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PROGRESS IN THE APPLICATION OF THE PHASE CLOSURE TECHNIQUE TO HIGH RESOLUTION PASSIVE SONAR IMAGING

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ABSTRACT

A significant improvement has been developed in the first few months of this program that allows the phase closure technique to deal more correctly with the case of distributed inhomogeneities. Our modification of the phase closure technique, to which we refer as "cross-correlated subarrays" involves processing the hydrophone signals in groups, corresponding to operating the array as a series of smaller subarrays. The beamwidth of a subarray is chosen to restrict the response of the system to the angular field over which the assumptions of phase closure are valid. If we let $\Delta x_c$ be the coherence scale of the system (a function of oceanic parameters, together with range $R$ and wavelength $\lambda$), then for phase closure to be valid (with or without cross-correlated subarrays) we require $\Delta x_c < (\lambda R)^{1/2}$. Cross-correlated subarrays are required explicitly if the maximum source separation, $s$, is such that $s > \Delta x_c$. These expectations have been tested by means of a series of numerical experiments. The results are consistent with the predictions, giving us confidence that the main features of our algorithm are correct. The next step is to apply the algorithm to actual sonar patrol data, to be obtained from Autonetics Marine Systems Division of Rockwell International. Pending the outcome of these tests, we expect that the algorithm should provide a practical and effective method for improving the quality of passive sonar images.
INTRODUCTION

The purpose of this program is to explore the possibility of applying the phase closure principle to passive sonar imaging in order to overcome the distorting effects caused by both the propagation medium and phase errors in the instrument itself. The phase closure technique was originally developed for Very Long Baseline radio astronomy in order to overcome the phase distortion caused by the Earth's atmosphere (Readhead and Wilkinson, 1978). An important difference between this case and passive sonar is that for radio astronomy, the distorting medium represents a thin sheet at one end of the propagation path, whereas for the sonar case, the inhomogeneities responsible for phase distortion are distributed continuously between the source and receiver.

Prior to the beginning of this program, Rockwell International Science Center had, under the auspices of its Independent Research and Development funds, already laid some of the foundation for the application of the phase closure technique to passive acoustic imaging. In this work (Marsh, Richardson, and Martin 1985) the theoretical limits of the standard phase closure technique were determined, and these limits demonstrated by performing a series of numerical experiments. A closely related technique for overcoming phase distortion effects has recently been discussed by Paulraj and Kailath (1985). Their technique models the phase errors in exactly the same way as the phase closure technique, although the solution to the imaging problem is accomplished by eigenvector methods. Also closely related is the concept of phase recovery from bispectra (Bartelt, Lohmann, and Wirnitzer, 1984). As in the case of the conventional phase closure method, both of these methods would be applicable to instrumental phase errors, but would not be able to handle the case of distributed inhomogeneities without some modification.

Our efforts since the beginning of the present DARPA-funded project have been concerned with extending the validity of the phase closure technique to the case of distributed inhomogeneities. Our principal accomplishments have been:
1. Extend the range of validity of the phase closure technique using cross-correlated subarrays.

2. Theoretical understanding of the behavior of the algorithm in the context of distributed inhomogeneities.


We will discuss the theoretical basis of this work, and present the results of numerical testing with synthetic data.

THEORETICAL CONSIDERATIONS

As discussed by Marsh, Richardson and Martin (1985), the far-field intensity distribution $I(e_n)$ of a spatially incoherent sound source in the case of an inhomogeneous ocean is related to the measured visibility $V_{ij}^{\text{meas}}$ by:

$$V_{ij}^{\text{meas}} = \sum_n g_{in} g_{jn}^* I(e_n) e^{-2\pi i u_{ij} \cdot e_n} \Delta \Omega_n$$  \hspace{1cm} (1)

where $e_n$ is a unit vector in the direction of the $n$th source component as viewed from the receiver, $\Delta \Omega_n$ is the solid angle subtended by the $n$th source, $u_{ij}$ is the baseline vector joining hydrophones $i$ and $j$ in units of wavelengths, and $g_{in}$ and $g_{jn}$ are complex gain factors representing the effect of propagation anomalies in the medium.

