SIDELOBE SECTOR NULLING WITH MINIMIZED PHASE PERTURBATIONS(U) ROME AIR DEVELOPMENT CENTER GRIFFISS AFB NY R A SHORE ET AL. MAR 85 RADC-TR-85-56
SIDELOBE SECTOR NULLING WITH MINIMIZED PHASE PERTURBATIONS

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A new phase-only pattern synthesis method has been developed to lower the sidelobes of a given antenna pattern in a wide sector, while reducing pattern distortion elsewhere. The method consists of finding the phase perturbations that minimize a performance measure defined to be a weighted sum of the average sector sidelobe power and the sum of the squared weight perturbations. By varying the weights assigned to the sector power and the weight perturbations, it is possible to shift the relative emphasis placed on the two objectives of reduced sidelobe power and small pattern distortion. The phase perturbations are obtained numerically using a nonlinear computer optimization code, since the equations obtained by equating to zero the partial derivatives are nonlinear and cannot be solved analytically. Curves are plotted showing the variation of the average sector sidelobe power, sum of the squared weight perturbations, gain in the look direction, and pattern null locations, with the ratio of the weights (over)
19. Abstract (Contd)

in the performance measure. Examples are shown of the patterns with reduced sidelobes that are obtained using this method.
6. Unperturbed Uniform 21-Element Array Pattern (----) and Perturbed Pattern (——) With Lowered Sidelobes in the Sector \([20^\circ, 30^\circ]\). \(\mu_2/\mu_1 = 100\)

7. Unperturbed Uniform 41-Element Array Pattern (----) and Perturbed Pattern (——) With Lowered Sidelobes in the Sector \([20^\circ, 30^\circ]\). \(\mu_2/\mu_1 = 100\)

8. Unperturbed Uniform 41-Element Array Pattern (----) and Perturbed Pattern (——) With Lowered Sidelobes in the Sector \([20^\circ, 30^\circ]\) Obtained With Combined Phase and Amplitude Weight Perturbations. \(\mu_2/\mu_1 = 100\)

9. Unperturbed Uniform 41-Element Array Pattern (----) and Perturbed Pattern (——) With Lowered Sidelobes in the Sector \([20^\circ, 30^\circ]\). \(\mu_2/\mu_1 = \infty\)
Sidelobe Sector Nulling
With Minimized Phase Perturbations

1. INTRODUCTION

Reduction of the sidelobes in extended sectors of an antenna pattern is often required to minimize the effects of clutter or of wide-bandwidth point interferences. Such enhanced sidelobe protection may, however, degrade desirable features of the pattern such as gain and beam width, or an already low average sidelobe level. A tradeoff exists, of course, between the two goals of lowered sidelobes in certain pattern sectors, and the preservation of the integrity of a design antenna pattern. Preservation of pattern integrity demands that the perturbations of the complex array weights required to achieve lowered sidelobes be kept as small as possible. This suggests that a useful performance measure in sidelobe sector nulling is the weighted sum of the average power in a specified sidelobe region and the squared weight perturbations. By varying the weights assigned to the average power in the sidelobe sector and the weight perturbations, and minimizing the performance measure, it is then possible to shift the relative emphasis placed on the two principal objectives.

(Received for Publication 21 March 1985)
When both the amplitude and phase of the array weights can be freely varied, an analytic solution can be obtained for the array weights that minimize the performance measure described above. The purpose of this report is to present a sidelobe sector null synthesis method based on the minimization of the same performance measure when perturbations of the complex weights are restricted to be of the array weight phases only. Interest in such phase-only pattern control methods has been stimulated by the growing importance of phased array antennas, since the required phased controls are already available as part of a beam steering system. Unlike the case of combined phase and amplitude control, a set of nonlinear equations is obtained by setting the derivatives of the performance measure with respect to the array weight phases equal to zero. This set of equations cannot be solved analytically, but the phases that minimize the performance measure can be found numerically using a nonlinear optimization computer code. Curves are plotted showing the variation of the average sector sidelobe power, sum of the squared weight perturbations, gain in the look direction, and pattern null locations, with the ratio of the weights in the performance measure. Examples are shown of the patterns with reduced sidelobes that are obtained using this method.

