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Finite Temperature Stabilization of the Gradient Drift Instability in Barium Clouds

J. F. Drake
Science Applications International Corp.
McLean, VA 22102

J. D. Huba and S. T. Zalesak
Geophysical and Plasma Dynamics Branch
Plasma Physics Division

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NAVAL RESEARCH LABORATORY
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**Title:** Finite Temperature Stabilization of the Gradient Drift Instability in Barium Clouds

**Authors:** Drake, J.F., Huba, J.D., and Zalesak, S.T.

**Abstract:**
We present a relatively simple analysis of the gradient drift instability in barium clouds which includes the effects of both finite temperature and finite parallel length. It is found that short wavelength modes are stabilized as the electrons redistribute parallel to the magnetic field and neutralize the charge imbalance set up by the instability. An analytical expression for the critical wavenumber for stabilization is given, as well as numerical results. We discuss the application of these results to the structuring of barium clouds.
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INTRODUCTION

The study of the dynamic evolution of artificial plasma clouds (e.g., barium clouds) in the earth's ionosphere and magnetosphere has been an active area of research for more than two decades. The initial motivation for these active space experiments was to use the plasma cloud as a diagnostic to determine ambient plasma conditions (e.g., electric fields, neutral winds). However, it was soon discovered that the interaction of an artificial plasma cloud and the near-earth space plasma was very complex, and that plasma clouds undergo a complicated and dynamic evolution. One of the prominent features of cloud evolution in the ionosphere is the development of field-aligned striations on the cloud's steepened "backside". These striations have been attributed to the onset of the gradient drift instability (Linson and Workman, 1970). This instability can occur in a weakly collisional plasma which contains a density gradient and a neutral wind (or ambient electric field) (Simon, 1963; Hoh, 1963).

A substantial amount of theoretical and computational research has been invested in understanding the linear and nonlinear development of the gradient drift instability and its relevance to plasma cloud structure (Volk and Haerendel, 1971; Perkins et al., 1973; Zabusky et al., 1973; Shiau and Simon, 1972; Perkins and Doles, 1975; Scannapieco et al., 1976; Chaturvedi and Ossakow, 1979; Keskinen et al., 1980; McDonald et al., 1980; McDonald et al., 1981; Husa et al., 1983). The bulk of analyses to date have neglected the effects of plasma dynamics parallel to the ambient magnetic field \( B_0 \) (i.e., considered perturbations only in the plane transverse to \( B_0 \)) or have incorporated parallel effects in a crude way.

Francis and Perkins (1975), for example, assume that the ambient and perturbed potentials map uniformly along $B_0$ thereby connecting the cloud dynamics to that of the conducting background at different altitudes. In this model finite thermal effects have a stabilizing influence on short wavelength modes. However, several studies have been performed which attempt to include parallel dynamics self-consistently in the stability analysis (Goldman et al., 1976; Sperling and Glassman, 1984; Sperling et al., 1984). These studies incorporate the parallel length of the cloud along the ambient field into the stability analysis and have shown that parallel effects can play an important role in the development of the gradient drift instability as it relates to ionospheric plasma clouds.

In particular, Sperling et al. (1984) have recently shown that when the finite size of the cloud along $B_0$ is self-consistently incorporated in the linear stability analysis, the long wavelength modes tend to have much smaller growth rates than short wavelength modes. This reduction in growth due to finite cloud length occurs because the integrated Pedersen conductivity of the background plasma over the extent of the mode along the field becomes greater than the cloud. The distance the mode extends along the field is proportional to the perpendicular wavelength of the mode so that long wavelength modes are more strongly affected. However, this analysis neglected the effects of finite temperature. Sperling and Glassman (1984) included finite temperature in the analysis and found that short wavelength modes became propagating rather than purely growing modes. Moreover, for sufficiently short wavelengths there was some evidence that the modes were completely stabilized. A drawback of this work though is that a relatively complex, second-order differential equation is solved numerically to obtain results. The underlying physics of the stabilization mechanism is therefore somewhat obscure and not
addressed in the paper. Nonetheless, it is evident that finite temperature
and finite cloud length effects can impact the development of the gradient
drift instability.

The purpose of this paper is to present a linear stability analysis of
the gradient drift instability which incorporates both finite temperature
and finite parallel length effects. A simple plasma model is used (similar
to the one used in Sperling et al. (1984)) which permits an analytical
solution to the dispersion equation. It is found that the short wavelength
modes are stabilized by redistribution of electrons parallel to \( B_0 \) (i.e.,
parallel electron diffusion or parallel electron response to the perturbed
fields) which neutralize the charge imbalance set up by the instability.
An analytical expression for the critical wavenumber for the stabilization
of instability is given, as well as numerical results.

The organization of the paper is as follows. In the next section we
derive a set of general nonlinear equations which describe the evolution
of a three dimensional plasma cloud. In Section III we discuss the
equilibrium to be used in the instability analysis. In Section IV we
derive a general dispersion equation, and present both analytical and
numerical results. Finally, in the last section we summarize our findings
and discuss the relevance of our results to cloud structure.

