IMPLEMENTING WAVEGUIDE BANDWIDTH SIX-PORT REFLECTOMETER
CIRCUITS AT MILLI. (U) ROYAL SIGNALS AND RADAR
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IMPLEMENTING WAVEGUIDE BANDWIDTH SIX-PORT REFLECTOMETER CIRCUITS AT MILLIMETRIC WAVELENGTHS

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Title: IMPLEMENTING WAVEGUIDE BANDWIDTH SIX-PORT REFLECTOMETER CIRCUITS AT MILLIMETRIC WAVELENGTHS

Author: L D Hill

Date: May 1984

SUMMARY

This memorandum concerns the implementation of two WG22 six-port reflectometer circuits for the measurement of complex voltage reflection coefficient, using commercially available directional couplers. The means for establishing the phase conditions necessary for each circuit to be used over the 26.5 to 40 GHz band without further adjustment are described.
IMPLEMENTING WAVEGUIDE BANDWIDTH SIX-PORT REFLECTOMETER CIRCUITS AT MILLIMETRIC WAVELENGTHS

L D Hill

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1 INTRODUCTION

Two six-port reflectometer circuits for the measurement of complex voltage reflection coefficient ($\Gamma_L$) at millimetric wavelengths have been devised at RSRE(1,2,3). Each comprises waveguide thermistor power sensors, directional couplers and short circuits. Using external standards the imperfections in a six-port reflectometer are calibrated out prior to use. This suggests that the circuits could be designed with the simplifying assumptions that each directional coupler is: (a) lossless; (b) perfectly matched; and (c) has a phase relationship between the coupled and transmitted paths which is 90° at all frequencies in the band. In practice commercially available directional couplers give a large variation in the phase relationship over the waveguide bandwidth and this variation causes the six-port calibration method used at RSRE(4,5) to fail at some frequencies.

This memorandum describes the circuit adjustments required to correct the phase relationship observed in WG22 directional couplers supplied by Flann Microwave Instruments. These initial adjustments have succeeded in that they allow both circuits to be calibrated at 0.5 GHz intervals over the 26.5 to 40 GHz band without further adjustment.
Operation of a six-port reflectometer depends on computing the co-ordinates, in the $\Gamma = x + jy$ plane, of the intersection of three circles to give the real and imaginary components of $\Gamma_L$. These circles are the loci of constant ratios of the outputs from three detectors of the instrument to that of the fourth. For a circuit comprising ideal couplers, the co-ordinates of the centres of these circles (hereafter called Q centres) are invariant with $\Gamma_L$. The relative positions of these Q centres are heavily dependent upon the phase relationship of the directional couplers employed and will deviate from the theoretical positions if the simplifying assumptions are not valid. The effectiveness of the circuit adjustments have been assessed by investigating the variation with frequency of the relative positions of the Q centres determined from measured data taken during a calibration.

For each six-port circuit the following is presented:

i. Theory predicting the relative Q centre positions that would be obtained from ideal directional couplers.

ii. A description of the measurement of the variation in coupled/transmitted phase relationship in the directional couplers available and the means adopted to compensate for this.

iii. Plot of the relative Q centre positions over the 26.5 to 40 GHz range obtained from an assembled circuit to compare practice with theory.

Conclusions are then drawn on the tolerance of phase tracking needed from directional couplers, for their use in these six-port circuits.

2 THE "THREE-COUPLER" SIX-PORT CIRCUIT

An advantage of this first circuit design is in its simplicity, which allows it to be constructed from "off the shelf" components; it employs just three four-port directional couplers, two short circuits and four thermistor power meters. It was first reported in references 1 and 2 and the circuit arrangement is shown in Figure 1.

2.1 SOME THEORY OF THE "THREE-COUPLER" DESIGN

![Figure 1. The "Three-Coupler" Six-Port Circuit](image)

A signal flow graph for the above circuit, assuming perfect components, is shown in Figure 2 where the voltage transmission and coupling coefficients for the directional couplers are given by the notation $te^{-j\theta}$ and $ce^{-j(\theta-90)}$ respectively.
Figure 2. Signal Flow Graph for the "Three-Coupler" Circuit

