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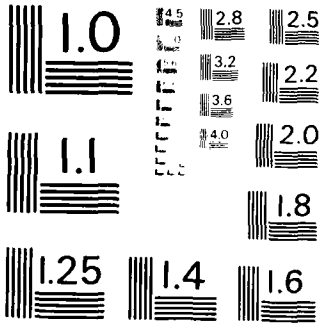


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# DEMONSTRATION OF A JACOBI POLYNOMIAL REPRESENTATION OF A KRONECKER DELTA FUNCTION


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# DEMONSTRATION OF A JACOBI POLYNOMIAL REPRESENTATION OF A KRONECKER DELTA FUNCTION

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# ABSTRACT

A formula derived by Noble has been generalized to obtain an expansion of the Kronecker delta function as an infinite series involving the products of two Jacobi polynomials.

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# INTRODUCTION

In our investigations of the inviscid, incompressible flow of rotating fluid shells confined between concentric, spherical, rigid, co-rotating boundaries, we encountered the need for a proof of a Kronecker delta function representation in terms of a series of products of two Jacobi polynomials. In particular, we required a proof that

$$\delta_{l,0} = \frac{(\mu + \tfrac{1}{2})(1-\Lambda)^{\mu+\tfrac{1}{2}}}{2^{\mu+\tfrac{1}{2}}} F_{\mu+l,\mu-l}(\Lambda) , \tag{1}$$

where

$$F_{\mu+l,\mu-l}(\Lambda) = \sum_{k=0}^{\infty} \frac{(k+\mu/2+l/2+\tfrac{1}{4})}{(k+\mu+l+\tfrac{1}{2})(k+\mu+\tfrac{1}{2})}$$

$$\times P_k^{(\mu+l+\tfrac{1}{2},\,\mu-l-\tfrac{1}{2})}(\Lambda)\, P_{k+l}^{(\mu-l+\tfrac{1}{2},\,\mu+l-\tfrac{1}{2})}(\Lambda) , \; 0 \le |\Lambda| \le 1. \tag{2}$$

For $(\alpha,\beta) > -1$, the $P_n^{(\alpha,\beta)}(\Lambda)$ are the Jacobi polynomials which are orthogonal over the interval $(-1,+1)$ with the integration weight factor $w(x) = (1-x)^{\alpha}(1+x)^{\beta}$. They are normalized by the relation $P_n^{(\alpha,\beta)}(1) = \Gamma(\alpha+n+1)/[\Gamma(n+1)\Gamma(\alpha+1)]$, with $\Gamma(z)$ the gamma (factorial) function. For $(\alpha,\beta) \le -1$, the Jacobi polynomials[1] $P_n^{(\alpha,\beta)}(\Lambda)$ are defined in terms of the polynomials for $(\alpha,\beta) > -1$ through repeated applications of the contiguous relations[1,2]

$$(2n+\alpha+\beta)\, P_n^{(\alpha-1,\beta)}(\Lambda) = (n+\alpha+\beta)\, P_n^{(\alpha,\beta)}(\Lambda) - (n+\beta)\, P_{n-1}^{(\alpha,\beta)}(\Lambda) \tag{3a}$$

and

$$(2n+\alpha+\beta)\, P_n^{(\alpha,\beta-1)}(\Lambda) = (n+\alpha+\beta)\, P_n^{(\alpha,\beta)} + (n+\alpha) P_{n-1}^{(\alpha,\beta)}(\Lambda) \tag{3b}$$

with $P_0^{(\alpha,\beta)}(\Lambda) = 1$ and $P_{-1}^{(\alpha,\beta)}(\Lambda) = 0$.

Certain sums involving products of two Jacobi polynomials have been previously evaluated (see, for example, Refs. 3 – 6). Our method for evaluating the sum in Eq. 2 employs mathematical procedures analogous to those which Noble[5] used to show that[6]