II. GENERAL EQUATIONS

We first derive a set of nonlinear equations to describe the evolution
of a warm plasma cloud in a uniform magnetic field \( \mathbf{B} = B_0 \mathbf{z} \) with a
background neutral wind \( \mathbf{V}_n = v_n \mathbf{x} \) [see Fig. 1a]. For simplicity we
consider only low frequency \( \partial/\partial t \ll v_\alpha \) motion of the cloud and take the
electron collisions to be sufficiently weak so that \( v_e/\Omega_e \ll 1 \) but
allow \( v_\perp/\Omega_\perp \) to be arbitrary. The collision frequency and gyrofrequency of
the α species are given by \( v_\alpha \) and \( \Omega_\alpha \) respectively. Both electrons and ions are taken to be warm and for simplicity we consider the isothermal limit. In this case the fundamental equations of our analysis are continuity, momentum transfer, charge neutrality and Ampere's law:

\[
\frac{\partial n}{\partial t} + \nabla \cdot (nv_n) = 0
\] (1)

\[
0 = -eE - \frac{e}{c} v_e \times B - m_e v_e (v_e - \langle v_n \rangle) - m_e v_{ei} (v_e - \langle v_i \rangle) - \frac{T_e}{n} v_n
\] (2)

\[
0 = eE + \frac{e}{c} v_i \times B - m_i v_{in} (v_i - \langle v_n \rangle) - m_i v_{ie} (v_i - v_e) - \frac{T_i}{n} v_n
\] (3)

\[
\nabla \cdot \mathbf{j} = \nabla \cdot [n(v_i - v_e)] = 0
\] (4)

\[
\nabla \times \mathbf{B} = \frac{4\pi}{c} \mathbf{j}
\] (5)

where \( v_\alpha \) and \( T_\alpha \) are the fluid velocity and temperature of species \( \alpha \), respectively, \( v_{an} \) is the α specie-neutral collision frequency, \( v_{ei} \) is the electron-ion collision frequency, \( v_{ie} \) is the ion-electron collision frequency, and \( \alpha \) refers to electrons (e) or ions (i). We take the electric and magnetic fields to be represented by potentials as

\[
E = -\nabla \phi - \frac{1}{c} \frac{\partial A_z}{\partial t} \mathbf{\hat{z}}
\] (6)

and

\[
B = B_0 \mathbf{\hat{z}} + vA_z \times \mathbf{\hat{z}}
\] (7)
where $\phi$ is the electrostatic potential and $A_z$ is the vector potential associated with the magnetic field produced by the self-consistent plasma currents. We consider only $A_z$ since $J_\parallel \gg J_\perp$ and assume $|v A_z \times \hat{z}| \ll B_0$.

The electron cross-field motion is given by

$$v_{e\perp} = -\frac{c}{B} \nabla_\perp \phi \times \hat{z} + \frac{c T_e}{e B n} \nabla_\perp n \times \hat{z}$$

(8)

while the parallel motion is given by

$$v_{e\parallel} = -[e E_\parallel + T_e n_{\parallel} \ln(n)] + f[v_e/v_n] T_e n_{\parallel} \ln(n)]/[m_e (v_e + f v_{e\parallel})]$$

(9)

with

$$E_\parallel = -\nabla_\parallel \phi - \frac{1}{c} \frac{\partial A_z}{\partial t}$$

(10)

where $v_e = v_{e\parallel} + v_{e\perp}$, $\nabla_\parallel = \hat{b} \cdot \nabla$, $\hat{b} = B/[B_0]$, and $T = T_e + T_i$.

The assumption that $v_e / \Omega_e \ll 1$ while $v_{i\parallel} / \Omega_{i\parallel} \sim 1$ requires that

$$f \equiv m_e v_{e\perp} / m_i v_{i\parallel} \ll 1.$$  

The ion cross-field motion is given by

$$v_{i\perp} = \delta_i \left[-\frac{c}{B} \nabla_\perp \phi \times \hat{z} + \frac{v_{i\parallel}}{n_i} \nabla_\perp n \times \hat{z} - \frac{c T_i}{e B n} \nabla_\perp n \times \hat{z} \right.$$

$$\left. - \frac{v_{i\parallel}}{n_i} \frac{c}{B} \nabla_\perp \phi + \left(\frac{v_{i\parallel}}{n_i}\right)^2 \nabla_\perp n - \frac{v_{i\parallel}}{n_i} \frac{c T_i}{e B n} \nabla_\perp n \right],$$

(11)

where $\delta_i = (1 + v_{i\parallel}^2 / n_i^2)^{-1}$, and the parallel motion is given by

$$v_{i\parallel} = \left[[e E_\parallel + T_e n_{\parallel} \ln(n)] v_{e\parallel} - e n_{\parallel} \nabla_\parallel \ln(n) \right]/[m_i v_{i\parallel} (v_e + f v_{e\parallel})].$$

(12)
In (11) we have included both the Pedersen and Hall responses to the electric field, neutral wind and pressure.

Substituting (8)-(12) into (1), (4) and (5) we find that

\[ \frac{dn}{dt} - \frac{c}{B} \nabla \phi \times \nabla n + \nabla \left( \frac{c}{4 \pi e} \nabla^2 n_{\parallel} - D_{\perp} \nabla \phi \right) = 0, \]  

(13)

\[ \delta_1 \frac{c}{B} \nabla \phi \times \nabla n + \delta_1 \frac{c}{B} \nabla \phi \times \nabla n + \frac{\nu_{in}}{\Omega_i} \nabla^2 n_{\perp} z \times \nabla n \]

\[ + D_{\perp} \nabla^2 n_{\perp} + \frac{c}{4 \pi e} \nabla \phi \nabla^2 n_{\perp} = 0, \]  

(14)

\[ \nabla^2 n_{\perp} = \frac{4 \pi}{c \eta} \left( \nabla \phi \right)^2 + \frac{1}{c} \frac{dA_z}{dz}, \]  

