DYNAMIC MODELING OF NYLON AND POLYESTER DOUBLE BRAID LINE

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FINAL REPORT
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Technical Director
U.S. Coast Guard Research and Development Center
Avery Point, Groton, Connecticut 06340
The dynamic stiffness (called the apparent spring constant) and hysteresis of nylon and polyester double braid lines 1-1/2 and 2 inches in diameter are determined in a series of laboratory tests. The experimentally determined apparent spring constant and hysteresis are treated analytically to calculate the viscoelastic parameters of the three-parameter model of a synthetic line. The dynamic behavior of nylon double braid line of 1/2 to 2 inches in diameter are represented by a single equation. The dynamic behavior of 1-1/2 inch and 2 inch diameter polyester double braid line is presented. The apparent spring constant and hysteresis stabilize (i.e., change by less than 10%) after approximately 10,000 cycles of loading. The hysteresis decreases by approximately 30-50% between approximately 100 cycles and 10,000 cycles of loading. The apparent spring constant increases by approximately 20% between approximately 100 and 10,000 cycles of loading. The load-elongation curves for the lines tested are not significantly affected by the loading conditions applied in these tests.
**METRIC CONVERSION FACTORS**

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*1 in = 2.54 (exactly). For other exact conversions and more detailed tables, see NBS Misc. Publ. 286, Units of Weights and Measures. Price $2.25. SD Catalog No. C13.10.286.
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1.0 BACKGROUND AND OBJECTIVE

Synthetic lines are being applied in engineering systems that are of advancing complexity. Some of these systems perform functions in severe environments where life and equipment is at risk. To improve the design and operation of these systems, they are numerically modeled so that the response of the system to the environment can be better understood. In modeling the system as a whole, a model of the synthetic line must also be developed that accurately describes its stiffness and damping properties.

The Coast Guard Research and Development Center was among the first to develop the coefficients for parametric models of the dynamic properties of synthetic lines and prove they change with loading conditions and exposure to the environment. This work shows that the dynamic stiffness of a line is several times greater than the quasi-static stiffness (the stiffness observed in a slow, unidirectional loading). In modeling synthetic lines and their systems, this difference must be understood. Depending on the type of system, errors in modeling the dynamic stiffness of the synthetic lines can result in large errors in estimating the response of the system to the conditions.

The three-parameter model of viscoelastic lines (see next section) has been used to model synthetic lines because it possesses the elastic and damping properties necessary to simulate the behavior of synthetic line under dynamic loading.

The use of the three-parameter model requires estimating the response coefficients of the two springs and dashpot that comprise the model. Previously, the Coast Guard Research and Development Center (Bitting, 1980), determined the coefficients for the viscoelastic model using a combination of experimental and analytical techniques. The coefficients were determined for selected lines up to 1-1/4 inches in diameter.

The objective of the work in this report is to determine the coefficients for nylon and polyester double braid line of 1-1/2 inch and 2 inch diameter and combine them with the previous work to create one model for all diameters tested of one material.

2.0 SUMMARY

The dynamic stiffness (called the apparent spring constant) and hysteresis of nylon and polyester double braid lines 1-1/2 and 2 inches in diameter are determined in a series of laboratory tests. The experimentally determined apparent spring constant and hysteresis are treated analytically to calculate the viscoelastic parameters of the three-parameter model of a synthetic line. The dynamic behavior of nylon double braid line of 1/2 to 2 inch diameter can be represented by a single equation.

The dynamic behavior of 1-1/2 inch and 2 inch diameter polyester double braid line is presented in individual equation form, rather than one combined equation for both diameters. This is done because the viscoelastic parameters of the polyester double braid line tested do not follow the same trend as does the nylon double braid line.
The apparent spring constant and hysteresis stabilize (i.e. change by less than 10%) after approximately 10,000 cycles of loading. The hysteresis decreases by approximately 30-50% between approximately 100 cycles and 10,000 cycles of loading. The apparent spring constant increases by approximately 20% between approximately 100 and 10,000 cycles of loading.

The concept that the apparent spring constant is two to four times greater than the quasi-static spring constant is confirmed by the data presented in this report.

The load-elongation curves for the lines tested are not significantly affected by the loading conditions applied to these tests. The tensile strength of nylon 1-1/2 inch and polyester 2 inch lines is not significantly affected by that dynamic loading whereas the strength of nylon 2 inch and polyester 1-1/2 inch diameter lines do exhibit a significant loss of strength.

3.0 THREE-PARAMETER MODEL OF A SYNTHETIC LINE

The dynamic behavior of nylon and polyester line can be represented by the Maxwell three-parameter model. The following brief explanation of this model will assist the reader in understanding the difference between dynamic and quasi-static elasticity.

Synthetic lines are a viscoelastic material. To the designer of systems that include synthetic line, two important characteristics of a viscoelastic material are: a) sensitivity to the rate of loading, and b) dissipation of energy during cyclic loading. A mechanical analog of a viscoelastic material, called the Maxwell three-parameter model, is shown in Figure 3-1. It consists of two springs and a dashpot. The spring with spring constant $K_0$, the quasi-static spring constant, is the stiffness of the line during a very slow, unidirectional loading. It represents the slope of the load-elongation curve and it is typical of the information usually furnished by synthetic line manufacturers.

The energy dissipation portion of the model is provided by a combination of spring, having stiffness $K_1$ called the dissipation spring constant, and the dashpot having a damping coefficient $N$.

The expression (Reid, 1968)

$$\tau = \frac{N}{K_1}$$

is the characteristic time constant.

The two springs and dashpot have a combined effect called the apparent spring constant ($K_{dp}$) which is the stiffness of the line during cyclic loading. Whereas $K_0$ is a quasi-static stiffness, $K_{dp}$ is the dynamic stiffness. The apparent spring constant is two to four times larger than the quasi-static spring constant; that is to say, a line is two to four times stiffer in dynamic loading conditions than in static and quasi-static loading
Figure 3-1  Three-Parameter Model of a Viscoelastic Material
conditions. To the designer of synthetic line systems, this is a very important concept regarding synthetic line physical behavior. The spring constant value assigned to a line in an analytical model can directly affect the system loads predicted by the model. If the spring constant is in error by two to four times, the predicted loads may well be in error by two to four times. In estimating the spring constant of a line, some system designers or modelers use the slope of the load-elongation curve which appears in manufacturers' catalogues. This is actually the quasi-static spring constant and should not be used in dynamic load conditions.

