A COMPILATION OF FIELD STRENGTH FORMULAS FOR ELF (EXTREMELY LOW FREQUENCY. (U) NAVAL UNDERWATER SYSTEMS CENTER NEW LONDON CT NEW LONDON LAB. P R BANNISTER

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A Compilation of Field Strength Formulas for ELF Radio Wave Propagation in the Earth-Ionosphere Waveguide

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Preface

This report was prepared under NUSC Project No. A59007, "ELF Propagation RDT&E" (U), Principal Investigator, P. R. Bannister (Code 3411), Navy Program Element No. 1140IN and Project No. XD792, Space and Naval Warfare Systems Command (SPAWARSYSCOM), Capt. R. Koontz (Code PDW 110-3), Program Manager ELF Communications.

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This report presents new formulas for ELF electric and magnetic dipole propagation in the earth-ionosphere waveguide. These new formulas extend the Wait and Galejs results, which are valid for distances greater than approximately three ionospheric reflecting heights, down to the quasi-nearfield range. Simplified versions of these general formulas that are valid for four specific ranges are also presented.
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Electric dipole moment (ampere meters)

\[ S = \sqrt{\frac{\sigma/a}{\sin(\rho/a)}} \], spherical earth spreading factor

\[ ikS_0 \] ELF earth-ionosphere waveguide propagation constant

\[ \frac{u}{(c/v)^2} = \frac{\pi \rho}{2h(c/v)^2} \]

\[ T = \frac{G(u)e^{-\alpha \rho}}{ik \rho} \left( \frac{-i\pi x}{2} \right)^{2} H_{1}^{(2)}(x) \]

\[ \frac{-\pi}{2h} \]

Earth-ionosphere waveguide propagation velocity (meters/second)

Vertical electric dipole

\[ W = \frac{e^{-\alpha \rho}}{(k \rho)^2} \left[ V(t) + \frac{i\pi}{2} G(u)x^2H_{0}^{(2)}(x) \right] \]

\[ x = k \rho(c/v) \]

Vertical distance in a cylindrical coordinate system (meters)

Earth-ionosphere waveguide attenuation rate (nepers/meter)

\[ ik, \text{ free-space propagation constant (meters}^{-1}) \]

\[ \gamma_e = [i\omega_0(\sigma_e + i\omega_0) ]^{1/2} \], propagation constant in the earth (meters}^{-1}) \]

\[ \gamma_i = [i\omega_0(\sigma_i + i\omega_0) ]^{1/2} \], propagation constant in the ionosphere (meters}^{-1}) \]

\[ \epsilon_0 = 10^{-9}/36\pi \text{ farads/meter, permittivity of free space} \]

\[ \epsilon_e \] Effective permittivity of the earth (farads/meter)

\[ \epsilon_i \] Effective permittivity of the ionosphere (farads/meter)

\[ 120\pi, \text{ impedance of free space (ohms)} \]

\[ \eta_e = \sqrt{\frac{i\omega_0}{\sigma_e + i\omega_0 \epsilon_e} } \], impedance of the earth (ohms)

Free-space wavelength (meters)

\[ \lambda_e \] Wavelength in the earth (meters)
\[ \mu_0 = 4\pi \times 10^{-7} \text{ henries/meter, permeability of free space} \]

\[ \rho \]
Radial distance in a cylindrical coordinate system (meters)

\[ \rho_{mv} \]
Distance where the minimum value of the VED radial wave impedance occurs (kilometers)

\[ \sigma_e \]
Effective conductivity of the earth (Siemens/meter)

\[ \sigma_i \]
Effective conductivity of the ionosphere (Siemens/meter)

\[ \phi \]
Azimuth angle in a cylindrical coordinate system

\[ \omega \]
\(2\pi f\) radians/second, angular frequency
A COMPILATION OF FIELD-STRENGTH FORMULAS FOR ELF RADIO-WAVE PROPAGATION IN THE EARTH-IONOSPHERE WAVEGUIDE

INTRODUCTION

It is the purpose of this report to present new formulas for horizontal electric dipole (HED), horizontal magnetic dipole (HMD), and vertical electric dipole (VED) extremely low frequency (ELF) radio-wave propagation in the earth-ionosphere waveguide. These new formulas extend the results of Wait and Galejs, which are valid for measurement distances, \( \rho \), greater than approximately three ionospheric reflecting heights, \( h \), down to the quasi-nearfield range, which is defined as the range where \( \rho \) is greater than an earth wavelength, \( \lambda_e \), but much less than a free-space wavelength, \( \lambda_0 \). For the sake of completeness, the abovementioned previously derived formulas also will be included.

The three dipole antennas (VED, HED, and HMD) are situated at zero height with respect to a cylindrical coordinate system \((\rho, \phi, z)\) and are assumed to carry a constant current, \( I \). The axes of the VED and HED (of dipole moment \( p \)) are oriented in the \( z \) and \( x \) directions, respectively, while the axis of the HMD (of dipole moment \( m \)) is oriented in the \( y \) direction. The ionosphere is located at height \( z > h \), while the earth is located at height \( z < 0 \). The propagation constant in the air is denoted by

\[
\gamma_0 = \frac{ik = i2\pi/\lambda_0}{},
\]

whereas the propagation constants in the earth and ionosphere are denoted by

\[
\gamma_e = \sqrt{i\omega\mu (\sigma + i\omega\varepsilon)}
\]

and

\[
\gamma_i = \sqrt{i\omega\mu (\sigma + i\omega\varepsilon)}
\]

respectively. The magnetic permeability of the whole space is assumed to equal \( \mu_0 \), the permeability of free space. Meter-kilogram-second (NKS) units are employed and a suppressed time factor of \( \exp(i\omega t) \) is assumed.

DERIVATION PROCEDURE

Accounting for ionospheric reflection effects out to distances of approximately three ionospheric reflecting heights, \( h \), is a tedious process involving an infinite sum of images. However, by following the procedure outlined by Martin and Bannister and Williams, we find that each VED, HED, and HMD field-component expression can be multiplied by one of the following four functions:
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\[ G(u) = \left(\frac{2u}{\pi}\right) \coth u + \left(1 - \frac{2}{\pi}\right) u^2 \text{csch}^2 u \]  
(1)

\[ G(t) = \left(\frac{2t}{\pi}\right) \coth t + \left(1 - \frac{2}{\pi}\right) t^2 \text{csch}^2 t \]  
(2)

\[ V(t) = t^3 \coth t \text{csch}^2 t \]  
(3)

and

\[ H(t) = G(t) + V(t), \]  
(4)

where

\[ u = \frac{\pi \sigma}{2h} \]  
(5)

and

\[ t = \frac{u}{(c/v)^2}, \]  
(6)

where \(c/v\) is the earth-ionosphere waveguide phase-velocity ratio

\[ c = 3 \times 10^8 \text{ m/s}. \]

The functions \(G(t), H(t),\) and \(V(t)\) are plotted versus \(t\) in figure 1.*

Note that the plot of \(G(t)\) versus \(t\) is also a plot of \(G(u)\) versus \(u\). When \(u < 0.5\), \(G(u) \approx 1\); when \(t < 0.5\), \(G(t) \approx V(t) \approx 1\). However, when \(t < 2\), \(H(t) \approx 2\). Furthermore, when \(u > 2.5\), \(G(u) \approx 2u/\pi\); when \(t > 2.5\), \(G(t) \approx 2t/\pi\). However, when \(t > 4.5\), \(V(t) \approx 0\) and \(H(t) \approx 2t/\pi\).

When the measurement distance, \(\varepsilon\), is greater than approximately three ionospheric reflecting heights from the source, each VED, HED, and HMD field-component expression varies as a Hankel function,

\[ H_0^{(2)}(kS_{\varepsilon}) \]

or

\[ H_1^{(2)}(kS_{\varepsilon}) \]

or a combination of the two, where \(k = 2\pi/\lambda\) and \(iS_{\varepsilon}\) is the propagation constant in the earth-ionosphere waveguide. \(S_{\varepsilon}\) is related to the phase velocity \(v\) and attenuation rate \(\alpha\) by the formulas \(c/v = \text{Re}S_{\varepsilon}\) and \(\alpha = -8.7k \text{ Im}S_{\varepsilon}\).

At ELF (i.e., 30 to 300 Hz), \(\text{Re}(kS_{\varepsilon}) > \text{Im}(kS_{\varepsilon})\). Therefore,

\[ H_0^{(2)}(kS_{\varepsilon}) \approx H_0^{(2)}(x)e^{-\alpha x} \]  
(7)

*All figures have been placed together at the end of this report.
and
\[ H_1^{(2)}(k_0\rho) = H_1^{(2)}(x)e^{-\alpha\rho}, \]
where \( x = k\rho(c/v) \).

In this report, we will use previously derived\(^5\) \(-^8\) quasi-nearfield range formulas \((\rho > \lambda_e, \phi << \lambda_0, \text{ and } \rho < h/3)\), as well as the Wait\(^1\) and Galejs\(^2\) formulas \((\rho > 3h)\), to find VED, HED, and HMD formulas valid at ELF for \( \rho > \lambda_e \) with no restrictions on the ratio of \( \rho \) to \( h \).

