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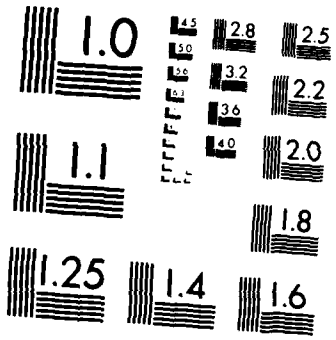

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A VARIABLE-COMPLEXITY NORM MAXIMIZATION PROBLEM

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ABSTRACT

The problem of finding a point with *largest norm* in a bounded polyhedral set is shown to have a considerable range of complexity depending on the norm employed. For a p-norm with integer $p \geq 1$, the problem is shown to be NP-complete. For the ∞-norm, the problem can be solved in polynomial time. The problem of finding an upper bound to the largest norm for any $p \in [1,\infty]$ can be solved in polynomial time by solving a single linear program.

AMS (MOS) Subject Classifications: 03D15, 90C05, 90C30.

Key Words: Optimization, maximum norm, complexity theory, NP-complete.

Work Unit Number 5 - Optimization and Large Scale Systems

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SIGNIFICANCE AND EXPLANATION

The solution set to many important constrained optimization problems is a set that is bounded by planes. When such a set is bounded it is useful to find the size of a largest element in that set. In this work we show that this problem may be extremely easy or difficult depending on the measure of size (norm) employed. For one such measure the problem is relatively easy while for all other measures it is intractable. However the problem of merely finding an upper bound for the size of the largest element turns out to be a surprisingly simple problem that can be solved by a single linear program.

The responsibility for the wording and views expressed in this descriptive summary lies with MRC, and not with the authors of this report.

A VARIABLE-COMPLEXITY NORM MAXIMIZATION PROBLEM

O. L. Mangasarian and T.-H. Shiau

1. Introduction

The problem of obtaining bounds for polyhedral sets has received considerable attention in mathematical programming [14, 15, 16, 12, 8, 9]. Part of the significance of this problem stems from the fact that the solution set to a linear program [4, 10] and to a monotone linear complementarity problem [2] is such a polyhedral set. Bounding the solution set to such problems when possible is then of practical interest. In this work we shall consider the polyhedral set X in R^n defined by

$$X := \{x \mid x \in R^n,\ Ax \geq b\} \tag{1.1}$$

where A is a given $m \times n$ real matrix and b is a given $m \times 1$ real vector. We assume throughout this work that X is bounded. It is easy to show that a necessary and sufficient condition for X to be bounded is that

$$Y = \{y \mid y \in R^n,\ Ay \geq 0,\ y \neq 0\} = \phi \quad . \tag{1.2}$$

The problem we wish to consider here is

$$\max_{x \in X} \|x\|_p \tag{1.3}$$

where $\|\cdot\|_p$ denotes the p-norm on R^n, $1 \leq p < \infty$, defined by $\|x\|_p = \left(\sum_{i=1}^{n} |x_i|^p\right)^{1/p}$ and $\|x\|_\infty = \max_{1 \leq i \leq n} |x_i|$.

We will show that while (1.3) can be solved in polynomial time for $p = \infty$, it is NP-complete [6, 11] for integer $p \geq 1$. Since it is widely believed that <u>no</u> NP-complete problem can be solved in polynomial time (the famous conjecture $P \neq NP$ in computational complexity theory), the difference in the difficulty between $p = \infty$ and all other integer $p \geq 1$ is enormous. (The standard complexity theory terms used here are defined in Section 4.) In fact we can summarize the complexity situation for our problem (1.3) as shown in Table 1.

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Problem	Complexity	Method of Solution
1. Find an upper bound to $\max_{x \in X} \|x\|_p$ for any $p \in [1,\infty]$	P (Deterministic polynomial time)	Single linear program
2. $\max_{x \in X} \|x\|_\infty$	P	2n linear programs
3. $\max_{x \in X} \|x\|_1$	NP-complete (Nondeterministic polynomial time)	2^n linear programs
4. $\max_{x \in X} \|x\|_p$ Integer $p \geq 2$	NP-complete	Vertex enumeration

Table 1. Complexity of $\max_{x \in X} \|x\|_p$ and its method of solution.

