INTERNAL FLOW STUDIES OF A CLASS OF BALLISTIC LAUNCHERS

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ABSTRACT

A simple time-dependent model of the internal ballistics of a two-stage light gas gun is briefly described. The gasdynamic flow field between the driving piston and the driven sabot is singled out for special study. This flow regime is idealized as a quasi-one-dimensional flow of an inviscid, compressible, real gas with arbitrarily moving longitudinal boundaries. A method of numerical solution of the governing partial differential equations is suggested. In addition, a rezoning technique for describing the time-dependent computational grid is presented.
ACKNOWLEDGEMENT

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INTRODUCTION

Ballistic testing in an instrumented free-flight range is an important part of the broad field of experimental aerodynamics. Launcher technology, in turn, is a vital part of ballistic testing. The flight speed, initial acceleration history and exit flow environment are all dependent on launcher design parameters. It is important in ballistic testing, therefore, to have the capability of predicting launcher performance.

The two-stage light gas gun is one example of a large class of compressed gas launchers. Such a gun is to be used at the Air Force Armament Laboratory as a primary launcher for a variety of free-flight model configurations. A satisfactory performance prediction program for this launcher is required. One of the objectives of the research pursued during this summer research program is to develop such a procedure. A second objective is to investigate the complex gasdynamic processes which occur within the launcher during a firing cycle.

A literature survey of DOD, NTIS and NASA data bases revealed only sparse information on important aspects of launcher performance such as piston extrusion behavior, diaphragm rupture dynamics, launcher experimental data and internal flow field descriptions. Gun analysis programs
for other facilities have been developed (see References 1 through 4) which yield adequate predictions of internal pressure and muzzle velocity. However, these codes are either outdated or unavailable, or both. Also, they apparently do not provide performance without considerable computation expense, nor do they provide detailed information on the internal flow field.

This fundamental data deficiency suggests that the formulation of a very simple performance prediction model would be beneficial and that detailed studies of the internal flows would add to the understanding of launcher performance.

In the present study, a simple, time-dependent volumetric model of the gas gun operation is incorporated into a computer program in a modular form which will permit investigation of various aspects of the launcher cycle in an independent manner. In addition, more detailed models of the basic flow kinematics and thermodynamics are derived which have promise in revealing the gas property distributions during gun operation.

II. OBJECTIVES

The principal objective of the present work is to develop a performance prediction model for a class of ballistic launchers which includes the two-stage light gas gun. A typical configuration of such a launcher is shown in Figure 1. It is intended not only to develop a model
which can be useful immediately in assessing launcher performance as a function of launcher and projectile geometric characteristics and propellant charge weight, but also to investigate means of providing a more detailed description of the gasdynamics within the gun.

The author has assisted his graduate student, Mr. Raymond Patin, in the formulation of a baseline launcher performance model which can be used in the initial predictions. Parametric studies and comparisons with experimental data indicate that this model is reasonable. Details of the basic model are presented in Reference 5. However, essential model features are outlined in this report to provide background for the discussion of flow analysis methods to be discussed here. In the present report details are given of the analytical procedures necessary to determine the quasi-one-dimensional distributions of thermodynamic and kinematic properties of a flow field confined by two boundaries moving at different speeds.

III. THE BASELINE LAUNCHER PERFORMANCE MODEL

In Figure 1 are shown the physical details of a typical two-stage light gas gun. The gun operation sequence begins with the ignition of a single-perforation propellant charge in the combustion chamber. The pressure rise created by gas generation during combustion forces the piston to move toward the diaphragm, compressing the
helium gas in the process. At some point during this compression process, the scored metal diaphragm ruptures, exposing the base of the sabot to high-pressure gas. The sabot is then propelled at high speed through the launch tube.

In Reference 5 a simple model of this launching process is described in detail. Only an outline of this performance model will be given here. The time-dependent equations which are integrated to provide a simulation of the launcher operation are listed in functional form in Table 1. The pressure variation in the combustion chamber is described by a differential equation derived by O. K. Heiney in Reference 6. The kinematics of the piston and sabot are determined from the conventional translational momentum equations. The gas properties within the pump tube and launch tube chambers are determined by isentropic pressure/volume relationships.

This basic prediction program appears to describe the trends of launcher operation correctly. However, to date, insufficient comparisons of the model predictions with experimental data have been made to declare it a valid and versatile model. One of the weaknesses of this baseline prediction method is that it cannot account for the details of the gasdynamics within the pump tube of the launcher. In the following sections of this report methods for
detailed analysis of this flow regime will be discussed.