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[1] W. Magnus, F. Oberhettinger, and R. P. Soni, *Formulas and Theorems for the Special Functions of Mathematical Physics*, Springer-Verlag, New York (1966).
[2] I. S. Gradshteyn and I. M. Ryzhik, *Tables of Integrals, Series, and Products*, Alan Jeffrey, ed., Academic Press, New York (1965).
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$$2^{c-1}(1-y)^{a}(1+x)^{b}(x-y)^{a+b-c-1}\Delta(x,y)$$

$$= \Gamma(a+b-c)\sum_{k=0}^{\infty}\frac{k!\Gamma(k+c+1)(2k+c+1)}{\Gamma(k+a+1)\Gamma(k+b+1)}\,P_k^{(c-b,b)}(x)\,P_k^{(a,c-a)}(y)\,, \quad (4)$$

where

$$\Delta(x,y) = \begin{cases} 1 \text{ if } -1 \le y < x \le 1 \\ 0 \text{ if } -1 \le x \le y \le 1\,. \end{cases} \quad (5)$$

## PROOF

The proof is done in two stages: first the $l = 0$ case is treated, and then the $l \neq 0$ case is analyzed. Both stages use the expansion of a function of two variables in an infinite series of Jacobi polynomials of one variable and coefficients that depend on the other variable. In particular, the coefficients in the expansion are obtained by respresenting the Jacobi polynomial as a finite series, evaluating the resulting elementary integrals, and re-summing the resultant series.

In proving Eq. 1, it is convenient to use the relationship

$$P_n^{(\alpha,-1)}(x) = \left(\frac{\alpha+n}{2n}\right)(1+x)\,P_{n-1}^{(\alpha,1)}(x) \qquad \text{for } n \ge 1, \quad (6)$$

in order to write Eq. 2 as

$$F_{\mu+l,\mu-l} = \frac{1}{(2\mu+1)}\,P_l^{(\mu-l+1/2,-1)}(\Lambda)$$

$$+\frac{(1+\Lambda)^2}{8}\sum_{k=1}^{\infty}\frac{(2k+\mu+l+1/2)}{k(k+l)}\,P_{k-1}^{(\mu+l+1/2,1)}(\Lambda)\,P_{k+l-1}^{(\mu-l+1/2,1)}(\Lambda)\,, \quad (7)$$

where the first term is the $k=0$ term of the sum appearing in Eq. 2.

**CASE OF $l = 0$**

For $l=0$, we expand

$$H^{(0)}(x,y) \equiv \frac{2^{\mu+5/2}}{(1+x)(1+y)} \int_{-1}^{w} (1-t)^{-(\mu+3/2)} dt$$

$$= \sum_{k=0}^{\infty} C_k^{(0)}(y)\, P_k^{(\mu+1/2,1)}(x), \tag{8a}$$

where

$$w = \begin{cases} x \text{ if } y > x \\ y \text{ if } y \le x\,. \end{cases} \tag{8b}$$

Multiplying Eq. 8a by $(1+x)(1-x)^{\mu+1/2}\, P_m^{(\mu+1/2,1)}(x)$ and integrating from $x = -1$ to 1 yields

$$\frac{(m+1)2^{\mu+5/2}C_m^{(0)}(y)}{(m+\mu+3/2)(2m+\mu+5/2)} = \frac{2^{\mu+5/2}}{(1+y)}$$

$$\times \left\{ \int_{-1}^{y} dx(1-x)^{\mu+1/2}\, P_m^{(\mu+1/2,1)}(x) \int_{-1}^{x} dt(1-t)^{-(\mu+3/2)} \right.$$

$$\left. + \left[ \int_{y}^{1} dx(1-x)^{\mu+1/2}\, P_m^{(\mu+1/2,1)}(x) \right] \left[ \int_{-1}^{y} (1-t)^{-(\mu+3/2)} dt \right] \right\}$$

$$= (1+y)^{-1}(\mu+1/2)^{-1} \left[ -4 \int_{-1}^{1} dx(1-x)^{\mu+1/2}\, P_m^{(\mu+1/2,1)}(x) \right.$$

$$+ 2^{\mu+5/2} \int_{-1}^{y} dx P_m^{(\mu+1/2,1)}(x)$$

$$\left. + \; 2^{\mu+5/2}(1-y)^{-(\mu+1/2)} \int_{y}^{1} dx(1-x)^{\mu+1/2}\, P_m^{(\mu+1/2,1)}(x) \right]. \tag{9}$$