We note in passing that the <u>minimization</u> problem $\min_{x \in X} \|x\|_p$ is by contrast a much simpler convex programming problem for $p \in [1,\infty]$. In fact for $p = 1$ and ∞ it can be solved by standard linear programming techniques [4, 10] or by a polynomial time algorithm e.g. [7]. For $p = 2$ the problem is a convex quadratic program which can be solved by standard techniques e.g. [2] or by a polynomial time algorithm [3].

In the following sections of this paper we will show how each of the problem of Table 1 is solved and its complexity. Section 2 deals with finding an upper bound to (1.3) for $p \in [1,\infty]$. Section 3 deals with problem (1.3) for $p = 1$ and ∞ while Section 4 deals with the cases of integer $p \geq 1$.

2. Bounding $\max_{x \in X} \|x\|_p$

It is somewhat surprising that for any $p \in [1,\infty]$, an upper bound to the solution of the nonconvex problem $\max_{x \in X} \|x\|_p$ can be obtained by solving a single linear program (Theorem 2.1 below). This is especially so since we show (Section 4) that the problems

$\max_{x \in X} \|x\|_p$ for integer $p \geq 1$ are intractable NP-complete problems. When X is contained in the nonnegative orthant $R^n_+ := \{x \mid x \in R^n, x \geq 0\}$ it is evident that a solution to the 1-norm problem $\max_{x \in X} \|x\|_1$ is easily obtained by the single linear progrm

$$\max_{x \in X \cap R^n_+} ex \tag{2.1}$$

where e is a vector of ones. However when $X \not\subset R^n_+$, as may be the case here, solution of $\max_{x \in X} \|x\|_1$ will take 2^n linear programs, as shown in Section 3. In fact we will show in Section 4 that the problem $\max_{x \in X} \|x\|_1$ is NP-complete. However, merely obtaining an upper bound to $\max_{x \in X} \|x\|_p$ for any $p \in [1,\infty]$ will take at most a single linear program as shown by the following result.

2.1 <u>Theorem</u>. Let X be nonempty and bounded, let

$$B := (A^TA)^{-1}A^T \quad , \quad d := Bb \tag{2.2}$$

and let $B_{\cdot j}$ denote the jth column of B. Then for any $p \in [1,\infty]$ and any $x \in X$

$$\|x\|_p \leq \max_{1 \leq j \leq m} \{\|d\|_p, \|\gamma B_{\cdot j} + d\|_p\} \tag{2.3}$$

where γ is the maximum value of the following solvable linear program

$$\gamma := \max_{x,y}\{ey \mid x \in R^n, y \in R^m, Ax - y = b, y \geq 0\} \quad . \tag{2.4}$$

<u>Proof</u>. Note first that the boundedness condition (1.2) implies the linear independence of the columns of A and hence the nonsingularity of A^TA. In addition the nonemptiness and boundedness of X implies the solvability of the linear program (2.3). Hence

$$\begin{aligned}
\max_{x \in X} \|x\|_p &= \max_{x,y}\{\|x\|_p \mid x \in R^n, y \in R^m, Ax - y = b, y \geq 0\} \\
&= \max_{x,y}\{\|x\|_p \mid x = By + d, (AB-I)(y+b) = 0, y \geq 0, ey \leq \gamma\} \\
&\leq \max_{x,y}\{\|x\|_p \mid x = By + d, y \geq 0, ey \leq \gamma\} \\
&= \max_{y}\{\|By + d\|_p \mid y \geq 0, ey \leq \gamma\} \\
&= \max_{1 \leq j \leq m} \{\|d\|_p, \|\gamma B_{\cdot j} + d\|_p\}
\end{aligned}$$

where the last equality follows from the fact that the maximum of a convex function on a bounded polyhedral set is attained at a vertex [13, Corollary 32.3.4].

□

Note that if a lower bound to $\max_{x \in X} \|x\|_p$ is also desired, then we have the following.

2.2 <u>Corollary</u>. Under the assumptions of Theorem 2.1 we have that

$$\|B\bar{y} + d\|_p \leq \max_{x \in X} \|x\|_p$$

where $\bar{y}$ is a solution of the linear program (2.4).

Since by Khachian's result [7] a linear program is solvable in polynomial time in the size of the problem, and since the algebraic operations prescribed in (2.3) can all be performed in polynomial time, the following holds.

2.3 <u>Corollary</u>. The bound (2.3) can be computed in time which is polynomial in the size of A and b.

