Gain Anisotropy in Low Pressure Chemical Lasers

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**GAIN ANISOTROPY IN LOW PRESSURE CHEMICAL LASERS**

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**Abstract:**

The gain coefficient in a low pressure chemical laser, during lasing, differs when viewed along the optical resonator axis and when viewed normal to the axis. Expressions for axial and normal gain coefficients are deduced. The ratio of normal to axial gain coefficient is evaluated for a saturated multimode chemical laser employing a Fabry-Perot resonator. The results are useful for correlating numerical code calculations with experimental power on gain measurements made normal to the optical axis.
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I. INTRODUCTION

Continuous-wave (cw) chemical lasers are frequently operated at pressures of the order of 1 Torr to achieve efficient lasing. At these pressures, the medium is inhomogeneously (Doppler) broadened, and the line shape (gain coefficient versus frequency), under power on operation, is a function of the viewing angle. For example, when viewed along the optical resonator axis, the line shape may have "holes" because of the radiation field; when viewed from a direction orthogonal to the resonator axis, it is free of holes. This anisotropy of the gain medium is the subject of the present study. The purpose is to aid in the experimental validation of numerical cw chemical laser codes wherein the line shape, as viewed along the resonator axis, is predicted, and experimental measurements of power on gain are made from an orthogonal direction.

In the first portion of this study, general expressions for line shape are deduced. These expressions are then applied to determine both the axial and orthogonal gain distribution in a low pressure multimode cw chemical laser with a Fabry-Perot (F-P) resonator.
II. THEORY

A. Line shape

Consider a gain medium with mean velocity \( u \) in the \( x \) direction and thermal velocities \( v_x, v_y, v_z \) in the \( x, y, z \) directions, respectively (Fig. 1). The resonator axis is considered to be parallel to the \( y \) axis. Let \( v_o \) denote the resonant frequency for particles at rest. The particles with velocities \( v_y \) and \( v_z \) are resonant with frequencies \( v \) (i.e., Doppler effect)

\[
\left( \frac{v}{v_o} - 1 \right)_y = \frac{v_y}{c} \quad (1a)
\]
\[
\left( \frac{v}{v_o} - 1 \right)_z = \frac{v_z}{c} \quad (1b)
\]

for radiation in the \( y \) and \( z \) directions, respectively. Particle velocity and resonant frequency can be used interchangeably. Consider a two-level model and introduce the notation

\[
\Delta n \equiv n_2 - n_1 \quad (2a)
\]
\[
[\Delta n(v)]_y \equiv [n_2(v) - n_1(v)]_y \quad (2b)
\]
\[
[\Delta n(v)]_z \equiv [n_2(v) - n_1(v)]_z \quad (2c)
\]

where \( \Delta n \) is the net difference between the number of particles, per unit volume, in the upper (subscript 2) and lower (subscript 1) lasing levels. The quantities \( [\Delta n(v)]_y \, dv \) and \( [\Delta n(v)]_z \, dv \) denote the number of particles in the frequency interval \( v \) to \( v + dv \) that are resonant with radiation in the \( y \) and \( z \) directions, respectively. The Doppler broadened gain coefficient in the \( y \) and \( z \) directions is then
Fig. 1. Coordinate system.
\[
g_{y}(v) = \sigma_o \int_{-\infty}^{\infty} L(v-v')[\Delta n(v')]_y dv' \tag{3a}
\]
\[
g_{z}(v) = \sigma_o \int_{-\infty}^{\infty} L(v-v')[\Delta n(v')]_z dv' \tag{3b}
\]

where

\[
L(v-v') = \frac{\sigma(v,v')}{\sigma_o} = \left[1 + 4\left(\frac{v-v'}{\Delta v_h}\right)^2\right]^{-1} \tag{3c}
\]

Here \(\Delta v_h\) is the homogeneous width (FWHM) for the medium, \(\sigma(v,v')\) is the cross section for stimulated emission at \(v\) resulting from particles with "velocity" \(v'\) and \(\sigma_o\) denotes \(\sigma(v_o,v_o)\).