As an example of our derivation procedure, consider the VED \( H_\phi \) component. When \( \rho > 3h \),
\[ H_\phi^{VE} = \frac{ipk(c/v)}{4h} H_1^{(2)}(x)e^{-\alpha\rho} = \frac{pe^{-\alpha\rho}(\rho)}{2\pi\rho^2} \left( \frac{-i\pi x}{2} \right) H_1^{(2)}(x). \]  
(10)

For \( x \leq 0.25 \), equation (10) reduces to
\[ H_\phi^{VE} = \frac{p}{2\pi\rho^2} \left( \frac{\rho}{h} \right). \]  
(11)

When \( \rho < h/3 \), the quasi-nearfield range formula is
\[ H_\phi^{VE} = \frac{p}{2\pi\rho^2}. \]  
(12)

Since \( G(u) = 1 \) for \( z < h/3 \) and \( z/h \) for \( z > 3h \), then, for \( x \leq 0.25 \),
\[ H_\phi^{VE} = \frac{pG(u)}{2\pi\rho^2}. \]  
(13)

Because the range of validity of equations (10) and (13) overlap when \( \rho > 3h \) and \( x \leq 0.25 \), we can substitute \( G(u) \) for \( z/h \) in equation (10) to obtain the general formula valid for \( \rho > \lambda_e \) (with no restrictions on the ratio of \( \rho \) to \( h \)). It is
\[ H_\phi^{VE} = \frac{pG(u)e^{-\alpha\rho}}{2\pi\rho^2} \left( \frac{-i\pi x}{2} \right) H_1^{(2)}(x). \]  
(14)

Wait's expression\(^1\) for \( H_\phi^{VE} \) is
\[ H_\phi^{VE} = \frac{ik}{2}\phi \left[ \left( \frac{-i\pi x}{2} \right) H_1^{(2)}(x) \right]. \]  
(15)

Therefore,
\[ \text{T} = \frac{G(u)e^{-\alpha\rho}}{ik} \left[ \left( \frac{-i\pi x}{2} \right) H_1^{(2)}(x) \right], \]  
(16)

which, for \( x \leq 0.25 \), reduces to
The magnitude of $T$ (from equation (16)) is compared with Wait's infinite sum-of-images result in figure 2 as a function of distance and frequency. For this particular comparison, $h = 90$ km and $a = \infty$ (i.e., $c/v = 1.0$ and $\alpha = 0.0$). Note that the agreement is excellent.

We know from previous results that there is a substantial amplitude dip in the VED $E_z$ component in the range of 100 to 300 km (depending on frequency). Therefore, we will let

$$E_{z1}^V = E_z^1 - E_z^2.$$ \hspace{1cm} (18)

For $\rho < h/3$, the quasi-nearfield range formula is

$$E_{z1} = \frac{i60\rho V(t)}{k\rho^3}.$$ \hspace{1cm} (19)

For $\rho$ comparable to $h$, we can easily show that

$$E_{z1} = \frac{i60\rho V(t)}{k\rho^3},$$ \hspace{1cm} (20)

which reduces to equation (19) when $t < 0.5$ and vanishes when $t > 4.5$.

When $\rho > 3h$,

$$E_{z2} = \frac{30pk(c/v)^2H^2_2(x)e^{-\alpha\rho}}{k^3} = \frac{i60pG(u)x^2}{k^3} - i\mu\pi \begin{pmatrix} \mu \ln \left( \frac{1.123}{x} \right) - \frac{i\pi}{2} \end{pmatrix}.$$ \hspace{1cm} (21)

When $\alpha < 0.25$, equation (21) reduces to

$$E_{z2} = \frac{i60pG(u)x^2}{k^3} \begin{pmatrix} \mu \ln \left( \frac{1.123}{x} \right) - \frac{i\pi}{2} \end{pmatrix}. \hspace{1cm} (22)$$

Therefore,

$$E_{z2} = \frac{i60pG(u)x^2}{k^3} \begin{pmatrix} \mu \ln \left( \frac{1.123}{x} \right) - \frac{i\pi}{2} \end{pmatrix}.$$ \hspace{1cm} (23)

Employing equations (18), (20), and (23) results in

$$E_z = \frac{i60\rho e^{-\alpha \rho}}{k^3} \begin{pmatrix} \mu V(t) + \frac{i\mu}{2} \ln \left( \frac{G(u)x^2}{H^2_2(x)} \right) \end{pmatrix},$$ \hspace{1cm} (24)

which is the final result.

Wait's expression for $E_{z1}$ is

$$E_{z1} = \frac{i60\rho V(t)}{k^3}.$$ \hspace{1cm} (25)
Therefore,

\[ W = \frac{e^{-\alpha_0}}{(k_0)^2} \left[ V(t) + \frac{i\pi}{2} G(u)x^2H_0^{(2)}(x) \right], \]  

(26)

which, for \( x \leq 0.25 \), reduces to

\[ W = \frac{1}{(k_0)^2} \left[ V(t) - G(u)x^2 \left[ \ln(\frac{1.123}{x}) - \frac{i\pi}{2} \right] \right]. \]  

(27)

The magnitude of \( W \) (from equation (26)) is compared with Wait's infinite sum-of-images result in figure 3 as a function of distance and frequency. For this particular comparison, \( h = 90 \) km and \( \sigma_i = \infty \) (i.e., \( c/v = 1.0 \) and \( \alpha = 0.0 \)). Note that the agreement is excellent.

From equation (27), we see that the minimum value of \( W \) will occur when \( V(t) - G(u)x^2 \ln(1.123/x) \). That is,

\[ W_{\text{MIN}} = \frac{i\pi}{2} G(u)(c/v)^2. \]  

(28)

When \( u > 2.5 \),

\[ W_{\text{MIN}} = iu(c/v)^2 = i(\frac{\pi}{2h})(c/v)^2. \]  

(29)

FIELD-STRENGTH FORMULAS

In this section, we will present new formulas for HED, HMD, and VED ELF radio-wave propagation in the earth-ionosphere waveguide. All of these formulas have been obtained by following the procedure outlined in the previous section. They are valid for \( \rho > \lambda_e \), with no restrictions on the ratio of \( \omega \) to \( h \).

It should be noted that for \( 2 \text{ Mm} \leq \rho < 19 \text{ Mm} \), all of the field-strength-component formulas presented in this report should be multiplied by the spherical earth spreading factor \( S \), which is equal to

\[ S = \left( \frac{\rho/a}{\sin(\rho/a)} \right)^n, \]  

(30)

where \( n = 1/2 \) for all \( E_\rho \), \( E_z \), and \( H_z \) components; \( n = 3/2 \) for all \( E_\phi \) and \( H_\phi \) components; and \( a \) is the radius of the earth (\( \approx 6.37 \text{ Mm} \)).

FOR THE VED

Expressions for the VED are

\[ E_{V\rho} = \frac{\eta e^{\mu \rho} G(u)e^{-\alpha_0}}{2\pi \rho} \left[ \left( \frac{-i\pi}{2} \right) H_1^{(2)}(\lambda) \right], \]  

(31)
The various VED components are related by

\[
\frac{E_{V, E}}{H_{V, E}} = -\frac{n_e}{n_0} \left( \frac{i\omega_0}{\sigma_e + i\omega\varepsilon_e} \right)^{1/2}, \tag{32}
\]

\[
\frac{E_{V, E}}{H_{V, E}} = \frac{\eta_0 (c/\nu)}{xG(u)} \left[ V(t) + \frac{i\pi}{2} G(u) x^2 H^{(2)}_0(x) \right], \tag{33}
\]

and

\[
\frac{E_{V, E}}{E_{V, E}} = \frac{\eta_0 (c/\nu)}{n_e xG(u)} \left[ V(t) + \frac{i\pi}{2} G(u) x^2 H^{(2)}_0(x) \right], \tag{34}
\]

where \( n_0 (= 120\pi) \) is the free-space impedance and \( n_e \) is the earth impedance.

FOR THE HED

Expressions for the HED are

\[
E_{H, E} = \frac{pG(t) \cos \phi e^{-\alpha c}}{2\pi (\sigma_e + i\omega\varepsilon_e) e^3} \left[ \left( -\frac{i\pi}{2} \right) H^{(2)}_0(x) f(x) \right], \tag{35}
\]

\[
E_{H, E} = \left( -\frac{c}{2\pi} \right) \left[ \left( -\frac{i\pi}{2} \right) H^{(2)}_0(x) \right], \tag{36}
\]

\[
E_{H, E} = \left( -\frac{i\omega_0 pG(u) \cos \phi e^{-\alpha c}}{2\pi \nu e^3} \right) \left[ \left( -\frac{i\pi}{2} \right) H^{(2)}_0(x) \right], \tag{37}
\]

\[
H_{H, E} = \frac{pH(t) \sin \phi e^{-\alpha c}}{2\pi e^3} \left[ \left( -\frac{i\pi}{2} \right) H^{(2)}_0(x) \right], \tag{38}
\]

and

\[
H_{H, E} = \left( -\frac{c}{2\pi} \right) \left[ \left( -\frac{i\pi}{2} \right) H^{(2)}_0(x) f(x) \right], \tag{39}
\]

where
The magnitude of \( f(x) \) is plotted in figure 4 versus \( x \). For small values of \( x \), \( f(x) - 1 \), while for large values of \( x \), \( f(x) - x \).

