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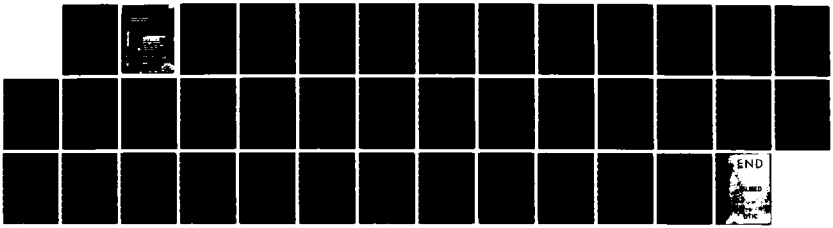

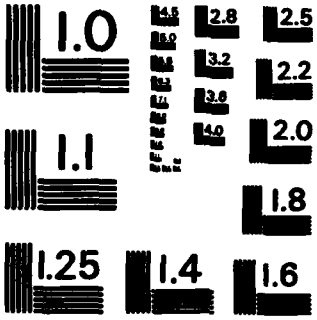

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Given that the examinee knows the answer to item i if and only if he knows the answer to both item g and item h, a 'conjunctive' item response model is found such that items g, h, and i all have the same mathematical form of response function. Since such items may occur in practice, it is desirable that item response models satisfy this condition. For models with two parameters per item, the most general functional form satisfying this condition is found. A third, 'guessing' parameter may be added. The corresponding disjunctive model is also derived.

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Frederic M. Lord

Educational Testing Service

October 1984

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Conjunctive and Disjunctive Item Response Functions

Abstract

Given that the examinee knows the answer to item i if and only if he knows the answer to both item g and item h, a 'conjunctive' item response model is found such that items g, h, and i all have the same mathematical form of response function. Since such items may occur in practice, it is desirable that item response models satisfy this condition. For models with two parameters per item, the most general functional form satisfying this condition is found. A third, 'guessing' parameter may be added. The corresponding disjunctive model is also derived.

Conjunctive and Disjunctive Item Response Functions*

Consider two free-response spelling items that ask the examinee to spell, respectively, the word wife and the word house. Now consider a free-response spelling item that asks the examinee to spell the word housewife. Assume that an examinee will answer the third item correctly if and only if the examinee would answer both of the first two items correctly. If so, then

$$P_3(\theta) = P_1(\theta)P_2(\theta) \tag{1}$$

where θ is the ability of the examinee and $P_i(\theta)$ is the item response function for (probability of giving a correct answer to) item i.

Kristof (1968) pointed out that it is highly desirable that P_1, P_2, and P_3 should all have the same mathematical form, but that this condition is not satisfied by the usual logistic or normal ogive models in item response theory (IRT). He derived one-parameter families of item response function, $P(\theta)$, satisfying (1), stating that this result "is attainable if and only if all item [response] functions are powers of each other.... The model

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rules out normal ogives as well as logistic functions as possible item [response] functions...."

More recently Yen (1984) discussed certain difficulties encountered in vertical equating using a logistic model. She "hypothesized that [these difficulties] occur because the items increase in complexity as they increase in difficulty." Such problems might be avoided if the model $P(\theta)$ satisfying (1) were used.

In Kristof's study, each item is characterized by a single item parameter. Here, Kristof's result is generalized, using a different approach, so as to allow items to differ in each of two parameters.

Necessary Condition for Solving the Functional Equation

Suppose the item response function for any item has the form $P(a^*,b^*,\theta)$: it is a function of θ characterized by two item parameters, a^* and b^* . Denote the values of a^* and b^* for item 1 by a and b ; for item 2, by A and B ; for item 3 by α and β . Then (1) becomes

$$P(a,b,\theta)P(A,B,\theta) \equiv P(\alpha,\beta,\theta) \quad . \tag{2}$$

If such an identity is to hold for all θ , a , b , A , B , α , and β , it is necessary that the item parameters α and β

be functions of a , b , A , and B (item parameters by definition do not depend on θ):

$$\alpha \equiv \alpha(a,b,A,B) \quad \text{and} \quad \beta \equiv \beta(a,b,A,B) \quad . \tag{3}$$

Define $L(a^*,b^*,\theta) \equiv \log P(a^*,b^*,\theta)$. From (2)

$$L(a,b,\theta) + L(A,B,\theta) \equiv L(\alpha,\beta,\theta) \quad . \tag{4}$$

This is a functional equation in five variables and three unknown functions, $L(\ )$, $\alpha(\ )$, and $\beta(\ )$.

Take the derivative of this with respect to a , to b , and to A :

$$L^a(a,b,\theta) \equiv L^\alpha(\alpha,\beta,\theta)\alpha^a + L^\beta(\alpha,\beta,\theta)\beta^a \quad , \tag{5}$$

$$L^b(a,b,\theta) \equiv L^\alpha(\alpha,\beta,\theta)\alpha^b + L^\beta(\alpha,\beta,\theta)\beta^b \quad ,$$

$$L^A(A,B,\theta) \equiv L^\alpha(\alpha,\beta,\theta)\alpha^A + L^\beta(\alpha,\beta,\theta)\beta^A \quad ,$$

where each superscript denotes a derivative: for example, $\alpha^a \equiv \partial\alpha(a,b,A,B)/\partial a$. Continuity and differentiability of functions are assumed as needed here and in the following derivations.

Given any fixed set of values of the three variables α , β , and θ , both $L^\alpha(\alpha,\beta,\theta)$ and $L^\beta(\alpha,\beta,\theta)$ are fixed. Equations (5) are then three nonhomogeneous linear equations relating L^α

and L^{β} . These three equations will be inconsistent unless the augmented matrix

$$\begin{bmatrix} \alpha^{a}(a,b,A,B) & \beta^{a}(a,b,A,B) & L^{a}(a,b,\theta) \\ \alpha^{b}(a,b,A,B) & \beta^{b}(a,b,A,B) & L^{b}(a,b,\theta) \\ \alpha^{A}(a,b,A,B) & \beta^{A}(a,b,A,B) & L^{A}(a,b,\theta) \end{bmatrix}$$

is singular. Therefore the corresponding determinant must vanish:

$$(\alpha^{a}\beta^{b} - \alpha^{b}\beta^{a})L^{A} + (\alpha^{b}\beta^{A} - \alpha^{A}\beta^{b})L^{a} + (\alpha^{A}\beta^{a} - \alpha^{a}\beta^{A})L^{b} \equiv 0 \quad .$$

Rewrite this as

$$h(a,b,A,B)L^{A} \equiv f(a,b,A,B)L^{a} + g(a,b,A,B)L^{b} \quad . \qquad (6)$$

This result accomplishes the elimination L^{a} and L^{β} from immediate consideration.