During one cycle of sinusoidal loading, the load and elongation of a viscoelastic material traced a closed path called the hysteresis loop. The apparent spring constant is approximated by the slope of the hysteresis loop (Figure 3-2) and is given by (Reid, 1968)

\[ K_{ap} = \left[ \frac{K_0^2 + (K_0 + K_1)^2(\omega \tau)^2}{1 + (\omega \tau)^2} \right]^{1/2} \]  

(1)

where:

- \( K_0 \) = quasi-static spring constant (lbs/in./in.)
- \( K_1 \) = dissipation spring constant (lbs./in./in.)
- \( \tau \) = characteristic time constant (seconds)
- \( \omega \) = frequency of excitation (radians/second)

Hysteresis (H) is the amount of energy dissipated by the line during one cycle of loading. It is defined by the equation (Reid, 1968).

\[ H = \int T \partial \epsilon \]

or

\[ H = \frac{\pi K_1 (\Delta T)^2 \omega \tau}{K_0 + (K_0 + K_1)^2(\omega \tau)^2} \]  

(2)

where:

- \( \Delta T \) = load amplitude (lbs)
- \( T \) = mean load (lbs)
- \( \epsilon \) = strain (in./in.)

Graphically, hysteresis, H, is the area inside the hysteresis loop shown in Figure 3-2.

Equations 1 and 2 have been used in the past to model synthetic lines in several computer models of systems that include synthetic lines; however, there had been no effort to reliably determine the value of \( K_1 \) and \( \tau \) for lines of different materials, constructions and diameters. Determining those two constants directly is not a trivial matter experimentally. However, it is possible to determine \( K_1 \) and \( \tau \) analytically using some experimental data. The procedure involves experimentally measuring \( K_{ap} \), \( H \), and \( K_0 \) for a given
Figure 3-2 Theoretical Hysteresis Loop
Figure 6 - 4  Hysteresis of 1 1/2- and 2-inch Polyester Double Braid Line
Figure 6-3 Characteristic Time Constant, $\tau$, for 1 1/2- and 2-inch Diameter Polyester Double Braid Line
Figure 6.2 Dissipation Spring Constant, $K_1$, and Quasi-Static Spring Constant, $K_0$, for 1-1/2- and 2-inch Diameter Polyester Double Braid Line
Figure 6-1 Apparent Spring Constant and Quasi-Static Spring Constant of Polyester Double Braid Line, 1-1/2- and 2-inch Diameter
Table 6-1  Parametric Equation coefficients for $K_1$, $K_0$ and for $\tau$
Polyester Double Braided Lines, 1-1/2- and 2-inch Diameter

$$K_1 = A + BT$$
$$K_0 = C + DT$$
$$\tau = E + FT$$

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range (lbs.) 7,035 - 14,079 10,995 - 21,991

$T$: mean line tension (lbs.)
$r^2$: Pierson Correlation Coefficient
range: range of mean line tension over which the equation is valid
This behavior of polyester line has not been explained; there is no other data on polyester double braid lines of similar diameters that could verify this trend. Because of this unexplained trend, the viscoelastic parameters of 1-1/2 inch and 2 inch diameter polyester line are not combined; the parameters are estimated from equations for the individual line diameters. The coefficients for the equations appear in Table 6-1. The correlation coefficients in Table 6-1 indicate that, in general, the equations do predict the viscoelastic parameters very well. The viscoelastic parameters are graphed in Figures 6-2 and 6-3.

Perhaps the best method of evaluating the accuracy of the equations is to compare the apparent spring constant and hysteresis values calculated from them and compare that to the experimentally measured data. The graph of the experimental and calculated values of apparent spring constant and hysteresis appear in Figures 6-1 and 6-4. It is observed that both data sets are essentially coincident.

The concept that the apparent spring constant is several times larger than the quasi-static spring constant is observed (Figure 6-1) to be correct for polyester double braid line as it was for nylon line in Section 5.5.

7.0 THE EFFECT OF THE NUMBER OF CYCLES OF LOADING ON THE DYNAMIC BEHAVIOR

This experiment was not designed to investigate in detail the effect of the number of cycles of loading on the apparent spring constant and hysteresis. This would require including another variable in the experimental design which would lengthen the experiment substantially. However, general conclusions can be drawn that contribute to the understanding of this effect.

The apparent spring constant and hysteresis both stabilized (i.e., change by less than 10%) by approximately 10,000 cycles. The hysteresis decreases by approximately 30-50% from very low number of cycles (i.e. approximately 100 cycles) to stabilization at 10,000 cycles. The apparent spring constant increases by approximately 20% at the 10,000 cycle level. The general trends are indicated in Appendix D.

8.0 STRENGTH CHANGE AND LOAD-ELONGATION CURVES

The tensile strength of nylon 1-1/2 inch and polyester 2 inch line is not statistically reduced as a result of the cyclic loading applied in this test procedure. Nylon 2 inch and polyester 1-1/2 inch diameter lines exhibit a reduction in strength as a result of cyclic loading. The results appear in Table 8-1.

The load-elongation curves of the lines tested are not significantly changed by cycle loading in the load region tested. The coefficients of the load-elongation curves appear in Table 8-2 and the curves appear in Figure 8-1, 8-2, 8-3, 8-4. In the figures, "baseline" refers to the tensile strength of new lines that have not been subjected to cyclic loading (refer to Appendix A-1.); "final" refers to lines that were cyclically loaded during the recording of the viscoelastic data (refer to Section A-2.0).
Figure 5-2  Hysteresis of Nylon and Polyester Double Braid Lines
Figure 5 - 1  Apparent Spring Constant and Quasi-Static Spring Constant of Nylon Double Braid Line - 1 1/2 and 2 inch Diameter
5.4 Comparison of Equation Results

Statistical treatment of the above three representations (Sections 5.1, 5.2, 5.3) of the viscoelastic behavior of nylon double braid line indicated that there are only minor statistical differences between the ability of these methods to predict the behavior of the line. The equations for individual diameters (Section 5.1) do produce the most accurate results. However, there is no difference of practical significance among any of the three methods.