The various HED components are related by

\[
\frac{E_{\phi}^{HE}}{H_{\phi}^{HE}} = -\frac{E_{\phi}^{HE}}{H_{\phi}^{HE}} - \eta e, \quad (41)
\]

\[
\frac{E_{z}^{HE}}{H_{\phi}^{HE}} = -\frac{i\omega x G(u)}{(c/v)f(x)G(t)}, \quad (42)
\]

\[
\frac{E_{z}^{HE}}{E_{\phi}^{HE}} = \frac{i\omega x G(u)}{\eta e(c/v)f(x)G(t)}, \quad (43)
\]

and

\[
\frac{H_{\phi}^{HE}}{H_{\phi}^{HE}} = \frac{E_{\phi}^{HE}}{E_{\phi}^{HE}} - G(t)f(x)\cot \phi \left[ \frac{\sin(\rho/a)}{(\rho/a)} \right]. \quad (44)
\]

FOR THE HMD

Expressions for the HMD are

\[
E_{\phi}^{HM} = \frac{mG(t) \cos \phi e^{-\alpha \phi}}{2\pi \rho^3} \left[ \frac{-i\pi x}{2} \right] \left[ \frac{(-i\pi x)H_1^{(2)}(x)}{2} \right] f(x), \quad (45)
\]

\[
E_{\phi}^{HM} = \frac{mG(t) \sin \phi e^{-\alpha \phi}}{2\pi \rho^3} \left[ \frac{-i\pi x}{2} \right] \left[ \frac{(-i\pi x)H_1^{(2)}(x)}{2} \right], \quad (46)
\]

\[
E_{z}^{HM} = \frac{i\omega \rho G(u) \cos \phi e^{-\alpha \phi}}{2\pi \rho^2} \left[ \frac{-i\pi x}{2} \right] \left[ \frac{(-i\pi x)H_1^{(2)}(x)}{2} \right], \quad (47)
\]

\[
H_{\phi}^{HM} = \frac{mH(t) \sin \phi e^{-\alpha \phi}}{2\pi \rho^3} \left[ \frac{-i\pi x}{2} \right] \left[ \frac{(-i\pi x)H_1^{(2)}(x)}{2} \right], \quad (48)
\]

and

\[
H_{z}^{HM} = \frac{mG(t) \cos \phi e^{-2\alpha \phi}}{2\pi \rho^3} \left[ \frac{-i\pi x}{2} \right] \left[ \frac{(-i\pi x)H_1^{(2)}(x)}{2} \right] f(x), \quad (49)
\]
The various HMD field components are related by

\[
\frac{E_{\phi}}{H_{\rho}} = -\frac{E_{\phi}}{H_{\rho}} = -\eta_e, \quad (50)
\]

\[
\frac{E_{z}}{H_{\phi}} = \frac{i\eta_0 G(u)}{(c/v)f(x)G(t)}, \quad (51)
\]

\[
\frac{E_{z}}{E_{\rho}} = \frac{i\eta_0 G(u)}{\eta_e(c/v)f(x)G(t)}, \quad (52)
\]

and

\[
\frac{H_{\phi}}{H_{\rho}} = -\frac{E_{\phi}}{E_{\rho}} = -\frac{G(t) f(x) \cot \phi \left\{ \sin(c/a) \right\}}{(\rho/a)}. \quad (53)
\]

FIELD-STRENGTH FORMULAS FOR \( x < 0.25 \)

When \( x = kp(c/v) \leq 0.25 \), \( f(x) \approx 1.0 \), and the Hankel functions can be approximated by

\[
\left( \frac{-i\pi x}{2} \right) H_{1}^{(2)}(x) \approx 1.0 \quad (54)
\]

and

\[
H_{0}^{(2)}(x) = \frac{i2}{\pi} \left[ \ln \left( \frac{1.123}{x} \right) - \frac{i\pi}{2} \right]. \quad (55)
\]

Thus, the VED expressions listed in the previous section reduce to

\[
E_{\rho} = -\frac{\eta_0 G(u) e^{-\alpha_0}}{2\pi \rho^2}, \quad (56)
\]

\[
E_{z} = \frac{i60pe^{-\alpha_0}}{k_0^2} \left\{ V(t) - G(u)x^2 \left[ \ln \left( \frac{1.123}{x} \right) - \frac{i\pi}{2} \right] \right\}, \quad (57)
\]

\[
H_{\rho} = \frac{pG(u)e^{-i\gamma \rho}}{2\pi \gamma}, \quad (58)
\]

\[
E_{z} = \frac{i\eta_0}{(k_0) G(u)} \left\{ V(t) - G(u)x^2 \left[ \ln \left( \frac{1.123}{x} \right) - \frac{i\pi}{2} \right] \right\}, \quad (59)
\]

and
When \( x < 0.25 \), the HED expressions listed in the previous section reduce to

\[
\begin{align*}
\frac{E_{\text{HE}}}{E_{\text{DO}}} & = -\frac{i\kappa_0}{\kappa (\kappa^{*} + i\omega)} \left[ V(t) - G(u)x^2 \left[ \ln \left( \frac{1.123}{x} \right) - \frac{\ln x}{2} \right] \right]. \\
\frac{H_{\text{HE}}}{H_{\text{DO}}} & = \frac{\kappa G(u) \cos \phi e^{-\alpha \phi}}{2\pi \kappa_{\text{DO}}^{2}}, \\
\frac{H_{\text{HE}}}{H_{\text{DO}}} & = \frac{\kappa G(u) \sin \phi e^{-\alpha \phi}}{2\pi \kappa_{\text{DO}}^{2}}, \\
\frac{E_{\text{HE}}}{E_{\text{DO}}} & = \frac{\kappa G(u)}{G(t)}, \quad \frac{E_{\text{HE}}}{E_{\text{DO}}} = \frac{\kappa G(u)}{G(t)}, \\
\frac{E_{\text{HE}}}{E_{\text{DO}}} & = \frac{\kappa G(u)}{G(t)}, \quad \frac{E_{\text{HE}}}{E_{\text{DO}}} = \frac{\kappa G(u)}{G(t)}.
\end{align*}
\]

and

\[
\begin{align*}
\frac{H_{\text{HE}}}{H_{\text{HE}}} & = \frac{E_{\text{HE}}}{E_{\text{HE}}} - \frac{G(t) \cot \phi}{H(t)}. \\
\frac{E_{\text{HE}}}{E_{\text{HE}}} & = \frac{\kappa G(u)}{G(t)}, \quad \frac{E_{\text{HE}}}{E_{\text{HE}}} = \frac{\kappa G(u)}{G(t)}.
\end{align*}
\]

When \( x < 0.25 \), the HMD expressions listed in the previous section reduce to

\[
\begin{align*}
\frac{E_{\text{HM}}}{E_{\text{DO}}} & = \frac{\kappa G(t) \cos \phi e^{-\alpha \phi}}{2\pi \kappa_{\text{DO}}^{2}}, \\
\frac{E_{\text{HM}}}{E_{\text{DO}}} & = \frac{\kappa G(t) \sin \phi e^{-\alpha \phi}}{2\pi \kappa_{\text{DO}}^{3}}, \\
\frac{E_{\text{HM}}}{E_{\text{DO}}} & = \frac{i\omega \kappa G(u) \cos \phi e^{-\alpha \phi}}{2\pi \kappa_{\text{DO}}^{2}},
\end{align*}
\]
When $\rho < h/3$, $G(u) = G(t) - V(t) = 1.0$, $H(t) = 2$, and $\alpha = 0$. For this case, the formulas presented in this section (equations (56) through (76)), reduce to the familiar quasi-nearfield range results. They will not be repeated here since they are already given by equations (56) through (76) with $G(u) = G(t) - V(t) = 1.0$, $H(t) = 2.0$, and $\alpha = 0$.

FIELD-STRENGTH FORMULAS FOR $z > 3h$

When $z > 3h$,

$$G(u) = \frac{2u}{\pi} = \frac{\rho}{h},$$

$$G(t) - H(t) - \frac{2t}{\pi} = \frac{\rho}{h(c/v)^2},$$

and

$$V(t) \approx 0.$$

For this case, the general field-strength formulas presented in equations (31) through (53) reduce to those of Wait and Galejs. For the sake of completeness, they will be repeated here.
FOR THE VED

Expressions for the VED are

\[ E^\text{VE}_0 = -\frac{n_e e\alpha}{2\pi\hbar} \left[ \frac{-i\pi x}{2} \right] H_1^{(2)}(x), \quad (80) \]

\[ E^\text{VE}_z = -\frac{n_0 p(c/v)}{4\pi\hbar} (x)e^{-\alpha_0} \]

\[ H^\text{VE}_\phi = \frac{e^{-\alpha_0}}{2\pi\hbar} \left[ \frac{-i\pi x}{2} \right] H_1^{(2)}(x), \quad (82) \]

\[ E^\text{VE}_z = -\frac{n_0 (c/v) x H_0^{(2)}(x)}{H_1^{(2)}(x)}, \quad \frac{E^\text{VE}_z}{E^\text{VE}_0} = \frac{n_0 (c/v)}{\hbar \left[ H_1^{(2)}(x) \right]} \quad \text{(83)} \]

and

\[ \frac{E^\text{VE}_z}{H^\text{VE}} = \frac{n_0 (c/v)}{H_1^{(2)}(x)} \quad \text{(84)} \]

FOR THE HED

Expressions for the HED are

\[ E^\text{HE}_c = \left[ \frac{p \cos \beta e^{\alpha_0}}{2\pi(\varepsilon_1 + i\omega_1)h(c/v)^2} \right] \left[ \frac{-i\pi x}{2} \right] H_1^{(2)}(x)f(x), \quad (85) \]

\[ E^\text{HE}_\phi = \left[ \frac{p \sin \beta e^{-\alpha_0}}{2\pi(\varepsilon_1 + i\omega_1)h(c/v)^2} \right] \left[ \frac{-i\pi x}{2} \right] H_1^{(2)}(x), \quad (86) \]

\[ E^\text{HE}_z = \left[ \frac{i\omega_0 \rho \cos \beta e^{-\alpha_0}}{2\pi\gamma e\hbar} \right] \left[ \frac{-i\pi x}{2} \right] H_1^{(2)}(x), \quad (87) \]

\[ H^\text{HE}_0 = \left[ \frac{p \sin \beta e^{-\alpha_0}}{2\pi\gamma e h(c/v)^2} \right] \left[ \frac{-i\pi x}{2} \right] H_1^{(2)}(x), \quad (88) \]

\[ H^\text{HE}_z = \left[ \frac{p \cos \beta e^{-\alpha_0}}{2\pi\gamma e h(c/v)^2} \right] \left[ \frac{-i\pi x}{2} \right] H_1^{(2)}(x)f(x), \quad (89) \]

\[ \frac{E^\text{HE}_z}{H^\text{HE}_z} = \frac{i\omega_0 (c/v)x}{f(x)}, \quad (90) \]
FOR THE HMD