Given any fixed set of values of a , b , and θ , both L^{a} and L^{b} are fixed. ıtion (6) expresses a linear relationship between L^{a} and L^{b} . This relationship must hold as (A,B) takes on different values (A_1,B_1), (A_2,B_2), (A_3,B_3), ... Thus (6) represents an (infinite) set of linear nonhomogeneous equations relating L^{a} and L^{b} . These equations will be inconsistent if the augmented matrix,

$$\begin{bmatrix} f(a,b,A_1,B_1) & g(a,b,A_1,B_1) & h(a,b,A_1,B_1)L^A(A_1,B_1,\theta) \\ f(a,b,A_2,B_2) & g(a,b,A_2,B_2) & h(a,b,A_2,B_2)L^A(A_2,B_2,\theta) \\ f(a,b,A_3,B_3) & g(a,b,A_3,B_3) & h(a,b,A_3,B_3)L^A(A_3,B_3,\theta) \\ \vdots & \vdots & \vdots \end{bmatrix}$$

is of rank 3. Thus (using an obvious notation) the determinant

$$(f_1g_2 - f_2g_1)h_3L_3^A + (f_3g_1 - f_1g_3)h_2L_2^A + (f_2g_3 - f_3g_2)h_1L_1^A \equiv 0 \quad . \tag{7}$$

This result accomplishes the elimination of L^a and L^b from immediate consideration.

Given any fixed set of values of a, b, A_1, B_1, A_2, B_2, A_3, and B_3, the f's, g's, and h's in (7) are fixed. Equation (7) must still hold for $\theta = \theta_1, \theta_2, \theta_3, \ldots$. Thus (7) represents an (infinite) set of homogeneous linear equations in the three fixed quantities $(f_1g_2 - f_2g_1)h_3$, $(f_3g_1 - f_1g_3)h_2$, and $(f_2g_3 - f_3g_2)h_1$. Such equations can be consistent only if the matrix

$$\begin{bmatrix} L^A(A_1,B_1,\theta_1) & L^A(A_2,B_2,\theta_1) & L^A(A_3,B_3,\theta_1) \\ L^A(A_1,B_1,\theta_2) & L^A(A_2,B_2,\theta_2) & L^A(A_3,B_3,\theta_2) \\ L^A(A_1,B_1,\theta_3) & L^A(A_2,B_2,\theta_3) & L^A(A_3,B_3,\theta_3) \\ \vdots & \vdots & \vdots \end{bmatrix}$$

is of less than full rank. If the equations are consistent, the third row of the matrix must be linearly dependent on the first two rows: in other words $L^A(A,B,\theta_3)$ must be a linear function of $L^A(A,B,\theta_2)$ and $L^A(A,B,\theta_1)$.

This conclusion is a contradiction, since θ_3 is arbitrary and cannot be expressed as a function of θ_1 and θ_2 . Thus the premise that the equations are 'consistent' must be false. This proves that when θ varies, (7) represents a set of 'inconsistent' equations, according to common terminology. This does not mean, however than (7) is invalid: the 'inconsistent' homogeneous linear equations will be satisfied provided

$$(f_1g_2 - f_2g_1)h_3 \equiv (f_3g_1 - f_1g_3)h_2 \equiv (f_2g_3 - f_3g_2)h_1 \equiv 0 \quad . \quad (8)$$

This proves that if differentiable functions satisfying (4) exist, then the f , g , and h must satisfy (8). According to (8) and (7), either

(Case 1) $h(a,b,A,B) \equiv 0$, or

(Case 2) $h(a,b,A,B)$ is nonzero but $g(a,b,A,B) \equiv 0$ and/or $f(a,b,A,B) \equiv 0$, or

(Case 3) f/g is independent of A,B :

$$\frac{f(a,b,A_1,B_1)}{g(a,b,A_1,B_1)} \equiv \frac{f(a,b,A_2,B_2)}{g(a,b,A_2,B_2)} \equiv F_1(a,b) \quad , \tag{9}$$

where $F_1(a,b)$ denotes a (more or less) arbitrary function of a and b only.

If Case 1 were to hold, then, from the definition of h in (6),

$$\begin{vmatrix} \alpha^a & \beta^a \\ \alpha^b & \beta^b \end{vmatrix} \equiv 0 \quad .$$

But for fixed A , B , this is the Jacobian of the transformation $\alpha \equiv \alpha(a,b)$, $\beta \equiv \beta(a,b)$. When the Jacobian is zero, α and β do not vary independently. In the present problem, this would mean that for fixed A , B , $L(\alpha,\beta,\theta)$ is only a one-parameter family, and thus that $P(\alpha,\beta,\theta)$ is only a one-parameter family of item response functions. This is contrary to the original requirements, so Case 1 is inappropriate.

Case 2 will be treated later. The remaining possibility is given by (9). Substitute (9) into (6), obtaining

$$h(a,b,A,B)L^A(A,B,\theta) \equiv g(a,b,A,B)[F_1(a,b)L^a(a,b,\theta) + L^b(a,b,\theta)] \quad .$$

Denote the quantity in brackets by $F_2(a,b,\theta)$. Take the logarithm of the absolute value of both sides:

$$\log|h| + \ell_A \equiv \log|g| + f_2 \tag{10}$$

where $\ell_A \equiv \log|L^A|$ and $f_2 \equiv \log|F_2|$. Differentiate (10) on θ :

$$\ell_A^\theta(A,B,\theta) \equiv f_2^\theta(a,b,\theta) \quad . \tag{11}$$

Now the left side of (11) is not a function of a or b , the right side is not a function of A or B . It follows that each side must be independent of all four variables a , b , A , B . Thus

$$\ell_A^\theta(A,B,\theta) \equiv \phi_1(\theta) \quad , \tag{12}$$

a (more or less) arbitrary function of θ only.

