INTERFEROMETRIC DETERMINATION OF GPS (GLOBAL POSITIONING SYSTEM) SATELLITE. (U) MASSACHUSETTS INST OF TECH CAMBRIDGE DEPT OF EARTH ATMOSPHERI.

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**Authors:** R.I. Aboot, Y. Bock, C.C. Counselman III, R.W. King, S.A. Gourevitch, and B.J. Rosen

**Performing Organization:** Massachusetts Institute of Technology Dept. of Earth, Atmospheric, & Planetary Sciences, Cambridge, MA 02139

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**Abstract:** See reverse side.
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INTERFEROMETRIC DETERMINATION OF GPS SATELLITE ORBITS

R.I. Abbot, Y. Bock, C.C. Counselman III, R.W. King
Department of Earth, Atmospheric, and Planetary Sciences
Massachusetts Institute of Technology
Cambridge, Massachusetts 02139

S.A. Gourevitch and B.J. Rosen
Steinbrecher Corporation
185 New Boston St
Woburn, Massachusetts 01801

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INTRODUCTION

If the NAVSTAR Global Positioning System (GPS) is to be useful for crustal motion monitoring, the orbits of the GPS satellites will need to be known with uncertainties of the order of 1 part in $10^7$ or less. This level of accuracy has not been achieved. A major problem has been the instability of the cesium-beam frequency standards which are employed in most of the present tracking stations.

To show that interferometric observations by stations equipped with hydrogen-maser frequency standards can yield better accuracy, we have installed the dual-band tracking receivers of the Air Force Geophysics Laboratory at the NEROC Haystack Observatory (HO) in Massachusetts, at the U.S. Naval Observatory Timing Service Substation (NOTSS) in Florida, and at Harvard College Observatory's George R. Agassiz Station (GRAS) in Texas.

THE OBSERVATIONS AND DATA ANALYSIS

Observations made by these receivers on four days in January, 1984 and six days in August, 1984, have been analyzed to determine the orbits of the GPS satellites then available. As the GRAS station was not installed until July, 1984, the January analysis utilized a single band (L1 only) Macrometer tracker (Bock et al., 1984) operated by Aero Service in Phoenix, Arizona where a hydrogen maser was lacking (a cesium-beam standard was used). The GRAS station replaced the Arizona station in the August analysis. Figure 1 shows the ground tracks for NAVSTARs 1, 3, 4, 6, 8, and 9. Also shown are projections of the 25 degree elevation cutoffs for HO, NOTSS and GRAS. (This figure would change only slightly if Phoenix were shown instead of GRAS.) Highlighted are the regions of mutual visibility and the ground tracks of the satellites through these regions. For illustrative purposes we have assumed a 25 degree minimum-elevation cutoff. This assumption was intended to account for possible obstructions of the satellites; the receivers are able to track to fifteen degrees above the horizon.

The observations consisted of samples of the phases of the reconstructed carriers of the L1 and L2 signals received from each satellite, each relative to the phase of a local reference oscillator at each site. The time span of the daily observations of a given satellite was between 3 and 5 hours, depending on the visibilities of the satellite from the three tracking stations (Figure 1). After combining the L1 and L2 phase observations at each epoch in order to remove ionospheric effects, we combined the data from different stations to make single-difference observations (between stations, and nonredundant) and double-difference observations (differenced again between satellites). (See Bock et al., these proceedings.)

The orbits of the GPS satellites were computed by direct numerical integration of the equations of motion. For the January analysis the WGS72 gravity field (Seppelin, 1972) was used, and for August, the GEM L2

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Macrometer is a registered trademark of Aero Service Division, Western Geophysical Company of America ("Aero Service"), which purchased the business and substantially all the assets of Macrometrics, Inc., on March 8, 1984.
field (Lerch et al., 1982), each to degree and order eight. The effect of the direct solar radiation pressure was accounted for with an isotropic model and with a free parameter for each satellite. We also included, with a free parameter, a force along the y-axis of the spacecraft.

The coordinates of our GPS tracking stations were determined differently for the January and August analyses. For the January analysis, we obtained WGS72 coordinates of NGS monuments which were close to each station (L. D. Hothem, private communication). These WGS72 coordinates had estimated uncertainties of about 0.5 m. We then used local survey ties, performed by NGS, to obtain positions of our antennas relative to the NGS marks. For the August analysis we defined the origin of our reference system by the Lageos-derived coordinates of the McDonald Laser Ranging Station (MLRS) (B. D. Tapley et al., private communication, 1985), located 6 km from the GRAS receiver. We then used Mark III VLBI determinations of the baseline vectors to HO and NOTSS from GRAS (D. S. Robertson, private communication, 1984) and local area surveys by NGS to relate the GPS and the VLBI antenna positions at HO and NOTSS. At GRAS, the vector between the GPS antenna and the nearest NGS mark has not yet been accurately surveyed by NGS. However, when we were at GRAS installing the antenna, we made measurements (with a steel tape) with respect to four NGS survey marks from which we have estimated our antenna's position. We estimate that this crude determination has an uncertainty of about 20 cm.