(15)

where \( n_e = m_e v_e / n e^2 \) is the parallel resistivity, \( D_{\parallel} = T / m_i \nu_{in} \) and \( D_{\perp} = \delta_1 (v_{in} / \Omega_i) c T / e B \) are the parallel and perpendicular ion transport coefficients, \( T = T_e + T_i \), and

\[ \phi \equiv \phi - \frac{T_e}{e} \ln(n) - \frac{B}{c} \frac{\nu_{in}}{\Omega_i} \nabla n \times z, \]  

(16)

\[ \frac{\partial}{\partial t} \equiv \frac{\partial}{\partial t} + \frac{\nu_{in}}{\Omega_i} \nabla n \times \nabla n \]  

(17)

\[ \nabla \phi = \frac{\partial}{\partial z} + \delta_0^{-1} \nabla A_z \times \nabla n. \]  

(18)

Equation (13) is the electron continuity equation, (14) is the charge conservation equation and (15) is Ampere's law. Note that the electron pressure has been absorbed into \( \phi \) in (16) and that terms of order \( f \ll 1 \) have been discarded compared with those of order unity. Equations (13)-(18) constitute a complete description of the evolution of a three-dimensional warm plasma cloud.
III. EQUILIBRIUM

We will consider the linear stability of a two-dimensional cloud which is localized both along and across the magnetic field $B_0$: $n_c = n_c(x, z)$ with $n_c \neq 0$ for $|x| < x_c$ and $|z| < z_c$. The background plasma is taken to be uniform throughout the region $|z| < z_b$ between two insulating plates at $z = \pm z_b$. The location of the plates, enables us to control the ratio of the total magnetic-field-line integrated Pedersen conductivity of the cloud to that of the background, $n_c z_c / (n_c z_c + n_b z_b)$. This ratio is an important parameter of the equilibrium configuration.

The equations describing the two dimensional plasma cloud are given by

\begin{align}
\frac{\partial n}{\partial t} - D \frac{\partial^2 n}{\partial z^2} + \frac{c}{4\pi e} \frac{\partial}{\partial z} \left( \frac{2A_z}{\partial x^2} \right) & = 0, \quad (19) \\
\frac{\partial \phi}{\partial t} + \frac{c}{B} \frac{\partial \phi}{\partial z} - \frac{\partial}{\partial x} \left( \frac{\partial}{\partial z} \left( \frac{1}{2} V_{\phi} \right) \right) & = 0, \quad (20) \\
\frac{\partial^2 A_z}{\partial t^2} & = \frac{4\pi}{c n_e} \left( \frac{\partial \phi}{\partial z} + \frac{1}{c} \frac{\partial A_z}{\partial t} \right). \quad (21)
\end{align}

When there are no density gradients in the $x$ direction, the solution to these equations is $\phi = -(B/c)(V_{\phi} / n_e) V_{\phi} x$ so that

$$
\phi = (T_e / e) \ln \left[ n(z) / n_b \right] 
$$

(22)

with $A_z = 0$, i.e., the potential adjusts itself so that the electrons are in force balance along the magnetic field as the cloud diffuses along $B_0$. In writing the solution for $\phi$ in (22) we have required $\phi = 0$ for $|z| > z_c$ since we have assumed that only the charge accumulations associated with the motion of the plasma cloud itself are responsible for the development of $\phi$. There is no charge at $z = \pm z_b$. The density satisfies
\[ \frac{\partial n}{\partial t} - D_{||} \frac{\partial^2 n}{\partial z^2} = 0 \]  

so that \( n \) diffuses along \( B_0 \). In the opposite limit, where there are no density gradients in the \( z \) direction \((\partial/\partial z = 0)\), \( A_z = 0 \) and

\[ \dot{\phi} = -\left(\frac{T}{e}\right) \ln(n) - (B/c) \frac{\nu_{in}}{\Omega_i} v_n \int_{-\infty}^x dx (n_b/n) \]  

so that

\[ \phi = -\frac{T}{e} \ln\left(\frac{n(x)}{n_b}\right) + \frac{B}{c} \frac{\nu_{in}}{\Omega_i} v_n \int_{-\infty}^x dx \left(1 - \frac{n_b}{n}\right), \]  

where we have required \( \phi \rightarrow 0 \) as \( |x| \rightarrow \infty \). In Eq. (24) the potential adjusts itself so that the perpendicular ion pressure gradient is balanced by the electrostatic field. In comparing the expressions for \( \phi \) in (22) and (24), it is important to note that the electron and ion pressures push the potential in the opposite direction (compare the signs of the terms proportional to \( T_e \) and \( T_i \)).

We now return to the more general two dimensional equations in (19)-(21). These equations describe three time scales: the resistive flux diffusion time, \( \tau_r = 4\pi \frac{x_c^2}{\eta_e c^2} \); the parallel diffusion time, \( \tau_{||} = \frac{z_c^2}{D_{||}} \); and the perpendicular diffusion time, \( \tau_\perp = \frac{x_c^2}{D_{\perp}} \). We assume that the flux diffusion time is the shortest time scale so that inductive effects are not important in the equilibrium, i.e., in Eq. (21) \( \partial A_z / \partial t \ll c \partial \phi / \partial z \). The equations then simplify to

\[ \frac{\partial n}{\partial t} - D_{||} \frac{\partial^2 n}{\partial z^2} - D_{\perp} \frac{\partial^2 n}{\partial x^2} - \delta \frac{c}{B} \frac{\nu_{in}}{\Omega_i} v_n \frac{\partial \phi}{\partial x} = 0, \]  

\[ \delta \frac{c}{B} \frac{\nu_{in}}{\Omega_i} v_n \frac{\partial \phi}{\partial x} + \frac{1}{e n_e \partial x} \frac{\partial^2 \phi}{\partial z^2} + D_{\perp} \frac{\partial^2 n}{\partial x^2} = 0. \]  

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1. CONCLUDING REMARKS

We have presented a relatively simple analysis of the gradient drift instability in barium clouds which includes both finite temperature and finite parallel length effects. We have derived a general set of nonlinear equations which describe the evolution of a three-dimensional plasma cloud in the ionosphere. We then investigate the stability of a two-dimensional plasma cloud in which the density varies in the direction of the neutral wind and along the ambient magnetic field $B_0$. By modeling the density variation along $B_0$ as a waterbag, we are able to obtain an analytic dispersion relation for the gradient drift instability. There is no static equilibrium since the cloud diffuses both perpendicular and parallel to $B_0$ so that we restrict our analysis to time scales short compared to the cloud diffusion time scales.