Expressions for the HMD are

\[
\begin{align*}
\frac{E_{\phi}}{E_\phi} &= \frac{\nu_0 (c/v)x}{n_\phi f(x)}, \\
\frac{H_{\phi}}{H_\phi} &= -f(x) \cot \phi \left[ \frac{\sin(\rho/a)}{(\rho/a)} \right].
\end{align*}
\] (91)

\[
\begin{align*}
\frac{E_{z}}{E_\phi} &= \frac{i \nu_0 (c/v)x}{n_\phi f(x)}, \\
\frac{H_{z}}{H_\phi} &= -f(x) \cot \phi \left[ \frac{\sin(\rho/a)}{(\rho/a)} \right].
\end{align*}
\] (92)

\[
\begin{align*}
\frac{E_{\phi}}{E_\phi} &= \left[ \frac{m e \cos \phi e^{-\alpha \rho}}{2 \pi h (c/v)^2 \rho^2} \right] \left[ \frac{-i \pi x}{2} \right] H_1^{(2)}(x) f(x), \\
\frac{H_{\phi}}{H_\phi} &= \left[ \frac{m e \sin \phi e^{-\alpha \rho}}{2 \pi h (c/v)^2 \rho^2} \right] \left[ \frac{-i \pi x}{2} \right] H_1^{(2)}(x), \\
\frac{E_{z}}{E_\phi} &= \left[ \frac{i \omega_0 m \cos \phi e^{-\alpha \rho}}{2 \pi h (c/v)^2 \rho^2} \right] \left[ \frac{-i \pi x}{2} \right] H_1^{(2)}(x) f(x), \\
\frac{H_{z}}{H_\phi} &= \left[ \frac{m \sin \phi e^{-\alpha \rho}}{2 \pi h (c/v)^2 \rho^2} \right] \left[ \frac{-i \pi x}{2} \right] H_1^{(2)}(x), \\
\frac{E_{\phi}}{E_\phi} &= \left[ \frac{m \cos \phi e^{-\alpha \rho}}{2 \pi h (c/v)^2 \rho^2} \right] \left[ \frac{-i \pi x}{2} \right] H_1^{(2)}(x) f(x), \\
\frac{H_{\phi}}{H_\phi} &= -f(x) \cot \phi \left[ \frac{\sin(\rho/a)}{(\rho/a)} \right].
\end{align*}
\] (93)
(94)
(95)
(96)
(97)
(98)
(99)

\[
\begin{align*}
\frac{E_{z}}{E_\phi} &= \frac{i \nu_0 (c/v)x}{n_\phi f(x)}, \\
\frac{H_{z}}{H_\phi} &= \frac{i \nu_0 (c/v)x}{n_\phi f(x)}.
\end{align*}
\] (100)
FIELD-STRENGTH FORMULAS FOR $\rho > 3h$ AND $x > 1.6$

For $x > 1.6$, the Hankel functions can be approximated by

$$H_0^{(2)}(x) = \sqrt{\frac{2}{\pi x}} e^{-i(x-\pi/4)}, \quad (101)$$

$$H_1^{(2)}(x) = i\sqrt{\frac{2}{\pi x}} e^{-i(x-\pi/4)}, \quad (102)$$

$$\left(\frac{-i\pi x}{2}\right)H_1^{(2)}(x) = \sqrt{\frac{\pi x}{2}} e^{-i(x-\pi/4)}, \quad (103)$$

and

$$\left(\frac{-i\pi x}{2}\right)H_1^{(2)}(x)f(x) = i\sqrt{\frac{\pi x}{2}} e^{-i(x-\pi/4)}. \quad (104)$$

Thus, the VED expressions listed in the previous section reduce to

$$E_{\phi}^{VE} = -\frac{n_e\rho e^{-\alpha \phi}}{2\pi\hbar \rho} \sqrt{\frac{\pi x}{2}} e^{-i(x-\pi/4)}, \quad (105)$$

$$E_z^{VE} = -\frac{n_0\rho (c/v)e^{-\alpha \phi}}{2\pi\hbar \rho} \sqrt{\frac{\pi x}{2}} e^{-i(x-\pi/4)}, \quad (106)$$

$$H_{\phi}^{VE} = \frac{pe^{-\alpha \phi}}{2\hbar z} \sqrt{\frac{\pi x}{2}} e^{-i(x-\pi/4)}, \quad (107)$$

$$E_{\phi}^{VE} = -\frac{n_0(c/v)}{\hbar \rho}, \quad (108)$$

and

$$\frac{E_{\phi}^{VE}}{E_z^{VE}} = -\frac{n_0/n_e(c/v)}{c/v} \quad (109)$$

When $x = 1.6$, the HED expressions listed in the previous section reduce to

$$E^{HE} = \frac{p \cos \phi e^{-\alpha \phi}(ix)}{2\pi(\epsilon_e + i\omega_e)\hbar (c/v)^2} \sqrt{\frac{\pi x}{2}} e^{-i(x-\pi/4)}, \quad (110)$$

$$E^{HE} = \frac{p \sin \phi e^{-\alpha \phi}}{2\pi(\epsilon_e + i\omega_e)\hbar (c/v)^2} \sqrt{\frac{\pi x}{2}} e^{-i(x-\pi/4)}, \quad (111)$$

E^{HE} = \frac{p \cos \phi e^{-\alpha \phi}(ix)}{2\pi(\epsilon_e + i\omega_e)\hbar (c/v)^2} \sqrt{\frac{\pi x}{2}} e^{-i(x-\pi/4)}, \quad (110)$$

$$E^{HE} = \frac{p \sin \phi e^{-\alpha \phi}}{2\pi(\epsilon_e + i\omega_e)\hbar (c/v)^2} \sqrt{\frac{\pi x}{2}} e^{-i(x-\pi/4)}, \quad (111)$$

15
When $x > 1.6$, the HMD expressions listed in the previous section reduce to

\begin{align*}
E_{HE}^z &= \frac{i\omega_0 \rho \cos \phi e^{-\alpha \rho}}{2\pi \gamma \rho_0} \sqrt{\frac{\pi x}{2}} e^{-i(x-\pi/4)}, \\
H_{HE}^z &= \frac{p \sin \phi e^{-\alpha \rho}}{2\pi \gamma \rho_0 (c/v)^2 \rho^2} \sqrt{\frac{\pi x}{2}} e^{-i(x-\pi/4)}, \\
H_{HE}^\rho &= \frac{p \cos \phi e^{-\alpha \rho}(ix)}{2\pi \gamma \rho_0 (c/v)^2 \rho^2} \sqrt{\frac{\pi x}{2}} e^{-i(x-\pi/4)}, \\
\frac{E_{HE}^z}{H_{HE}^\rho} &= -\frac{\eta_0}{\eta_0} \frac{(c/v)}{(c/v)}, \\
\frac{E_{HE}^z}{H_{HE}^\rho} &= -\frac{\eta_0}{\eta_0} \frac{(c/v)}{(c/v)},
\end{align*}

and

\begin{align*}
\frac{E_{HE}^z}{H_{HE}^\rho} &= \frac{E_{HE}^z}{H_{HE}^\rho} = \frac{ix \cot \phi \left[ \sin(\phi/a) \right]}{\left( \phi/a \right)}.
\end{align*}

When $x > 1.6$, the HMD expressions listed in the previous section reduce to

\begin{align*}
E_{HM}^z &= \frac{m x_0 \rho \cos \phi e^{-\alpha \rho}(ix)}{2-h_0 (c/v)^2 \rho^2} \sqrt{\frac{\pi x}{2}} e^{-i(x-\pi/4)}, \\
E_{HM}^\rho &= \frac{-m x_0 \rho \sin \phi e^{-\alpha \rho}}{2\pi h_0 (c/v)^2 \rho^2} \sqrt{\frac{\pi x}{2}} e^{-i(x-\pi/4)}, \\
E_{HM}^\rho &= \frac{i\omega_0 \rho \cos \phi e^{-\alpha \rho}}{2\pi \rho_0} \sqrt{\frac{\pi x}{2}} e^{-i(x-\pi/4)}, \\
H_{HM}^z &= \frac{m \sin \phi e^{-\alpha \rho}}{2\pi h_0 (c/v)^2 \rho^2} \sqrt{\frac{\pi x}{2}} e^{-i(x-\pi/4)}, \\
H_{HM}^\rho &= \frac{m \cos \phi e^{-\alpha \rho}(ix)}{2-h_0 (c/v)^2 \rho^2} \sqrt{\frac{\pi x}{2}} e^{-i(x-\pi/4)}, \\
\frac{E_{HM}^z}{H_{HM}^\rho} &= -\frac{\eta_0}{\eta_0} \frac{(c/v)}{(c/v)}, \\
\frac{E_{HM}^z}{H_{HM}^\rho} &= -\frac{\eta_0}{\eta_0} \frac{(c/v)}{(c/v)}.
\end{align*}
DISCUSSION

We have used the recently developed theory of Greifinger and Greifinger and the Wait exponential ionospheric-conductivity profile to determine average ELF propagation constants for both daytime and nighttime propagation conditions. The resulting average values of ELF attenuation rate, phase-velocity ratio, and ionospheric reflection height (presented in table 1) are in excellent agreement with measured data.