The integrals are readily evaluated by expanding the Jacobi polynomial as[1]

$$P_m^{(\alpha,\beta)}(x) = \sum_{r=0}^{m} \gamma(m,\alpha,\beta;r)(1-x)^r, \tag{10a}$$

with

$$\gamma(m,\alpha,\beta;r) \equiv \Gamma(\alpha+m+1)\Gamma(\alpha+\beta+m+r+1)(-\tfrac{1}{2})^r/$$

$$[r!(m-r)!\Gamma(\alpha+r+1)\ \Gamma(\alpha+\beta+m+1)]; \tag{10b}$$

integrating each term; and re-summing the series. In particular,

$$\frac{(m+1)2^{\mu+5/2}}{(m+\mu+3/2)(2m+\mu+5/2)}\ C_m^{(0)}(y) = (1+y)^{-1}(\mu+\tfrac{1}{2})^{-1}$$

$$\times \sum_{r=0}^{m} \gamma(m,\mu+\tfrac{1}{2},1;r) \left\{ \frac{-2^{r+\mu+7/2}}{(r+\mu+3/2)} - \frac{2^{\mu+5/2}}{(r+1)}[(1-y)^{r+1}-2^{r+1}] \right.$$

$$\left. + 2^{\mu+5/2}\frac{(1-y)^{r+1}}{(r+\mu+3/2)} \right\} = (1+y)^{-1}(\mu+\tfrac{1}{2})^{-1}\sum_{r=0}^{m}\gamma(m,\mu+\tfrac{1}{2},1;r)$$

$$\times \left[ 2^{r+\mu+7/2} - 2^{\mu+5/2}(1-y)^{r+1} \right] \frac{(\mu+\tfrac{1}{2})}{(r+1)(r+\mu+3/2)}$$

$$= -2^{\mu+7/2}(1+y)^{-1}(m+\mu+3/2)^{-2}\left[ P_{m+1}^{(\mu+\frac{1}{2},-1)}(-1) - P_{m+1}^{(\mu+\frac{1}{2},-1)}(y) \right]$$

$$= \frac{2^{\mu+5/2}}{(m+1)(m+\mu+3/2)}\ P_m^{(\mu+\frac{1}{2},1)}(y)\ , \tag{11}$$

where the fact that

$$\gamma(m+1,\mu+\tfrac{1}{2},-1;r+1) =$$

$$-(m+\mu+3/2)^2\gamma(m,\mu+\tfrac{1}{2},1;r)/[2(r+\mu+3/2)(r+1)]$$

is used to arrive at the next to the last line, and Eq. 6 is used in the final step. Inserting this result in Eq. 8a yields

$$H^{(0)}(x,y) = \frac{2^{\mu+5/2}}{(1+y)(1+x)} \left[ \frac{(1-w)^{-(\mu+1/2)} - 2^{-(\mu+1/2)}}{(\mu+1/2)} \right]$$

$$= \sum_{k=0}^{\infty} \frac{(2k+\mu+5/2)}{(k+1)^2} P_k^{(\mu+1/2,1)}(y)\, P_k^{(\mu+1/2,1)}(x) \tag{12a}$$

or

$$\frac{1}{(2\mu+1)} \left(\frac{2}{1-w}\right)^{\mu+1/2} = \frac{1}{(2\mu+1)} + \frac{(1+y)(1+x)}{8}$$

$$\times \sum_{k=1}^{\infty} \frac{(2k+\mu+1/2)}{k^2} P_{k-1}^{(\mu+1/2,1)}(y)\, P_{k-1}^{(\mu+1/2,1)}(x) , \tag{12b}$$

with $w$ defined by Eq. 8b. Since $P_0^{(\alpha,\beta)}(\Lambda) = 1$, comparing Eq. 7 with $l = 0$ to Eq. 12b with $x=y=\Lambda$ yields

$$F_{\mu,\mu} = \frac{1}{(2\mu+1)} \left(\frac{2}{1-\Lambda}\right)^{\mu+1/2} , \tag{13}$$

which demonstrates that Eq. 1 holds for $l = 0$.