In recent work Sperling et al. (1984) found that the finite length of the cloud suppressed the growth of long wavelength modes because of "good" coupling to the background plasma. However, the growth rate of short wavelength modes is unaffected by the finite size of the cloud since they do not couple to the background in a zero temperature plasma. In contrast to this result, we find that in a finite temperature plasma the short wavelength modes do couple to the background plasma and that for sufficiently short wavelengths the modes are completely stabilized. Stabilization results as the electrons redistribute parallel to $B_0$ and neutralize the charge imbalance set up by the instability. The parallel electron motion results from diffusion and/or the response to the perturbed fields. We have also derived a simple analytic expression (see (54)) for the wavenumber corresponding to marginal stability (i.e., $\gamma = 0$). Our results are consistent with previous numerical computations of Sperling and
V. NUMERICAL RESULTS

We now present quantitative results for the wave frequency of the gradient drift instability by solving (43) numerically. In Fig. 2 we plot $\gamma/\gamma_0$ (solid curve) and $\omega_r/\gamma_0$ (dashed curve) vs. $k_y\rho_s$ where $\omega = \omega_r + i\gamma$ is the wave frequency. We consider the following typical parameters for a barium cloud at 180 km: $n_c/n_b = 10.0$, $T_e = T_i = 0.1$ eV, $m_i = 16 m_p$ ($0^+$ background), $z_c = 10$ km, $B = 0.3$ G, $\alpha = 10^6$, $D_\perp = 100$ m$^2$/sec, $V_n = V_{eff} = 20$ m/sec and $L_n = 1$ km. From these values we note that $c_s = 10^3$ m/sec and $\rho_s = 6$ m. The main features of Fig. 2 are described as follows. First, the growth rate has a maximum value at $k_y\rho_s = k_{ym}\rho_s = 0.08$. For $k_y < k_{ym}$ the growth rate decreases because of coupling to the background plasma as described in Sperling et al. (1984). For $k_y > k_{ym}$ the growth rate decreases rapidly and becomes stable (i.e., $\gamma < 0$). This is due to parallel electron motion as described in the preceding section and is the dominant finite temperature effect. Second, the critical wavenumber for stabilization as given by (54) is denoted by the arrow along the $k_y\rho_s$ axis ($k_y\rho_s = 0.26$). It is seen that (54) gives a very good approximation to the critical wavenumber obtained numerically ($k_y\rho_s = 0.28$). For the parameters used the critical wavelength is given by $\lambda_c = 135$ m. Third, the real frequency is linear in $k_y$ and is proportional to the diamagnetic drift velocity $V_d = (cT/eB)(r_c/n_c)$.

The results shown in Fig. 2 appear to be in qualitative agreement with the numerical work of Sperling and Glassman (1984). They consider a plasma cloud with a Gaussian distribution along the magnetic field rather than a waterbag model. The shape of the curve of $\gamma$ vs $k_y$ shown in Fig. 2 is similar to corresponding curves presented in their report. Moreover, they also find that $\omega_r = k_y$ for large $k_y$. 

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\[ \omega_r = \omega_n \left( 1 + \frac{n_c^2}{n_b^2} \frac{2\gamma_0 z_c^2}{\omega_0} - 1 \right) \]  

where \( L_n = -\left( n_c^2 / n_c^2 \right)^{-1} \) and \( \rho_s = c_s / \Omega_i \) is the Larmor radius based on the sound speed \( c_s = (T/m_i)^{1/2} \). Note from (54) that in the limit \( T \to 0 \), then \( k_c \to \infty \) so the mode is not stabilized for a zero temperature plasma [Sperling et al. (1984)]. Equation (54) also appears to imply that \( k_c \) becomes very large as the integrated cloud density \( n_c \) increases. This result is misleading. In a cloud which is finite in the \( y \) direction, the neutral wind velocity \( V_n \) in (54) should be replaced by the effective slip velocity \( V_{eff} \) of the cloud and the neutral wind. For a circular cloud, the effective velocity is given by [Ossakow and Chaturvedi, 1978]

\[ V_{eff} = 2V_n n_b z_b / (n_b z_b + n_c z_c). \]  

In the limit of very large \( n_c z_c \), Eq. (54) yields the critical wavelength

\[ \frac{k^2 L_n^2}{c_n} = \frac{4\gamma_0^2 z_c^2}{2D_2} \left( 1 + \frac{4\gamma_0^2 V_n^2}{\omega_s^2 c_s^2} \right), \]  

which is independent of \( n_c z_c \). Thus, short wavelength modes are stable even in large ionospheric clouds.
Two significant features of (52) are first that $\gamma_0$ does not enter the equation, and second that there appears to be an instability even in the absence of $\gamma_0$. However, the unstable root is invalid. In order for $\phi$ to remain bounded as $|z|$ becomes large, we require $\text{Re}(k_b) > 0$. Thus, for $\bar{\gamma} = i\omega_\kappa$ we find

$$k_b = (\omega_\kappa/\alpha D)_1^{1/2} e^{i\pi/4}.$$  \hspace{1cm} (53)

The root with the opposite sign must be discarded. Inserting this expression for $k_b$ into (46), we obtain the damped root in (52). Thus, when thermal effects are retained, the gradient-drift mode is stable for large $k_y$.