The principal assumption underlying the standard phase closure technique is:

$$g_{in} = g_i$$  \hspace{1cm} (2)
i.e., the gain factors vary more slowly with source position than with receiver position. If we assume that the principal effect of the medium is a phase distortion, i.e.,

\[ g_i = e^{i \psi_i} \quad (3) \]

then Eq. (2) implies that we can express the measured phase \( \phi_{ij}^{\text{meas}} = \arg(v_{ij}^{\text{meas}}) \) in terms of the true phase \( \phi_{ij}^{\text{true}} \) (corresponding to a homogeneous ocean) as:

\[ \phi_{ij}^{\text{meas}} = \phi_{ij}^{\text{true}} + \psi_i - \psi_j \quad (4) \]

The standard phase closure technique attempts to obtain the phase errors \( \phi_i \) using a measurement model of the form (4), and correct the data accordingly.

For a given set of physical conditions, one can define an angular field-of-view \( \Delta \theta_{pc} \) over which (2) is satisfied. Provided all of the acoustic sources are located within this narrow field, the standard phase closure technique is valid. Unfortunately, this is unlikely to be true in practice, since sources occur typically over a wide range of azimuths. Recent work has been devoted to the problem of extending the phase closure technique to handle this situation. A conceptually simple way of accomplishing this is to divide the array up into a number \( N \) of identical subarrays, and operate each as a small phased array, with the beams all pointed in the same direction. The beam pattern of each subarray will, of course, be much broader than the resolution of the full array. We will refer to the beam pattern of each subarray as the primary beam. We will treat each subarray as an individual element in a larger scale array, and cross-correlate the signals from each subarray as if we were dealing with an array of single hydrophones. The effect of this is that the angular response of the larger array will have been multiplied by the primary beam pattern corresponding to an individual subarray. By an appropriate choice of the number of elements in a subarray, we could, provided certain conditions are met, restrict the primary beamwidth \( \Delta \theta_{pri} \) to the field \( \Delta \theta_{pc} \) over which (2) is satisfied. In the case of a homogeneous ocean, \( \Delta \theta_{pri} \) can be obtained simply...
from the Fourier transform of the aperture distribution for a single subarray. In the case of an inhomogeneous ocean, however, the phase distortions will broaden the response, ultimately setting a lower limit $\Delta \theta_b$ on the primary beamwidth obtainable.

Both $\Delta \theta_{pc}$ and $\Delta \theta_b$ can be related to the phase structure function $D(x - x')$, defined by Flatte et al. (1979) as:

$$D(x - x') = E[(\phi(x) - \phi(x'))^2]$$

where $x$ and $x'$ represent the positions of two receivers located at a distance $R$ from a point source and displaced perpendicular to the direction of propagation, $\phi(x)$ represents the spatial variation of the phase of the received signal and $E$ is the expectation operator. We can then express $\Delta \theta_{pc}$ and $\Delta \theta_b$ in terms of some convenient criterion limiting the amount of permissible phase variation $\Delta \phi$ over the source and receiver, respectively. Provided the inhomogeneities have the same statistical character throughout the region between source and receiver, and taking $\Delta \phi$ to be 1 radian, we can then write

$$R \frac{\Delta \theta_{pc}}{\Delta \theta_b} = \frac{\lambda}{\Delta \theta_b} = \Delta x_c$$

where $\lambda$ is the wavelength and $\Delta x_c$ is defined by

$$D(\Delta x_c) = 1$$

Provided $\Delta \theta_{pc} > \Delta \theta_b$, the primary beamwidth can be restricted to $\Delta \theta_{pc}$, enabling the phase closure technique to produce an image over the restricted field $\theta_r - \Delta \theta_{pc}/2$ to $\theta_r + \Delta \theta_{pc}/2$ where $\theta_r$ represents the center of the restricted field. A mosaic of subimages could then be produced, corresponding to various values of $\theta_r$, and the final image produced by a combination of these subimages. In order to produce the $r$th subimage, the phase closure technique would be applied to the set of visibilities $V_{ij}^{(r)}$ defined
\[ V_{ij}(r) = \sum_{k=k_0(i)+1}^{k_0(i)+n_s} \sum_{\lambda=\lambda_0(j)+1}^{\lambda_0(j)+n_s} a_k(i) a_\lambda(j) E(S_k S_\lambda^*) e^{-2\pi i u_{k\lambda} \sin \theta_r} \] 