One effective method of comparing the three equations is to compare the results calculated from them. It is, after all, the objective to obtain estimates of Kap and H that characterize the overall behavior of the line. Graphs of the apparent spring constant calculated by the three methods appear in Appendix C. It is observed that the data points all fall close to the regression line, and that a good estimate of the apparent spring constant is provided by these equations.

5.5 General Behavior of Nylon Double Braid Line

The apparent spring constant of all line diameters follows a linear relationship with increasing line tension (at a constant load amplitude and frequency.) This means that only one equation is required to characterize apparent spring constant. This trend is observed in Figure 5-1 which contains a plot of all the experimentally measured apparent spring constants.

The important concept that the apparent spring constant is several times larger than the quasi-static spring constant is observed in Figure 5-1.

Hysteresis increases with line diameter but decreases with increasing tension for a particular line diameter. This is observed in Figure 5-2.

6.0 VISCOELASTIC PARAMETERS FOR POLYESTER DOUBLE BRAID LINE

The viscoelastic behavior of polyester lines does not follow completely the same trend as does nylon line. As discussed in Section 5.0, the apparent spring constant of nylon line increases with mean tension without regard to diameter. This is observed in Figure 5-1. The apparent spring constant of polyester line also displays the general trend of increasing stiffness with increasing load (Figure 6-1). The hysteresis of nylon line displays a general trend that increases with line diameter. The hysteresis of polyester line, however, does not show the same trend (Figure 5-2). For the two line diameters tested (i.e. 1-1/2 and 2 inch), the hysteresis values at the designated mean load levels (i.e. 10%, 15%, 20% of rated break strength) are not significantly different. In Figure 5-2, it is observed that the hysteresis of 2 inch nylon line is substantially greater than that of 1-1/2 inch diameter nylon line. The hysteresis for both polyester lines, also shown in Figure 5-2, is substantially the same where the load region of the two lines overlap.
Table 5-1  Parametric Equation Coefficients for $K_1$ and $K_0$ for Nylon Double Braid Lines

Test condition: wet

<table>
<thead>
<tr>
<th>Diameter (inches)</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>Correlation coefficient</th>
<th>range (lbs)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1/2</td>
<td>-7,283</td>
<td>20.16</td>
<td>3,783</td>
<td>10.47</td>
<td>.998</td>
<td>.997</td>
</tr>
<tr>
<td>1</td>
<td>-46,600</td>
<td>28.48</td>
<td>-18,980</td>
<td>15.83</td>
<td>.998</td>
<td>.997</td>
</tr>
<tr>
<td>1 1/4</td>
<td>52,900</td>
<td>12.38</td>
<td>21,800</td>
<td>9.269</td>
<td>1.00</td>
<td>1.00</td>
</tr>
<tr>
<td>1 1/2</td>
<td>-34,067</td>
<td>27.32</td>
<td>58,167</td>
<td>7.820</td>
<td>.997</td>
<td>.995</td>
</tr>
<tr>
<td>2</td>
<td>-98,133</td>
<td>27.85</td>
<td>85,167</td>
<td>8.779</td>
<td>.993</td>
<td>.965</td>
</tr>
</tbody>
</table>

$K_1, K_0$: lbs./in./in.  
$K_1 = A + BT$  
$K_0 = C + DT$  
$T$: mean load (lbs.)  
$A, C$: lbs./in./in.  
$B, D$: 1/in./in.  

10
where $T$ is the mean line tension (lbs.)

The coefficients for the linear multi-variate regression equations are shown in Table 5-1. One advantage of using individual equations is usually greater accuracy; the correlation coefficients in Table 5-1 indicate that the equations fit the data very well.

5.2 All Diameters Together, as a Function of Tension and Diameter

This equation is of the same general form as that used in the previous work (Bitting, 1980) which has the form of Equation 3. Since frequency was found to have an insignificant effect on the line's dynamic properties, and the load amplitude is held constant according to the test plan, and since no significant curvature was indicated, Equation 3 can be simplified to include only first-order effects of tension. Diameter was added as a factor to conform to the current experimental design (Section 4.0). This equation is of the form:

\[
K_1 = -7396 + 25.79T - 20,665D \\
K_0 = -14,015 + 10.17T + 33,246D
\]

where: \(D\) is the diameter of nylon double braid line (in.)
\(T\) is the mean load (lbs.)

The above equations were derived from the original data set as were those in Section 5.1; however, rather than performing a regression analysis on data sets for individual diameters, the data for all diameters were used and diameter was included as a variable in the regression analysis.

5.3 All Diameters Together, as a Function of Only Tension

When \(K_0\) and \(K_1\) for all diameters of a line are plotted as a function of mean load, the parameters tend to form a straight line. This suggests that one equation could be used for all diameters of line without regard to diameters. This would greatly reduce the number of equations required to calculate the parameters for all of the line diameters. The correlation coefficients below indicate that, considering the large variation of diameters tested, the single-equation form fits the data almost as well as the individual equation.

Using these equations, the dynamic properties of all nylon double braid lines tested can be expressed as:

\[
K_1 = -19430 + 23.94T \quad \text{(correlation coefficient: .989)} \\
K_0 = 8,109 + 12.39T \quad \text{(correlation coefficient: .977)}
\]

This equation is valid for the conditions:

- mean load (T) = 640 lbs. - 26200 lbs.
- diameters = 1/2 in. - 2 in.
- period of excitation = 5 seconds per cycle
- line condition = wet
- load amplitude = +/- 5% of tensile strength
Discrete Variables

material/construction: nylon double braid
    polyester double braid
diameter: 1-1/2 in.
    2 in.

Fixed Test Parameters

load amplitude: +/- 5% BS
excitation wave form: sinusoidal
number of cycles (n): up to 20,000
number of samples: 4

5.0 VISCOELASTIC PARAMETERS FOR NYLON DOUBLE BRAID LINE

The viscoelastic parameter, $K_0$, $K_1$, and $\tau$ are expressed in three different equations, each reflecting an increased level of generality. All three methods will be presented here because the final use of this data is not known, and one method may be more convenient than another to the final user; by presenting all three, the user can select the method of calculating the parameters that is most compatible with the computer model of the overall engineering system being modeled.