**CASE $l \neq 0$**

For $l \neq 0$, we consider

$$H^{(l)}(x,y) \equiv \begin{cases} 0 \text{ for } 1 \geq y \geq x \geq -1, \\ \dfrac{\Gamma(\mu+5/2)}{\Gamma(\mu-l+5/2)\, l!\, (1+x)} \dfrac{\partial^{l-1}}{\partial y^{l-1}} \left[(1-y)^{-(\mu+3/2)}(x-y)^l\right] \\ \text{for } 1 \geq x > y \geq -1 . \end{cases} \tag{14}$$

Application of Leibnitz's rule for differentiating a product yields

$$H^{(l)}(x,y) = \sum_{l=0}^{l-1} S_l \frac{(x-y)^{l+1}}{(1+x)} U(x,y) , \tag{15}$$

$$\text{with } S_t \equiv \frac{(\mu+3/2)\Gamma(\mu+t+3/2)\Gamma(l)(-1)^{l-t-1}(1-y)^{-(\mu+3/2+t)}}{\Gamma(\mu-l+5/2)\Gamma(t+2)\Gamma(t+1)\Gamma(l-t)} ,$$

and $U(x,y) = 1$ if $1 > x > y \geq -1$, and $U(x,y) = 0$ otherwise. The function $H^{(l)}(x,y)$ is expanded in terms of Jacobi polynomials

$$H^{(l)}(x,y) = \sum_{k=0}^{\infty} C_k^{(l)}(y)\, P_k^{(\mu-l+\frac{1}{2},1)}(x) , \tag{16}$$

with the coefficients determined from

$$\begin{aligned} I(m,l) &\equiv \frac{(m+1)2^{\mu-l+5/2}}{(m+\mu-l+3/2)(2m+\mu-l+5/2)} C_m^{(l)}(y) \\ &= \int_{-1}^{1} H^{(l)}(x,y)\ (1-x)^{\mu-l+\frac{1}{2}}(1+x)P_m^{(\mu-l+\frac{1}{2},1)}(x)\, dx \\ &= \sum_{t=0}^{l-1} S_t \sum_{r=0}^{m} \gamma(m,\ \mu-l+\tfrac{1}{2},1;r) \int_y^1 dx(1-x)^{\mu-l+\frac{1}{2}+r}(x-y)^{t+1} . \end{aligned} \tag{17}$$

The integral is evaluated by changing the variable from $x$ to $z \equiv (x-y)/(1-y)$, which leads to

$$\begin{aligned} I(m,l) &= \sum_{r=0}^{m} \gamma(m,\mu-l+\tfrac{1}{2},1;r)(1-y)^{\mu-l+\frac{1}{2}+r+2}\Gamma(\mu-l+r+3/2) \\ &\quad \times \sum_{t=0}^{l-1} \left[ \frac{\Gamma(t+2)S_t(1-y)^t}{\Gamma(\mu-l+r+t+7/2)} \right] . \end{aligned} \tag{18}$$

Substituting for $S_t$, the inner summation appearing in Eq. 18 reduces to

$$\begin{aligned} \sigma(r,l) &= \frac{(\mu+3/2)(-1)^{l-1}}{\Gamma(\mu-l+5/2)} \sum_{t=0}^{l-1} \left\{ \frac{\Gamma(\mu+3/2+t)}{\Gamma[\mu+3/2+t-(l-2-r)]} \right\} \\ &\quad \times \left[ \frac{(l-1)!}{t!(l-1-t)!} \right] (-1)^t . \end{aligned} \tag{19}$$

For $l \geq r+2$, we note that

$$\sigma(r,l) = \frac{(\mu+3/2)(-1)^{r+1}}{\Gamma(\mu-l+5/2)} \frac{\partial^{l-2-r}[z^{\mu+1/2}(1-z)^{l-1}]}{\partial z^{l-2-r}}\bigg|_{z=1} = 0 \text{ for } r \geq 0. \tag{20}$$

The derivative is zero at $z = 1$ because $(l-2-r) < l-1$. The fact that the derivative reproduces the series in Eq. 19 can be readily verified by writing the factor $(1-z)^{l-1}$ as a polynomial in $z$, differentiating each term, and then setting $z=1$. For $l < r+2$, we note that

$$\sigma(r,l) = \frac{(-1)^{l-1}(\mu+3/2)}{\Gamma(\mu-l+5/2)} \int_0^1 ds_1 \int_0^{s_1} ds_2 \ldots \int_0^{s_{j-1}} ds_j s_j^{\mu+1/2}(1-s_j)^{l-1}\bigg|_{j=r+2-l}$$

$$= \frac{(-1)^{l-1}\Gamma(\mu+5/2)\Gamma(r+1)}{\Gamma(\mu-l+5/2)\Gamma(r+2-l)\Gamma(\mu+5/2+r)} . \tag{21}$$