We note that the bound (2.3) of Theorem 2.1 may be sharp as evidenced by the following example.

2.4. <u>Example</u>

$$A = \begin{pmatrix} -2 & 1 \\ 5 & 1 \\ 1 & -4 \end{pmatrix} \qquad b = \begin{pmatrix} -10 \\ -10 \\ -2 \end{pmatrix}$$

For this example it is easy to verify that

$$\max_{x \in X} \|x\|_p = 10 \quad \text{for} \quad p = 1,\ 2 \ \text{ and } \infty \quad , \ \gamma = 42$$

$$B = \begin{pmatrix} -.0649 & .1688 & .0260 \\ .0519 & .0649 & -.2208 \end{pmatrix} \quad , \quad d = \begin{pmatrix} -1.0909 \\ -.7273 \end{pmatrix}$$

Computing the bound (2.3) of Theorem 2.1 gives for $p = 1,\ 2$ and ∞

$$\max_{1 \leq j \leq 3} \{\|d\|_p,\ \|\gamma B_{\cdot j} + d\|_p\} = 10 \quad .$$

3. $\max_{x \in X} \|x\|_p$ for $p = \infty$ and 1

It is rather obvious that the problem $\max_{x \in X} \|x\|_\infty$ can be solved by maximizing the absolute value of each component of x separately subject to x being X. This leads to the following.

3.1 Proposition

The problem $\max_{x \in X} \|x\|_\infty$ can be solved by solving the $2n$ linear programs

$$\max_{1 \leq i \leq n} \max\{\pm x_i \mid x \in R^n,\ Ax \geq b\} \quad . \tag{3.1}$$

Since each linear program can be solved in polynomial time [7] we have the following.

3.2 Corollary

The problem $\max_{x \in X} \|x\|_\infty$ can be solved in time which is polynomial in the size of A and b.

Since the problem $\max_{x \in X} \|x\|_1$ is equivalent to $\max_{x \in X} \sum_{i=1}^{n} |x_i|$, its solution can be obtained by solving 2^n linear programs as follows.

3.3 Proposition

The problem $\max_{x \in X} \|x\|_1$ can be solved by solving the 2^n linear programs

$$\max_{v \in V} \max_{x} \{vx \mid x \in R^n,\ Ax \geq b\} \tag{3.2}$$

where V is the set of 2^n vertices of the cube in R^n defined by

$$\{v \mid v \in R^n,\ -e \leq v \leq e\} \quad . \tag{3.3}$$

While $2n$ linear programs can be solved in a reasonable amount of time for intermediate-sized problems, solving 2^n linear programs is intractable even for n as small as 15. It is even worse for general $p \in (1,\infty)$ if we try to enumerate the vertices of X for finding the maximal p-norm, for the number of vertices can be much larger than 2^n. One may try to find other algorithms that are computationally effective. Unfortunately, as shown in the next section, problem (1.3) with $p \neq \infty$ is no easier than the partition problem (see (4.1) below) which is <u>inherently intractable</u>.

4. The intractability of the norm maximization problem for $p \neq \infty$

We begin this section with some basic concepts of complexity theory [6, 11]. Problem A reduces (in polynomial time) to problem B, denoted by $A \propto B$, iff the following holds: If there is a polynomial time algorithm for B, then one can construct a polynomial time algorithm for A using the algorithm for B as a subroutine. Problems A and B are polynomially equivalent iff $A \propto B$ and $B \propto A$. An NP-complete problems is one which is polynomially equivalent to any one of the standard intractable problems such as the satisfiability, partition, or travelling salesman problems [6, 11]. These problems are considered intractable because any algorithm which solves any one of them requires, in the worst case, an amount of time which is exponential in the problem size. An NP-hard problem is any problem such that some NP-complete problem reduces to it in polynomial time. Thus an NP-hard problem is at least as difficult as an NP-complete problem. We will now shown that our norm maximization problem (1.3) is NP-hard for $p \neq \infty$ by reducing the following NP-complete partition problem to it:

(4.1) Given integers $c_1, c_2, \ldots, c_n$, is then a set $S \subset \{1,2,\ldots,n\}$ such that $\sum_{j \in S} c_j = \sum_{j \notin S} c_j$?