An analysis of the above shows that the ratios of the power absorbed by the detectors at the individual ports 4, 5 and 6 to that at port 3 are related to the $\Gamma_L$ of a load terminating the measurement port by the following equations:

\[
\frac{|V_4|}{|V_3|} = t_1^2 t_2^4 t_3^4 e^{-2j(\theta_1 + 2\theta_2 + 2\theta_3)} \left[ \frac{c_2^2}{t_3^2} + j \frac{c_2^2}{t_2^2} \right]^2 \\
\frac{|V_5|}{|V_3|} = \left( \frac{c_2}{c_1} \right)^2 t_1^2 t_2^4 t_3^4 e^{-2j(\theta_2 + 2\theta_3)} \left[ \frac{c_3^2}{t_3^2} + j \frac{c_3^2}{t_2^2} - \frac{1}{t_3^2} \right]^2 \\
\frac{|V_6|}{|V_3|} = \left( \frac{c_3}{c_1} \right)^2 t_1^2 t_2^4 t_3^4 e^{-2j(\theta_2 + 2\theta_3)} \left[ \Gamma_L - e^{-j\beta} \right]^2
\]

The above equations can be written (using an obvious notation) in the form:

\[
\frac{|V_{k+3}|}{|V_3|}^2 = |D_k|^2 |W_k|^2
\]

If the short circuit on coupler No 3 is positioned so that $B = 0$ and a length of waveguide introduced at coupler No 2 so that the short circuit position will make $\alpha = (2\theta_3 - 90)$, i.e., the length of coupler No 3 transmission path less $\lambda_0/8$, the magnitude of the $W$ vectors related to the above power ratios will be: 3
\[ |w_1| = |\Gamma_L + \frac{c_3^2}{t_3^2} + j \frac{c_2^2}{t_2^2t_3^2}| \]
\[ |w_2| = |\Gamma_L + \frac{c_3^2}{t_3^2} - j \frac{1}{t_3^2}| \]
\[ |w_3| = |\Gamma_L - 1| \]

where \[ |w_k|^2 = \left| \frac{1}{D_k} \left( \frac{v_{k+3}}{v_3} \right) \right|^2 \]

These equations represent circles of the form \[ |R| = |x + jy - (a + jb)| \]
where \[ |R| \] is the radius of a circle centred at \((a + jb)\). Clearly the three circles intersect at \(\Gamma_1 = x + jy\). If 3 dB coupling values are chosen for directional couplers Nos 2 and 3 then the relative Q centre positions obtained from the above equations will be as shown in Figure 3.

![Figure 3. Predicted relative Q centre positions for the "Three-Coupler" Circuit](image-url)
The Q centres for vectors \( W_1 \) and \( W_2 \) will move with frequency because the \( \lambda_g/8 \) condition imposed at coupler No 2 is only strictly true, at one frequency. However the circuit should give Q centre displacement which will ensure near orthogonal intersection of at least two of the vectors over the full waveguide bandwidth.

2.2 THE CONSTRUCTION OF A "THREE-COUPLER" CIRCUIT IN WG22

In practice the waveguide components are not ideal and assembly of a circuit which complies with the foregoing design theory will require a knowledge of the phase characteristics of directional couplers Nos 2 and 3 across the frequency band.

Typical results from measurements on Flann Model 22131-03-4P directional coupler are given in Table 1. Where the electrical lengths of the transmitted and coupled paths were determined from \( \Gamma_L \) measurements made using an existing WG22 six-port reflectometer.

![Diagram of three coupler circuit](image)

Table 1.

<table>
<thead>
<tr>
<th>Frequency (GHz)</th>
<th>Phase Change (deg)</th>
<th>Electrical Length (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Transmission Path Ports 1 to 2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>28</td>
<td>-357.72</td>
<td>194.94</td>
</tr>
<tr>
<td>33.25</td>
<td>-238.19</td>
<td>194.15</td>
</tr>
<tr>
<td>38</td>
<td>-164.96</td>
<td>193.89</td>
</tr>
<tr>
<td>Coupled Path Ports 1 to 4</td>
<td></td>
<td></td>
</tr>
<tr>
<td>28</td>
<td>-110.21</td>
<td>183.76</td>
</tr>
<tr>
<td>33.25</td>
<td>-294.48</td>
<td>184.32</td>
</tr>
<tr>
<td>38</td>
<td>-178.37</td>
<td>184.77</td>
</tr>
</tbody>
</table>

For a directional coupler with the above characteristics to have a quadrature phase relationship between the coupled and transmitted paths, the coupled path length will need to be \( 194.15 - \lambda_g/4 = 191.24 \) mm (using the mid-band values). Therefore to construct this circuit, the side arm on coupler No 3 will have to be lengthened by 6.92 mm to make \( \beta = 0 \). Similarly, the side arm of coupler No 2 will need to be lengthened by \( ((2 \times 194.15 - \lambda_g/4)/2) + 6.92 = 199.61 \) mm to make \( \alpha = (2\theta_3 - 90) \).
The waveguide arrangement for the circuit then becomes as shown in Figure 4 (for illustration, the typical values quoted in Table 1 have been used for couplers No 2 and No 3).