2. ANALYSIS

We consider a linear array of N equispaced, isotropic elements with inter-element spacing d and phase reference at the array center. Let

\[ \mathbf{w}_0 = [w_{01}, w_{02}, \ldots, w_{0N}]^T \]

and

\[ \mathbf{w} = [w_1, w_2, \ldots, w_N]^T \]

denote the vectors of the original and perturbed complex weights respectively, and let

\[ w_{on} = a_n \exp(j\phi_{on}), \quad n = 1, 2, \ldots, N \].

Because of the large number of references cited above, they will not be listed here. See References, page 13.
Then, since the perturbations are assumed to be of the phases only,

\[ w_n = w_n^o \exp(j\phi_n) = a_n \exp[j(\phi_n^o + \phi_n)], \quad n = 1, 2, \ldots, N. \]

The original and perturbed field patterns are given respectively by

\[ p_o(u) = \sum_{n=1}^{N} w_n^o \exp(jd_n u) \]

and

\[ p(u) = \sum_{n=1}^{N} w_n \exp(jd_n u) \]

with

\[ d_n = (N - 1)/2 - (n - 1), \quad n = 1, 2, \ldots, N \]

and

\[ u = (2\pi/\lambda) \sin(\theta), \]

where \( \lambda \) is the wavelength, and \( \theta \) the pattern angle measured from broadside to the array.

The sum of the squared weight perturbations is given by

\[ \sum_{n=1}^{N} |w_n - w_n^o|^2 = 2 \sum_{n=1}^{N} a_n^2 [1 - \cos(\phi_n^o)] = 4 \sum_{n=1}^{N} [a_n \sin(\phi_n^o/2)]^2. \]

Let the sidelobe sector in which the power is to be minimized be specified by the interval \([u_o - \epsilon, u_o + \epsilon]\). Then the average power, \( P_{av}(u_o', \epsilon) \), in the sidelobe sector is given by

\[ P_{av}(u_o', \epsilon) = (1/2\epsilon) \int_{u_o - \epsilon}^{u_o + \epsilon} |p(u)|^2 \, du \]

\[ = \sum_{n=1}^{N} \sum_{m=1}^{N} a_n a_m \cos[\phi_n^o - \phi_m^o + \phi_n - \phi_m + (d_n - d_m)u_o] \cdot \text{sinc}[(d_n - d_m)\epsilon], \quad (1) \]
where \( \text{sinc}(x) = \frac{\sin(x)}{x} \). The performance measure, \( P \), is now defined to be

\[
P = \mu_1 \sum_{n=1}^{N} |w_n - w_{on}|^2 + \mu_2 P_{\text{av}}(u_0, \epsilon),
\]

where \( \mu_1 \) and \( \mu_2 \) are the respective weights assigned to the sum of the squared weight perturbations and the average power in the sidelobe sector. A necessary condition for \( P \) to have a minimum is that the partial derivatives of \( P \) with respect to the phase perturbations be equal to zero; that is

\[
\frac{\partial P}{\partial \phi_q} = 0, \quad q = 1, 2, \ldots, N.
\]

It is a straightforward matter to calculate the partial derivatives, thus obtaining the set of equations

\[
\mu_1 a_q^2 \sin(\phi_q) - \mu_2 a_q \sin[\phi_{qo} - \phi_{on} + \phi_q - \phi_n + (d_q - d_n)u_0] 
\cdot \text{sinc}[(d_q - d_n)\epsilon] = 0.
\]

This set of equations is nonlinear in the variables \( \phi_n \) and cannot be solved analytically. It is possible, however, to find a set of phase perturbations that minimizes \( P \) using a nonlinear optimization computer code.