IV. GASDYNAMICS ANALYSIS METHODS

The flow field in the pump tube section of a two-stage light gas gun can be idealized as a quasi-one dimensional flow in a tube of variable area with longitudinal boundaries which are moving at different rates. Of particular interest in this class of flows are the characteristics in any regions of variable area and in the vicinity of the moving boundaries. In the case of the light gas gun this latter aspect is further complicated by the fact that one of the boundaries (the piston) may intrude into a region of decreasing area (extrude), resulting in a surface velocity which is not the same as the velocity of the center of mass of the piston. In any flow involving high pressures and temperatures the effects of departures of gas behavior from the ideal gas law model must also be evaluated.

In this section of the report an analytical model for the flow of a real gas in a variable area tube with moving longitudinal boundaries is presented. No attempt is made to solve the equations at this time, but a method of solution is presented and discussed.

The quasi-one-dimensional equations of flow for an inviscid, compressible fluid can be written in conservative form as follows:

CONSERVATION OF MASS

25-8
\[
\frac{\partial \rho}{\partial t} + \frac{\partial \rho m}{\partial x} + \frac{m}{A} \frac{dA}{dx} = 0 \quad (1)
\]

CONSERVATION OF MOMENTUM

\[
\frac{\partial \rho m}{\partial t} + \frac{\partial \rho m^2}{\partial x} + \frac{m^2}{\rho A} \frac{dA}{dx} = 0 \quad (2)
\]

CONSERVATION OF ENERGY

\[
\frac{\partial \rho e}{\partial t} + \frac{\partial \rho (e+p)}{\partial x} + \frac{m(e+p)}{\rho A} \frac{dA}{dx} = 0 \quad (3)
\]

where \( \rho \) is the mass density (mass per unit volume), \( m \) is the translational momentum per unit volume, \( e \) is the total energy per unit volume, \( p \) is the thermodynamic pressure, \( x \) is the axial coordinate, \( t \) is the time and \( A \) is the cross-sectional area of the tube. The total energy, \( e \), is given by

\[
e = \rho (i + \frac{m^2}{2 \rho^2}) \quad (4)
\]

where \( i \) is the internal energy per unit mass of the gas. The gas in the confined chamber is assumed to be helium. An appropriate real gas model for helium at high pressure and temperature, given by Harrison in Reference 7, is proposed for use here. This model is a fourth order virial formulation given as follows:

\[
p = \rho RT \left[ 1 + B(T) \rho + C(T) \rho^2 + D(T) \rho^3 \right] \quad (5)
\]

where \( R \) is the engineering gas constant and \( B(T), C(T) \) and \( D(T) \) are empirical functions of the temperature.

Equations (1) through (5) govern the quasi-one-dimensional, unsteady, inviscid flow of a real gas.
(helium). The boundary conditions for the flow of interest here are as follows:

At $X = X_p$, the boundary moves with velocity $X_p$
At $X = L + X_s$, the boundary moves with velocity $X_s$

where $L$ is the initial separation of piston and sabot.

The solution of the partial differential equations and boundary conditions which govern this flow situation must be effected by numerical methods. In the following section an appropriate method of numerical integration for the equations is given. Also, a procedure for dealing with computational grid dynamics by rezoning is presented.

IV. NUMERICAL METHODS

In 1969 MacCormack published a second-order accurate numerical technique for the solution of a class of partial differential equations (see Reference 8). The method, used successfully by the author in a multidimensional gasdynamic analysis (Reference 9), is applicable to the set of hyperbolic partial differential equations which govern the present flow study. In general terms, the two-level technique can be written:

Governing Equation

$$\frac{\partial U}{\partial t} + \frac{\partial F}{\partial x} + G = 0$$

(6)
where $U$ is the conserved function vector and $F$ and $G$ are the flux vectors.

Numerical Solution

Predictor

$$U_j^{n+1} = U_j^n - \Delta t \left( \frac{F_j^n - F_{j-1}^n}{\Delta x} + G_j^n \right)$$

Corrector

$$U_j^{n+1} = \frac{1}{2} \left[ U_j^n + U_j^{n+1} - \Delta t \left( \frac{F_j^{n+1} - F_{j-1}^{n+1}}{\Delta x} + G_j^{n+1} \right) \right]$$

where $n$ is the time index and $j$ is the space index.

