ESTIMATION OF WATER VAPOR PROFILES FROM TOTAL PRECIPITABLE WATER (U) COLORADO STATE UNIV FORT COLLINS DEPT OF ATMOSPHERIC SCIENCE. A E LIPTON ET AL. JUL 83
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Scientific Report No. 3

July 1983

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Estimation of Water Vapor Profiles from Total Precipitable Water.

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Monitor - Vincent Falcone - LYS

**Report Date:** July 1983

**Number of Pages:** 11 pages

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**Key Words:** Precipitable Water
Water Vapor
Empirical Orthogonal Functions

A method is described for converting a value of total precipitable water into an approximate mixing ratio profile. Mixing ratios are provided at the six radiosonde mandatory reporting levels. The method uses an approximation based on the empirical orthogonal functions (EOF's) of mixing ratio profiles. The approximate profile depends on the precipitable water, latitude, and time of year at the site of interest.
1.0 INTRODUCTION

Most atmospheric analysis routines and numerical models require
water vapor information to be input at specific atmospheric levels.
However, when water vapor data are minimal it may be necessary to rely
on total precipitable water values as the only moisture information
available. Such a condition is especially likely to occur over the
oceans, where all upper-air data are scarce. The analyst then faces the
problem of assigning values of mixing ratio (or some related quantity)
to specific levels of the atmosphere based on an approximation to the
integral of water vapor over all levels.

Several papers have discussed the relationship between precipitable
water and surface dew point temperature. Smith (1966) suggested that
the mixing ratio profile can be represented by the power law

\[ w = w_0 \left( \frac{p}{p_0} \right)^\lambda \]  

(1)

where \( w \) is mixing ratio, \( p \) is pressure, \( \lambda \) is an empirically derived
constant, and the subscript \( o \) refers to the surface. Equation 1 can be
used to relate precipitable water to \( w \) at all levels. While \( \lambda \) is
computed for a given latitude and season, the solution for \( w \) is
constrained to a power law distribution.
2.0 EIGENVECTOR APPROACH

An alternative method may be used to relate total water to mixing ratio at specific levels. This method is based on computing the empirical orthogonal functions (eigenvectors) of mixing ratio profiles. Given a data base of profiles for a particular latitude band and season, eigenvectors can be computed which represent the basic modes of vertical variation in water vapor. The first eigenvector represents the primary mode, which describes how variations in total water translate into mixing ratio profile variations. Kendall (1975) gives a complete discussion of eigenvectors and the associated principal components. Wachtmann (1975) gives an example of the use of eigenvectors to represent atmospheric profiles.

Any mixing ratio profile vector \( \mathbf{y} \) can be represented as a linear combination of the eigenvectors and a vector \( \mathbf{a} \) of coefficients.

\[
\mathbf{y} = \mathbf{y}_m + \mathbf{Ea}
\]  

(2)

where \( \mathbf{E} \) is a square matrix with each column composed of one eigenvector, and \( \mathbf{y}_m \) is the mean mixing ratio profile. If we ignore all minor modes of mixing ratio variation, we can approximate \( \mathbf{y} \) using only the first eigenvector \( \mathbf{a} \) and its corresponding coefficient \( e \).

\[
\mathbf{y} \approx \mathbf{y}_m + \mathbf{a}e
\]  

(3)

From Equation 3 it is clear that an approximation to the mixing ratio profile can be obtained by finding the appropriate coefficient \( e \),
given that the first eigenvector and the mean profile are specified in advance.