Although in the above analysis we have considered a single sidelobe sector only, the nulling method presented generalizes immediately to any number of sidelobe sectors. The expression for \( P_{\text{av}}(u_0, \epsilon) \) given in Eq. (1) entering into the performance measure defined by Eq. (2) is simply replaced by a sum of such expressions with different values of \( \mu_0 \) and \( \epsilon \). Different weights can be assigned to the various sidelobe sectors if desired. It is also possible to let the width \( \epsilon \) of a sidelobe sector become zero, in which case the method synthesizes a null at a point location.

3. RESULTS

The phase perturbations required to minimize the performance measure defined by Eq. (2) were calculated using the nonlinear optimization computer code LPNLP.\(^{34}\) Calculations were performed for uniform arrays of 11, 21, and 41 elements.

elements with interelement spacing $\lambda/2$. The look direction of the array was made zero degrees by setting all phases initially to zero, and the pattern sector for reduced sidelobe power was taken to be the interval $[20^\circ, 30^\circ]$. The ratio, $\mu_2/\mu_1$, of the weights assigned to the average sector sidelobe power and the sum of the squared weight perturbations respectively, was varied from 0.0001 to 100,000. In Figures 1, 2, and 3, respectively, the average sector sidelobe power, sum of the squared weight perturbations, and gain in the look direction are plotted. The ability to lower sidelobes in a wide pattern sector without significantly affecting the look direction gain increases rapidly with the number of array elements. For an 11-element array, a loss of 1.75 dB in gain is associated with a 40-dB reduction in average power over the nulling sector, while for a 41-element array only a 0.18 dB loss in gain is required to achieve a 40 dB reduction in average sector sidelobe power. The phase perturbations required to lower the sector sidelobe level, also cause a shift in the mainbeam direction that decreases with increasing number of array elements. For the 11-element array, and the weight ratio $\mu_2/\mu_1$ equal to 100,000, the peak shifted to $2.47^\circ$ with the peak gain 0.75 dB higher than the gain in the look direction, while for the 41-element array and the same weight ratio, the mainbeam shifted by only $-0.03^\circ$ with the peak gain only 0.001 dB higher than the look direction gain.

![Figure 1](image-url)

**Figure 1. Average Sidelobe Power in the Sector $[20^\circ, 30^\circ]$ for Arrays of 11, 21, and 41 Elements**
Figure 2. Sum of Squared Weight Perturbations for Arrays of 11, 21, and 41 Elements

Figure 3. Look Direction Gain of Perturbed Pattern for Arrays of 11, 21, and 41 Elements
In Figure 4 the locations of the pattern nulls closest to the sector \([20^\circ, 30^\circ]\) are plotted as a function of \(\log_{10}(\mu_2/\mu_1)\) for a uniform amplitude array of 41 elements. It is clearly seen how an increasing number of nulls are moved into, and closer to, the sector \([20^\circ, 30^\circ]\) as \(\mu_2/\mu_1\) increases and hence as increasing weight is placed on lowering the sidelobes in the sector as compared with preserving the original pattern. It is interesting, however, that even for values of \(\mu_2/\mu_1\) much larger than those shown in Figure 4, a maximum of only nine nulls are moved into the sector \([20^\circ, 30^\circ]\). (For uniform arrays of 11 and 21 elements, the maximum number of nulls moved into the sector \([20^\circ, 30^\circ]\) was 4 and 6 respectively.) This behavior contrasts strongly with that found when both the amplitude and phase of the array weights can be varied, in which case more and more nulls up to the maximum number of N-1 are moved into the nulling sector as \(\mu_2/\mu_1\) increases to \(\infty\). The maximum number of pattern nulls that can be moved into the nulling sector with phase-only weight control is, moreover, considerably less than the number \(N/2\) that might be expected, in view of the fact that in phase-only nulling in real antenna patterns of linear arrays there are \(N/2\) degrees of freedom. The

![Figure 4. Location of Nulls in Perturbed Pattern of 41-Element Array in the Vicinity of the Sector \([20^\circ, 30^\circ]\)](figure)

'Sector Sidelobe Nulling', RADC-TR-81-326, AD A112628.
restriction of the perturbations of the array weights to be of the phases only, thus results in a considerable loss of pattern control compared with the full control available with combined phase and amplitude weight variation.