To shed further light on the nature of the ELF field strengths in the earth-ionosphere waveguide, the radial impedance of the wave \((E_z/H_z)\) now is

\[
\frac{H^H M \phi}{H_D^H M} - \frac{E^H M}{E_D^H M} = -ix \cot \phi \left[ \frac{\sin(\rho/a)}{(\rho/a)} \right].
\]  

(125)

Table 1. Typical ELF Propagation Parameters

<table>
<thead>
<tr>
<th>Frequency (Hz)</th>
<th>Time of Day</th>
<th>h (km)</th>
<th>c/v</th>
<th>(\alpha) (dB/Mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>30</td>
<td>Day</td>
<td>46.1</td>
<td>1.34</td>
<td>0.6</td>
</tr>
<tr>
<td>50</td>
<td>Day</td>
<td>47.8</td>
<td>1.30</td>
<td>1.0</td>
</tr>
<tr>
<td>75</td>
<td>Day</td>
<td>49.1</td>
<td>1.27</td>
<td>1.5</td>
</tr>
<tr>
<td>100</td>
<td>Day</td>
<td>50.1</td>
<td>1.25</td>
<td>1.9</td>
</tr>
<tr>
<td>150</td>
<td>Day</td>
<td>51.4</td>
<td>1.22</td>
<td>2.8</td>
</tr>
<tr>
<td>200</td>
<td>Day</td>
<td>52.4</td>
<td>1.20</td>
<td>3.7</td>
</tr>
<tr>
<td>300</td>
<td>Day</td>
<td>53.7</td>
<td>1.18</td>
<td>5.4</td>
</tr>
<tr>
<td>30</td>
<td>Night</td>
<td>72.0</td>
<td>1.12</td>
<td>0.6</td>
</tr>
<tr>
<td>50</td>
<td>Night</td>
<td>73.3</td>
<td>1.11</td>
<td>0.8</td>
</tr>
<tr>
<td>75</td>
<td>Night</td>
<td>74.3</td>
<td>1.10</td>
<td>1.0</td>
</tr>
<tr>
<td>100</td>
<td>Night</td>
<td>75.0</td>
<td>1.09</td>
<td>1.2</td>
</tr>
<tr>
<td>150</td>
<td>Night</td>
<td>76.0</td>
<td>1.09</td>
<td>1.6</td>
</tr>
<tr>
<td>200</td>
<td>Night</td>
<td>76.8</td>
<td>1.08</td>
<td>2.0</td>
</tr>
<tr>
<td>300</td>
<td>Night</td>
<td>77.8</td>
<td>1.07</td>
<td>2.7</td>
</tr>
</tbody>
</table>
considered. By definition, this quantity is the impedance of the wave looking in the radial, or \( \phi \), direction.

For \( x >> 1 \),

\[
\frac{E_z}{H_\phi} = -n_0 (c/v) ,
\]

while, for \( x << 1 \) and \( \phi < h/3 \),

\[
\frac{E_z}{H_\phi} = \frac{n_0}{ik\phi} \quad (126)
\]

and

\[
\frac{E_{HE}}{H_{HE}} = \frac{E_{HE}}{H_{HE}} = -ik\phi n_0 . \quad (128)
\]

For the intermediate, and most interesting, range,

\[
\frac{E_z}{H_\phi} = \frac{i n_0 (c/v)}{xG(u)} \left[ V(t) + \frac{i\pi}{2} G(u) x^2 H_0^2(x) \right]
\]

and

\[
\frac{E_{HE}}{H_{HE}} = \frac{E_{HE}}{H_{HE}} = \frac{i n_x G(u)}{(c/v) f(x)} . \quad (33)
\]

while, for \( x \leq 0.25 \), equations (33) and (42) reduce to

\[
\frac{E_z}{H_\phi} = \frac{i n_0}{(kz)G(u)} \left[ V(t) - G(u) x^2 \left[ \ln \left( \frac{1.125}{x} \right) - \frac{i\pi}{2} \right] \right]
\]

and

\[
\frac{E_{HE}}{H_{HE}} = \frac{E_{HE}}{H_{HE}} = \frac{i n_{xz} G(u)}{G(t)} . \quad (59)
\]

Referring to equation (59), we see that the minimum value of \( E_{VE}/H_{VE} \) will occur when \( V(t) = -G(u) x^2 \left[ \ln \left( \frac{1.125}{x} \right) - \frac{i\pi}{2} \right] \). That is,
Alternatively, the approximate distance from the VED source where the minimum value of radial wave impedance occurs, \( \omega_{mv} \), can be expressed as (from equation (129))

\[
\omega_{mv} = \frac{80.63 |E_Z^{VE}/H_{\phi}^{VE}|_{\text{min}}}{f(c/v)^2} \text{ km}.
\]  

(130)

Presented in figures 5 and 6 are plots of the VED radial wave impedance versus distance for frequencies of 30 to 300 Hz. Equation (33) and the values of \( h \) and \( c/v \) listed in table 1 were used in the calculations. Note that, for frequencies of 30 to 100 Hz, there is a unique distance where the minimum value of radial wave impedance occurs. Presented in table 2 are values of \( \omega_{mv} \) calculated from equation (130). Comparing these values with the curves of figures 5 and 6 reveals that the table-2 \( \omega_{mv} \) calculations are accurate within 10 km.

Presented in figures 7 and 8 are plots of the HED and HMD radial wave impedance versus distance for frequencies of 30 to 300 Hz. Equation (42) and the values of \( h \) and \( c/v \) listed in table 1 were used in the calculations.

Table 2. Approximate Distance Where the Minimum Value of the VED Radial Wave Impedance Occurs

<table>
<thead>
<tr>
<th>Frequency (Hz)</th>
<th>Time of Day</th>
<th>Minimum ( E_Z^{VE} ) (ohms)</th>
<th>Approximate ( \omega_{mv} ) (km)</th>
</tr>
</thead>
<tbody>
<tr>
<td>30</td>
<td>Day</td>
<td>120</td>
<td>180</td>
</tr>
<tr>
<td>50</td>
<td>Day</td>
<td>170</td>
<td>162</td>
</tr>
<tr>
<td>75</td>
<td>Day</td>
<td>220</td>
<td>147</td>
</tr>
<tr>
<td>100</td>
<td>Day</td>
<td>270</td>
<td>139</td>
</tr>
<tr>
<td>30</td>
<td>Night</td>
<td>95</td>
<td>204</td>
</tr>
<tr>
<td>50</td>
<td>Night</td>
<td>140</td>
<td>153</td>
</tr>
<tr>
<td>75</td>
<td>Night</td>
<td>190</td>
<td>169</td>
</tr>
<tr>
<td>100</td>
<td>Night</td>
<td>230</td>
<td>156</td>
</tr>
</tbody>
</table>
Referring to figures 5 through 8, we see that there is a substantial variation in both the VED and HED radial wave impedance. For example, at 30 Hz, the VED wave impedance is equal to 30,000 ohms to 20 km, 120 ohms at 180 km, and 505 ohms (120πc/ν) at 5,000 km (figure 5). On the other hand, the 30-Hz HED (or HMD) wave impedance varies from 5 ohms at 20 km to 505 ohms at 5,000 km (figure 7).

As Wait\(^1\) has pointed out, the observed variation of the magnitude and/or phase of the radial wave impedance as a function of frequency should provide a basis for distance measuring (provided the atmospheric source could be represented by an equivalent VED or HED). Such a scheme, while admittedly crude, requires only one receiving station equipped with a vertical whip and loop antenna.

CONCLUSIONS

In this report, we have presented new formulas for HED, HMD, and VED ELF radio-wave propagation in the earth-ionosphere waveguide. These new formulas extend the results of Wait\(^1\) and Galejs,\(^2\) which are valid for measurement distances greater than approximately three ionospheric reflecting heights, down to the quasi-nearfield range, which is defined as the range where the measurement distance is greater than an earth wavelength but much less than a free-space wavelength. For the sake of completeness, the abovementioned previously derived formulas also have been included.

Plots of the VED, HED, and HMD radial wave impedance versus distance have been presented for both daytime and nighttime propagation conditions. These plots show a substantial variation in the radial wave impedances for distances less than 1,000 km. Also, we have shown that there is a unique distance where the minimum value of the VED radial wave impedance occurs.
Figure 1. Magnitude of the Functions $G(t)$, $H(t)$, and $V(t)$ Versus $t$

Figure 2. Magnitude of the Function $T$ Versus Distance
Figure 3. Magnitude of the Function $W$ Versus Distance

Figure 4. Magnitude of the Function $|f(x)|$ Versus $x$
Figure 5. VED Radial Wave Impedance Versus Distance for Daytime Propagation Conditions

Figure 6. VED Radial Wave Impedance Versus Distance for Nighttime Propagation Conditions
Figure 7. HED and HMD Radial Wave Impedance Versus Distance for Daytime Propagation Conditions

Figure 8. HED and HMD Radial Wave Impedance Versus Distance for Nighttime Propagation Conditions
REFERENCES


Appendix

E/H CALCULATION PROGRAMS

Five programs were written to solve for E/H fields at various distances from a transmission source. The following cases were considered:

1. \( E_z/H_o \) of an HED source, nearfield approximation;
2. \( E_z/H_o \) of a VED source, nearfield approximation;
3. \( E_z/H_o \) of an HED source, nearfield approximation;
4. \( E_z/H_o \) of an HED source, farfield approximation; and
5. \( E_z/H_o \) of a VED source, farfield approximation.

The programs are written in VAX-11 FORTRAN. They reside as executable modules on the Naval Underwater Systems Center (NUSC) VAX system. They can be invoked by keying RUN 'filename' where the five files shown below correspond to the five cases listed above:

1. v703::DUOI:[ETC.EHNEARFLD]EHHED;
2. v703::DUOI:[ETC.EHNEARFLD]EHVED;
3. v703::DUOI:[ETC.EHNEARFLD]EHNHO;
4. v703::DUOI:[ETC.EHFARFLD]EHRED; and
5. v703::DUOI:[ETC.EHFARFLD]EHVED.

Each of the programs contains an interactive section in which the user enters parameters of interest. These parameters involve the range of distance from the source for which \( E/H \) is to be calculated, the frequency of interest, the time of day, and, sometimes, the angle from the zenith.

Listings are provided for each of the programs created. Program structure and flow are clearly indicated in the listings. Most of the program variables, such as \( T \), \( GT = G, t \), \( X \), \( RHO \), \( RFLHT \) (= ionospheric reflection height), \( CIEL \) (= c i), \( JD = night or day values \), etc., correspond exactly to the variables in the equations. It is suggested that the reader have the equations at hand, particularly when tracing the flow of the actual calculations, as the calculations follow the equations in a logical and consistent manner. These programs are pretty much 'number crunchers;' their flow is not complex.