The remaining task is to show how \( c \) can be determined from precipitable water \( U \). Precipitable water is defined as

\[
U = \frac{\int P \, dp}{P_0}
\]

If the trapezoidal approximation is used for the integral then this becomes

\[
U = \frac{1}{L} \sum_{i=1}^{L} V_i \Delta P_i
\]

where \( V_i = (v_i + v_{i-1})/2; \Delta P_i = P_i - P_{i-1} \), and \( L \) is the number of levels at which measurements are made. Substituting from Equation 3 gives

\[
U = \frac{1}{L} \sum_{i=1}^{L} (\bar{v}_i + \bar{w}_i) \Delta P_i,
\]

where \( \bar{v}_i \) and \( \bar{w}_i \) are defined similarly to \( \bar{v}_i \). Solving for \( e \) gives

\[
e = \frac{L - \sum_{i=1}^{L} \bar{v}_i \Delta P_i}{L - \sum_{i=1}^{L} \bar{w}_i \Delta P_i}
\]

which can be written as

\[
e = \frac{sU - L}{L}
\]
since \( a = \sum_{i=1}^{L} \frac{\Delta p_i}{\sum_{i=1}^{L} \Delta p_i} \) and \( b = \sum_{i=1}^{L} \frac{\Delta p_i}{\sum_{i=1}^{L} \Delta p_i} \) are constants for a given latitude band and season. The water vapor profile can thus be easily approximated from precipitable water by using Equation 8 and then Equation 3.

3.0 RESULTS

The mixing ratio means and the first eigenvectors were computed at the six mandatory levels for five domains of season and latitude using a data base of oceanic radiosonde measurements of mixing ratio. Table 1 defines the five domains and gives the computed values of \( a \) and \( b \). The first eigenvectors are given in Table 2 and the mean mixing ratios are given in Table 3.

Table 1. The five domains for computation.

<table>
<thead>
<tr>
<th>Domain**</th>
<th>Latitude</th>
<th>Season*</th>
<th>Number in Data Base</th>
<th>( a ) (kg m(^{-1}) s(^{-2}))</th>
<th>( b ) (kPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 - TR</td>
<td>30(^\circ)S-30(^\circ)N</td>
<td>All</td>
<td>173</td>
<td>680.2</td>
<td>42.50</td>
</tr>
<tr>
<td>2 - MLS</td>
<td>30(^\circ)-60(^\circ)N</td>
<td>Summer</td>
<td>184</td>
<td>508.1</td>
<td>42.90</td>
</tr>
<tr>
<td>3 - MLW</td>
<td>30(^\circ)-60(^\circ)N</td>
<td>Winter</td>
<td>55</td>
<td>325.9</td>
<td>36.08</td>
</tr>
<tr>
<td>4 - FOS</td>
<td>60(^\circ)-90(^\circ)N</td>
<td>Summer</td>
<td>97</td>
<td>255.7</td>
<td>49.67</td>
</tr>
<tr>
<td>5 - POW</td>
<td>60(^\circ)-90(^\circ)N</td>
<td>Winter</td>
<td>116</td>
<td>157.8</td>
<td>38.34</td>
</tr>
</tbody>
</table>

*Summer = 1 May - 31 October. Winter = 1 November - 30 April
**TR = Tropical, ML = Mid-latitude, PO = Polar

Table 2. First Eigenvectors.

<table>
<thead>
<tr>
<th>Domain</th>
<th>Pressure Level (kPa)</th>
<th>Explained Variance (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>30</td>
<td>40</td>
</tr>
<tr>
<td>1 - TR</td>
<td>.011</td>
<td>.038</td>
</tr>
<tr>
<td>2 - MLS</td>
<td>.009</td>
<td>.031</td>
</tr>
<tr>
<td>3 - MLW</td>
<td>.005</td>
<td>.036</td>
</tr>
<tr>
<td>4 - FOS</td>
<td>.003</td>
<td>.059</td>
</tr>
<tr>
<td>5 - POW</td>
<td>.000</td>
<td>.013</td>
</tr>
</tbody>
</table>
Table 3. Mean Mixing Ratios (g/kg)