4.1 Theorem. The norm maximization problem (1.3) is NP-hard for $p \in [1,\infty)$.

Proof. We will show this by reducing (4.1) to (1.3). Let $p \in [1,\infty)$. We first reduce (4.1) to the following problem:

(4.2) Given integers $c_1, c_2, \ldots, c_n$, is there an $x \in R^n$ such that:

$$\sum_{i=1}^{n} c_i x_i = 0,\ -1 \leq x_i \leq 1,\ 1 \leq i \leq n,\ \|x\|_p^p \geq n \ ?$$

It is easy to see that (4.1) has a solution S iff (4.2) has a solution x with $|x_i| = 1$ for $1 \leq i \leq n$ and $x_i = 1$ for $i \in S$ and $x_i = -1$ for $i \notin S$. Now it is easy to see that (4.2) can be reduced to an instance of problem (1.3) by defining

$$A := \begin{bmatrix} I \\ -I \\ c^T \\ -c^T \end{bmatrix}, \quad b := \begin{bmatrix} -e \\ -e \\ 0 \\ 0 \end{bmatrix}$$

and answering the question:

(4.3) $$\text{Is } \max\{\|x\|_p \mid x \in R^n,\ Ax \geqq b\} \geqq n^{1/p} \text{ ?}$$

Hence if we can solve (1.3) in polynomial time we can solve each of (4.3), (4.2) and (4.1) in polynomial time. Hence (4.1) $\propto$ (1.3) and (1.3) is NP-hard.

□

We go on to show now that our norm maximization problem (1.3) is in fact NP-complete for integer $p \neq \infty$. In order to do this we introduce additional concepts from complexity theory. A <u>nondeterministic</u> algorithm is an algorithm which at each step has a finite number of moves from which to choose (instead of only one for deterministic algorithms) and it solves a problem in a finite sequence of choices leading to a correct answer. <u>NP</u> is the class of problems solvable by a nondeterministic algorithm in polynomial time, including (4.1) and all other NP-complete problems. In fact NP-complete problems are the class of most difficult problems in NP in the sense that each problem in NP reduces in polynomial time to each NP-complete problem. By Cook's theorem [1, 6, 11], all we need to show for (1.3) to be NP-complete is that it is NP-hard (which we already have done in Theorem 4.1) and that it is in the class NP, which we proceed to do now. In order to do that we introduce the following <u>decision</u> problem related to our optimization problem (1.3):

(4.4) Given A, b with integer entries satisfying (1.2), and nonzero integers r, s, p, is there a vector x in R^n such that

$$Ax \geqq b, \quad \|x\|_p^p \geqq \frac{r}{s} \text{ ?}$$

Note that in the proof of Theorem 4.1 we have already established that the decision problem (4.4) in NP-hard, because we reduced the partition problem (4.1) to (4.2) which is an instance of (4.4). We will now first show that (4.4) is in NP and hence it is NP-complete. Then we will show that an optimization problem (1.3) is polynomially equivalent to the NP-complete decision problem (4.4). Note that condition (1.2) which is imposed on problem

.4) which is a necessary and sufficient condition for the boundedness of X, plays an sential role in Proposition (4.2) below which establishes that (4.4) is in NP.

2 <u>Proposition</u>. Problem (4.4) is in NP for integer $p \geq 1$.

<u>oof</u>. It follows by the convexity of the norm and the boundedness of X by (1.2) [13], at $\|x\|_p^p \geq \frac{r}{s}$ for some $x \in X$ iff $\|v\|_p^p \geq \frac{r}{s}$ for some vertex v of X. Moreover, v a vertex iff there is a $J \subset \{1,2,\ldots,m\}$, $|J| = n$ such that v is the unique solution $A_i x = b_i$, $i \in J$, and $A_j x \geq b_j$ for $j \notin J$. Consequently we can prescribe the llowing nondeterministic algorithm for solving (4.4).

3 <u>Algorithm</u>

(i) <u>choose</u> J, a subset of $\{1,2,\ldots,m\}$ with cardinality n.

(ii) Solve $A_i x = b_i$, $i \in J$ for one x, or conclude that the system is inconsistent.