Integrate (12) on θ :

$$\ell_A(A,B,\theta) \equiv \phi_2(\theta) + F_3(A,B) \quad ,$$

where $\phi_2(\theta) \equiv \int\phi_1(\theta)d\theta$ and $F_3(A,B)$ is the 'constant of integration.' Exponentiate, obtaining

$$L^A(A,B,\theta) \equiv \exp[\phi_2(\theta) + F_3(A,B)] \quad ,$$

$$L^A(A,B,\theta) \equiv F_4(A,B)\phi_3(\theta)$$

where F_4 and ϕ_3 are (more or less) arbitrary nonnegative functions. Integrate on A to find, finally

$$L(A,B,\theta) \equiv F_5(A,B)\phi_3(\theta) + G_1(B,\theta) \quad , \qquad (13)$$

where $F_5(A,B) \equiv \int F_4(A,B)\, dA$ and $G_1(B,\theta)$ is the constant of integration, both (more or less) arbitrary functions.

The only remaining case, Case 2, also leads to (13), as will now be shown. If both $g(a,b,A,B)$ and $g(a,b,A,B)$ were zero in Case 2, (6) would become

$$h(a,b,A,B)L^A(A,B,\theta) \equiv 0 \quad .$$

Since $h(\ )$ is nonzero in Case 2, this would imply $L^A = 0$. But $L^A \equiv 0$ means that the log of the item repsonse function does not vary with A , a special limiting situation of no general interest here. The only interesting alternatives under Case 2 is that either $f = 0$ or else $g = 0$, but not both.

When $g \equiv 0$, $h \neq 0$, and $f \neq 0$, then (16) becomes

$$hL^A \equiv fL^a \quad .$$

Take the logarithm of the absolute value of both sides and then differentiate on θ to find

$$\ell_A^\theta \equiv \ell_a^\theta \quad .$$

This is equivalent to (11). Thus Case 2 with $g \equiv 0$ also leads to (13).

The remaining possibility, that $f \equiv 0$, $h \neq 0$, and $g \neq 0$ (in Case 2) also leads to (13), except that a and b , also A and B , and also α and β are interchanged. Since the original problem is invariant under this interchange, (13) will still apply, given an appropriate initial choice of parameter assignment. Thus with suitable parameter assignment, (13) determines a specific form of $L(\ ,\ ,\ )$ that is necessary (but not yet sufficient) for satisfying (4).

Necessary and Sufficient Condition for a Solution

Using (13), dropping numerical subscripts and rearranging, (4) can now be written

$$[F_5(a,b) + F_5(A,B) - F_5(\alpha,\beta)]\phi_3(\theta) \equiv G_1(\beta,\theta) - G_1(b,\theta) - G_1(B,\theta) \quad . \tag{14}$$

Now, $\phi_3(\theta)$ is not identically zero, since by (13) this would make make $L(A,B,\theta)$ and hence $L(\alpha,\beta,\theta)$ independent of A. Divide by (14) by $\phi_3(\theta)$:

$$F_5(a,b) + F_5(A,B) - F_5(\alpha,\beta) \equiv G_2(\beta,\theta) - G_2(b,\theta) - G_2(B,\theta) \quad , \tag{15}$$

where $G_2(\ ) \equiv G_1(\ )/\phi_3(\theta)$. Differentiate on θ :

$$G_2^{\theta}(\beta,\theta) \equiv G_2^{\theta}(b,\theta) + G_2^{\theta}(B,\theta) \quad . \tag{16}$$

Since the right side is independent of a and A , $\beta(a,b,A,B)$ must be also. Hereafter the notation $\beta(b,B)$ will be used.

Differentiate (16) on b to find:

$$G_2^{\theta\beta}(\beta,\theta)\beta^{b} \equiv G_2^{\theta b}(b,\theta) \quad .$$

Take the logarithm of the absolute value of both sides:

$$\log|\beta^{b}| + g_{\theta\beta}(\beta,\theta) \equiv g_{\theta b}(b,\theta) \quad .$$

where $g_{\theta b} \equiv \log|G_2^{\theta b}|$, and $g_{\theta\beta}$ likewise. Differentiate on θ :

$$g^{\theta}_{\theta\beta}(\beta,\theta) \equiv g^{\theta}_{\theta b}(b,\theta) \quad .$$

Repeat the foregoing operations on (15), switching the roles of b and B to find

$$g^{\theta}_{\theta\beta}(\beta,\theta) \equiv g^{\theta}_{\theta B}(B,\theta) \quad .$$

Eliminate $g^{\theta}_{\theta\beta}$ from the last two equations, obtaining

$$g^{\theta}_{\theta b}(b,\theta) \equiv g^{\theta}_{\theta B}(B,\theta) \quad .$$

Since the left side is not a function of B and the right side is not a function of b , it is seen that each side is a function of θ only; so

$$g^{\theta}_{\theta b}(b,\theta) \equiv \psi_1(\theta) \quad ,$$

say.

Integrate this on θ :

$$g_{\theta b}(b,\theta) \equiv \psi_2(\theta) + k_1(b) \quad ,$$

where $\psi_2(\theta) \equiv \int\psi_1(\theta)d\theta$ and $k_1(b)$ is the constant of integration. Exponentiate:

$$|G_2^{\theta b}(b,\theta)| \equiv \exp[\psi_2(\theta) + k_1(b)]$$

$$G_2^{\theta b}(b,\theta) \equiv k_2(b)\psi_3(\theta) \quad .$$

Integrate on b and then on θ :

$$G_2^{\theta}(b,\theta) \equiv k_3(b)\psi_3(\theta) + \chi_1(\theta) \quad ,$$

$$G_2(b,\theta) \equiv k_3(b)\psi_4(\theta) + \chi_2(\theta) + k_4(b) \quad .$$

Thus, finally, by the relation of G_2 to G_1 ,

$$G_1(b,\theta) \equiv k_3(b)\psi_5(\theta) + k_4(b)\phi_3(\theta) + \psi_6(\theta) \quad . \qquad (17)$$

From (13) and (17),

$$L(A,B,\theta) \equiv F_5(A,B)\phi_3(\theta) + k_3(B)\psi_5(\theta) + k_4(B)\phi_3(\theta) + \psi_6(\theta)$$

$$\equiv F_6(A,B)\phi_4(\theta) + k_3(B)\psi_5(\theta) + \psi_6(\theta) \quad .$$