The stabilization mechanism can be understood in terms of our earlier discussion of the local dispersion relation. When $\omega_\kappa$ is discarded in (45) and (46), the parallel wavevector $k_b$ remains real (the mode is evanescent in the outer region) and the instability survives even as $k_y \rightarrow \infty$. However, when $\omega_\kappa$ is retained, the parallel wavevector $k_b$ becomes complex (the mode is evanescent and propagating in the outer region) and the mode is stable for large $k_y$. In other words, $\omega_\kappa$ causes the mode to become oscillatory along $B$, and the electrons can now move along rather than across $B$ to neutralize the ions; the mode is therefore stable.

To obtain an expression for the critical wavelength $k_c$ where the mode becomes stable, we take $\bar{\gamma} = -i\omega_\tau$ and solve (46) directly. The critical wavelength is given by

$$k_c^2 = \frac{\gamma_\tau^2}{\frac{2}{c_s^2} \left[ \frac{1}{n_b^2} \left( \frac{2}{n_b^2} + \frac{\tau_n}{\alpha D_1 L_n} \right)^2 + \frac{4 n_b^2}{c_s^2} \frac{\rho_s^2}{\alpha D_1^2 L_n^2} \right]}. \hspace{1cm} (54)$$
\[
(\gamma_0 - \gamma)^2 = \alpha \frac{n_b^2 D r}{n_c^2 z_c} \gamma,
\]

(47)

and is independent of the plasma temperature. For \( z_c^2 \gg \alpha D r n_b^2 / n_c^2 \gamma_0 \),

\[
\gamma = \gamma_0
\]

(48)

while in the opposite limit \( z_c^2 \ll \alpha D r n_b^2 / n_c^2 \gamma_0 \),

\[
\gamma = \frac{\gamma_0 n_c^2 z_c^2}{\alpha n_b^2 D r}.
\]

(49)

These results have been obtained previously [Sperling et al. (1984)].

When \( k_y \) becomes very large, \( k_y^2 = \gamma / \alpha D \) and the dispersion relation becomes

\[
(\gamma - i \omega_* - \gamma_0)^2 = \alpha D \frac{n_b^2}{n_c^2 z_c^2} \gamma.
\]

(50)

For large \( k_y \), we have \( \omega_* \gg \gamma_0 \) since \( \omega_* = k_y \) and therefore to lowest order

\[
\gamma = i \omega_*.
\]

(51)

the mode simply propagates at the diamagnetic frequency. To investigate whether the mode is stable or unstable in this limit, corrections to this result must be calculated. These corrections arise from the right side of (50) and yield the eigenvalue

\[
\gamma = i \omega_* + \frac{n_b}{n_c^2 z_c} (\alpha D \omega_*)^{1/2} e^{i \pi / 4}.
\]

(52)
where

\[
\frac{k_b}{k_c} = \left[ \frac{n_b}{n_c} \frac{-\gamma}{\gamma_0 - \gamma + i\omega_*} \right]^{1/2}. \tag{44}
\]

In Sperling et al. (1984), the dispersion relation was carefully solved in all possible limits of the various parameters. In order to clearly emphasize the role of finite temperature in stabilizing short wavelength modes, we will focus our analysis primarily on modes with growth rates less than \(\gamma_0\) for which \(k_b/k_c \ll 1\). In this limit (43) becomes

\[
k_z^2 c_z = k_b \tag{45}
\]

or

\[
n_c z_c (\gamma - i\omega_* - \gamma_0) + n_b k_b^{-1} \gamma = 0. \tag{46}
\]

In the form shown in (46), the nature of the dispersion relation can be simply understood. The first (second) term is the magnetic field line integrated contribution from the cloud (background). The background contribution arises only from the region \(|z| < k_b^{-1}\) since the perturbed potential \(\tilde{\psi}\) is small outside of this interval. The result in (46) can be directly obtained by integrating (33) for \(\tilde{\psi}\) along \(z\).

The dispersion relation in (45) or (46) can be solved analytically for various values of \(k_y\). In the limit \(k_y \to 0\), \(k_b^2 = \gamma/\omega D_\perp\) and the dispersion relation is given by
discontinuity in the density at \( z = \pm z_c \), we find that \( \phi \) and \( A_z \) must be continuous. For the even \( \phi \) solution \( (\phi_c^2 = 0) \), we obtain the dispersion equation

\[
k_c z_c = \tan^{-1} \left[ \frac{k_b \gamma + k_c^2 D_{rb}}{k_c \gamma + k_c^2 D_{rc}} \right] + m \pi
\]  

(42)

where \( m \) is an integer. The dispersion relation for the odd \( \phi \) mode \( (\phi_c^1 = 0) \) is similar to (42) except \( \tan^{-1} \) is replaced by \( -\cot^{-1} \). The result in (42) is identical to the dispersion relation obtained previously by Sperling et al. (1984) except that the expressions for \( k_b \) and \( k_c \) given in (40d) and (41d) now contain thermal effects which were previously neglected.