(8)

where

\[ k_0(i) = (i - 1)n_s \]

\[ \lambda_0(j) = (j - 1)n_s \]

and \( S_k, S_\lambda \) represent the signals received by the \( k^{th} \) and \( \lambda^{th} \) hydrophones; \( a_k(i), a_\lambda(j) \) are weights representing the apodizing function for each subarray, \( n_s \) is the number of elements in each subarray.

The new algorithm sketched above would enable phase-closure imaging over a wide field-of-view, and should provide a substantial improvement in imaging performance over that obtained by conventional beamforming. We will refer to it subsequently as "phase closure with cross-correlated subarrays." We now consider the question as to the physical regime over which this algorithm will be valid. A useful quantity to bear in mind in this regard is \( \Delta x_c \) defined by (7), which represents the coherence scale for the system at the particular range and frequency. The condition that \( \Delta \theta_{pc} > \Delta \theta_\theta \) is equivalent to

\[ \Delta x_c > (\lambda R)^{1/2} \]  

(9)

Equation (9) thus represents a criterion by which one can determine whether or not phase closure (with cross-correlated subarrays) is capable of improving a distorted image. In order to relate it to more fundamental physical parameters, however, requires a knowledge of the form of the phase structure function \( D \), which in turn is related to the spatial correlation function of the refractive index. If we assume that

\[ E\phi(x) \phi(x') = \phi^2 \exp(-(x-x')^2/L^2) \]  

(10)
then provided $|x - x'| \ll L$

$$D(x - x') = 2\phi^2 (x - x')^2 / L^2$$

(11)

and hence

$$\Delta \chi_c = \frac{L}{\sqrt{2} \phi} \ .$$

(12)

If the inhomogeneities are statistically uniform and isotropic, then from Flatte et al., (1979):

$$\phi^2 = 0.4 \left(\frac{2\pi}{\lambda}\right)^2 <\mu^2> RL$$

(13)

where $<\mu^2>$ represents the variance of the refractive index fluctuations. Equations (12) and (13) then give

$$\Delta \chi_c = 0.2 \lambda \left(\frac{L}{<\mu^2> R}\right)^{1/2} \ .$$

(14)

Thus from (9), we find that the maximum range over which the phase closure technique (with cross-correlated subarrays) can be applied to a distributed medium is

$$R_{max} = 0.2 \left(\frac{\lambda L}{<\mu^2>}\right)^{1/2} \ .$$

(15)

In the case of horizontal imaging in the ocean at 500 Hz (assuming $L = 5$ km and $<\mu^2>^{1/2} = 5 \times 10^{-4}$) we obtain $R_{max} = 49$ km, which suggests that substantial improvement in sonar images from horizontal arrays should be possible over ranges up to this value.
NUMERICAL EXPERIMENTS

In order to test the concepts discussed in the previous section, a series of numerical experiments was performed. In these experiments, an assumed source consisting of 4 pointlike components spread over a distance of 200 m was observed at a distance of 1 km and a frequency of 300 Hz, using a regularly spaced array of length 200 m containing 21 hydrophones. The sound velocity was assumed to be 1500 m/s. Ocean inhomogeneities were simulated by placing four equally spaced phase screens between the source and receivers. Along each phase screen, the phase was made to vary randomly, with a correlation length \( L \). The rms phase deviation through the entire system was chosen to be approximately 1 radian, so that \( L \) corresponds to \( \Delta x_c \). Various values of \( L \) were assumed, in some cases chosen to purposely violate the assumptions for phase closure. The propagation was calculated according to the Fresnel approximation, as discussed by Marsh, Richardson and Martin (1985). In the case of the cross-correlated subarrays algorithm, each subarray had 5 elements, weighted with a rectangular function, i.e., no apodizing was used. A mosaic of 3 subimages was produced in each case, spaced by the half-power widths of the corresponding primary beams (sinc functions). Simple addition was used in order to construct the final image from the 3 subimages.