From this and (4)

$$[F_6(a,b) + F_6(A,B)]\phi_4(\theta) + [k_3(b) + k_3(B)]\psi_5(\theta)$$

$$\equiv F_6(\alpha,\beta)\phi_4(\theta) + k_3(\beta)\psi_5(\theta) - \psi_6(\theta) \quad .$$

This shows that $\psi_6(\theta)$ is a linear function of ϕ_4 and $\psi_5(\theta)$:

$$\psi_6(\theta) \equiv C\phi_4(\theta) + K\psi_5(\theta) \quad ,$$

where C and K are constants (since $\psi_6(\theta)$ does not depend on a , b , A , B). Substitute this into each of the two preceding equations, replace $\phi_4(\theta)$ by $\Phi(\theta)$, $\psi_5(\theta)$ by $\Psi(\theta)$, and $k_3(\ )$ by $G(\ )$, dropping numerical subscripts to obtain finally

$$L(A,B,\theta) \equiv F(A,B)\Phi(\theta) + G(B)\Psi(\theta) \quad , \tag{18}$$

$$[F(a,b) + F(A,B)]\Phi(\theta) + [G(b) + G(B)]\Psi(\theta)$$

$$\equiv F(\alpha,\beta)\Phi(\theta) + G(\beta)\Psi(\theta) \quad . \tag{19}$$

For equality to hold in (19), it is necessary that $\beta(b,B) \equiv G^{-1}[G(b) + G(B)]$ where $G^{-1}(\ )$ is the inverse function of $G(\ )$. Also that

$$\alpha(a,b,A,B) \equiv F^{-1}_{\beta(b,B)}[F(a,b) + F(A,B)]$$

where $F^{-1}_{\beta(b,B)}$ is the inverse function defined for each given b by $F^{-1}_{b}[F(a,b)] \equiv a$.

The solution to (2) is found by exponentiating (18):

$$P(A,B,\theta) \equiv \exp[F(A,B)\Phi(\theta) + G(B)\Psi(\theta)]$$

$$\equiv [f(A,B)]^{\Phi(\theta)}[g(B)]^{\Psi(\theta)}$$

$$\equiv [\phi(\theta)]^{F(A,B)}[\psi(\theta)]^{G(B)} \quad , \tag{20}$$

where $\Phi(\theta) \equiv \log \phi(\theta)$, $\Psi(\theta) \equiv \log \psi(\theta)$, $F(A,B) \equiv \log f(A,B)$, and $G(B) \equiv \log g(B)$.

The Conjunctive Item Response Function

A reparameterization of the IRT model will simplify (20) for present purposes. Define new item parameters by $b' \equiv G(B)$, $a' \equiv F(A,B)$. The general conjunctive item response function (20) is now simply

$$P(a',b',\theta) \equiv [\phi(\theta)]^{a'}[\psi(\theta)]^{b'} \quad . \tag{21}$$

Without loss of generality, since $\phi(\theta)$ and $\psi(\theta)$ remain to be chosen, take $a' > 0$ and $b' > 0$. To be an item response function (IRF), it is necessary that $P(a',b',\theta)$ be a monotonic increasing function of θ and that $P(a',b',-\infty) = 0$, $P(a',b',\infty) = 1$. If it is possible to have $a' \to 0$, then $\psi(\theta)$ must satisfy corresponding conditions. If it is possible to have $b' \to 0$, then $\phi(\theta)$ must satisfy corresponding condition. It will be assumed hereafter that both $\psi(\theta)$ and $\phi(\theta)$ satisfy such conditions.

For convenience, the primes in (21) will now be dropped. If $\psi(\theta) \propto \phi(\theta)$, it would follow that $\psi(\theta) \equiv \phi(\theta)$, since otherwise $\psi(\infty) = \phi(\infty) = 1$ could not hold. If $\psi(\theta) \equiv \phi(\theta)$, then $P(a,b,\theta) \equiv [\phi(\theta)]^{a+b}$. In this case, the reparameterization $a'' \equiv a + b$ would reduce $P(\ )$ to a one-parameter family. Thus it is essential that $\psi(\theta)$ and $\phi(\theta)$ should not be proportional.

A plausible model is obtained by choosing $\psi(\theta)$ and $\phi(\theta)$ to be some familiar two-parameter IRF. If the two-parameter logistic function is used, for example, the IRF for item i is

$$P(a_i,b_i,\theta) = (1 + e^{-\sigma\theta-\mu})^{-a_i}(1 + e^{-\theta})^{-b_i} \tag{22}$$

where σ and μ are arbitrary constants. These constants must be the same for all items in a given test, but they can vary from one test to another. Ideally σ and μ should be constant for all items of a single type: they are test or item-type parameters, not item parameters.

Up to this point, the argument has dealt with the probability that a given examinee knows the correct answer to an item or to a component part of an item. If the final item is presented in multiple-choice form, however, the examinee may answer correctly simply by random guessing. To deal with this situation, a 'guessing' parameter c_i can plausibly be introduced into (22), so that now

$$P(a_i,b_i,c_i,\theta) = c_i + (1 - c_i)(1 + e^{-\sigma\theta-\mu})^{-a_i}(1 + e^{-\theta})^{-b_i} \quad . \tag{23}$$

This IRF does not satisfy the functional equation (2). Nevertheless, (23) may be quite appropriate; for example, when guessing does not contribute to the probability that an examinee knows the correct answer to the component parts of the item.

The Disjunctive Item Response Function

The original reasoning dealt with situations where an examinee would know the correct answer to the actual item only if he knew the correct answer to two separate subproblems in the actual item. One can also imagine situations where the examinee will know the answer to the actual item whenever he knows the answer to either one of two separate subproblems: In other words, there are two independent routes to knowing the answer to the actual item. This is the disjunctive case.