<table>
<thead>
<tr>
<th>Domain</th>
<th>Pressure Level (kPa)</th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>30</td>
<td>40</td>
<td>50</td>
<td>70</td>
<td>85</td>
<td>100</td>
<td></td>
</tr>
<tr>
<td>1 - TR</td>
<td>.22</td>
<td>.63</td>
<td>1.43</td>
<td>3.96</td>
<td>9.08</td>
<td>13.91</td>
<td></td>
</tr>
<tr>
<td>2 - MLS</td>
<td>.19</td>
<td>.59</td>
<td>1.17</td>
<td>3.18</td>
<td>6.30</td>
<td>10.47</td>
<td></td>
</tr>
<tr>
<td>3 - MLW</td>
<td>.15</td>
<td>.47</td>
<td>.87</td>
<td>1.77</td>
<td>4.03</td>
<td>6.97</td>
<td></td>
</tr>
<tr>
<td>4 - POS</td>
<td>.09</td>
<td>.36</td>
<td>.76</td>
<td>1.91</td>
<td>3.04</td>
<td>4.39</td>
<td></td>
</tr>
<tr>
<td>5 - POW</td>
<td>.02</td>
<td>.13</td>
<td>.33</td>
<td>.97</td>
<td>1.75</td>
<td>2.56</td>
<td></td>
</tr>
</tbody>
</table>

The mean mixing ratios and the eigenvectors can be compared using the graphs in Figures 1 and 2, respectively. As expected, there is more total water in summer than in winter, and more in lower latitudes. For each domain the first eigenvector shows the vertical distribution of mixing ratio variance within the first mode of water vapor variability.

The influences of the means and eigenvectors for the five domains are demonstrated by the curves in Figure 3. The profiles shown were derived by solving Equations 8 and 3 for an arbitrary value of $U = 2.22 \text{ g/cm}^2$, and using the appropriate values of $g$, $\eta$, a, b for each domain. Note that the shapes of the polar summer and winter (POS and POW) profiles are unrealistic in this case because 2.22 cm is a very high value of precipitable water for the polar region. The profile shapes are actually a different function of precipitable water for each domain.

4.0 COMPUTER PROGRAM

A FORTRAN computer program has been written which performs the conversion of precipitable water values into approximate mixing ratio profiles. All of the necessary coefficients are stored in the program, so the only needed inputs are latitude, Julian date, and precipitable water. The code is brief and includes ample documentation. The Appendix to this report provides a program listing and sample output.
Figure 1. Mean mixing ratio profiles computed from the five data base sets indicated in Table 1.
Figure 2. For each of the five domains listed in Table 1, the plot shows the first eigenvector of the mixing ratio covariance matrix.
Profile Domain Comparison
2.22cm Precipitable Water

Figure 3. Example of a single value of total precipitable water (2.22 cm) translated into a mixing ratio profile for each of the five domains listed in Table 1.
REFERENCES