The quantity \([\Delta n(v')]_y\), in Eq. (3a) is found from a consideration of the optical resonator and may exhibit holes in the vicinity of optical mode frequencies. However, \([\Delta n(v')]_z\) has a Maxwellian distribution because there are no external factors that disturb the equilibrium in the \(z\) direction. Thus

\[
\frac{[\Delta n(v)]_z}{\Delta n} = \frac{\bar{p}_o}{\Delta v_d} \exp(-X^2) \tag{4a}
\]

where

\[
\bar{p}_o \equiv \left[(4 \ln 2)/\pi\right]^{1/2}/\Delta v_d \tag{4b}
\]
\[
X \equiv (4 \ln 2)^{1/2} \left(v-v_o\right)/\Delta v_d \tag{4c}
\]

and \(\Delta v_d\) is the Doppler width (FWHM). *

\[\Delta v_d/v_o = [8(\ln 2)kT/m]^{1/2}/c.\]
We now consider the limit $\Delta v_h/\Delta v_d \rightarrow 0$. The quantity $\Delta n(v')$ can be evaluated at $v' = v$ in Eqs. (3a), and (3b); these become

$$g_y(v) = (\pi/2)G_0 G_{\Delta v_h} \Delta n(v)$$  \hfill (5a)

$$g_z(v) = (\pi/2)G_p \Delta n \exp(-x^2)$$ \quad \hfill (5b)

Also,

$$g_y(v)/g_z(v) = \frac{[\Delta n(v)]_y}{p_0 \Delta n \exp(-x^2)}$$ \quad \hfill (6)

Thus, in the present limit, the ratio of the gain at frequency $v$ in the $y$ and $z$ direction is simply equal to the ratio of the number density of the particles that are resonant with the radiation in $y$ and $z$ directions, respectively. Numerical estimates for $g_y(v)/g_z(v)$, for a multimode chemical laser, are obtained in the next section using the results of References 3 and 4.

B. Multimode Chemical Laser

We consider a multimode low pressure cw chemical laser in the limit $\Delta v_h/\Delta v_d \rightarrow 0$. An F-P resonator is assumed. The laser is illustrated in Fig. 2. It is assumed that the F-P resonator is in perfect alignment and that no diffractive coupling occurs between the optical modes at adjacent streamwise stations. It is assumed further that the mirror separation $L$ is sufficiently large that the longitudinal mode separation $\Delta v_c = c/(2L)$ is of order $\Delta v_h$ or smaller. The latter assumption results in a large number of longitudinal
Fig. 2. Low pressure cw chemical laser with F-P resonator.
modes that truncate the line shape, when viewed along the optical axis, as indicated in Fig. 3. The performance of this laser was evaluated in Reference 3 using a two-level model. The corresponding gain, when viewed in the z direction, is obtained here.

Let $\zeta = k_{cd} x/u$ be a normalized streamwise distance, where $k_{cd}$ is a characteristic deactivation rate, and let $\nu_f$ denote the highest longitudinal mode frequency which is excited at each streamwise station. Introduce

$$X_f = (4 \ln 2)^{1/2} \frac{(\nu_f - \nu_o)}{\Delta \nu_c}$$

which is a function of $\zeta$. The number of longitudinal modes, at each streamwise station, equals $(\nu_f - \nu_o)/\Delta \nu_c$ and is large because $\Delta \nu_c$ is small. The resultant line shape, when viewed from the y and z directions, is illustrated in Fig. 3. The variation of $[\Delta n(\nu)]_z/(\overline{p_0} \Delta n)$ with frequency is Maxwellian as indicated in Eq. (5b). The variation of $[\Delta n(\nu)]_y/(\overline{p_0} \Delta n)$ with frequency is flat, in the range $|X| < X_f$ because lasing suppresses the gain coefficient to the resonator threshold value, $g_\text{c}$. However, the variation of $[\Delta n(\nu)]_y/(\overline{p_0} \Delta n)$ with frequency is Maxwellian in the range $|X| > X_f$. The condition that the net number density difference $\Delta n$ be the same, when viewed from either the y or z directions, indicates that the area under each of the curves, in Fig. 3, is the same. Hence these curves are related by the expressions.*

*Note from Fig. 3 that there are more upper level particles with large thermal velocities in the $\pm y$ direction than there are upper level particles with large thermal velocities in the $\pm z$ direction. This is because upper level particles with velocities $v_y$ that correspond to $|X| > X_f$ are unaffected by the radiation field whereas upper level particles with velocity $v_z$ are "homogeneously" deactivated by the radiation field.
Fig. 3. Line shape in multimode low pressure cw chemical laser.
\[
\frac{[\Delta n(v)]_z}{p_0 \Delta n} = \exp(-x^2)
\] (8a)

\[
\frac{[\Delta n(v)]_y}{p_0 \Delta n} = \frac{2X_f}{\pi^{1/2}} + \exp(X_f^2) \text{erfc} X_f |x| < X_f
\]

\[
= [ ]^{-1} \exp(X_f^2 - x^2) |x| > X_f
\] (8b)