The corresponding functional equation is the same as (2) except that $Q(a,b,\theta) \equiv 1 - P(a,b,\theta)$ is substituted for $P(a,b,\theta)$:

$$Q(a,b,\theta)Q(A,B,\theta) \equiv Q(\alpha,\beta,\theta) \quad . \tag{24}$$

The corresponding IRF is thus

$$Q(a,b,\theta) \equiv [1 - \phi(\theta)]^{a_i}[1 - \psi(\theta)]^{b_i} \quad , \tag{25}$$

where $\phi(\theta)$ and $\psi(\theta)$ satisfy the same conditions as before. In the logistic case with guessing,

$$Q(a_i,b_i,c_i,\theta) \equiv (1 - c_i)(1 + e^{\sigma\theta+\mu})^{-a_i}(1 + e^{\theta})^{-b_i} \quad . \tag{26}$$

Practical Implementation and Conclusion

The conjunctive (disjunctive) IRF's have one convenient feature: When there is no guessing ($c_i = 0$) , the probability of answering all of a set of m items correctly (incorrectly) has a much simpler mathematical form than is usually the case. For conjunctive items,

$$\text{Prob}(u_1 = 1,\ u_2 = 1,\ldots,u_m = 1) = [\phi(\theta)]^{\sum^m a_i}[\psi(\theta)]^{\sum^m b_i} \quad . \tag{27}$$

The parameters a_i and b_i of conjunctive or disjunctive items are not at all readily interpreted in terms of item difficulty and item discriminating power. It is hard to find even a complicated function, let alone a simple function, of a_i and and b_i that can be comfortably interpreted as a satisfactory measure of item difficulty or of item discriminating power.

Figure 1 shows six plots of IRF's from (22) illustrating various degrees of skewness. One can, of course, obtain a pure logistic curve from (22) by setting either a_i or b_i equal to zero. It is also possible to obtain double ogives with three points of inflexion (not illustrated here).

A new computer program has been written for simultaneous maximum likelihood estimation of the item parameters a_i and b_i, the test parameters σ and μ, and the ability parameters θ for the conjunctive model (23). The new program uses fixed c_i values determined in advance by a run of the data on the computer program LOGIST (Wingersky, 1983). Although the estimates converge in any one computer run from a given set of trial values, the result is apparently not yet useful because from different sets of trial values the computer reaches different local maxima corresponding to different assignments of zero values to a_i or to b_i for different subsets of items.

The fit of the model to the responses of a group of examinees could probably be much improved by substituting (26) for (23) whenever an item is estimated to have a zero value of a_i or b_i, thus assuming that such items are disjunctive items. Subsequent iterations of the estimation process might then lead to a

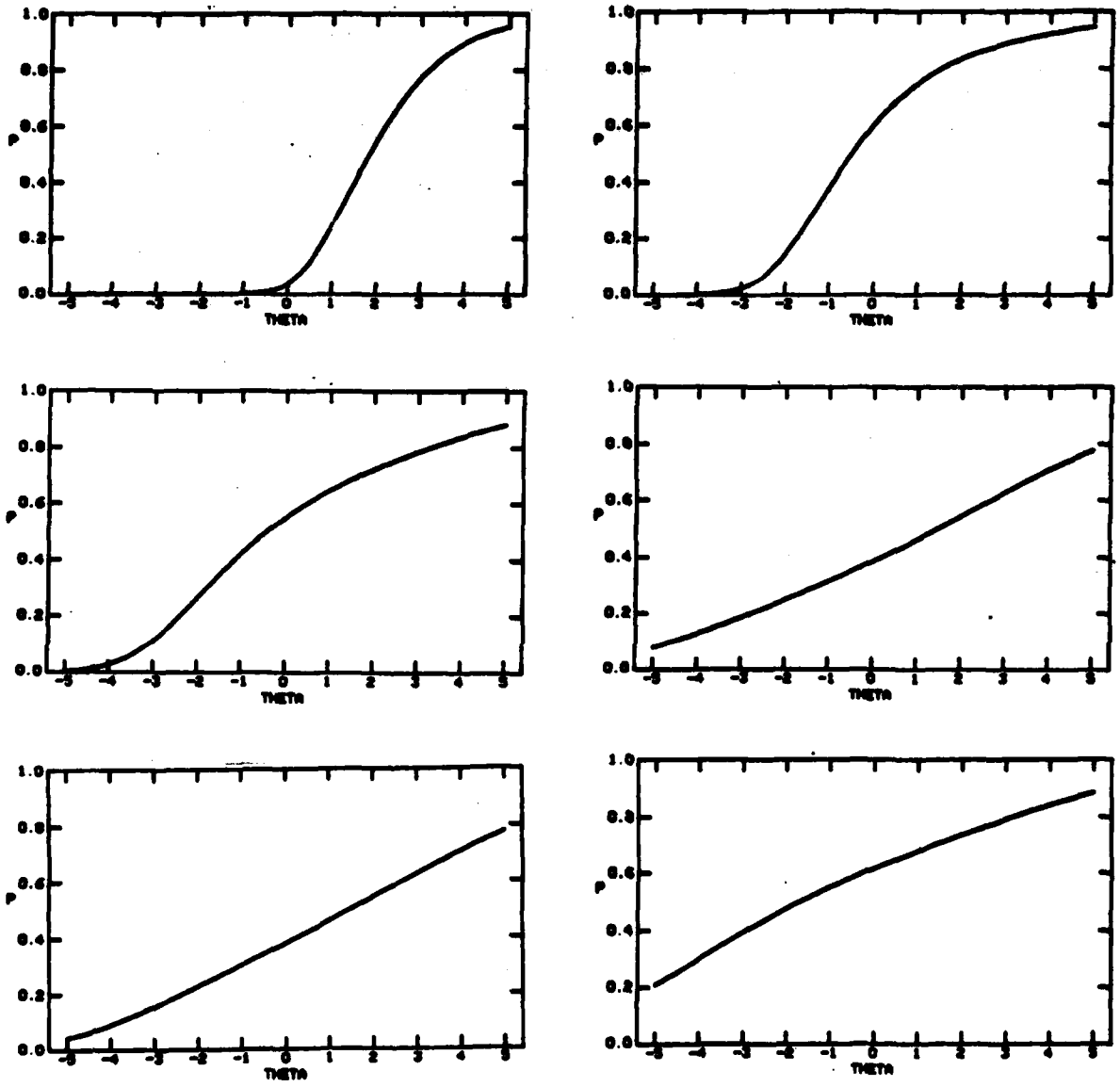

Figure 1. Illustrative conjunctive item response functions with c = 0 (22).

reassignment of some of these items to the conjunctive model while other items might be transferred to the disjunctive model. Ultimately, such a process might successfully classify all items as conjunctive or disjunctive in such a way as to find a global maximum of the likelihood function and an optimal fit of the models to the data. It is not at all clear, however, how much computer time such a process might require.

It is possible that the conjunctive and disjunctive models, based as they are on relevant psychological considerations, may provide a better fit to real data than the usual logistic or normal ogive models.