For each of the methods presented below, the characteristic time constant, $\tau$, is calculated from the same equation because that parameter is relatively constant for all diameters of a particular material/construction combination.

The equation for $\tau$ is:

$$\tau = 6.0 + 2 \sin\left(\frac{\pi D^2}{2}\right)$$

where $D$ is the diameter of the line.

This expression for $\tau$ is not intended as a generalized expression; rather, it is a convenient expression describing $\tau$ for only the data discussed here. It cannot be generalized and used for diameters other than those detailed in this report. The derivation of this equation is discussed in Appendix B; it was developed as part of the effort to combine data sets.

The three methods of describing the viscoelastic parameters are discussed in the following sections.

5.1 Individual Diameters, as a Function of Tension

In this form, the parameters for each diameter of line are calculated from a separate equation. These equations are of the form:

$$K_1 = A + BT$$

and

$$K_0 = C + DT$$
Bitting, 1980) with the purpose of formulating the experimental design for this experiment.

The synthetic line parameters developed in the previous report (Bitting, 1980) are of the form shown in Equation 3. It is the intent of this work to determine the coefficients for these equations for nylon and polyester double braid lines of 1-1/2 inch and 2 inch diameter.

4.0 GENERAL TECHNICAL APPROACH

Nylon and polyester double braid lines, 1-1/2 and 2 inches in diameter, are cyclically loaded at specific combinations of mean load, load amplitude, and frequency, and the response (i.e., the hysteresis and apparent spring constant) measured. These experimental values of hysteresis and apparent spring constant are used to calculate values of dissipation spring constant $(K_1)$ and the characteristic time constant $(\tau)$ as described in Section 3.0. Parametric models are developed for each line constant (i.e., $K_0$, $K_1$, and $\tau$) as described in Sections 3.0 and 3.1. These parametric models are to be combined with the previous models (Bitting 1980) to form one generalized parametric model for nylon double braid line of 1/2, 3/4, 1, 1-1/4, 1-1/2, and 2 inch diameter. If the generalized model is found to contain excessive error, separate models for each line diameter will be developed. The viscoelastic parameters for the polyester double braid lines will be put in parametric form, also.

In a preliminary test, the apparent spring constant and hysteresis were found to vary by less than 20% when the frequency is varied between 5 seconds per cycle and 10 seconds per cycle. These results allowed holding the frequency constant at 5 seconds per cycle for all subsequent tests (as per the Experimental Design and Test Procedure, Section A-4.0).

It is also necessary to determine how the response of the line changes during cyclic loading. Therefore, apparent spring constant and hysteresis are measured at 1000, 5000, 10,000, and 20,000 cycles during the test. Apparent spring constant and hysteresis are plotted against the number of cycles of loading to investigate any trends.

All line samples used in the cyclic tests are tensile tested to determine the amount of strength loss due to cyclic loading.

The details of the experimental design and the test procedures are discussed in Appendix A. A summary of the general test variables is shown below.

<table>
<thead>
<tr>
<th>Independent Variables</th>
<th>Dependent Variables</th>
</tr>
</thead>
<tbody>
<tr>
<td>period: 5 sec/cycle</td>
<td>apparent spring constant</td>
</tr>
<tr>
<td>mean load: 10%-20% BS</td>
<td>hysteresis</td>
</tr>
</tbody>
</table>
frequency, mean load, and load amplitude and then solving Equations 1 and 2 simultaneously to determine $K_1$ and $\tau$. This yields a set of synthetic line viscoelastic property constants (i.e. $K_0$, $K_1$, $\tau$) for a set of loading conditions (i.e. mean load, load amplitude and frequency).

Theoretically, the apparent spring constant is only a function of $K_0$, $K_1$, $\omega$ and $\tau$ as shown in Equation 1. However, early work at the R&D Center (Bitting, 1980) shows it to be a function of mean load and load amplitude also. Hysteresis (Equation 2) is theoretically a function of only $K_1$, $K_0$, load amplitude, frequency and $\tau$. The same work also shows that hysteresis is a function of mean load also. These so-called "constants" (i.e. $K_1$, $K_0$, and $\tau$) are really less constant than originally thought. Rather than improving the theoretical equations to account for these new factors, parametric models for $K_1$ and $\tau$ can be developed directly from experimental data. The theoretical equations of apparent spring constant (Equation 1) and hysteresis (Equation 2) can still be used in simulations of a system with synthetic lines in it, but the constants in the equation must first be determined from the parametric equations developed from experimental data. The form of the parametric model is discussed in the following section.

3.1 Parametric Model of Synthetic Line Properties

The parametric equation discussed here is the core of the Box-Behnken experimental design technique. Briefly, the levels of the independent variables (i.e., mean load, load amplitude and frequency) are selected and the experiment conducted at those levels to measure the response (i.e. the hysteresis and apparent spring constant) and the constants are calculated as described above. This produces a set of line constants for a known loading condition. Then each constant is regressed into an equation relating it to independent variables (i.e. loading conditions).

The form of the equation is:

$$Y = B_1 + B_2 f^2 + B_3 T^2 + B_4 \Delta T^2 + B_5 f + B_6 T + B_7 f \Delta T + B_8 T + B_9 f (\Delta f) + B_{10} T (\Delta T)$$

(3)

where $Y$ (the response) can be $K_1$ or $\tau$, depending on which set of data is used in the regression.

Coefficient $B_1$ is the mean value (within the experimental range tested) of property, coefficients $B_5$-$B_7$ are the first order effects of the frequency, mean load, and load amplitude (called factors); coefficients $B_2$-$B_4$ are second order effects of the factors on the line property; coefficients $B_8$-$B_{10}$ are the interactions, or combined effects, of the factors. It must be emphasized that the model is only valid within the bounds of the factors tested.