This explicit method is very simple to set up for the particular set of equations to be solved in the present case. These equations are:

CONSERVATION OF MASS

Predictor

$$\xi_j^{n+1} = \xi_j^n - \Delta t \left( \frac{m_j^n - m_{j-1}^n}{\Delta x} + \frac{m_j^n}{A_j} \frac{dA}{dx_j} \right)$$

Corrector

$$\xi_j^{n+1} = \frac{1}{2} \left[ \xi_j^n + \xi_j^{n+1} - \Delta t \left( \frac{m_j^{n+1} - m_{j-1}^{n+1}}{\Delta x} + \frac{m_j^{n+1}}{A_j} \frac{dA}{dx_j} \right) \right]$$

CONSERVATION OF MOMENTUM

Predictor

$$m_j^{n+1} = m_j^n - \Delta t \left[ \frac{(p_t^2 + \tau^2)}{\Delta x} - \left( \frac{p_t}{\tau} \right)_j^{n+1} \right] + \left( \frac{m_j^2}{\tau A_j} \right)_j^{n+1} \frac{dA}{dx_j}$$

Corrector

$$m_j^{n+1} = \frac{1}{2} \left\{ m_j^n + m_j^{n+1} - \Delta t \left[ \frac{(p_t^2 + \tau^2)_j^{n+1} - (p_t^2 + \tau^2)_j^n}{\Delta x} + \left( \frac{m_j^2}{\tau A_j} \right)_j^{n+1} \frac{dA}{dx_j} \right] \right\}$$

CONSERVATION OF ENERGY

Predictor

$$\varepsilon_j^{n+1} = \varepsilon_j^n - \Delta t \left[ \frac{(\varepsilon^2 + \tau^2)}{\Delta x} - \left( \frac{\varepsilon}{\tau} \right)_j^{n+1} \right] + \left[ \frac{m(\varepsilon + \tau)}{\tau A_j} \right]_j^{n+1} \frac{dA}{dx_j}$$

Corrector

$$\varepsilon_j^{n+1} = \frac{1}{2} \left( \varepsilon_j^n + \varepsilon_j^{n+1} - \Delta t \left[ \frac{(\varepsilon^2 + \tau^2)_j^{n+1} - (\varepsilon^2 + \tau^2)_j^n}{\Delta x} + \left[ \frac{m(\varepsilon + \tau)}{\tau A_j} \right]_j^{n+1} \frac{dA}{dx_j} \right] \right)$$

The particular solution of interest here involves a flow field between boundaries which are moving at
arbitrary, different speeds. This presents some computational difficulties. The computational grid which must be established to effect the numerical solution of Equations (8) through (10) must, necessarily, be in motion. In order to produce a result which is accurate the grid must deform and move simultaneously. This effect can be achieved by the formulation of an "adaptive" grid. One method of constructing such a grid for a one-dimensional problem is by the rezoning technique which will be used in the present solution. "Rezoning" implies that at the end of each computation step, the computational grid is adjusted to conform to the new boundary conditions. Such adjustments may involve tedious calculations at each time step to prevent the transmission of numerically-induced flow perturbations, but they ensure a solution with appropriate resolution. Consider the sketches of Figure 2. In part (a) is shown the grid structure which was used to advance the solution from time step "n" to time step "n+1". In that time interval the computational boundaries were altered by the boundary motion. In part (b) is shown the adjusted structure (the rezoned structure) for time step, "n+1". The equations which describe the grid distribution and the corresponding values of the state variables can be given as follows (refer to Figure 2):

**COMPUTATION DOMAIN**

\[ L^{n+1} = L^n - (X_p - X_s)^{n+1} \]  \hspace{1cm} (11)

25-12
CELL SIZE
\[ \Delta X_j^{n+1} = \Delta X_j^n - \left( \frac{X_p^{n+1} - X_s^{n+1}}{N} \right) \]  \hspace{1cm} (12)

CELL BOUNDARY LOCATION
\[ X_j^{n+1} = X_p^{n+1} + \sum_{k=1}^{j-1} \Delta X_k^{n+1} \]  \hspace{1cm} (13)

STATE VARIABLE ALLOCATION (Typical Internal Cell)
\[ \phi_j^{n+1} = \left( a_j^{n+1} \phi_{j-1}^{n+1} + b_j^{n+1} \phi_j^{n+1} \right) / \Delta X_j^n \]  \hspace{1cm} (14)

where
\[ a_j^{n+1} = X_p^{n+1} + \sum_{k=1}^{j-1} (\Delta X_k^{n+1} - \Delta X_k) \]
\[ b_j^{n+1} = \sum_{k=1}^{j} \Delta X_k^n - \sum_{k=1}^{j-1} \Delta X_k^{n+1} - X_p^{n+1} \]

These equations are a typical set. A different set is required for each different pattern of boundary motion. These equations have been established based on a uniform grid. Grid clustering, which may be necessary in some cases, will be discussed in the next section of the report.