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Navy

1 Dr. Nick Bond
Office of Naval Research
Liaison Office, Far East
APO San Francisco, CA 96503

1 Lt. Alexander Bory
Applied Psychology
Measurement Division
NAMRL
NAS Pensacola, FL 32508

1 Dr. Robert Breaux
NAVTRAEQUIPCEN
Code N-095R
Orlando, FL 32813

1 Dr. Robert Carroll
NAVOP 115
Washington , DC 20370

1 Dr. Stanley Collyer
Office of Naval Technology
800 N. Quincy Street
Arlington, VA 22217

1 CDR Mike Curran
Office of Naval Research
800 N. Quincy St.
Code 270
Arlington, VA 22217

1 Dr. John Ellis
Navy Personnel R&D Center
San Diego, CA 92252

1 DR. PAT FEDERICO
Code P13
NPRDC
San Diego, CA 92152

1 Mr. Paul Foley
Navy Personnel R&D Center
San Diego, CA 92152

1 Mr. Dick Hoshaw
NAVOP-135
Arlington Annex
Room 2834
Washington , DC 20350

Navy

1 Dr. Norman J. Kerr
Chief of Naval Education and Training
Code 0CA2
Naval Air Station
Pensacola, FL 32508

1 Dr. Leonard Kroeker
Navy Personnel R&D Center
San Diego, CA 92152

1 Daryll Lang
Navy Personnel R&D Center
San Diego, CA 92152

1 Dr. William L. Maloy (02)
Chief of Naval Education and Training
Naval Air Station
Pensacola, FL 32508

1 Dr. James McBride
Navy Personnel R&D Center
San Diego, CA 92152

1 Dr William Montague
NPRDC Code 13
San Diego, CA 92152

1 Library, Code P201L
Navy Personnel R&D Center
San Diego, CA 92152

1 Technical Director
Navy Personnel R&D Center
San Diego, CA 92152

6 Personnel & Training Research Group
Code 442PT
Office of Naval Research
Arlington, VA 22217

1 Dr. Carl Ross
CNET-PDCD
Building 90
Great Lakes NTC, IL 60088

1 Mr. Drew Sands
NPRDC Code 62
San Diego, CA 92152

1 Mary Schratz
Navy Personnel R&D Center
San Diego, CA 92152

Navy

1 Dr. Robert G. Smith
Office of Chief of Naval Operations
OP-987H
Washington, DC 20350

1 Dr. Alfred F. Smode
Senior Scientist
Code 7B
Naval Training Equipment Center
Orlando, FL 32813

1 Dr. Richard Snow
Liaison Scientist
Office of Naval Research
Branch Office, London
Box 39
FPO New York, NY 09510

1 Dr. Richard Sorensen
Navy Personnel R&D Center
San Diego, CA 92152

1 Dr. Frederick Steinheiser
CNO - OP115
Navy Annex
Arlington, VA 20370

1 Mr. Brad Sympson
Navy Personnel R&D Center
San Diego, CA 92152

1 Dr. Frank Vicino
Navy Personnel R&D Center
San Diego, CA 92152

1 Dr. Ronald Weitzman
Naval Postgraduate School
Department of Administrative
Sciences
Monterey, CA 93940

1 Dr. Douglas Wetzel
Code 12
Navy Personnel R&D Center
San Diego, CA 92152

1 DR. MARTIN F. WISKOFF
NAVY PERSONNEL R& D CENTER
SAN DIEGO, CA 92152

1 Mr John H. Wolfe
Navy Personnel R&D Center
San Diego, CA 92152

Navy

1 Dr. Wallace Wulfeck, III
Navy Personnel R&D Center
San Diego, CA 92152

Marine Corps

1 Jerry Lehnus
CAT Project Office
HQ Marine Corps
Washington , DC 20380

1 Headquarters, U. S. Marine Corps
Code MPI-20
Washington, DC 20380

1 Special Assistant for Marine
Corps Matters
Code 100M
Office of Naval Research
800 N. Quincy St.
Arlington, VA 22217

1 Major Frank Yohannan, USMC
Headquarters, Marine Corps
(Code MPI-20)
Washington, DC 20380

Army

1 Dr. Kent Eaton
Army Research Institute
5001 Eisenhower Blvd.
Alexandria , VA 22333

1 Dr. Clessen Martin
Army Research Institute
5001 Eisenhower Blvd.
Alexandria, VA 22333

1 Dr. William E. Nordbrock
FMC-ADCO Box 25
APO, NY 09710

1 Dr. Harold F. O'Neil, Jr.
Director, Training Research Lab
Army Research Institute
5001 Eisenhower Avenue
Alexandria, VA 22333

1 Commander, U.S. Army Research Institute
for the Behavioral & Social Sciences
ATTN: PERI-BR (Dr. Judith Orasanu)
5001 Eisenhower Avenue
Alexandria, VA 22333

1 Mr. Robert Ross
U.S. Army Research Institute for the
Social and Behavioral Sciences
5001 Eisenhower Avenue
Alexandria, VA 22333

1 Dr. Robert Sasmor
U. S. Army Research Institute for the
Behavioral and Social Sciences
5001 Eisenhower Avenue
Alexandria, VA 22333

1 Dr. Joyce Shields
Army Research Institute for the
Behavioral and Social Sciences
5001 Eisenhower Avenue
Alexandria, VA 22333

1 Dr. Hilda Wing
Army Research Institute
5001 Eisenhower Ave.
Alexandria, VA 22333

Air Force

1 Dr. Earl A. Alluisi
HQ, AFHRL (AFSC)
Brooks AFB, TX 78235

1 Mr. Raymond E. Christal
AFHRL/MOE
Brooks AFB, TX 78235

1 Dr. Alfred R. Fregly
AFOSR/NL
Bolling AFB, DC 20332

1 Dr. Patrick Kyllonen
AFHRL/MOE

Brooks AFB, TX 78235

1 Dr. Randolph Park
AFHRL/MOAN
Brooks AFB, TX 78235

1 Dr. Roger Pennell
Air Force Human Resources Laboratory
Lowry AFB, CO 80230

1 Dr. Malcolm Ree
AFHRL/MP
Brooks AFB, TX 78235

1 Major John Welsh
AFHRL/MOAN
Brooks AFB , TX 78223

Department of Defense

12 Defense Technical Information Center
Cameron Station, Bldg 5
Alexandria, VA 22314
Attn: TC

1 Dr. Clarence McCormick
HQ, MEPCOM
MEPCT-P
2500 Green Bay Road
Noprth Chicago, IL 60064

1 Military Assistant for Training and
Personnel Technology
Office of the Under Secretary of Defens
for Research & Engineering
Room 3D129, The Pentagon
Washington, DC 20301