As examples of the patterns obtained with the null synthesis method described in this report, Figures 5, 6, and 7 show the unperturbed pattern and the perturbed pattern obtained with $\mu_2/\mu_1 = 100$ and the sector for reduced sidelobes taken to be $[20^\circ, 30^\circ]$, for arrays of 11, 21, and 41 elements respectively. As the number of elements increases, the power in the sector $[20^\circ, 30^\circ]$ decreases and the perturbed pattern follows the original pattern more closely, especially in the near-in sidelobe region. The far-out sidelobes for the 41-element pattern, however, are still considerably higher than those of the original pattern. This pattern distortion is noticeably larger than that associated with combined phase and amplitude control, where for the same case the maximum increase in the perturbed sidelobe envelope above the unperturbed envelope is 2 dB as shown in Figure 8.

![Figure 5](image_url)

Figure 5. Unperturbed Uniform 11-Element Array Pattern (----) and Perturbed Pattern (-----) With Lowered Sidelobes in the Sector $[20^\circ, 30^\circ]$. $\mu_2/\mu_1 = 100$
Figure 6. Unperturbed Uniform 21-Element Array Pattern (----) and Perturbed Pattern (-----) With Lowered Sidelobes in the Sector \([20^\circ, 30^\circ]\). \(\mu_2/\mu_1 = 100\)

Figure 7. Unperturbed Uniform 41-Element Array Pattern (-----) and Perturbed Pattern (-----) With Lowered Sidelobes in the Sector \([20^\circ, 30^\circ]\). \(\mu_2/\mu_1 = 100\)
Figure 8. Unperturbed Uniform 41-Element Array Pattern (-----) and Perturbed Pattern (-----) With Lowered Sidelobes in the Sector [20°, 30°] Obtained With Combined Phase and Amplitude Weight Perturbations. $\mu_2/\mu_1 = 100$

Despite the significant pattern distortion exhibited in Figure 7, the inclusion of the weight perturbations in the performance measure affords considerable protection for the integrity of the original pattern. This can be clearly seen by comparing Figure 7 with Figure 9 in which we show the perturbed pattern obtained for the 41-element uniform amplitude array when $\mu_1 = 0$. Hence, zero weight is placed on minimizing the weight perturbations. Here a reduction of power in the sidelobe sector [20°, 30°] somewhat greater than that achieved when $\mu_2/\mu_1 = 100$ (-97 dB compared with -59 dB) is obtained at the expense of a wild distortion of the original pattern.

Calculations were also performed to investigate the uniqueness of the set of phase perturbations found by the nonlinear computer optimization code to minimize the performance measure. The required starting set of phase perturbations, normally set equal to zero, was varied widely over the interval [-\(\pi\), +\(\pi\)]. Although this resulted in an increase in convergence time, the solutions found did not differ significantly from one another. This suggests that the performance measure has only one local minimum and that this minimum is the global minimum for the problem.
4. CONCLUSIONS

A performance measure consisting of a weighted sum of sector sidelobe power and squared element weight phase perturbations has been shown to be effective in reducing sidelobes in a specified pattern sector while maintaining the mainbeam gain. Necessary conditions for the minimization of the performance measure were derived by equating to zero the partial derivatives of the performance measure with respect to the phase perturbations. Because these equations are nonlinear in the phase perturbations and cannot be solved analytically, an unconstrained minimization computer code was used to obtain the phase perturbations minimizing the performance measure. The fact that the same solution was obtained from a variety of different numerical starting points suggests that the performance measure has a unique minimum in the \([-\pi, +\pi]\) range. The number of iterations required to solve the problem increases with the number of array elements and also with increasing weight placed on reduction of sector sidelobe power compared with minimization of weight perturbations.
Pattern distortion in regions of the pattern other than the nulling sector can be reduced considerably from what it is if no weight is placed on minimizing the phase perturbations. Both average power in the nulling sidelobe sector and pattern distortion decrease as the number of array elements increases. The level of pattern distortion is, however, considerably higher than that produced when the weight perturbations are not restricted to be of the phases only.
References


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