The multiple integral in Eq. 21 is readily evaluated by recalling that integration by parts leads to

$$\int_0^1 ds_1 \int_0^{s_1} ds_2 \ldots \int_0^{s_{j-1}} ds_j f(s_j) = \int_0^1 \frac{(1-s_1)^{j-1}}{(j-1)!} f(s_1)ds_1 . \tag{22}$$

The fact that the multiple integral reproduces the series in Eq. 19 can be readily verified by writing $(1-s_j)^{l-1}$ as a polynomial in $s_j$ and integrating each term separately. Inserting Eqs. 20 and 21 into Eq. 18 yields

$$I(m,l) = \begin{cases} \dfrac{(-1)^{l-1}\Gamma(\mu+5/2)}{\Gamma(\mu-l+5/2)} \displaystyle\sum_{r=l-1}^{m} \gamma(m,\mu-l+1/2,1;r)(1-y)^{r-l+1} \\ \times \dfrac{\Gamma(\mu-l+r+3/2)r!}{\Gamma(\mu+5/2+r)(r+1-l)!} \quad \text{for } m \geq l-1 \\ 0 \qquad\qquad\qquad\qquad \text{otherwise.} \end{cases} \tag{23}$$

For $m \geq l-1$, using the definition of $\gamma(m,\mu-l+1/2,1;r)$ and introducing the indices $r' = r-(l-1)$ and $m' = m-(l-1)$ leads to

$$I(m,l) = \frac{\Gamma(\mu+5/2)(\tfrac{1}{2})^{l-1}}{(m'+\mu+\tfrac{1}{2})\,\Gamma(\mu+5/2-l)}$$

$$\times \sum_{r'=0}^{m'} \frac{[(y-1)/2]^{r'}\Gamma(m'+\mu+l+\tfrac{1}{2}+r')}{(m'-r')!r'!\Gamma(\mu+l+3/2+r')}$$

$$= \frac{\Gamma(\mu+5/2)(\tfrac{1}{2})^{l-1}}{(m+\mu+3/2)(m+\mu-l+3/2)\Gamma(\mu-l+5/2)} P_{m-(l-1)}^{(\mu+l+\frac{1}{2},-1)}(y)\ , \qquad (24)$$

where the definition of $\gamma(m-l+1,\ \mu+l+\tfrac{1}{2},-1;r)$ was employed to obtain the last line. Inserting Eqs. 23 and 24 into Eqs. 17 and 16 yields

$$H^{(l)}(x,y) = \frac{2^{-(\mu+3/2)}\Gamma(\mu+5/2)}{\Gamma(\mu+5/2-l)} \sum_{m=l-1}^{\infty} \frac{(2m+\mu-l+5/2)}{(m+1)(m+\mu+3/2)}$$

$$\times P_{m-(l-1)}^{(\mu+\frac{1}{2}+l,-1)}(y)\ P_m^{(\mu-l+\frac{1}{2},1)}(x)\ . \qquad (25)$$

Changing the summation index to $m' = m-(l-1)$, writing the $m'=0$ term separately, and using Eq. 6 in both the $m'=0$ and the $m'\neq 0$ terms lead to

$$H^{(l)}(x,y) = \frac{2^{-(\mu-\frac{1}{2})}\Gamma(\mu+5/2)}{(1+x)\Gamma(\mu+5/2-l)} \left[\frac{P_l^{(\mu-l+\frac{1}{2},-1)}(x)}{(2\mu+1)} + \frac{(1+x)(1+y)}{8}\right.$$

$$\left.\times \sum_{m'=1}^{\infty} \frac{(2m'+l+\mu+\tfrac{1}{2})}{m'(m'+l)} P_{m'-1}^{(\mu+l+\frac{1}{2},1)}(y)\ P_{m'+l-1}^{(\mu-l+\frac{1}{2},1)}(x)\right]\ . \qquad (26)$$

Setting $x=y=\Lambda$ in Eq. 26 and comparing with Eq. 7, one finds

$$H^{(l)}(\Lambda,\Lambda) = \frac{2^{-(\mu-\frac{1}{2})}\Gamma(\mu+5/2)}{(1+\Lambda)\Gamma(\mu+5/2-l)} F_{\mu+l,\mu-l} = 0,\ l\neq 0\ , \qquad (27)$$

where the vanishing of $H^{(l)}(\Lambda,\Lambda)$ follows directly from the form of Eq. 15. This completes the proof of Eq. 1.

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