It is important to establish the sequence of solution of the above set of equations. The computational steps to be employed at each grid point in advancing the solution from one time step to the next are:

1. Determine the computational domain and establish the grid size.
2. Determine the computational time step by invoking the Courant condition.
3. Advance the state variables to the next time by
application of the MacCormack algorithm.

4. Solve the dynamic equations of the boundaries for the given time step. (For the case of the light gas gun this involves solving the kinematic equations which are listed in Table 1.)

5. Perform the rezoning operations.

6. Continue with the solution.

V. ADVANCED CONSIDERATIONS

The foregoing computational methods have been functional, but not elegant. There are other methods which will result in a more efficient code and, consequently, give better and quicker results. The MacCormack integration algorithm, like all second order systems, has a serious deficiency in regions of rapid change of the state variables (such as near shock waves) ... it becomes divergent. The use of artificial diffusive terms in the equations prevents the divergence, but it results in an oscillatory solution and tends to smear the effective discontinuities over several grid points. The flux-corrected transport (FCT) algorithm (see Reference 10) circumvents this difficulty by a clever computational scheme. The application of this method to the present problem would provide benefits in dealing with shock waves and flow perturbations induced by area change.

The grid structure of the present problem could be streamline by constructing a truly adaptive grid. This would involve transforming the governing equations with a time-dependent grid transformation equation which
adequately describes the grid evolution to be expected for
the present flow field. An algorithm to cluster grid
points at locations of large change could be included here.
These grid calculations would require the solution of an
additional partial differential equation, assuring the
continuity of the transformation. These concepts are
discussed in References 11 and 12.

VI. RECOMMENDATIONS

The internal flow fields of the light gas gun are of
interest from both operational and physical viewpoints.
It is recommended that the baseline performance
prediction model discussed in Reference 5 be developed
to its full potential by comprehensive comparisons with
experimental data for many test conditions. Further,
detailed studies of the flow fields within the various
launcher chambers, in the manner suggested in this report,
should be continued.

VI. REFERENCES

1. Murphy, J. R. B., L. R. Badhwar and G. A. Lavoie,
"Interior Ballistics Calculation Systems for Light
Gas Guns and Conventional Guns," AGARD C. P. No.
10, The Fluid Dynamic Aspects of Ballistics,
September 1966.

Two-Stage Light Gas Gun," Sandia Corporation
Report SAND-75-0323, Albuquerque, N. M., July,
1975.


(1) Method of Reference 2.

(2) Volumetric analysis in baseline model. Finite difference analysis in gasdynamic model.

(3) Characteristics solution.

FIGURE 1. TYPICAL ARRANGEMENT OF TWO-STAGE LIGHT GAS GUN
<table>
<thead>
<tr>
<th>COMBUSTION PRESSURE RATE</th>
<th>[ \frac{dP_c}{dt} = f_1(P_c, N_b, \dot{x}_p) ]</th>
</tr>
</thead>
<tbody>
<tr>
<td>PROPELLANT WEIGHT RATE</td>
<td>[ \frac{dN_b}{dt} = f_2(P_c) ]</td>
</tr>
<tr>
<td>PISTON VELOCITY RATE</td>
<td>[ \frac{d\dot{x}_p}{dt} = f_3(P_c, P_h) ]</td>
</tr>
<tr>
<td>PISTON DISPLACEMENT RATE</td>
<td>[ \frac{dx_p}{dt} = f_4(\dot{x}_p) ]</td>
</tr>
<tr>
<td>SABOT VELOCITY RATE</td>
<td>[ \frac{d\dot{x}_s}{dt} = f_5(P_h, P_b) ]</td>
</tr>
<tr>
<td>SABOT DISPLACEMENT RATE</td>
<td>[ \frac{dx_s}{dt} = f_6(\dot{x}_s) ]</td>
</tr>
<tr>
<td>PRESSURE-VOLUME RELATIONS</td>
<td>[ P_h = f_7(X_p, Y') ]</td>
</tr>
</tbody>
</table>

- \( P_c \) = Combustion Chamber Pressure
- \( N_b \) = Propellant Weight Burned
- \( \dot{x}_p \) = Piston Velocity
- \( X_p \) = Piston Displacement
- \( \dot{x}_s \) = Sabot Velocity
- \( X_s \) = Sabot Displacement
- \( P_h \) = Pump Tube Pressure
- \( P_b \) = Launch Tube Pressure
- \( Y' \) = Gas Specific Heat Ratio

**TABLE 1. BASELINE PERFORMANCE PREDICTION EQUATIONS**
(a) Structure at time, $t^n$, for an $N$-cell grid.

(b) Structure at time, $t^{n+1}$, for an $N$-cell grid.

FIGURE 2. REZONING STRUCTURE FOR ONE-DIMENSIONAL GRID