To fit the phase observations, we adjusted simultaneously by least squares the six initial conditions and the coefficients of the solar radiation pressure and y-axis force models for each satellite, a clock rate parameter for each baseline on each day, and an additive phase bias for each series of observations of each satellite. The station coordinates were held fixed in the solutions.

THE JANUARY ANALYSIS

In January we observed five GPS satellites (NAVSTARs 1, 3, 4, 6, and 8). In the analysis we included one-way observables, single-differences and double-differences.

The estimated uncertainties in satellite orbital positions are given in Table 1. For each satellite we show the formal standard error for the most poorly determined Cartesian coordinate.

To test the accuracies of the derived orbits, we generated orbital ephemerides by reintegrating from the initial conditions estimated in our analysis; then we examined the geodetic results obtained when we used our orbits to process double-difference data from observations on a completely independent baseline. These observations were performed by NGS (C. Goad, private communication, 1984) with Macrometer® V-1000 Interferometric Surveyors on a 13 km baseline in Southern California. This baseline was measured three times: on January 26, 27, and 30, 1984. We processed the observations from each day separately in order to see the day-to-day consistency of the baseline determinations and, by implication, of the orbit determinations.

We first determined the California baselines using the observations of all five satellites. We noticed, however, that the residuals for NAVSTAR 6 with respect to the a priori coordinates of the baseline were significantly
larger than for the other five satellites. We attributed this to a relatively poor determination of the NAVSTAR 6 orbit. Our determination of this satellite’s orbit is suspect since NAVSTAR 6 passes far to the west of our tracking network (see Figure 1). We then repeated our California baseline determinations without the observations of NAVSTAR 6. Table 2 gives the three individual estimates of the baseline vector which we obtained using our orbits for the remaining satellites. Also shown, for each day, are the root-mean-square (rms) of the double-difference phase residuals from the least-squares adjustment of the baseline vector. The day-to-day consistency of the baseline estimates indicate that our orbit determinations are precise to within about 0.3 ppm. A precision of 0.3 ppm corresponds to about 6 m along-track or across-track at the orbital altitude of 20,000 km and is not inconsistent with the estimated uncertainties of Table 1.

THE AUGUST ANALYSIS

In August we observed NAVSTARS 1, 4, 6, 8, and 9. We had learned from our January analysis that the one-way phase observations do not contribute significantly to the orbit determination. For this reason only the single-differences and double-differences were included in the August analysis.

In the August analysis, we had no suitable independent observations to test the accuracy of our orbit determination. In order to make at least a consistency check we divided the August observations into 2 subsets: set A consisted of observations on days 215, 217, and 219; set B with observations on days 216, 218, and 220.

The estimated uncertainties in satellite orbital positions obtained from the two analyses A and B are given in Table 3. Again, the formal standard error for the most poorly determined Cartesian coordinate is shown.

To test the accuracy of our estimation, we re-integrated the orbits using the initial conditions estimated from data sets A and B, and then compared the orbits by plotting the radial, along-track, and across-track differences throughout the time span of overlap between the observation sets. The maximum of the difference in each component, for each satellite, is shown in Table 4. All of the maximum along-track differences are between 5 and 10 m, whereas the peak radial and across-track differences are typically about 2 m.

If we assume that the errors in the “A” and “B” orbits are equal, and recognize that the differences between them are the sums of peak (not rms) errors, then we conclude that the rms errors in the orbits are no worse than 3-4 m in any component. These errors, like the formal uncertainties, are about a factor of two smaller than the orbital errors deduced for the January analysis.

DISCUSSION AND CONCLUSIONS

The post-fit residuals from a typical set (day 219) of single-difference and double-difference observations from the August observations are shown in Figure 2. The appearance of systematic, common-mode variations in the single-difference residuals suggests that the largest source of error in our...
analyses is the unmodeled fluctuations in the local oscillators. Although we can't rule out a contribution from the receivers themselves, we have independent evidence that much of the fluctuation is due to the variations in the relative phases of the hydrogen maser frequency standards at the sites. These variations should cancel in double-differences. Other potentially significant sources of errors are variations in the tropospheric path delay, any uncorrected cycle slips, and unmodeled accelerations of the satellites.