This equation is very useful for two reasons: a) it can be used to calculate directly values of the line properties for use in Equations 1 and 2, and b) the relative magnitude of the coefficients (the B's) indicate which factors have the greatest effect on the line properties being modeled. This second usefulness will be used in Appendix A to evaluate previous data (in
Table 8 - 1  Tensile Strength of Nylon and Polyester Double Braid Line
1-1/2 and 2 inch Diameter

Condition: wet

<table>
<thead>
<tr>
<th>Diameter (inches)</th>
<th>(1) Baseline Tensile Strength</th>
<th>(2) Final Tensile Strength</th>
<th>Significant Loss</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Nylon</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1 1/2</td>
<td>61,524 (4,295)</td>
<td>54,183 (5,858)</td>
<td>No (3)</td>
</tr>
<tr>
<td>2</td>
<td>115,960 (6,120)</td>
<td>95,602 (4,843)</td>
<td>Yes</td>
</tr>
<tr>
<td>Polyester</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1 1/2</td>
<td>70,359 (2,805)</td>
<td>64,762 (1,805)</td>
<td>Yes</td>
</tr>
<tr>
<td>2</td>
<td>109,957</td>
<td>101,928 (3,075)</td>
<td>No (3)</td>
</tr>
</tbody>
</table>

(Standard Deviation)

(1) with no cyclic loading
(2) after cyclic loading
(3) no statistical strength loss at the 95% confidence level
Table 8 - 2 Load-Elongation Curve Equation Coefficients for Nylon and Polyester Double Braid Line, 1-1/2- and 2-inch Diameter

\[ \epsilon = A + BT + CT^2 + DT^3 \]

<table>
<thead>
<tr>
<th>Material</th>
<th>Diameter (in.)</th>
<th>A ((10^{-2}))</th>
<th>B ((10^6))</th>
<th>C ((10^{-10}))</th>
<th>D ((10^{16}))</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nylon</td>
<td>1 1/2 baseline</td>
<td>1.15</td>
<td>16.7</td>
<td>-3.94</td>
<td>36.2</td>
</tr>
<tr>
<td></td>
<td>final</td>
<td>2.12</td>
<td>16.1</td>
<td>-5.01</td>
<td>58.2</td>
</tr>
<tr>
<td></td>
<td>2 baseline</td>
<td>1.53</td>
<td>7.13</td>
<td>-.877</td>
<td>4.46</td>
</tr>
<tr>
<td></td>
<td>final</td>
<td>2.06</td>
<td>7.94</td>
<td>-1.20</td>
<td>6.94</td>
</tr>
<tr>
<td>Polyester</td>
<td>1 1/2 baseline</td>
<td>-1.40</td>
<td>6.06</td>
<td>-.397</td>
<td>99.2</td>
</tr>
<tr>
<td></td>
<td>final</td>
<td>-.277</td>
<td>4.95</td>
<td>-.326</td>
<td>.468</td>
</tr>
<tr>
<td></td>
<td>2 baseline</td>
<td>-.205</td>
<td>4.92</td>
<td>-.259</td>
<td>.0896</td>
</tr>
<tr>
<td></td>
<td>final</td>
<td>-7.02</td>
<td>2.99</td>
<td>-.0466</td>
<td>-.549</td>
</tr>
</tbody>
</table>

Where \( \epsilon \) = Strain \( \left( \frac{\text{in}}{\text{in}} \right) \)

\( T \) = Tension (lbs.)
Figure 8-1 Load-Elongation Curve of Nylon Double Braid Line, 1 1/2-inch Diameter
Figure 8-2 Load-Elongation Curve of Nylon Double Braid Line, 2-inch Diameter

S.E. = Standard Error
Figure 8-3 Load-Elongation Curve of Polyester Double Braid Line, 1 1/2-inch Diameter
Figure 8-4  Load-Elongation Curve of Polyester Double Braid Line, 2-inch Diameter
9.0 CONCLUSIONS

- The dynamic behavior of nylon double braid line of 1/2 to 2 inches in diameter can be represented by a single equation. This reduces the amount of data that must be entered into a computer performing a dynamic simulation of a system containing nylon double braid line. Instead of entering arrays of regression coefficients for a number of line diameters, just one simple linear equation is used.

- The dynamic behavior of 1-1/2 inch and 2 inch diameter polyester double braid lines are presented in individual equation form, rather than one combined equation for both diameters. This is done because the viscoelastic parameters of the polyester double braid lines tested do not follow the same trend as does the nylon double braid line.

- The apparent spring constant and hysteresis stabilize (i.e., change by less than 10%) after approximately 10,000 cycles of loading. The hysteresis decreases by approximately 30-50% between approximately 100 and 10,000 cycles of loading. The apparent spring constant increases by approximately 20% between approximately 100 and 10,000 cycles of loading. These property changes would occur over only one day of deployment in the ocean.

- The concept that the apparent spring constant is 2-4 times greater than the quasi-static spring constant is confirmed by the data presented in this report. To the designer of synthetic line systems, this is a very important concept regarding synthetic line physical behavior. The spring constant value assigned to a line in an analytical model can directly affect the system loads predicted by the model. If the spring constant is in error by 2-4 times, the predicted response may well be in error by 2-4 times.

- The load-elongation curves for the lines tested are not significantly affected by the loading conditions applied in these tests. The tensile strength of nylon 1-1/2 inch and polyester 2 inch lines is not significantly affected by the dynamic loading, whereas the strength of nylon 2 inch and polyester 1-1/2 inch diameter lines do exhibit a significant loss of strength.
REFERENCES


APPENDIX A

EXPERIMENTAL DESIGN AND TEST PROCEDURE

A-1.0 Baseline Tensile Tests

A-1.1 Purpose:

a. Experimentally determine the break strength (BS) of new, wet line samples.

b. Record the load-elongation curve.

A-1.2 Test Sample Preparation:

Each sample will have an eye spliced in each end as specified in Table A-1 and Figure A-1. The entire sample will be soaked in fresh water at room temperature for three days.

A-1.3 Test Procedure

The Cordage Institute Standard Test Method for Fiber Ropes (Reference 3) is the controlling document for all test procedures and sample dimensions. This procedure is followed rather than Federal Standard 191 because the Principal Investigator believes that the Cordage Institute method more precisely controls the test conditions and will lead to better quality, more reproducible data. All tests will be performed wet, but not submerged.

The Baseline Tensile Test setup is shown in Figure A-2. The sample is attached between a deadhead and a large hydraulic cylinder which applies a constant-displacement load to the sample. The tests are conducted in the Cyclic/Tensile Test Machine at the Research and Development Center.