(iii) <u>if</u> solution x found <u>and</u> $A_j x \geq b_j$ for $j \notin J$ <u>and</u> $\|x\|_p^p \geq \frac{r}{s}$ then <u>print</u> x; <u>success</u>; <u>else</u> <u>failure</u>; <u>endif</u>.

ep (ii) can be performed in polynomial time (e.g. by Gaussian elimination). Since we ve assumed that p is an integer, $\|x\|_p^p$ can be evaluated in $O(\log_2 p)$ multiplications d $O(\log_2 n)$ additions. Hence Algorithm 4.3 is a polynomial time algorithm and (4.4) is NP.

□

Now we show that the NP-complete decision problem (4.4) and our optimization problem .3) are polynomially equivalent, thus establishing the NP-completeness of (1.3). First is obvious that if one can solve the optimization problem (1.3), then one can answer the cision problem (4.4). The reverse is usually done by a binary search technique showing at the optimization problem can be solved by a polynomial number of decision problems. is is all rather obvious for discrete combinatorial problems, but not for our continuous oblem (1.3). To do this here, we shall use arguments similar to those of Khachian [7]. fine

$$L := \sum_{i,j=1}^{m,n} \log_2(|A_{ij}| + 1) + \sum_i \log_2(|b_i| + 1) + \log_2(nm+1) + \log_2(p+1) \quad .$$

L is the total length of binary digits representing the input A, b, n, m, p of problem (1.3).

4.4 <u>Theorem</u>. For any integer $p \geq 1$, problem (1.3) is in NP and hence it is NP-complete.

<u>Proof</u>. Since an optimal solution of (1.3) is at a vertex of X [13], such a vertex can be written by Cramer's rule as $\left(\frac{D_1}{D}, \frac{D_2}{D}, \ldots, \frac{D_n}{D}\right)^T$, where D and D_i are determinants of submatrices of $[A \quad b]$. Hence

(i) For any vertex $v = \left(\frac{D_1}{D}, \ldots, \frac{D_n}{D}\right)^T$, $|D| < 2^L$, $|D_i| < 2^L$, $\|v\|_p^p < 2^{pL}$. (See [5] for details.)

(ii) For any two distinct vertices $\|v\|_p \neq \|w\|_p$, $v = \left(\frac{D_1}{D}, \ldots, \frac{D_n}{D}\right)^T$, $w = \left(\frac{B_1}{B}, \ldots, \frac{B_n}{B}\right)^T$ it follows that

$$\left|\|v\|_p^p - \|w\|_p^p\right| = \left|\frac{|D_1|^p + \ldots + |D_n|^p}{|D|^p} - \frac{|B_1|^p + \ldots + |B_n|^p}{|B|^p}\right| \geq \frac{1}{|D|^p|B|^p} > 2^{-2pL}$$

Hence we can reduce (1.3) to (4.4) by binary search on the interval $[0, 2^{pL}]$ until the range is less than 2^{-2pL}. Since each iteration reduces range by half, $3pL$ iterations will do that if we have:

(i) $\ell \leftarrow 0$, $u \leftarrow 2^{pL}$

(ii) <u>for</u> $i = 1$ <u>to</u> $3pL$ <u>do</u>

(iii) solve the decision problem (4.4) for input A, b, $\frac{r}{s} = \frac{1}{2}(\ell+u)$

(iv) <u>if</u> answer is yes <u>then</u> $\ell \leftarrow \frac{r}{s}$ <u>else</u> $u \leftarrow \frac{r}{s}$ <u>endif</u>

(v) <u>end for</u>

If (iii) can be done in polynomial time, then (i) to (v) can be done in polynomial time. After (v), we known that there exists an $x \in X$ such that $\ell = u - 2^{-2pL}$, $\|x\|_p^p \geq \ell$, whereas there is no $x \in X$ such that $\|x\|_p^p \geq u$. Hnece if we now use Algorithm 4.3 with input $\frac{r}{s} = \ell$, A and b, the x printed in step (iii) of Algorithm 4.3 is an exact vertex

solution of (1.3) obtained in polynomial time. Hence (1.3) is in NP, and since by Theorem 4.1, (1.3) is NP-hard it follows that (1.3) is NP-complete.

□

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The problem of finding a point with largest norm in a bounded polyhedral set is shown to have a considerable range of complexity depending on the norm employed. For a p-norm with integer $p \geq 1$, the problem is shown to be NP-complete. For the ∞-norm, the problem can be solved in polynomial time. The problem of finding an upper bound to the largest norm for any $p \in [1,\infty]$ can be solved in polynomial time by solving a single linear program.

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