In general, the dispersion equation (42) has an infinite number of solutions for a given set of physical parameters, corresponding to eigenmodes with an increasing number of nodes \( (m) \) along \( z \). The dispersion equation (42) can be solved numerically for arbitrary values of the background and cloud parameters. However, to gain an understanding of the general scaling of the growth rate \( \gamma \) with the parallel extent of the cloud and temperature, it is useful to solve (42) analytically. To do this we make a number of simplifying assumptions. We consider only the lowest order mode (i.e., \( m = 0 \) which implies \( 0 < k_c z_c < \pi/2 \); it is easily shown that this mode has the largest growth rate); take \( v_{in}/N_1 \) to be small so that take all parameters but the density to be the same inside and outside of the cloud; assume \( n_c >> n_b \); and \( v_{ei} >> v_{en} \) so that the resistive diffusion coefficient is continuous across the boundary \( z = z_c \) (i.e. \( D_{rb} = D_{rc} \)). With these assumptions the dispersion equation becomes

\[
k_c z_c = \tan^{-1} (k_b/k_c)
\]  

(43)
with

\[ (\gamma + \frac{k^2 y_*}{y_r}) \tilde{A}_{zb} = k_c \phi_b \]

(40c)

\[ k_b^2 = \frac{\gamma (\gamma + \frac{k^2 y_*}{y_r})}{\alpha D_r (\gamma + \frac{k^2 y_D}{y_r})} \mid_b \]

(40d)

where the subscript \( b \) on a given parameter indicates that it is to be evaluated in the region \( |z| > z_c \). Note that we have assumed \( k_b z_b \gg 1 \) and that solutions which diverge as \( z \) becomes large have been omitted from (40). This assumption is consistent with the approximation \( z_b \ll a^{1/2} z_c \) made in deriving the equilibrium potential \( \phi \) as long as \( k_b z_b \gg 1 \).

In the region \( |z| < z_c \) the solutions for \( \phi \) and \( \tilde{A}_z \) are

\[ \tilde{A}_z = \tilde{A}_{zc} \sin (k_c z) - \tilde{A}_{zc} \cos (k_c z) \]

(41a)

\[ \phi = \phi_c \cos (k_c z) + \phi_c \sin (k_c z) \]

(41b)

with

\[ (\gamma + \frac{k^2 y_*}{y_r}) \tilde{A}_{zc} = k_c \phi_c \]

(41c)

\[ k_c^2 = \frac{\gamma - i \omega - \gamma_0 (\gamma + \frac{k^2 y_*}{y_r})}{\alpha D_r (\gamma + \frac{k^2 y_D}{y_r})} \]

(41d)

We note that in writing (40d) and (41d) we have assumed \( \gamma \gg k_b \gamma_* \) for simplicity.

To complete the dispersion relation, we must match the various plane wave solutions at \( z = \pm z_c \). The appropriate matching conditions are obtained from (32) and (33). Integrating these two equations across the
where \( D_{ll} = \delta (v_{in}/\Omega_i) c T_i/e B \). In the limit \( k_z = 0 \), \( \tilde{n}_e = (i k_y n_0 / \gamma B) \phi \)
and the usual growth rate of the instability is obtained by equating \( \tilde{n}_e \) and \( \tilde{n}_i \). In this case the density perturbation \( \tilde{n}_e \) and the electric field perturbation, \( \tilde{E}_y = -i k_y \phi \), are in phase and the usual physical description of instability applies. However, in the limit of large \( k_z \) with \( T_e \neq 0 \), \( \tilde{n}_e = n_0 e \phi / T_e \), i.e., the electrons are adiabatic and neutralize the ions by moving along \( B \) rather than across \( B \). In this case \( \tilde{n}_e \) is out of phase with \( \tilde{E}_y \) and there is no instability. When \( T_e = 0 \) and \( k_z \) is large, the mechanism is somewhat different although the mode is also stable. In this case, the electrons bunch parallel to \( B \) and \( \tilde{n}_e = (k_z^2/\gamma e \eta_e) \phi \). For \( k_z \) large the ions can only neutralize the electrons if \( \eta = -k_y D_{ll} + i k_y n_0 v_{in} / \Omega_i \).
Namely, the electrons bunch parallel to \( B \) and the ions diffuse across \( B \) to neutralize the charge imbalance. Finally, we note that while electron parallel diffusion or ion perpendicular diffusion stabilizes the mode when the perturbation is periodic along \( B \), when the spatial dependence of the mode is exponential, i.e., \( \tilde{p}(z) \sim \tilde{p} \exp \left( \pm \frac{k_z z}{m} \right) \), the thermal effects are not stabilizing. The distinction between a periodic and a nonperiodic solution is important in interpreting the dispersion relation which is obtained for the equilibrium shown in Fig. 1b.

We now solve (32) and (33) for the profile given by (19). The boundary conditions used are \( \phi, \tilde{A}_z \rightarrow 0 \) as \( |z| \rightarrow z_m \). For the step profile for \( n(x,z) \) the solutions to (32) and (33) in the region \( |z| > z_c \) can be written as plane waves

\[
\tilde{A}_z = \tilde{A}_{zb} \exp \left[ -k_B |z| \right] \quad (40a)
\]

\[
\phi = \tilde{\phi}_b \exp \left[ -k_B |z| \right] \quad (40b)
\]
\[ \gamma = \left( \gamma_0 - \frac{k^2 T}{m v e} \right) \left( \frac{k^2 D}{y r} \right) \] (36)

The Alfven wave has a stabilizing influence on the instability in the electromagnetic limit but thermal effects do not affect the growth rate. Of course, the gradient drift instability is not stable when \( k \frac{v^2}{\nu} > \gamma_0 \) as might be construed from (35) since the expression for \( \gamma \) given in (35) breaks down when \( \gamma < k^2 \frac{D}{y r} \). Thermal effects can have a significant influence on the growth rate when \( \gamma < k^2 \frac{D}{y r} \) (electrostatic limit). Namely, for

\[ k^2 \frac{T}{m v e} < \gamma_0 \] (37)

the electrons can move a wavelength along \( B \) during the growth time of the instability, and the mode is stable. In the limit of \( k_z \rightarrow \infty \) in (36),

\[ \gamma = -k^2 D \] (38)

so that the mode damps at the ion diffusion rate.