A-1.4 Data Collection and Presentation

The elongation of the sample is measured by a dual-sheave extensiometer (Figure A-2) which is attached to a gauge length near mid-span of the line sample. The extensiometer produces a DC voltage that is directly proportional to the elongation in the gauge length. This technique can measure the elongation to failure without end effects. The load is measured by a load cell attached to one of the clevises. The load and elongation are recorded simultaneously on a computer which collects data at the rate of 30 data pairs per second. A backup x-y recorder is also used.

At the end of a tensile test, the computer prints the break strength, regresses all load-elongation data into a third-order polynomial equation, prints the coefficients for the equation and plots the load-elongation curve. The load-elongation curve has the form

\[
\epsilon = A + BT + CT^2 + DT^3
\]

where: \( T \) is the load (lbs)

\( \epsilon \) is strain (in./in.)
<table>
<thead>
<tr>
<th>Line Diameter (D)</th>
<th>Pin Diameter (d)</th>
<th>Eye Length (l)</th>
<th>Sample Length (L)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1-1/2 in.</td>
<td>3 in.</td>
<td>12 in.</td>
<td>8 ft.</td>
</tr>
<tr>
<td>1 in.</td>
<td>4 in.</td>
<td>16 in.</td>
<td>8 ft.</td>
</tr>
</tbody>
</table>

All dimensions conform with Cordage Institute Standard Test Method, Reference 3. Refer to Figure 3 for sketch of sample dimensions.

Eye length (l) measured with legs in contact.

Sample length (L) is the distance between the ends of the splice tapers.
d = CLEVIS PIN DIAMETER
D = LINE DIAMETER
e = EYE LENGTH
L = SAMPLE LENGTH

Figure A-1
SAMPLE CONFIGURATION
The Baseline Tensile Test data is used as follows:

a. Break strength (BS) is measured and presented in tabular form as a fundamental property of the line.

b. The break strength is used in the experimental design (Section A-2.5) to convert nondimensional load levels in %BS to loads in pounds for the cyclic tests.

c. The coefficients to the load-elongation curve equation are calculated and presented in tabular form as a fundamental property of the line.

d. The quasi-static spring constant, $K_0$, is calculated using the coefficients to the load-elongation curve from c. above. The quasi-static spring constant, $K_0$, is used as a known in the analysis technique to calculate the dissipation spring constant and characteristic time constant as described in Section A-2.4.

\[ K_0 = \frac{1}{(8+2CT+3T^2)} \]

where $T$ is the load (lbs).

A-2.0 Dynamic Tests

A-2.1 Purpose

a. Record hysteresis and apparent spring constant data at various levels of mean load and frequency for use in determining $K_1$ and $\tau$.

b. Record the change in hysteresis and apparent spring constant over time to determine when/if these dynamic properties stabilize.

A-2.2 General Experimental Design Considerations

An analysis of the trends observed in the coefficients to the parametric models developed in Reference 1, Tables 3 and 11, can provide guidance in the formulation of the experimental design for these investigations.

At the outset of this discussion, the reader should understand that the coefficients in Tables 3 and 11 of Reference 1 are actually Factor Effects in the "coded" Box-Behnken regression equation. In order to calculate actual values of apparent spring constant and hysteresis in engineering units, the simple conversion process must be followed in Reference 1, Appendix C. The following discussion does not directly apply to any equations appearing in this Appendix or the main body of this report.

The data can be used to answer three questions. First, does the hysteresis and apparent spring constant exhibit significant curvature (i.e. non-linearity)? If curvature is not indicated, then a shorter experimental plan will suffice resulting in reduced laboratory time. Second, does either of the independent variables (i.e. mean load or frequency) exhibit a weak
effect on the hysteresis and apparent spring constant? If so, then one could be held constant, thus eliminating one variable and reducing laboratory time. Third, is diameter such a strong factor that it should be treated separately (i.e. as a discrete variable rather than a continuous variable)?

The question of curvature in hysteresis and apparent spring constant can be evaluated by summing the coefficients for the first-order terms (B5-B7) and summing the coefficients of the second-order terms (B2-B4) and comparing them to the mean value (B1). If this is done for nylon double braid 1-1/4 inch diameter line (Reference 1, Table 3), it will be observed that the first-order effects of apparent spring constant are approximately 20% of the mean value of apparent spring constant (B1). This means that Kap will change 20% above the mean value and 20% below the mean value if only the first-order terms are used in the parametric model (Equation 3). If only the second-order terms are used in the model, Kap varies by 10% of the mean value. It appears that the first-order effects: a) are more important than the second-order effects and, b) exert a significant effect on apparent spring constant. If the same analysis is applied to the hysteresis coefficients (Reference 1, Table 11), it is observed that the first-order effects are 37% of the mean value and the second order effects are 57%. Therefore, both seem to be quite important in calculating the hysteresis. The conclusion: since the second-order terms are important in either the hysteresis or apparent spring constant models, a three-level experimental design must be used so that curvature can be modeled with a quadratic equation.

The sensitivity of hysteresis and apparent spring constant to mean load and frequency can be assessed using the coefficients in Reference 1, Table 11. The effect of frequency can be determined by summing the frequency effect coefficients, B2 and B5, and comparing that to the mean value, B1. The effect of mean load is determined by summing coefficients B3 and B6. In doing so, it is observed that frequency has a small effect on Kap (approximately 6% of the mean) and a larger effect on H (approximately 20% of mean). Results indicate that mean load has a 27% and 31% effect on Kap and H, respectively. The conclusion: both mean load and frequency are significant effects and must be included initially as independent variables in any future tests.