1 Dr. W. Steve Sellman
Office of the Assistant Secretary
of Defense (MRA & L)
2B269 The Pentagon
Washington, DC 20301

1 Dr. Robert A. Wisher
OUSDRE (ELS)
The Pentagon, Room 3D129
Washington, DC 20301

Civilian Agencies

1 Dr. Patricia A. Butler
NIE-BRN Bldg, Stop # 7
1200 19th St., NW
Washington, DC 20208

1 Dr. Vern W. Urry
Personnel R&D Center
Office of Personnel Management
1900 E Street NW
Washington, DC 20415

1 Mr. Thomas A. Warm
U. S. Coast Guard Institute
P. O. Substation 18
Oklahoma City, OK 73169

1 Dr. Joseph L. Young, Director
Memory & Cognitive Processes
National Science Foundation
Washington, DC 20550

Private Sector

1 Dr. Erling B. Andersen
Department of Statistics
Studiestraede 6
1455 Copenhagen
DENMARK

1 Dr. Isaac Bejar
Educational Testing Service
Princeton, NJ 08450

1 Dr. Menucha Birenbaum
School of Education
Tel Aviv University
Tel Aviv, Ramat Aviv 69978
Israel

1 Dr. Werner Birke
Personalstammamt der Bundeswehr
D-5000 Koeln 90
WEST GERMANY

1 Dr. R. Darrell Bock
Department of Education
University of Chicago
Chicago, IL 60637

1 Mr. Arnold Bohrer
Section of Psychological Research
Caserne Petits Chateau
CRS
1000 Brussels
Belgium

1 Dr. Robert Brennan
American College Testing Programs
P. O. Box 168
Iowa City, IA 52243

1 Dr. Glenn Bryan
6208 Poe Road
Bethesda, MD 20817

1 Dr. Ernest R. Cadotte
307 Stokely
University of Tennessee
Knoxville, TN 37916

1 Dr. John B. Carroll
409 Elliott Rd.
Chapel Hill, NC 27514

Private Sector

1 Dr. Norman Cliff
Dept. of Psychology
Univ. of So. California
University Park
Los Angeles, CA 90007

1 Dr. Hans Crombag
Education Research Center
University of Leyden
Boerhaavelaan 2
2334 EN Leyden
The NETHERLANDS

1 Lee Cronbach
16 Laburnum Road
Atherton, CA 94205

1 CTB/McGraw-Hill Library
2500 Garden Road
Monterey, CA 93940

1 Mr. Timothy Davey
University of Illinhois
Department of Educational Psychology
Urbana, IL 61801

1 Dr. Dattpradad Divgi
Syracuse University
Department of Psychology
Syracuse, NE 33210

1 Dr. Emmanuel Donchin
Department of Psychology
University of Illinois
Champaign, IL 61820

1 Dr. Hei-Ki Dong
Ball Foundation
800 Roosevelt Road
Building C, Suite 206
Glen Ellyn, IL 60137

1 Dr. Fritz Drasgow
Department of Psychology
University of Illinois
603 E. Daniel St.
Champaign, IL 61820

1 Dr. Stephen Dunbar
Lindquist Center for Measurement
University of Iowa
Iowa City, IA 52242

Private Sector

1 Dr. John M. Eddins
University of Illinois
252 Engineering Research Laboratory
103 South Mathews Street
Urbana, IL 61801

1 Dr. Susan Embertson
PSYCHOLOGY DEPARTMENT
UNIVERSITY OF KANSAS
Lawrence, KS 66045

1 ERIC Facility-Acquisitions
4833 Rugby Avenue
Bethesda, MD 20014

1 Dr. Benjamin A. Fairbank, Jr.
Performance Metrics, Inc.
5825 Callaghan
Suite 225
San Antonio, TX 78228

1 Dr. Leonard Feldt
Lindquist Center for Measurement
University of Iowa
Iowa City, IA 52242

1 Univ. Prof. Dr. Gerhard Fischer
Liebiggasse 5/3
A 1010 Vienna
AUSTRIA

1 Professor Donald Fitzgerald
University of New England
Armidale, New South Wales 2351
AUSTRALIA

1 Dr. Dexter Fletcher
University of Oregon
Department of Computer Science
Eugene, OR 97403

1 Dr. John R. Frederiksen
Bolt Beranek & Newman
50 Moulton Street
Cambridge, MA 02138

1 Dr. Janice Gifford
University of Massachusetts
School of Education
Amherst, MA 01002

Private Sector

1 Dr. Robert Glaser
Learning Research & Development Center
University of Pittsburgh
3939 O'Hara Street
PITTSBURGH, PA 15260

1 Dr. Marvin D. Glock
217 Stone Hall
Cornell University
Ithaca, NY 14853

1 Dr. Bert Green
Johns Hopkins University
Department of Psychology
Charles & 34th Street
Baltimore, MD 21218

1 DR. JAMES G. GREENO
LRDC
UNIVERSITY OF PITTSBURGH
3939 O'HARA STREET
PITTSBURGH, PA 15213

1 Dipl. Pad. Michael W. Habon
Universitat Dusseldorf
Erziehungswissenshaftliches Inst. II
Universitatsstr. 1
D-4000 Dusseldorf 1
WEST GERMANY

1 Dr. Ron Hambleton
School of Education
University of Massachusetts
Amherst, MA 01002

1 Prof. Lutz F. Hornke
Universitat Dusseldorf
Erziehungswissenschaftliches Inst. II
Universitatsstr. 1
Dusseldorf 1
WEST GERMANY