Calls are made for calculation of hyperbolic functions and Hankel functions. The hyperbolic function routines also are written in VAX-11 FORTRAN; the listings for these routines have been included. The Hankel-function
calculation consists of calling the appropriate IMSL Bessel functions and combining them in the calling program, as the Hankel function is a combination of Bessel functions. The IMSL routines are available through the 'IMSLIBS' library on the VAX.

C**********************************************************
C CREATED:  27FEB85
C LAST UPDATE:  07MAR85
C BY:  A. KUZEL
C PURPOSE:  FORTRAN PROGRAM TO CALCULATE THE VALUE
C OF E/H FOR AN ELF WAVEFORM OF A SET OF
C ALLOWED FREQUENCIES AT VARIOUS
C DISTANCES, <= 200 KM, FROM A HIS SOURCE.
C THIS IS THE NEAR FIELD APPROXIMATION.
C**********************************************************

C*DECLARE VARIABLES*
COMPLEX  I,H0,H1,H2,RAZ
REAL K,J0,J1,PI,RHO,FREQ,INC,MAXDIS,T,U,V,X,YO(1)
REAL CVEE,RFLHT,MMBSJ0,MMBSJ1,RAZA,RA2PH,YI(1)
CHARACTER TOO

C*INITIALIZE CONSTANTS*
K=3.00E+05
PI=4*ATAN(1.0)
I=CMPLX(0.0,1.0)

C**********************************************************
C*SELECT PROPER IONOSPHERIC REFLECTION HEIGHT AND
C/V CONSTANTS BASED ON USER'S INPUT DATA*

IF(FREQ.EQ.30) THEN
  IF(TOD.EQ.'D') THEN
    RFLHT=46.1
    CVEE=1.34
  END IF
  IF(TOD.EQ.'N') THEN
    RFLHT=72.0
    CVEE=1.12
  END IF
END IF
IF(FREQ.EQ.50) THEN
  IF(TOD.EQ.'D') THEN
    RFLHT=47.8
    CVEE=1.30
  END IF
  IF(TOD.EQ.'N') THEN

A-2
RFLHT=73.3
CVEE=1.11
END IF
END IF
IF(FREQ.EQ.75) THEN
IF(TOD.EQ. '0') THEN
RFLHT=49.1
CVEE=1.27
END IF
IF(TOD.EQ. 'N') THEN
RFLHT=74.3
CVEE=1.10
END IF
END IF
IF(FREQ.EQ.100) THEN
IF(TOD.EQ. '0') THEN
RFLHT=50.1
CVEE=1.25
END IF
IF(TOD.EQ. 'N') THEN
RFLHT=75.0
CVEE=1.09
END IF
END IF
IF(FREQ.EQ.150) THEN
IF(TOD.EQ. '0') THEN
RFLHT=51.4
CVEE=1.22
END IF
IF(TOD.EQ. 'N') THEN
RFLHT=76.0
CVEE=1.09
END IF
END IF
IF(FREQ.EQ.200) THEN
IF(TOD.EQ. '0') THEN
RFLHT=52.4
CVEE=1.20
END IF
IF(TOD.EQ. 'N') THEN
RFLHT=76.8
CVEE=1.08
END IF
END IF
IF(FREQ.EQ.300) THEN
IF(TOD.EQ. '0') THEN
RFLHT=53.7
CVEE=1.18
END IF
IF(TOD.EQ. 'N') THEN
RFLHT=77.8
CVEE=1.07
END IF
END IF

*WRITE HEADERS FOR OUTPUT TABLE*
WRITE(10,220)
WRITE(10,208)
WRITE(10,207)FREQ
IF(TOD.EQ. 'D') WRITE(10,211)
IF(TOD.EQ.'N') WRITE(10,212)
WRITE(10,208)
WRITE(10,208)
WRITE(10,208)
C
*PERFORM THE ACTUAL CALCULATION*
400  X=2*PI*FREQ*RHO*CVEE/K
T=ROPI/(2*RFLHT*CVEE**2)
U=T
V=T
CALL COTH(U)
CALL CSCH(V)
GT=GT+(1-2/PI)*T**2*V**2
JO=MBSJ0(X,IER)
J1=MBSJI(X,IER)
CALL MBSYN(X,0,1,Y0,IER)
CALL MBSYN(X,0.999,1,Y1,IER)
HO=CMPLX(J0,-Y0(I))
HI=CMPLX(J1,-Y1(I))
HR=HO/HI
RAZ=1+X*HR
RAZ=RAZ+120+PI*I*X
RAZ=RAZ
U=RFLHT/RHO
CALL TANH(U)
RAZ=RAZ/(CVEE*GT*U)
RAZA=CABS(RAZ)
RAZPH=ATAN2(AIMAG(RAZ),REAL(RAZ))
RAZPH=RAZPH+180.00/PI
WRITE(10,210)RHO,RAZA,RAZPH
RHO=RHO+INC
IF(RHO.LE.MAXOIS) GO TO 400
C
*FORMAT STATEMENTS*
200  FORMAT(4X,'ENTER STARTING DISTANCE FROM SOURCE: ')
201  FORMAT(4X,'ENTER DISTANCE INCREMENT: ')
202  FORMAT(4X,'ENTER MAXIMUM DISTANCE TO COMPUTE: ')
203  FORMAT(4X,'ENTER FREQUENCY: ')
204  FORMAT(F12.4)
205  FORMAT(4X,'NIGHT (N) OR DAY (D)?: ')
206  FORMAT(A2)
207  FORMAT(9X,'HORIZONTAL ELECTRIC DIPOLE FREQUENCY ',F7.2)
208  FORMAT(5X,'; ')
209  FORMAT(11X,'DISTANCE',8X,'MAGNITUDE',9X,'PHASE')
210  FORMAT(10X,F7.2,10X,F7.2,10X,F7.2)
211  FORMAT(9X,'TOD = DAYTIME')
212  FORMAT(9X,'TOD = NIGHT')
220  FORMAT(9X,'E/M NEAR FIELD APPROXIMATION')
STOP
END
*FORTRAN PROGRAM TO CALCULATE THE VALUE OF E/H FOR AN ELF WAVEFORM OF A SET OF ALLOWED FREQUENCIES F AT VARIOUS DISTANCES, <= 200 KM, FROM THE VED SOURCE. THIS IS THE NEAR FIELD APPROXIMATION.*

*DECLARE VARIABLES*

```fortran
COMPLEX I,H0,H1,HR,RAZ
REAL K,J0,J1,PI,RHO,FREQ,INC,MAXDIS,T,U,V,X,GT,YO(1)
REAL CVEE,RFLHMT,MMBSJ0,MMBSJ1,RAZA,RAZPH,Y1(1)
CHARACTER TOD
```

*INITIALIZE CONSTANTS*

```fortran
K=3.005E05
PI=4*ATAN(1.0)
I=CMPLX(0.0,1.0)
```

THE FOLLOWING SECTION IS THE INTERACTIVE PORTION OF THE PROGRAM*

THE USER Chooses STARTING DISTANCE, INCREMENT, AND ENDING DISTANCE TO CALCULATE, NIGHT OR DAY VALUES, AND FREQUENCY OF INTEREST.

```fortran
WRITE(5,200)
READ(5,204)RHO
WRITE(5,201)
READ(5,204)INC
WRITE(5,202)
READ(5,204)MAXDIS
WRITE(5,203)
READ(5,204)FREQ
WRITE(5,205)
READ(5,206)TOD
```

*CHOOSE PROPER VALUES OF IONOSPHERIC REFLECTION HEIGHT AND C/V CONSTANTS BASED ON USER'S INPUT DATA*

```fortran
IF(FREQ.EQ.30) THEN
  IF(TOD.EQ.'D') THEN
    RFLHMT=46.1
    CVEE=1.34
  END IF
  IF(TOD.EQ.'N') THEN
    RFLHMT=72.0
    CVEE=1.12
  END IF
END IF
END IF
IF(FREQ.EQ.50) THEN
  IF(TOD.EQ.'D') THEN
    RFLHMT=47.8
    CVEE=1.30
  END IF
  IF(TOD.EQ.'N') THEN
```
RFLHT=73.3
CVEE=1.11
END IF
END IF
IF(FREQ.EQ.75) THEN
IF(TOD.EQ. '0') THEN
RFLHT=49. 1
CVEE=1.27
END IF
IF(TOD.EQ. 'N') THEN
RFLHT=74.3
CVEE=1.10
END IF
END IF
IF(FREQ.EQ.100) THEN
IF(TOD.EQ. '0') THEN
RFLHT=50.1
CVEE=1.25
END IF
IF(TOD.EQ. 'N') THEN
RFLHT=75.0
CVEE=1.09
END IF
END IF
IF(FREQ.EQ.150) THEN
IF(TOD.EQ. '0') THEN
RFLHT=51.4
CVEE=1.22
END IF
IF(TOD.EQ. 'N') THEN
RFLHT=76.0
CVEE=1.09
END IF
END IF
IF(FREQ.EQ.200) THEN
IF(TOD.EQ. '0') THEN
RFLHT=52.4
CVEE=1.20
END IF
IF(TOD.EQ. 'N') THEN
RFLHT=76.8
CVEE=1.08
END IF
END IF
IF(FREQ.EQ.300) THEN
IF(TOD.EQ. '0') THEN
RFLHT=53.7
CVEE=1.18
END IF
IF(TOD.EQ. 'N') THEN
RFLHT=77.8
CVEE=1.07
END IF
END IF