The stabilization mechanism can be understood by examining the electron and ion density perturbations in the electrostatic limit,

\[ \tilde{n}_e = \frac{(i k_y c n^2 / B + k^2 / e n_e)}{\gamma + k^2 T / m v e} \] (39a)

\[ \tilde{n}_i = \frac{(i k_y c n^2 / B - k^2 c (v / n_i)}{\gamma + k^2 D_i - i k_y v / n_i} \] (39b)

13
In deriving (32) and (33) we have neglected parallel ion diffusion as discussed previously and assumed \( n_c \gg n_b \). The primes denote a derivative with respect to \( x \). The important finite temperature effects that appear in (32) and (33) are the diamagnetic drift frequency \( (\omega_x) \), perpendicular ion diffusion \( (D_\perp) \), and modification of the equilibrium \( (\phi_0') \). In the limit \( T_i = T_e = 0 \), Eqs. (32) and (33) reduce to those previously derived by Sperling et al. (1984). The eigenvalue \( \bar{\gamma} \) is in a frame of reference moving with the electron fluid.

Prior to solving (32) and (33) for the density profile shown in Fig. 1b, we first consider a cloud of infinite extent \( [z_c + \infty] \) and Fourier expand in the direction parallel to \( B_0 \), i.e., \( \hat{p}(z) \sim \hat{p} \exp [ik_z z] \). This allows insight into the influence of finite temperature and parallel dynamics on the instability. The local dispersion equation is given by

\[
(\bar{\gamma} - i\omega_x - \gamma_0)(\bar{\gamma} + k^2 D_r) = - \frac{k^2 v_A^2}{v_{in} \delta_1} (\bar{\gamma} - i k \bar{V}_y + k^2 D_\perp),
\]

where \( V_A = B/(4\pi m_i) \) is the Alfvén velocity. This dispersion relation illustrates the coupling between the gradient drift instability and the Alfvén wave when \( k_z \neq 0 \).

For simplicity, we neglect the terms in (34) which cause the mode to propagate \( (\omega_x, k_y \bar{V}_y \rightarrow 0 \) and \( \bar{\gamma} \rightarrow \gamma \) and note that \( D_r \gg D_\perp \) in ionospheric applications. The growth rate can be easily obtained in two limits: in the electromagnetic limit \( (\gamma \gg k^2 D_r) \),

\[
\gamma = \gamma_0 - \frac{k^2 v_A^2}{v_{in} \delta_1},
\]

while in the electrostatic limit \( (\gamma \ll k^2 D_r) \)
When the electron parallel conductivity is large, \( k_i \) is typically quite small so that these inequalities are easily satisfied. Finally, the time evolution of the equilibrium should have a negligible influence on the stability analysis providing

\[
|\gamma T_1|, |\gamma T_1| \ll 1
\]

where \( \gamma \) is the growth (damping) rate of the mode of interest.

IV. LINEARIZED EQUATIONS AND DISPERSION EQUATION

To determine the influence of finite temperature and parallel dynamics on the gradient drift instability we linearize (13)-(15) using the equilibrium discussed in the preceding section. We assume perturbed quantities to vary as \( \tilde{p} \sim \tilde{p}(z) \exp (\gamma t + ik y) \). After eliminating \( \tilde{n} \) algebraically from (13)-(15), we obtain two coupled differential equations for \( \tilde{\phi} \) and \( \tilde{A}_z \):

\[
(\gamma + k_i^2 D_r) \tilde{A}_z = -c_2 \frac{\partial \tilde{\phi}}{\partial z}
\]

(32)

\[
(\gamma - i\omega_a - \gamma_0) \tilde{\phi} = -\omega D_r (\gamma - ik \overline{V}_i + k_i^2 D_{yz}) \frac{1}{c} \frac{\partial \tilde{A}_z}{\partial z}
\]

(33)

where \( \tilde{\gamma} = \gamma + i(c/B)k_i \tilde{\phi}_0 \), \( \tilde{\phi} = \tilde{\phi} - (T_e/e) \ln \tilde{n} \), \( D_r = \nu e^2 / \omega pe^2 \), \( \overline{V}_i = (\nu_{in}/n_i) \delta_i \overline{V}_n \), \( \alpha = (\Omega / \nu_e)(n_i/\nu_{in}) \delta_i^{-1} \), \( \omega_a = -k_y(cT/eB)(n_c/n_c) \), \( \overline{V}_n = V_n + (c/B)(\nu_{in}/n_i) \tilde{\phi}_0 \), \( \gamma_0 = -\frac{n_c}{n_c} \overline{V}_n \) and

\[
\tilde{\phi}_0 = \left( \frac{B}{c} \frac{\nu_{in}}{\Omega} \frac{v_n}{e} \frac{T_n}{n_c} \frac{n_c c_c}{n_c d_z + n_b z_b} \right).
\]
\[
\frac{\partial \hat{n}}{\partial t} - \frac{\partial^2 \hat{n}}{\partial z^2} = - \frac{\partial^2 \Delta n}{\partial x^2} - \frac{1}{\eta_e} \frac{\partial^2 \Delta \phi}{\partial z^2} = 0,
\]

\[
\frac{1}{\eta_e} \frac{\partial \Delta \phi}{\partial z} = - \frac{\partial^2 \Delta n}{\partial x^2} - \frac{1}{\eta_e} \frac{\partial^2 \Delta \phi}{\partial z^2}.
\]

where \(\Delta \phi \ll \phi\). The equation for \(\Delta n\) has no steady state solution so \(\Delta n\) will generally evolve on a time scale of order \(\tau_1 \sim \tau_2\). The \(z\) dependent potential \(\Delta \phi\) drives an equilibrium current \(J_z = \eta_e^{-1} \partial \Delta \phi / \partial z\) which is required so that \(\nabla \cdot J = 0\). The time variation of the equilibrium and the equilibrium parallel current \(J_z\) will be neglected when we carry out the linear stability analysis.