Figure 4. WG22 "Three-Coupler" Circuit arrangement with adjustments

2.3 THE PERFORMANCE OF THE WG22 "THREE-COUPLER" CIRCUIT

A WG22 six-port reflectometer constructed broadly as described, has been calibrated at 0.5 GHz intervals from 26.5 to 40 GHz. The relative Q centre positions in the \( \Gamma \) plane are slightly dependent upon \( \Gamma_L \), but it can be seen from equations 2.14 and 2.15 of reference 5, that this effect can be eliminated by determining the Q centre co-ordinates from data taken when the measurement port is terminated by a matched load. Figure 5 shows a plot of the relative Q centre positions (in the \( \Gamma \) plane) obtained from data taken during a calibration. This shows that the Q centre relationship is reasonable at all of these frequencies and in practice the reflectometer gives accurate \( \Gamma_L \) measurements over the full waveguide bandwidth.
3 THE "FOUR-COUPLED" SIX-PORT CIRCUIT

A feature of this design(3) is that it should eliminate the variation in relative Q centre position with frequency. The electrical path lengths to the detectors P4 and P5, of the incident wave and the wave reflected by the termination at the measurement port, are balanced by the introduction of a compensating length of waveguide. This results in a quadrature relationship at P4 and P5 of the incident wave and the wave reflected from the measurement port.
3.1 SOME THEORY OF THE "FOUR-COUPLER" DESIGN

Figure 6. The "Four-Coupler" Six-Port Circuit

A signal flow graph of the above circuit, again assuming ideal components, is shown in Figure 7.

Figure 7. Signal Flow Graph for the "Four-Coupler" Circuit
The relationship between the power ratios given by this circuit and the $\Gamma_L$ of a load terminating its measurement port can be obtained from an analysis of the above flow graph; it is given by the following equations:

\[
\left| \frac{V_4}{V_3} \right|^2 = \left( \frac{C_2C_4}{C_1} \right)^2 \frac{t_1t_2t_3t_4}{e^{-2j(2\theta_2+2\theta_3+\theta_4+90)}} \left[ \Gamma + \frac{C_3^2}{t_3^2} e^{-j\beta} - \frac{t_4t_2}{C_4t_2t_3^2} e^{-j(\theta_L-\theta_2-2\theta_3-90)} \right]^2
\]

\[
\left| \frac{V_5}{V_3} \right|^2 = \left( \frac{C_2}{C_1} \right)^2 \frac{t_1t_2t_3t_4}{e^{-2j(2\theta_2+2\theta_3+\theta_4)}} \left[ \Gamma + \frac{C_3^2}{t_3^2} e^{-j\beta} - \frac{C_4t_2}{t_2t_3^2} e^{-j(\theta_L-\theta_2-2\theta_3+90)} \right]^2
\]

\[
\left| \frac{V_6}{V_3} \right|^2 = \left( \frac{C_3}{C_1} \right)^2 \frac{t_1t_2t_3}{e^{-2j(\theta_2+2\theta_3)}} \left[ \Gamma - e^{-j\beta} \right]^2
\]

If the short circuit on coupler No 3 is positioned so that $\beta = 0$ before and the compensating line length arranged so that $\theta_L = (\theta_2+2\theta_3)$, i.e. twice the electrical length of coupler No 3 plus the length of coupler No 2, the magnitude of the W vectors will be as follows:

\[
|W_1| = \left| \Gamma_L + \frac{C_3^2}{t_3^2} - j \frac{t_4t_L}{C_4t_2t_3^2} \right|
\]

\[
|W_2| = \left| \Gamma_L + \frac{C_3^2}{t_3^2} + j \frac{C_4t_L}{t_2t_3^2t_4} \right|
\]

\[
|W_3| = |\Gamma_L - 1|
\]

If 3 dB coupling values are chosen for directional coupler Nos 2, 3 and 4 the predicted relative Q centre positions for this circuit will be as shown in Figure 8 and should be invariant with frequency.
Figure 8. Predicted relative Q centre positions for the "Four-Coupler" Circuit

3.2 THE CONSTRUCTION OF A "FOUR-COUPLER" CIRCUIT IN WG22

The electrical path lengths of the Flann Model 22131-03-4P 3 dB directional couplers used for the assembly of this circuit were determined from $r_L$ measurements carried out at three frequencies as before, the values are given in Table 2.

Table 2.