PROGRAM PWPROFL; INPUT; OUTPUT, TAPE5=INPUT, TAPE6=OUTPUT
****** THIS PROGRAM DERIVES A
****** "MOST PROBABLE" MIXING RATIO PROFILE FROM A PRECIPITABLE WATER
****** VALUE. THE COEFFICIENTS USED ARE SPECIFIC TO
****** LATITUDE BAND AND TIME OF YEAR.
****** VARIABLE IDENTIFICATION
****** E = COEFFICIENTS OF THE FIRST EIGENVECTOR
****** TOP = NUMERATOR COEFFICIENT
****** DEN = DENOMINATOR COEFFICIENT
****** WM = MEAN MIXING RATIO PROFILES
****** W = DERIVED MIXING RATIO PROFILE
****** PW = PRECIPITABLE WATER
****** ISES = SEASON INDICATOR (WINTER=1, SUMMER=-1)
****** I = IDENTIFIES SEASON/LATITUDE-BAND DOMAIN
****** C = FIRST PRINCIPLE COMPONENT
***************************************************************
****** STORAGE ALLOCATION
****** DIMENSION TOP(5),DEN(5),WM(6,5),W(6),E(6,5),P(6)
****** DATA TOP/600.2*508.1*325.9*255.7*137.8*/DEN/42.50*41.90*36.08*+
+ 49.67*38.34*/P/30.40*50.70*85.00*/
****** DATA WM/00.00*02.45*04.09*06.01*09.91*/
****** + 01.79*03.59*05.17*08.28*10.67*/
****** + 00.19*00.47*00.87*01.77*03.03*04.29*/
****** + 00.09*00.76*01.01*02.03*04.04*04.39*/
****** + 00.02*00.13*00.33*00.97*01.75*02.56*/
****** DATA E/0.01*0.036*0.099*0.302*0.545*0.779*/
****** + 0.009*0.031*0.089*0.205*0.373*0.763*/
****** + 0.005*0.036*0.073*0.127*0.340*0.828*/
****** + 0.003*0.059*0.148*0.433*0.670*0.582*/
****** + 0.000*0.013*0.035*0.244*0.525*0.813*/
***************************************************************
****** WRITE A HEADER
****** WRITE(6,0800)(PL, L=1,6)
****** 080 FORMAT(1X,"LATITUDE JULIAN PW(CM)","10X","MIXING RATIO (G/KG)"
****** /13X,"DAY","13X",F6.2,"(F6.0,2X),"PRESSURE (KPA)"
***************************************************************
****** READ A DATA SET AND IDENTIFY CLASS
****** DO 420 J=1,500
****** READ(5,110)PLAT,JDAY,PW
****** 110 FORMAT(1X,F6.2,110,F10.2)
****** IF(EQ(0),MN.01600)440
****** ISES=-1
****** IF(JDAY.LT.121)OP.1)JDAY.GT.304)ISESN=1
****** IF(PLAT.GE.0)GOTO 210
****** PLAT=PLAT
****** ISES=ISESN
****** 210 IF(PLAT.GT.30.)GOTO 220
****** I=1
****** GOTO 290
****** 220 IF(PLAT.GT.60.)GOTO 240
****** IF(ISESN.GE.0)GOTO 230
****** I=2
****** GOTO 290
****** 230 I=3
****** GOTO 290
****** 240 IF(ISESN.GE.0)GOTO 250
****** I=4
****** GOTO 290
****** 250 I=5
****** 290 CONTINUE
***************************************************************
****** COMPUTE MIXING RATIOS AND PRINT
****** C=(196.2*PW-TOP(1))/DEN(1)
****** DO 320 L=1,6
****** ML=WML+L*V*C(L,1)
****** 320 CONTINUE
****** WRITE(6,410)PLAT,JDAY,PW(ML),L=1,6
****** 410 FORMAT(1X,F6.2,110,F10.2,2*F6.2)
****** 420 CONTINUE
***************************************************************
440 STOP
END
<table>
<thead>
<tr>
<th>LATITUDE</th>
<th>JULIAN DAY</th>
<th>PW (cm)</th>
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<th>40.</th>
<th>50.</th>
<th>70.</th>
<th>95.</th>
<th>100.</th>
</tr>
</thead>
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<td>10</td>
<td>3.93</td>
<td>.22</td>
<td>.64</td>
<td>1.48</td>
<td>4.03</td>
<td>9.24</td>
<td>14.14</td>
</tr>
<tr>
<td>71.11</td>
<td>159</td>
<td>1.10</td>
<td>.09</td>
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<td>.64</td>
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<td>4.38</td>
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<td>.25</td>
<td>.74</td>
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<td>16.21</td>
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<td>.62</td>
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<td>6.35</td>
<td>10.52</td>
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<tr>
<td>29.88</td>
<td>280</td>
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<td>.16</td>
<td>.41</td>
<td>.88</td>
<td>2.22</td>
<td>5.94</td>
<td>9.43</td>
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<tr>
<td>42.08</td>
<td>280</td>
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<td>.17</td>
<td>.54</td>
<td>1.03</td>
<td>2.09</td>
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<td>9.15</td>
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<td>.29</td>
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<td>2.07</td>
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END

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