in many diverse simple cases before asking them to apply the concept in more complex problems.

Many science and mathematics courses do not abide by these guidelines, for reasons which are easy to understand. Since instructors have large amounts of tacit knowledge and find it easy to interpret scientific concepts, they are unaware of the full cognitive complexities faced by students. Furthermore, they are not inclined to spend too much time on seemingly elementary scientific concepts when there are so many more interesting scientific issues to be discussed and when there is so much curiculum to be "covered". (Unfortunately, few instructors know about the recent studies, cited in the introduction, investigating students' actual knowledge of the scientific content "covered" in their courses. They might then realize how often students' knowledge is superficial, not effectively usable, and little more than a flimsy house of cards riddled with misconceptions.)

Specific instructional methods. The preceding guidelines can be used as a basis for designing specific instructional methods for teaching scientific concepts more effectively. For example, together with some collaborators, I have been studying an instructional model consisting of the following successive stages: (1) Introducing a new concept by specifying some of its general features and contrasting them with those of other pre-existing concepts. (2) Presenting an explicit procedure specifying the concept, and then teaching the student to interpret the concept systematically by invoking and implementing this procedure. (3) Letting the student actively apply this procedure in a variety of cases that are either typical or likely to lead to errors (because of the need for fine discriminations or because of the student's conflicting prior knowledge). The student thus gets carefully designed practice and begins to compile useful knowledge. (4) Teaching the student to detect and correct interpretation errors. (5) Finally, teaching the student how to interpret a concept rapidly by using his or her compiled knowledge, and then checking and modifying that knowledge if necessary.

This instructional model, elaborated in much greater detail than indicated in the preceding sketch, has been specifically worked out for the concept "acceleration". It has also been partially implemented in the form of a computer program (so as to permit testing and refinement of the instructional model with good control of the experimental variables). This work will be the topic of a future paper.

Irrespective of these particular teaching implementations, there is considerable general interest in translating a cognitive analysis of scientific concept interpretation into specific instructional models. (1) From a research point of view, experimental investigations of such models provide severe tests of the underlying cognitive assumptions upon which they are based. They also provide a good testbed, in a comparatively simple domain, for formulating and testing some general theoretical principles of instruction. (2) From a practical point of view, they promise to lead to more principled and reliably effective methods for teaching scientific or mathematical concepts. Progress toward this goal would be highly valuable since such concepts are basic prerequisites for any scientific understanding and are traditionally quite difficult to teach.

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