1 Dr. Paul Horst
677 G Street, #184
Chula Vista, CA 90010

1 Dr. Lloyd Humphreys
Department of Psychology
University of Illinois
603 East Daniel Street
Champaign, IL 61820

Private Sector

1 Dr. Steven Hunka
Department of Education
University of Alberta
Edmonton, Alberta
CANADA

1 Dr. Jack Hunter
2122 Coolidge St.
Lansing, MI 48906

1 Dr. Huynh Huynh
College of Education
University of South Carolina
Columbia, SC 29208

1 Dr. Douglas H. Jones
Advanced Statistical Technologies
Corporation
10 Trafalgar Court
Lawrenceville, NJ 08148

1 Professor John A. Keats
Department of Psychology
The University of Newcastle
N.S.W. 2308
AUSTRALIA

1 Dr. William Koch
University of Texas-Austin
Measurement and Evaluation Center
Austin, TX 78703

1 Dr. Thomas Leonard
University of Wisconsin
Department of Statistics
1210 West Dayton Street
Madison, WI 53705

1 Dr. Alan Lesgold
Learning R&D Center
University of Pittsburgh
3939 O'Hara Street
Pittsburgh, PA 15260

1 Dr. Michael Levine
Department of Educational Psychology
210 Education Bldg.
University of Illinois
Champaign, IL 61801

Private Sector

1 Dr. Charles Lewis
Faculteit Sociale Wetenschappen
Rijksuniversiteit Groningen
Oude Boteringestraat 23
9712BC Groningen
Netherlands

1 Dr. Robert Linn
College of Education
University of Illinois
Urbana, IL 61801

1 Dr. Robert Lockman
Center for Naval Analysis
200 North Beauregard St.
Alexandria, VA 22311

1 Dr. Frederic M. Lord
Educational Testing Service
Princeton, NJ 08541

1 Dr. James Lumsden
Department of Psychology
University of Western Australia
Nedlands W.A. 6009
AUSTRALIA

1 Dr. Gary Marco
Stop 31-E
Educational Testing Service
Princeton, NJ 08451

1 Mr. Robert McKinley
American College Testing Programs
P.O. Box 168
Iowa City, IA 52243

1 Dr. Barbara Means
Human Resources Research Organization
300 North Washington
Alexandria, VA 22314

1 Dr. Robert Mislevy
711 Illinois Street
Geneva, IL 60134

1 Dr. W. Alan Nicewander
University of Oklahoma
Department of Psychology
Oklahoma City, OK 73069

Private Sector

1 Dr. Melvin R. Novick
356 Lindquist Center for Measurment
University of Iowa
Iowa City, IA 52242

1 Dr. James Olson
WICAT, Inc.
1875 South State Street
Orem, UT 84057

1 Wayne M. Patience
American Council on Education
GED Testing Service, Suite 20
One Dupont Cirle, NW
Washington, DC 20036

1 Dr. James Paulson
Dept. of Psychology
Portland State Unive3rsity
P.O. Box 751
Portland, OR 97207

1 Dr. Mark D. Reckase
ACT
P. O. Box 168
Iowa City, IA 52243

1 Dr. Lawrence Rudner
403 Elm Avenue
Takoma Park, MD 20012

1 Dr. J. Ryan
Department of Education
University of South Carolina
Columbia, SC 29208

1 PROF. FUMIKO SAMEJIMA
DEPT. OF PSYCHOLOGY
UNIVERSITY OF TENNESSEE
KNOXVILLE, TN 37916

1 Frank L. Schmidt
Department of Psychology
Bldg. 66
George Washington University
Washington, DC 20052

1 Lowell Schoer
Psychological & Quantitative
Foundations
College of Education
University of Iowa
Iowa City, IA 52242

Private Sector

1 Dr. Kazuo Shigemasu
7-9-24 Kugenuma-Kaigan
Fujusawa 251
JAPAN

1 Dr. William Sims
Center for Naval Analysis
200 North Beauregard Street
Alexandria, VA 22311

1 Dr. H. Wallace Sinaiko
Program Director
Manpower Research and Advisory Services
Smithsonian Institution
801 North Pitt Street
Alexandria, VA 22314

1 Martha Stocking
Educational Testing Service
Princeton, NJ 08541

1 Dr. Peter Stoloff
Center for Naval Analysis
200 North Beauregard Street
Alexandria, VA 22311

1 Dr. William Stout
University of Illinois
Department of Mathematics
Urbana, IL 61801

1 Dr. Hariharan Swaminathan
Laboratory of Psychometric and
Evaluation Research
School of Education
University of Massachusetts
Amherst, MA 01003

1 Dr. Kikumi Tatsuoka
Computer Based Education Research Lab
252 Engineering Research Laboratory
Urbana, IL 61801

1 Dr. Maurice Tatsuoka
220 Education Bldg
1310 S. Sixth St.
Champaign, IL 61820

1 Dr. David Thissen
Department of Psychology
University of Kansas
Lawrence, KS 66044

Private Sector

1 Mr. Gary Thomasson
University of Illinois
Department of Educational Psychology
Champaign, IL 61820

1 Dr. Robert Tsutakawa
Department of Statistics
University of Missouri
Columbia, MO 65201

1 Dr. Ledyard Tucker
University of Illinois
Department of Psychology
603 E. Daniel Street
Champaign, IL 61820

1 Dr. V. R. R. Uppuluri
Union Carbide Corporation
Nuclear Division
P. O. Box Y
Oak Ridge, TN 37830

1 Dr. David Vale
Assessment Systems Corporation
2233 University Avenue
Suite 310
St. Paul, MN 55114

1 Dr. Howard Wainer
Division of Psychological Studies
Educational Testing Service
Princeton, NJ 08540

1 Dr. Ming-Mei Wang
Lindquist Center for Measurement
University of Iowa
Iowa City , IA 52242

1 Dr. Brian Waters
HumRRO
300 North Washington
Alexandria, VA 22314

1 Dr. David J. Weiss
N660 Elliott Hall
University of Minnesota
75 E. River Road
Minneapolis, MN 55455

1 Dr. Rand R. Wilcox
University of Southern California
Department of Psychology
Los Angeles, CA 90007

Private Sector

1 German Military Representative
ATTN: Wolfgang Wildegrube
Streitkraefteamt
D-5300 Bonn 2
4000 Brandywine Street, NW
Washington , DC 20016

1 Dr. Bruce Williams
Department of Educational Psychology
University of Illinois
Urbana, IL 61801

1 Ms. Marilyn Wingersky
Educational Testing Service
Princeton, NJ 08541

1 Dr. George Wong
Biostatistics Laboratory
Memorial Sloan-Kettering Cancer Center
1275 York Avenue
New York, NY 10021

1 Dr. Wendy Yen
CTB/McGraw Hill
Del Monte Research Park
Monterey, CA 93940

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