The gain ratio, at \( v_o \), is simply

\[
\frac{g_z(v_o)}{g_y(v_o)} = \frac{2X_f}{\pi^{1/2}} + \exp(X_f^2) \text{erfc} X_f
\] (9)

which is plotted in Fig. 4. The ratio is a function of \( X_f \), which, as previously noted, is a measure of the number of longitudinal modes excited at each streamwise station \( \zeta \).

An analytic solution can be obtained for the variation of \( g_z/g_y \) with streamwise distance by considering the limit

\[
R \equiv \frac{k_{cr}}{k_{cd}} \gg 1; \quad \frac{g_c}{g_{mzp}} \ll 1; \quad R \frac{g_c}{g_{mzp}} = 0(1)
\] (10)

Here \( R \) is the ratio of the translational cross relaxation rate to the collisional deactivation rate and has a value of order 100.\(^3\) The quantity \( g_c/g_{mzp} \) is the ratio of resonator threshold gain \( g_c \) to the maximum zero power gain that is achieved in the laser medium, \( g_{mzp} \). The latter is the line center gain at \( \zeta = 0.3051 \) under nonlasing conditions.\(^3\) The ratio \( g_c/g_{mzp} \) typically equals 0.01 in a highly saturated laser. Hence the assumption \( Rg_c/g_{mzp} = 0(1) \) is realistic. The variation of \( X_f \) with \( \zeta \) is then\(^4\) (for \( \zeta \neq 0 \)
Fig. 4. Effect of frequency range parameter $X_f$ on gain ratio.
\[ \zeta = \left( \frac{(2 + \beta^2)^{1/2} - \beta}{2} \right) /4 \]  

(11a)

\[ \beta = \frac{2}{\pi} \left( \frac{R}{1.804} \frac{g_c}{g_{mzp}} \right) \left[ \exp \left( \frac{x_f^2}{\pi} \right) \text{erf} \left( \frac{2 x_f}{\pi^{1/2}} \right) \right] \]  

(11b)

The variation of \( g_y(v_o)/g_z(v_o) \) with streamwise distance is obtained from Eqs. (9) and (11) and is plotted in Fig. 5 as a function of laser saturation level. The lasing region extends from \( \zeta = 0 \) to \( \zeta = 1/2 \). The ratio \( g_z(v_o)/g_y(v_o) \) varies from values of 1.5 to 2.5 near \( \zeta = 0 \) to the value 1, at \( \zeta = 1/2 \). The gain ratio increases as the degree of saturation increases (i.e., as \( Rg_c/g_{mzp} \) decreases). For an F-P resonator, \( g_y(v_o) = g_c \), so that the ordinate in Fig. 5 can be interpreted as \( g_z(v_o)/g_c \). The corresponding values of \( x_f \) vary from 1.0 to 2.0 near \( \zeta = 0 \) to 0 near \( \zeta = 1/2 \) as may be inferred from Figs. 4 and 5. The present results require perfect alignment of the F-P resonator and no diffractive coupling between adjacent streamwise stations, as was previously assumed.
Fig. 5. Variation of gain ratio with streamwise distance in saturated multimode low pressure cw chemical laser.
III. CONCLUSIONS

The present study has discussed gain anisotropy in low pressure cw chemical lasers and has provided numerical results for the case of a saturated multimode laser with an F-P resonator. Values of the gain ratio $g_z(v_o)/g_y(v_o)$ up to about 2.5 were obtained for the particular cases considered herein. It may be concluded that the gain ratio differs significantly from one during leasing.

The value of $g_z(v_o)/g_y(v_o)$ can be easily obtained in the course of numerical resonator calculations for low pressure chemical lasers. Such codes (e.g., Reference 2) presently compute $g_y(v_o)$ and the local net inversion. The corresponding value of $g_z(v_o)$ is obtained from the local net inversion by assuming a Maxwellian velocity distribution [e.g., Eq. (5b)]. Hence, experimental power on measurements of $g_z(v_o)$ can be used to provide a check of numerical code predictions.

The effect of source flow on gain anisotropy can be deduced from Ref. 5 and has not been considered herein.
IV. REFERENCES


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