Specifically, in the linear stability analysis we formally take the limit \(D_\perp \rightarrow 0\) and choose

\[
n_c(x) + n_b \quad |z| < z_c
\]

\[
n(x,z) = n_b \quad z_b > |z| > z_c
\]

(31)

as shown in Fig. 1b. The perpendicular diffusion coefficient is retained in the stability analysis. The step function model for the density is valid as long as the parallel wavelengths of the modes of interest are much longer than the actual parallel equilibrium density scale length, i.e.,

\[
k_\parallel^2 z_c^2 \ll 1.
\]

The neglect of the equilibrium parallel current and parallel diffusion can similarly be justified for

\[
k_\parallel J_z / ne \ll \omega
\]

\[
k_\parallel^2 D_\parallel \ll \omega.
\]
Equation (25) governs the time evolution of $n$ while (26) determines $\hat{\phi}$. For $z_b^2 < x_2^2 \omega \Omega / v ve^{-i \omega t}$, the second term in (26) is much larger than the remaining terms unless $\partial \hat{\phi} / \partial z = 0$. Thus, $\hat{\phi}$ is nearly constant along $B_0$.

An equation for $\phi$ can then be obtained by averaging (25) and (26) along $B_0$. The resulting equations are

$$\frac{\partial \bar{n}}{\partial t} = 0 \quad (27)$$

$$\frac{\partial}{\partial x} \bar{n} \frac{\partial \bar{\phi}}{\partial x} + \frac{T}{e} \frac{\partial^2 \bar{n}}{\partial x^2} = 0 \quad (28)$$

where the average $\bar{p}$ of a function $p(z)$ is defined as

$$\bar{p} = \int_{z_b}^{z_b} dz p(z) / 2z_b. \quad (29)$$

Thus, the integrated density $\bar{n}$ is a constant in time. From (28), we obtain an expression for the potential,

$$\phi = \frac{T}{e} \ln \left( \frac{n}{n_b} \right) - \frac{T}{e} \ln \left( \frac{n}{n_b} \right) + \frac{B \nu_{in}}{c \Omega} \int_{-\infty}^{x} dx \left( 1 - \frac{n_b}{n} \right). \quad (30)$$

The first and second terms in (30) compete in forcing the potential to balance the parallel electron pressure and ion perpendicular pressure, respectively. In the limit in which $n$ is constant along $B_0$, $\bar{n} = n$ and $\phi \sim - (T_i / e) \ln (n / n_b)$ so that $\phi$ balances the ion pressure. In opposite limit, where $z_b / z_c \rightarrow \infty$, $\bar{n} = n_b$ and $\phi \sim (T_e / e) \ln (n / n_b)$ so that $\phi$ balances the electron pressure.

Finally, subtracting the field line average of (25) and (26) from itself, we obtain an equation for $\Delta n = n - \bar{n}$ and $\Delta \hat{\phi} = \hat{\phi} - \bar{\phi}$,
Glassman (1984) who also noted the importance of finite temperature on the stability of the gradient drift mode.  

The extent to which the work described here may have relevance to the barium cloud "freezing" phenomenon deserves some discussion. Briefly, it is observed that barium ion clouds released at ionospheric altitudes will, through the nonlinear evolution of the gradient drift instability, break into smaller pieces. This process, known as bifurcation, continues until a certain minimum transverse dimension is reached, at which point further bifurcation ceases. This frozen scale size is observed to be approximately 400 m. Simple two dimensional models of barium cloud evolution fail to explain frozen scale sizes this large [McDonald et al, 1981]. One explanation which has been advanced to explain freezing is that of end-shorting [Francis and Perkins, 1975; Zalesak et al., 1984] which takes into account the distribution of plasma along magnetic field lines, finite plasma temperature, and parallel currents, as we have done here. However end-shorting depends on the "mapping" of transverse electric fields over relatively large distances along B; this becomes increasingly more difficult as k increases. Indeed, as shown here, and in Sperling et al. (1984), end-shorting fails to provide stabilization of the very high k modes (at least for the simple profiles of n_c, n_b and v_in considered). If nothing else, the present work shows the existence of a viable stabilization mechanism (i.e., finite temperature) in the gap missed by the end-shorting mechanism. It should also be pointed out, however, that we are not that far removed (135 m stabilization wavelength) from offering a mechanism that by itself may stabilize a 400 m diameter barium striation.
Given the uncertainties in applying a linear theory to explain a complex nonlinear phenomena, the stabilization of the gradient drift instability by parallel electron diffusion must be considered as a strong candidate to explain striation freezing.

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Fig. 1 Slab geometry and plasma model used in the analysis.

(a) Geometry and plasma configuration.

(b) Cloud and background density profile.
Fig. 2 Plot of $\omega_r/\omega_0$ (dashed curve) and $\gamma/\gamma_0$ (solid curve) vs. $k_y R_s$ where

$$\omega = \omega_r + i \gamma.$$ Parameters used are described in text.
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