*WRITE HEADERS FOR OUTPUT TABLE*

WRITE(10,220)
WRITE(10,208)
WRITE(10,207)FREQ
IF(TOD.EQ. 'D') WRITE(10,211)
C
*PERFORM THE ACTUAL CALCULATION*

400

x=2*PI*FREQ*RHO*CVEE/K
T=RHO*PI/(2*RFLHT*CVEE)**2
U=T
V=T
CALL COTH(U)
CALL CSCH(V)
G=(1-1/2)*T*2*V**2
V=(3*V**2-U)/2
H=G-V
J1=MMBSJ1(X,IER)
J0=MMBSJO(X,IER)
CALL MMBSYN(X,0.1,V0,IER)
CALL MMBSYN(X,0.9991,01,IER)
HO=CMPLX(J0,-Y0(1))
H1=CMPLX(J1,-Y1(1))
RAZ=HO*X**2*CVEE**2*H*(PI/2)*I
RAZ=V*V
U=RFLHT/RHO
CALL TANH(U)
RAZ=RAZ/U*V
RAZ=RAZ*I*120*PI*CVEE
RAZ=CABS(RAZ)
RAZPH=ATAN2(AIMAG(RAZ),REAL(RAZ))
WRITE(10,210) RHO,RAZA,RAZPH
RHO=RHO+INC
IF(RHO.LE.MAXDIS) GO TO 400

*FORMAT STATEMENTS*

201 FORMAT(4X,ENTER STARTING DISTANCE FROM SOURCE:)
202 FORMAT(4X,ENTER MAXIMUM DISTANCE TO COMPUTE:)
203 FORMAT(4X,ENTER FREQUENCY:)
204 FORMAT(F12.4)
205 FORMAT(4X,NIGHT (N) OR DAY (D)?)
206 FORMAT(A2)
207 FORMAT(10X,VERTICAL ELECTRIC DIPOLE FREQ = ',F7.2)
208 FORMAT(5X)
209 FORMAT(11X,DISTANCE',8X,MAGNITUDE',10X,PHASE')
210 FORMAT(10X,F7.2,9X,F9.2,10X,F7.2)
211 FORMAT(12X,TOD = DAYTIME')
212 FORMAT(12X,TOD = NIGHT')
220 FORMAT(9X,E/H NEAR FIELD APPROXIMATION')
STOP
END
FORTRAN PROGRAM TO CALCULATE THE VALUE OF $E_{sub Z}$ OVER $H_{sub Rho}$ FOR AN ELF WAVEFORM OF A SET OF ALLOWED FREQUENCIES AT VARIOUS DISTANCES, $\leq 200$ KM, FROM AN HED SOURCE, AT VARIOUS ANGLES FROM THE ZENITH.

This is the near field approximation.

*DECLARE VARIABLES*

```fortran
COMPLEX I,RAZ
REAL K,PI,RHO,FREQ,INC,MAXDIS,T,U,X,GT
REAL CVEE,RFLHT,RAZ,RAZPH,VT,HT,PHI
CHARACTER TOD
```

*INITIALIZE VARIABLES*

```fortran
K=3.00E-05
PI=4*ATAN(1.0)
I=CMPLX(0.0,0.0)
```

THE FOLLOWING SECTION IS THE INTERACTIVE PORTION OF THE PROGRAM.

THE USER CHOOSES STARTING DISTANCE, INCREMENT, AND ENDING DISTANCE TO CALCULATE, NIGHT OR DAY VALUES, FREQUENCY OF INTEREST, AND ANGLE OF INTEREST.

```fortran
WRITE(5,200)
READ(5,204)RHO
WRITE(5,201)
READ(5,204)INC
WRITE(5,202)
READ(5,204)MAXDIS
WRITE(5,203)
READ(5,204)FREQ
WRITE(5,205)
READ(5,206)TOD
WRITE(5,213)
READ(5,204)PHI
```

*SELECT PROPER IONOSPHERIC REFLECTION HEIGHT AND C/V VALUES BASED ON USER'S INPUT DATA.*

```fortran
IF(FREQ.EQ.30) THEN
  IF(TOD.EQ.'D') THEN
    RFLHT=46.1
    CVEE=1.34
  END IF
  IF(TOD.EQ.'N') THEN
    RFLHT=72.0
    CVEE=1.12
  END IF
END IF
```

```fortran
IF(FREQ.EQ.50) THEN
  IF(TOD.EQ.'D') THEN
    RFLHT=47.8
  END IF
```

CVEE=1.30
END IF
IF(TOD.EQ.‘N’) THEN
RFLHT=73.3
CVEE=1.11
END IF
END IF
IF(FREQ.EQ.75) THEN
IF(TOD.EQ.‘0’) THEN
RFLHT=49.1
CVEE=1.27
END IF
IF(TOD.EQ.‘N’) THEN
RFLHT=74.3
CVEE=1.10
END IF
END IF
IF(FREQ.EQ.100) THEN
IF(TOD.EQ.‘0’) THEN
RFLHT=50.1
CVEE=1.25
END IF
IF(TOD.EQ.‘N’) THEN
RFLHT=75.0
CVEE=1.09
END IF
END IF
IF(FREQ.EQ.150) THEN
IF(TOD.EQ.‘0’) THEN
RFLHT=51.4
CVEE=1.22
END IF
IF(TOD.EQ.‘N’) THEN
RFLHT=76.0
CVEE=1.09
END IF
END IF
IF(FREQ.EQ.200) THEN
IF(TOD.EQ.‘0’) THEN
RFLHT=53.7
CVEE=1.18
END IF
IF(TOD.EQ.‘N’) THEN
RFLHT=77.8
CVEE=1.07
END IF
END IF
*WRITE HEADERS FOR OUTPUT TABLE*
WRITE(*,207)FREQ
IF(TOD.EQ.'D') WRITE(10,211) PHI
IF(TOD.EQ.'N') WRITE(10,212) PHI
WRITE(10,208)
WRITE(10,209)
WRITE(10,208)
PHI=PHI*PI/180.00

C  *PERFORM THE ACTUAL CALCULATION*

400  X=2*PI*FREQ*RHO*CVEE/X
    T=RHO*PI/(2*RFLHT*CVEE**2)
    U=T
    V=T
    CALL COTH(U)
    CALL CSCH(V)
    GT=2*T/PI*U
    VT=T**3*V**2*U
    HT=VT+GT
    U=(1/TAN(PHI))
    V=RFLHT/RHO
    CALL TANH(V)
    RAZ=(*120*PI*X*U)
    RAZ=RAZ/(CVEE*HT+V)
    RAZA=CABS(RAZ)
    RAZPH=ATAN2(AIMAG(RAZ),REAL(RAZ))
    RAZPH=RAZPH*180.00/PI
    WRITE(10,210)RHO,RAZA,RAZPH
    RHO=RHO INC
    IF(RHO.LE.MAXDIS) GO TO 400

C  *FORMAT STATEMENTS*

200  FORMAT(4X,'ENTER STARTING DISTANCE FROM SOURCE:')
201  FORMAT(4X,'ENTER DISTANCE INCREMENT:')
202  FORMAT(4X,'ENTER MAXIMUM DISTANCE TO COMPUTE:')
203  FORMAT(4X,'ENTER FREQUENCY:')
204  FORMAT(F12.4)
205  FORMAT(4X,'ENTER SOURCE EZ/RHO FREQ = ',F7.2)
206  FORMAT(4X,'ENTER NIGHT (N) OR DAY (D)?:')
207  FORMAT(4X,'ENTER SOURCE EZ/RHO FREQ = ',F7.2)
208  FORMAT(5X,'')
209  FORMAT(11X,'DISTANCE',8X,'MAGNITUDE',10X,'PHASE')
210  FORMAT(11X,F7.2,9X,F9.2,10X,F7.2)
211  FORMAT(12X,'TOD = DAYTIME PHI = ',F5.2)
212  FORMAT(12X,'TOD = NIGHT PHI = ',F5.2)
213  FORMAT(4X,'ENTER PHI IN DEGREES:')
STOP
END
**Purpose:** This program calculates the value of the E/M field of an ELF waveform of one of a set of frequencies at various distances > 200 km from an NED source. This is the far field approximation.

---

**Declare Variables**

- COMPLEX I,Y,H0,H1,HR
- REAL JO,J1,Y1(1),YMBS0,YMBSJ1,MAXDIS
- REAL RHO,INC,P1,CVEE,FREQ,FUDGE,X,YA,PH
- CHARACTER TOD

**Initialize Constants**

- PI=4*ATAN(1.0)
- I=CMPLX(0.0,1.0)

The following section is the interactive portion of the program. The user chooses starting distance, increment, and ending distance to calculate, night or day values, and the frequency of interest.