We conclude that we have not yet reached the accuracy goal of \(1 \times 10^7\). With a more complete constellation of satellites, and perhaps additional tracking stations, it will be possible to use only double-difference observations to determine the orbits. Since double-differences are free of the effects of receiving-site frequency-standard instability, it should be possible to determine the orbits more accurately than we have been able to do. In our January and August experiments we had too little double-difference data because there was too little two-site, multiple-satellite, mutual visibility.

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REFERENCES


*INTERFEROMETRIC PHASE RESIDUALS*

**HAYSTACK — RICHMOND**

Figure 2. Single-difference and double-difference phase residuals from the Haystack–Richmond baseline, day 219.
Table 1: Estimated uncertainties in satellite orbital positions (one-sigma; worst of three coordinates) from the analysis of observations at HO, NOTSS, and Phoenix, January, 1984.

<table>
<thead>
<tr>
<th>Satellite:</th>
<th>NS1</th>
<th>NS3</th>
<th>NS4</th>
<th>NS6</th>
<th>NS8</th>
</tr>
</thead>
<tbody>
<tr>
<td>One-sigma Uncertainty:</td>
<td>10 m</td>
<td>4</td>
<td>2</td>
<td>6</td>
<td>2</td>
</tr>
</tbody>
</table>
Table 2. Results of baseline determinations. For each day the estimated latitude, longitude, and height of station MOJAVE 1 are given, assuming latitude 35 14 54.00722, longitude 116 47 27.95387, and height 1049.805 m for station MOBLAS which was at the origin of the baseline. The length of the baseline was about 13 km.

<table>
<thead>
<tr>
<th>Day</th>
<th>Latitude</th>
<th>Longitude</th>
<th>Height</th>
<th>rms of residuals (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>26</td>
<td>35°19'53.99396</td>
<td>116 53 23.94062</td>
<td>306.568</td>
<td>5.1</td>
</tr>
<tr>
<td>27</td>
<td>.99381</td>
<td>.94087</td>
<td>.520*</td>
<td>5.2</td>
</tr>
<tr>
<td>30</td>
<td>.99389</td>
<td>.94073</td>
<td>.564</td>
<td>7.8</td>
</tr>
<tr>
<td>Mean</td>
<td>.99389</td>
<td>.94074</td>
<td>.566</td>
<td>6.0</td>
</tr>
<tr>
<td>S.D.</td>
<td>0.00008</td>
<td>0.00013</td>
<td>0.003</td>
<td>---</td>
</tr>
</tbody>
</table>

* This height excluded from computations of mean and standard deviation (S.D.); we suspect a blunder in the field measurement of antenna height above survey mark. This same anomaly is seen when ephemerides from the Naval Surface Weapons Center are used to determine the baselines.
Table 3. Estimated uncertainties in satellite orbital positions (one sigma; worst of three coordinates) from two analyses of observations at HO, NOTSS, and GRAS, August 1984.

<table>
<thead>
<tr>
<th>Analysis</th>
<th>A</th>
<th>B</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Days 215,217,219</td>
<td>Days 216,218,220</td>
</tr>
<tr>
<td>Satellite</td>
<td></td>
<td></td>
</tr>
<tr>
<td>NS1</td>
<td>3 m</td>
<td>4</td>
</tr>
<tr>
<td>NS4</td>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td>NS6</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>NS8</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>NS9</td>
<td>3</td>
<td>3</td>
</tr>
</tbody>
</table>
Table 4. Differences in satellite position within span of overlap (days 216-219) between orbits determined from A (days 215, 217, 219) and B (days 216, 218, 220) data sets.

<table>
<thead>
<tr>
<th>Satellite</th>
<th>Radial</th>
<th>Along-track</th>
<th>Across-track</th>
</tr>
</thead>
<tbody>
<tr>
<td>NS1</td>
<td>1 m</td>
<td>8 m</td>
<td>4 m</td>
</tr>
<tr>
<td>NS4</td>
<td>2</td>
<td>11</td>
<td>2</td>
</tr>
<tr>
<td>NS6</td>
<td>2</td>
<td>11</td>
<td>1</td>
</tr>
<tr>
<td>NS8</td>
<td>1</td>
<td>7</td>
<td>1</td>
</tr>
<tr>
<td>NS9</td>
<td>1</td>
<td>4</td>
<td>2</td>
</tr>
</tbody>
</table>