Diameter is a very significant factor when evaluating the properties of line. This is apparent by that fact that the interval between the mean value (B1) of the various diameter lines (in Table 3 and 11, Reference 1) is much greater than any of the other factors in the Table 3. This is also expected because most properties increase with approximately the square of the diameter. Because diameter is such a strong factor, it will be evaluated as a separate, or discrete variable and combined in the parametric models if possible. It is possible to include diameter as another independent variable and run an experimental design with mean load, frequency and diameter. A Box-Behnken three-factor design could be used with only 15 trials rather than the 27 required by a full factorial design. This is accomplished because the Box-Behnken design uses a technique called hidden replication. However, if the results from this set of experiments is combined with those of the past experiments (Reference 1) and it is found that this generalized model contains too much error, it will be difficult to produce individual parametric models for the 1-1/2 inch and 2 inch lines separately because insufficient data was collected during the laboratory phase.
Figure C-4 Hysteresis vs Mean Load
Nylon Double Braid
Sizes: 1/2- thru 2-inch diameter
Figure C - 3 Kap vs Mean Load
Nylon Double Braid
Sizes: 1/2- thru 2-inch diameter
POLYNOMIAL ORDER = 1
STD ERROR = +.2911
CORR COEFF = +1.000

Figure C - 2  Kap vs Mean Load
Nylon Double Braid
Sizes: 1/2- thru 2-inch diameter
Figure C - 1  Kap vs Mean Load
Nylon Double Braid
Sizes: 1/2- thru 2-inch diameter
APPENDIX C

Results of Various Methods of Calculating $K_a$ and $H$ for Nylon Double Braid Line

The following figures are plots of the results of the various methods of calculating $K_a$ and $H$. They are:

Figure C-1: $K_a$ - Individual Diameters, as in Section 5.1.

Figure C-2: $K_a$ - All diameters together, as a function of diameter and tension, as in Section 5.2.

Figure C-3: $K_a$ - All diameters together, as a function of tension only, as in Section 5.3.

Figure C-4: $H$ - for the methods described in Section 5.1, 5.2, 5.3 and the experimentally data.
Figure B - 2 Hysteresis Adjustment
difference between the \( \text{Kap} \) from both sets of data (i.e. \( \text{Kap} \) is reasonably insensitive to the number of cycles of loading). Therefore, no steps will be taken to adjust the \( \text{Kap} \) of the previous data to the new data set. The smooth fit of both sets of data in Figure B-1 also suggests that estimating \( \text{Kap} \) within 10% can be accomplished.

3. Hysteresis has been observed, in the experience of the author, to decrease by as much as one-half of the first 100 cycles of loading. This can be used as a guideline in estimating the adjusted hysteresis from the actual data.

Figure B-2 contains a plot of the previous data (cycled to only 100 cycles) and the data from the current test (cycled to as many as 10,000 cycles). The data points for the two data sets are connected by a dotted line which is used only to assist the reader in observing that there is a very significant discontinuity between the two sets of data which is attributed to the number of cycles. The previous data is then adjusted downward (dashed line) to form a smooth curve with the new data; this makes all the data appear to have been collected under the same conditions (i.e., 10,000 cycles of loading.) This is consistent with paragraph 2 above. Reducing the level of the previous data is also justified by the observation in paragraph 3 above.

Having estimated from Figure B-2 what the adjusted value of \( \text{H} \) should be, \( \tau \) is then calculated using \( \text{Kap} \), \( \text{H} \) (adjusted) and \( \text{Ko} \) as explained in Section 3.0 of the main body of this report. If that calculated \( \tau \) value is consistent with the experimental value predicted by equation B-1, the \( \text{H} \) value is assumed to be reasonable; this is consistent with paragraph 1 above. If not, another iteration is executed by assuming another value of \( \text{H} \). This process results in a reasonably smooth curve of adjusted \( \text{H} \) values that conform to the expected \( \tau \) values. Comparison of the \( \tau \) values, in other words, is a check on the reasonableness of the adjusted \( \text{H} \) value.
Figure B-1  $K_{ap}$ for Various Line Diameters
Table B-1  Experimental and Calculated Values of Nylon Double Braided Line

Condition: new, wet

<table>
<thead>
<tr>
<th>Diameter (in.)</th>
<th>(sec)</th>
<th>(sec) (3)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1/2</td>
<td>7.07 (1)</td>
<td>6.76</td>
</tr>
<tr>
<td>3/4</td>
<td>7.0 (1)</td>
<td>7.54</td>
</tr>
<tr>
<td>1</td>
<td>7.83 (1)</td>
<td>8.00</td>
</tr>
<tr>
<td>1 1/4</td>
<td>8.2 (1)</td>
<td>7.26</td>
</tr>
<tr>
<td>1 1/2</td>
<td>5.41 (2)</td>
<td>5.23</td>
</tr>
<tr>
<td>2</td>
<td>6.08 (2)</td>
<td>6.00</td>
</tr>
</tbody>
</table>

(1) Average of values calculated for mean load of 10% rated break strength, 15% and 20%; from Bitting, 1980, Table 17.

(2) Experimentally determined from these experiments

(3) Calculated from equation (B-1)
APPENDIX B

Technique for Combining Data Sets

The objective of this report requires combining, if possible, the viscoelastic parameters from the previous work for nylon double braid lines of 1/2, 3/4, 1, 1-1/4 inch diameter (Bitting, 1980) with the parameters determined in this work for 1-1/2 inch and 2 inch diameter line. Its object, then, is to develop one equation for all six diameters of line. This would not be difficult except that the earlier lines were cycled for 100 cycles at each load level before the hysteresis loop was recorded and the new data for this project were cycled to as many as 10,000 cycles. Since hysteresis is affected by the number of cycles, it is not possible to combine the two sets of data without first adjusting the hysteresis to a value that would be expected if the line had been cycled for 10,000 cycles rather than just 100 cycles. In combining the data sets, the sponsor of the work requested that $K_a$ be estimated within 10% and the hysteresis be estimated within 25%.

The technique used to adjust the previous data to the new data draws on experience gained from previous experiments. The prime factors are:

1. The characteristic time constant, $\tau$, may be expected to be reasonable constant (with frequency and load amplitude held constant) for a particular line material/construction combination regardless of diameter. To make use of this concept, it is first necessary to derive an empirical equation that represents $\tau$ for all diameters of interest. This same equation is also used as one of the viscoelastic parameters, Section 5.0.

The average $\tau$ value (average of $\tau$ calculated to mean loads of 10%, 15%, and 20% of rated break strength (RBS)) for each diameter appears in Table B-1; the values for 1/2, 3/4, 1, 1-1/4 inch diameter lines are calculated from previous work (Bitting 1980, Table 17), and the values for 1-1/2 and 2 inch diameters are experimentally determined from the new data developed in the work discussed in this report. The equation:

$$\tau = 6.0 + 2 \sin\left(\frac{\pi D^2}{2}\right)$$

where $D =$ line diameter (inch)

is used to present the $\tau$ values of all the diameter lines in Table B-1. The $\tau$ values calculated from equation B-1 appear in Table (B-1). It must be emphasized that equation B-1 is used only as a convenient expression for the data presented in Table B-1, and it does not imply that other line diameters follow this equation, or that $\tau$ values in general follow a sine function.