**Pseudocode:**

1. WRITE(5,540)
2. READ(5,545) RHO
3. WRITE(5,547)
4. READ(5,560) INC
5. WRITE(5,548)
6. READ(5,545) MAXDIS
7. WRITE(5,549)
8. READ(5,550) FREQ
9. WRITE(5,205)
10. READ(5,206) TOD

**Select Proper Ionospheric Reflection Height and C/V Constants Based on Users Input Data**

- IF (FREQ.EQ.30) THEN
  - IF (TOD.EQ.'D') THEN
    - RFLMT=46.1
    - CVEE=1.34
  - END IF
  - IF (TOD.EQ.'N') THEN
    - RFLMT=72.0
    - CVEE=1.12
  - END IF
- END IF
- IF (FREQ.EQ.50) THEN
  - IF (TOD.EQ.'D') THEN
    - RFLMT=47.8
    - CVEE=1.30
  - END IF
  - IF (TOD.EQ.'N') THEN
    - RFLMT=73.3
CVC=1.11
END IF
END IF
IF(FREQ.EQ.75) THEN
IF(TOD.EQ. 'D') THEN
RFLHT=49.1
CVC=1.27
END IF
IF(TOD.EQ. 'N') THEN
RFLHT=74.3
CVC=1.10
END IF
END IF
IF(FREQ.EQ.100) THEN
IF(TOD.EQ. 'D') THEN
RFLHT=50.1
CVC=1.25
END IF
IF(TOD.EQ. 'N') THEN
RFLHT=75.0
CVC=1.09
END IF
END IF
IF(FREQ.EQ.150) THEN
IF(TOD.EQ. 'D') THEN
RFLHT=51.4
CVC=1.22
END IF
IF(TOD.EQ. 'N') THEN
RFLHT=76.0
CVC=1.09
END IF
END IF
IF(FREQ.EQ.200) THEN
IF(TOD.EQ. 'D') THEN
RFLHT=52.4
CVC=1.20
END IF
IF(TOD.EQ. 'N') THEN
RFLHT=76.8
CVC=1.08
END IF
END IF
IF(FREQ.EQ.300) THEN
IF(TOD.EQ. 'D') THEN
RFLHT=53.7
CVC=1.18
END IF
IF(TOD.EQ. 'N') THEN
RFLHT=77.8
CVC=1.07
END IF
END IF
END IF
IF(FREQ.EQ.400) CVC=1.15
IF(FREQ.EQ.800) CVC=1.11
IF(FREQ.EQ.1600) CVC=1.07

*WRITE HEADERS FOR OUTPUT TABLE*
WRITE(10,570)FREQ
IF(TOD.EQ. 'D') WRITE(10,211)
IF(TOD.EQ.'N') WRITE(10,212)
WRITE(10,575)
WRITE(10,580)
WRITE(10,575)
C  *PERFORM THE ACTUAL CALCULATION*
100  X=2*PI*FREQ*RHO*CVEE
    X=X/FUDGE
    JD=MMBSJ0(X,IER)
    JJ=MMBSJ1(X,IER)
    CALL MMBSYN(X,0.0,1,Y0,IER)
    CALL MMBSYN(X,0.999,1,Y1,IER)
    HO=CMPLX(JD,-Y0(I))
    H1=CMPLX(J1,-Y1(I))
    HR=HO/H1
    V=1.0-X*HR
    V=V*X/V
    V=V-PI*120*CVEE
    V=-I*V
    YA=CABS(V)
    PH=ATAN2(AIMAG(V),REAL(V))
    PH=PH*180.00/PI
    WRITE(10,600)RHO,YA,PH
    RHO=RHO+INC
    IF(RHO.LE.MAXD) GO TO 100
C  *FORMAT STATEMENTS*
205  FORMAT(4X,'NIGHT (N) OR DAY (D)?:')
206  FORMAT(A2)
211  FORMAT(9X,'TOD = DAYTIME')
212  FORMAT(9X,'TOD = NIGHT')
220  FORMAT(9X,'E/M FAR FIELD APPROXIMATION')
540  FORMAT(5X,'ENTER ORIGINAL DISTANCE FROM SOURCE:')
545  FORMAT(F9.2)
547  FORMAT(5X,'ENTER INCREMENT FOR THIS DISTANCE:')
548  FORMAT(5X,'ENTER MAX DISTANCE TO COMPUTE:')
550  FORMAT(2X,'ENTER FREQUENCY:')
560  FORMAT(F7.2)
570  FORMAT(9X,'HORIZONTAL ELECTRIC DIPOLE FREQ = ',F7.2)
575  FORMAT(4X,'')
580  FORMAT(12X,'DISTANCE',14X,'MAGNITUDE',15X,'PHASE')
600  FORMAT(10X,E12.5,10X,E12.5,10X,E12.5)
END
*DECLARE VARIABLES*

COMPLEX I,Y,M,HO,MR
REAL JO,J1,Y0(1),V1(1),MMBSJO,MMBSJ1,MAXDIS
REAL RHO,INC,PI,CVEE,FREQ,FUDGE,X,YA,PH
CHARACTER TOO

*INITIALIZE CONSTANTS*

PI=4*ATAN(1.0)
I=CMPLX(0.0,1.0)

THE FOLLOWING SECTION IS THE INTERACTIVE SECTION OF THE PROGRAM
OF THE PROGRAM. THE USER CHOOSES STARTING DISTANCE, INCREMENT, AND
ENDING DISTANCE TO CALCULATE, NIGHT OR DAY VALUES, AND THE FREQUENCY.

*SELECT PROPER IONOSPHERIC REFLECTION HEIGHT AND
C/V VALUES BASED ON USER'S INPUT DATA*

IF(FREQ.EQ.30) THEN
  IF(TOD.EQ. 'D') THEN
    RFLHT=46.1
    CVEE=1.34
  END IF
  IF(TOD.EQ. 'N') THEN
    RFLHT=72.0
    CVEE=1.12
  END IF
END IF

IF(FREQ.EQ.50) THEN
  IF(TOD.EQ. 'D') THEN
    RFLHT=47.9
    CVEE=1.30
  END IF
  IF(TOD.EQ. 'N') THEN
    RFLHT=72.0
    CVEE=1.12
  END IF
END IF

WRITE(5,540)
READ(5,545)RHO
WRITE(5,547)
READ(5,560)INC
FUDGE=3.0E-05
WRITE(5,548)
READ(5,545)MAXDIS
WRITE(5,550)
READ(5,560)FREQ
WRITE(5,205)
READ(5,206)TOD

THE PROGRAM CALCULATES THE VALUE OF E/H
OF AN ELF WAVEFORM OF ONE OF A SET OF
FREQUENCIES AT VARIOUS DISTANCES.
>= 200 KM. FROM THE VED SOURCE. THIS IS
THE FAR FIELD APPROXIMATION.

BY: A. KUZEL
CREATED: 21FEB85
LAST UPDATE: 07MAR85

/**/
RFLHT=73.3
CVEE=1.11
ENDIF
ENDIF
IF(FREQ.EQ.75) THEN
IF(TOD.EQ."0") THEN
RFLHT=49.1
CVEE=1.27
ENDIF
IF(TOD.EQ."N") THEN
RFLHT=74.3
CVEE=1.10
ENDIF
ENDIF
IF(FREQ.EQ.100) THEN
IF(TOD.EQ."0") THEN
RFLHT=50.1
CVEE=1.25
ENDIF
IF(TOD.EQ."N") THEN
RFLHT=75.0
CVEE=1.09
ENDIF
ENDIF
IF(FREQ.EQ.150) THEN
IF(TOD.EQ."0") THEN
RFLHT=51.4
CVEE=1.22
ENDIF
IF(TOD.EQ."N") THEN
RFLHT=76.0
CVEE=1.09
ENDIF
ENDIF
IF(FREQ.EQ.200) THEN
IF(TOD.EQ."0") THEN
RFLHT=52.4
CVEE=1.20
ENDIF
IF(TOD.EQ."N") THEN
RFLHT=76.8
CVEE=1.08
ENDIF
ENDIF
IF(FREQ.EQ.300) THEN
IF(TOD.EQ."0") THEN
RFLHT=53.7
CVEE=1.18
ENDIF
IF(TOD.EQ."N") THEN
RFLHT=77.8
CVEE=1.07
ENDIF
ENDIF
IF(FREQ.EQ.400) CVEE=1.15
IF(FREQ.EQ.800) CVEE=1.11
IF(FREQ.EQ.1600) CVEE=1.07

*WRITE HEADERS FOR OUTPUT TABLE*
WRITE(10,220)
WRITE(10,575)
WRITE(10,570)FREQ
IF(TOD.EQ.'D') WRITE(10,211)
IF(TOD.EQ.'N') WRITE(10,212)
WRITE(10,575)
WRITE(10,580)
WRITE(10,575)
C
*PERFORM THE CALCULATION*
100 X=2*PI*FREQ*RHO*VEE
X=X/FUDGE
JO=MMBSJ0(X,IER)
JI=MMBSJ1(X,IER)
CALL MMBSYN(X,0.0.1,Y0,IER)
CALL MMBSYN(X,0.999,1,Y1,IER)
HO=CMPLX(J0,-Y0(I))
HI=CMPLX(J1,-Y1(I))
HR=HO/H1
V=HR
V=V*PI*120*VEE
V=-V*V
VA=CABS(Y)
PH=ATAN2(AIMAG(Y),REAL(Y))
PH=PH*180.00/PI
WRITE(10,600)RHO,Y,A,PH
RHO=RHO+INC
IF(RHO.LE.MD) GO TO 100
C
*FORMAT STATEMENTS*
205 FORMAT(4X,'NIGHT (N) OR DAY (D)?:')
206 FORMAT(A2)
211 FORMAT(9X,'TOD = DAYTIME')
212 FORMAT(9X,'TOD = NIGHT')
220 FORMAT(9X,'E/H FAR FIELD APPROXIMATION')
540 FORMAT(9X,'ENTER DISTANCE FROM SOURCE:')
545 FORMAT(F9.2)
547 FORMAT(5X,'ENTER INCREMENT FOR DISTANCE:')
548 FORMAT(5X,'ENTER MAXIMUM DISTANCE TO BE COMPUTED:')
550 FORMAT(2X,'ENTER FREQUENCY:')
560 FORMAT(F7.2)
570 FORMAT(5X,'VERTICAL ELECTRIC DIPOLE FREQ = ',F7.2)
575 FORMAT(1X,'')
580 FORMAT(12X,'DISTANCE',14X,'MAGNITUDE',14X,'PHASE')
600 FORMAT(10X,E12.5,10X,E12.5,10X,E12.5)
END
SUBROUTINE COTH(ZZ)
REAL ZZ, XX
XX = EXP(ZZ) + EXP(-ZZ)
XX = XX / (EXP(ZZ) - EXP(-ZZ))
ZZ = XX
RETURN
END

SUBROUTINE CSCH(ZZ)
REAL ZZ, XX
XX = 2 - XX
XX = XX / (EXP(ZZ) - EXP(-ZZ))
ZZ = XX
RETURN
END

SUBROUTINE TANH(ZZ)
REAL ZZ, XX
XX = EXP(ZZ) - EXP(-ZZ)
XX = XX / (EXP(ZZ) + EXP(-ZZ))
ZZ = XX
RETURN
END
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