<table>
<thead>
<tr>
<th>Coupler Serial No</th>
<th>Electrical Length (mm)</th>
<th>Correction to coupled path needed for broadband quadrature relationship (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>144</td>
<td>194.33</td>
<td>183.85</td>
</tr>
<tr>
<td>144 Reversed</td>
<td>194.33</td>
<td>183.73</td>
</tr>
<tr>
<td>145</td>
<td>194.30</td>
<td>183.96</td>
</tr>
<tr>
<td>155</td>
<td>194.13</td>
<td>183.52</td>
</tr>
</tbody>
</table>
The directional couplers were arranged as shown in Figure 9 where the forward coupled path corrections for couplers Nos 144 and 145 are incorporated into the compensating line length (= 2 x 194.13 + 194.33 + 7.57 + 7.47).

![Diagram of the circuit arrangement](image)

**Figure 9.** The WG22 "Four-Coupler" Six-Port Circuit Arrangement

### 3.3 THE WG22 "FOUR-COUPLED" CIRCUIT PERFORMANCE

This circuit has been calibrated at various frequencies over the full waveguide bandwidth. The six to four port reduction process carried out during a calibration produces five real numbers, P, Q, R, A² and B² (using the notation of section 3 of reference 5), which characterise the network and are independent of \( r_L \). These network constants are determined from the power ratios obtained for 10 different terminations at the measurement port. \( P, Q \) and \( R \) are the square of the distances between the Q centres and \( A^2, B^2 \) are scaling factors for two of the power ratios. Table 3 lists the network constants produced by the above calibration, the theoretical values assuming perfect components are also given for comparison.

It can be seen from Table 3 that the network constants are consistent with the theoretical values by a factor within the range x0.3 to x1.6 throughout the frequency band. This shows that, with correction, commercially available couplers can be adequate for the construction of a six-port circuit which will operate over the full waveguide bandwidth.
Table 3.

<table>
<thead>
<tr>
<th>Frequency (GHz)</th>
<th>Network Constants</th>
</tr>
</thead>
<tbody>
<tr>
<td>(Theoretical values)</td>
<td>P</td>
</tr>
<tr>
<td>26.5</td>
<td>1.125</td>
</tr>
<tr>
<td>28.0</td>
<td>0.466</td>
</tr>
<tr>
<td>29.0</td>
<td>0.679</td>
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<tr>
<td>30.0</td>
<td>0.771</td>
</tr>
<tr>
<td>31.0</td>
<td>0.974</td>
</tr>
<tr>
<td>32.0</td>
<td>0.734</td>
</tr>
<tr>
<td>32.25</td>
<td>1.311</td>
</tr>
<tr>
<td>33.0</td>
<td>0.988</td>
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<tr>
<td>34.0</td>
<td>0.764</td>
</tr>
<tr>
<td>35.0</td>
<td>1.019</td>
</tr>
<tr>
<td>36.0</td>
<td>0.932</td>
</tr>
<tr>
<td>37.0</td>
<td>0.991</td>
</tr>
<tr>
<td>38.0</td>
<td>0.804</td>
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<tr>
<td>39.0</td>
<td>0.852</td>
</tr>
<tr>
<td>40.0</td>
<td>0.894</td>
</tr>
</tbody>
</table>

A plot of the relative Q centre positions (in the \( \Gamma \) plane) with a matched load terminating the measurement port is shown in Figure 10. The co-ordinates were determined from measured data taken at 1 GHz intervals.

Figure 10 shows that while the phase relationship of the Q centre co-ordinates is maintained reasonably well throughout the frequency band, their magnitudes are variable. However, the calibration is stable at those frequencies which give large variations. This departure from the predicted theoretical positions, may be due to resonances produced by the significant reflection coefficient (typically 0.2) of the detectors used, or any mismatch in the directional coupler.
4 CONCLUSION

The phase compensation required to enable these two six-port circuits to operate over the 26.5 to 40 GHz band, using commercially available directional couplers, has been reported. A consequence of this work is that an automated WG22 six-port reflectometer operated at 0.5 GHz intervals over the full waveguide bandwidth is now in use. It is concluded that to implement these circuits successfully in other millimetric bands, the directional couplers will need to have a nominal directivity of 30 dB and the coupled/transmitted phase relationship should track with frequency within ± 15° over the whole of the waveguide band.

5 ACKNOWLEDGEMENT

The author is indebted to Mr E J Griffin for his comments and helpful suggestions in preparing this memorandum.
REFERENCES


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15. Distribution statement

Abstract

This memorandum concerns the implementation of two WG22 six-port reflectometer circuits for the measurement of complex voltage reflection coefficient, using commercially available directional couplers. The means for establishing the phase conditions necessary for each circuit to be used over the 26.5 to 40 GHz band without further adjustment are described.
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