2. $H$ may be expected to increase with increasing diameter in a smooth, continuous fashion. This is based on the observation that $K_a$ and $K_0$ increase with increasing diameter and there has been no previous data to suggest that hysteresis does not exhibit a similar trend. Most properties of synthetic lines are a function of the square of the diameter. The plot in Figure B-1 demonstrates that $K_a$ is: a) a smooth increasing function, b) a function of the square of the diameter and, c) there is no distinguishable
TABLE A-2

DYNAMIC TESTS
EXPERIMENT LEVELS

<table>
<thead>
<tr>
<th>Frequency (cycles/sec) (Period: seconds/cycle)</th>
<th>.10</th>
<th>.133 (7.5)</th>
<th>.20 (5)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean load (T) (% BS)</td>
<td>10%</td>
<td>15%</td>
<td>20%</td>
</tr>
</tbody>
</table>

Load amplitude: ± 5%
**Fixed Test Parameters**

Load amplitude: +/- 5% BS  
Excitation wave form: sinusoidal  
Number of cycles (n): up to 20,000  
Number of samples: 4

A three-level factorial design will be run because significant curvature may occur (as discussed in Section A-2.2). The factor levels are shown in Table A-2; this scheme will be run for each material/construction combination and diameter.

**A-2.6 Data Reduction**

a. The Kap and H data collected in Section A-2.3 and the $K_0$ calculated in Section A-1.4 are used in the simultaneous solution of Equations 1 and 2 to calculate values of $K_1$ and characteristic time constant.

b. Values of Kap and H recorded at each n cycles will be graphed to illustrate the trend of these properties to stabilize at a particular number of cycles (Section A-3.0).

**A-3.0 Test for Stabilization of the Dependent Variables**

In all the trials, the dependent variables (i.e. Kap, H) will be measured at $n = 1000, 5000, 10000, 20000$ cycles. If a dependent variable is found to change by less than 10%, the properties will be considered stabilized and the trial terminated. The Kap and H at that number of cycles will be recorded as the results for that trial. This procedure will reduce laboratory time by shortening the tests; there is no point in running trials to 20,000 cycles if the properties are no longer changing.

**A-4.0 Test for Sensitivity to Frequency of Loading**

For the first sample of nylon double braid a complete series of trials will be run according to Table A-2. Then the data will be analyzed to determine if there is less than a 20% change in line properties as the frequency is varied between 5 seconds/cycle and 10 seconds/cycle. If the change is less than 20%, the dependent variables will be considered insensitive to frequency. All subsequent trials will be run only at the 5 second/cycle rate.

**A-5.0 Tensile Test of All Cyclic Samples**

At the conclusion of all cyclic testing, all line samples used in the cyclic tests will be tensile tested to determine the residual strength. The testing and data recording and reduction methods are the same as in Section A-1.3 and A-1.4.
A-2.3 Test Setup

The test is conducted in the Cyclic/Tensile Test Machine at the R&D Center. The general setup is shown in Figure A-3. The cyclic load is applied by a cyclic actuator which has a load capacity of 35,000 pounds and stroke of 36 inches. The actuator is controlled by a close-loop hydraulic servo-system. The frequency, mean load, load amplitude and load wave form are programmed in the control system. The load is sensed by a load cell mounted on the actuator. Because the system operates in load control, any slack in the line sample is automatically taken out by the actuator. The controller also counts the number of cycles executed by the actuator. The large hydraulic cylinder at the opposite end of the test machine applies a pretension to the sample as required.

The elongation in the line sample is measured using a dual-sheave extensiometer attached to a gauge length in mid-span as described in Section A-1.4. The load in the sample is measured by the same load cell that controls the hydraulic actuator.

The load and elongation data, in the form of hysteresis loops, are recorded digitally on a computer. A backup x-y recorder is also used to record the hysteresis loop. The computer displays the hysteresis loop on the screen real time and calculates the area inside each loop (i.e. the hysteresis).

A-2.4 Sample Preparation

Refer to Figure A-1.
Sample Length (L): 8 feet
Termination Method: Eye splice
Eye splice configuration as per Table A-1

All samples will be soaked in fresh water for 3 days (minimum).

A-2.5 Experimental Design

<table>
<thead>
<tr>
<th>Independent Variables</th>
<th>Dependent Variables</th>
</tr>
</thead>
<tbody>
<tr>
<td>Period: 5 sec/cycle - 10 sec/cycle</td>
<td>Apparent spring constant</td>
</tr>
<tr>
<td>Mean Load: 10%-20% BS</td>
<td>Hysteresis</td>
</tr>
</tbody>
</table>

Discrete Variables

<table>
<thead>
<tr>
<th>Material/construction:</th>
<th>Diameter:</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nylon double braid</td>
<td>1-1/2 inch</td>
</tr>
<tr>
<td>Polyester double braid</td>
<td>2 inch</td>
</tr>
</tbody>
</table>
APPENDIX D

The Change of $K_{ap}$ and $H$ with the Number of Cycles and Loading

$\%RBS = \text{mean load as percent of rated break strength.}$
Apparent Spring Constant vs Number of Cycles
Polyester Double Braid
Size: 1-1/2 Inch Diameter
Apparent Spring Constant vs Number of Cycles
Polyester Double Braid
Size: 2 Inch Diameter
Apparent Spring Constant vs Number of Cycles
Nylon Double Braid
Size: 1-1/2 Inch Diameter
Apparent Spring Constant vs Number of Cycles
Nylon Double Braid
Size: 2-Inch Diameter
Hysteresis vs Number of Cycles
Polyester Double Braid
Size: 1-1/2 inch Diameter
Hysteresis vs Number of Cycles
Polyester Double Braid
Size: 2 Inch Diameter
Hysteresis vs Number of Cycles
Nylon Double Braid
Size: 1-1/2 Inch Diameter
Hysteresis vs Number of Cycles
Nylon Double Braid
Size: 2 Inch Diameter