Since in these experiments \( L = \Delta x_c \), Eq. (9) implies that phase closure would be invalid if \( L < (\lambda R)^{1/2} \). In the case of \( L > (\lambda R)^{1/2} \), phase closure would be applicable, but cross-correlated subarrays would be required if the source components are separated by a distance \( s \) greater than \( L \). These two criteria form a rather crucial test of the theoretical basis of our algorithm, and values of \( L \) were chosen to test them. In these experiments, \( (\lambda R)^{1/2} \) corresponds to 71 m. We now discuss the results.

Case a: \( L > (\lambda R)^{1/2}, s = L \).

The assumed value of \( L \) was 200 m. In this case we expect phase closure to be valid, and cross-correlated subarrays should not be necessary. The results are shown in Fig. 1, which fulfills our expectations. The use of phase
Fig. 1 Imaging results for the case of mild phase distortion. The length scale of the inhomogeneities, L, was chosen such that \( L > (\lambda R)^{1/2} \) where \( \lambda \) is the wavelength and \( R \) is the range, thus phase closure was expected to be valid. In addition, the maximum source separation, \( s \), did not exceed \( L \), and hence, standard phase closure performs satisfactorily without the need for cross-correlated subarrays.

closure has substantially improved the image over that obtained by conventional beamforming, but the use of cross-correlated subarrays has produced only a marginal improvement over standard phase closure.
Case b: \( L > (\lambda R)^{1/2}, s = 2L \).

The assumed value of \( L \) was 100 m. We expect phase closure to be valid, but the sources are now so widely spaced in comparison to the length scale of the inhomogeneities that cross-correlated subarrays should be required. The results, shown in Fig. 2 support these expectations. Standard phase closure gave a very poor result, whereas cross-correlated subarrays gave a reasonable reconstruction.

Fig. 2 Imaging results for the case of moderate phase distortion. Parameters are the same as for Fig. 1, except that \( s = 2L \), i.e., the sources are now so widely spaced in comparison to the length scale of the inhomogeneities that cross-correlated subarrays are required.
Case c: \( L < (\lambda R)^{1/2} \).

The assumed value of \( L \) was 50 m. In this case we expect phase closure to be completely invalid, and the results in Fig. 3 show this to be true. It is interesting, however, that the cross-correlated subarrays image bears some resemblance to the assumed source, although the separation between components is incorrect.

![Graph of imaging results](image)

**Fig. 3** Imaging results for the case of strong phase distortion. In this case \( L < (\lambda R)^{1/2} \), and hence, phase closure is not able to restore the image.
DISCUSSION AND FUTURE PLANS

The numerical simulations discussed above give us confidence that the main features of our algorithm are correct, and that substantial image improvement should be possible in the case of towed arrays. The next step in the program is to apply the "phase closure + cross-correlated subarrays" algorithm to real data, taken from sonar patrols. Data for this purpose exists at Autonetics Marine Systems Division (AMSD) of Rockwell International. Initial tests will be performed on the assumption that sources are at a distance of the order of 10 km. For typical ocean parameters obtained from Flatte et al., (1979) ($\langle \mu^2 \rangle = 5 \times 10^{-4}$, $L = 5$ km), at a frequency of 500 Hz we obtain $\Delta \chi_\nu = 760$ m, corresponding to a subtended angle at the receivers of 4.4°. This means that hydrophone signals should be averaged so as to give a primary beam of this width. If the hydrophones are spaced at half-wavelength intervals (1.5 m), this means processing the hydrophone signals in groups of about 20.

Physical parameters corresponding to patrol data in hand will be obtained from AMSD in the near future. A numerical simulation using these parameters will be performed prior to testing with the patrol data.

REFERENCES


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