SCIENTIFIC ACHIEVEMENTS AND ACTIVITIES
SPONSORED BY U.S. AIR FORCE OFFICE OF SCIENTIFIC RESEARCH
UNDER GRANT AFOSR 80-0170

PERIOD: AUGUST 1, 1982 - JULY 31, 1983

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This report summarizes the scientific achievements and activities sponsored by the Air Force Office of Scientific Research under Grant AFOSR-80-0170 for the period August 1, 1982 - July 31, 1983. The report is divided into parts. Part I relates to the achievements and activities in the area of design of experiments and Part II relates to the achievements and activities in the area of reliability models.
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SUMMARY

This report summarizes the scientific achievements and activities sponsored by the Air Force Office of Scientific Research under Grant AFOSR 80-0170 for the period August 1, 1982 - July 31, 1983. The report is divided into parts. Part I relates to the achievements and activities in the area of design of experiments and Part II relates to the achievements and activities in the area of reliability models.
PART I: DESIGN OF EXPERIMENTS

I. INTRODUCTION

We have developed statistical design and methodology applicable to U.S. Air Force problems. Our research activities related directly to problems of data collection and analysis relevant to virtually all Air Force technical areas. There is a strong need in the Air Force to reduce costs and save time in the collection and analysis of large amounts of data, such as communication, engineering, equipment testing, and aerospace medicine data. The reduction in costs and time should be done clearly without any damage to the statistical quality of the data being collected and analyzed. Our new discoveries not only add to our store of knowledge about the multiple facets of data collection and data analysis in general, but they have immediate applications to many important problems which the United States Air Force is faced with. For example, our discoveries include, but are not limited to:

1. Repeated measurements.

One class of designs received special attention was repeated measurements designs, necessary and indispensable in such investigations as:

(i) Sometimes the experimental units are scarce and/or costly and thus have to be used in several experiments. This will frequently be the case in small clinics and in the development of large military systems such as aerospace vehicles, airplanes, radars, computers, etc.

(ii) In certain situations, the experimenter might be interested in discovering whether or not a trend can be traced among the responses obtained by successive applications of several treatments on a single experimental unit. For example, if one wants to measure the degree of adaptation to darkness over time, the most efficient use of subjects requires that each subject be treated at all points in the time of interest.

(iii) Human performance is an important factor in the safety of a system, but the effect of human performance decreases by non weapons' effects such as fatigue and crew task overload. Repeated measurements designs allow for measuring and evaluating the residual effects as the results of fatigue, etc.

Because of the sequential nature of exposing the units to various treatments, residual effects are likely to occur. To measure or eliminate residual effects, in contrast to direct effects, several models were considered which are capable of mimicking the true model of response under various experimental settings. We discovered and cataloged many efficient designs in
the area of repeated measurements designs.

2. **Optimal designs.**

A design describes the way a certain statistical experiment is to be conducted. It generally specifies where the observations are to be taken and in what proportion at each location. In many settings the design is, within limits, subject to choice by the experimenter. This is where a design specialist can help the scientist choose his design judiciously. One consideration is to identify a design which is efficient in some meaningful statistical sense and meet the budgetary constraints. We discovered several families of efficient designs along the lines indicated in our previous proposal. In addition, we characterized, constructed and cataloged optimal designs for comparing some special treatments called controls with several rival treatments. Whenever an existing system is considered for replacement by one of several sophisticated or modern systems, the primary question is whether any of the new systems is truly superior in performance to justify the expenditure involved in the replacement. The existing system could be any equipment used in airplanes or on the ground, any office system like a computer or any drug, etc. The existing systems are the controls and the more modern systems the rival treatments. We investigated the problem of finding most efficient designs for bringing out possible differences between the controls and the treatments. This area of research is relatively new and there are not many known results. We intend to extend the catalog of Hedayat and Majumdar and get efficient designs, at least for all parameter values useful in practice.

3. **The trade off methodology.**

The judicious choice of design points (e.g., stress levels) and the distributions of observations over the chosen points are the two most powerful methods for cutting the cost of experimentation without losing the essential statistical information needed for data analysis and modeling. The method of trade off introduced for the first time into the literature of design of experiments by Hedayat and developed by Hedayat and his several coauthors is a powerful technique for selecting and reducing the site and the number of design points. The method was successfully applied in the area of block designs and also in the area of finite population sampling by us. We cataloged the newly discovered designs. We believe there is a great promise in this method and we intend to perform more research in this area and eventually extend the methodology into other branches of experimentation useful to practitioners.
4. **Robust optimal designs.**

Another class of designs considered were robust optimal designs, necessary and useful in such studies as combat damage and loss data collection analysis. In general, such designs are needed for performing experiments in hostile (man-made or non-natural) environment which very often results in incomplete and/or questionable data. Our discoveries in this area are in preliminary stages. We intend to pursue this very useful area of research.

II. **RESEARCH ACCOMPLISHMENTS AND RELATED ACTIVITIES**

Our research activities can be classified into seven broad categories:

A. Production of twenty-one scientific reports;
B. Production of a catalog of efficient designs;
C. Production of a Ph.D. dissertation;
D. Presenting invited talks at international conferences held in the U.S. and abroad, presenting colloquium talks and inviting researchers to our campus;
E. Research work in progress.

We shall explain in some detail our effort in each of the seven categories.

A. **Production of twenty-one scientific reports**

In this category we have produced twenty-one scientific reports - the status of their publications in professional journals are as follows:

Twelve of these reports are already in print or in press, the remaining reports are either under consideration of publication or under revision for possible publication. The reports are:

**Papers in print**


**Papers in press**

1. Hedayat, A. A characterization of a universally optimal design within a class of block designs. *J. Statistical Planning and Inference*.


3. Hedayat, A. and Hwang, H.L. BIB(8,56,21,3,6) and BIB(10,30,9,3,2) designs with repeated blocks. *J. Combinatorial Theory, Series A*.


**Papers submitted**


B. Production of catalogs of efficient designs

In performing research we have always kept the applicability of our results in mind. We have never generalized or extended a result without justifying the practical importance of the problem under construction.

The results we have obtained may not be accessible to practitioners with limited knowledge of the subject at hand. In our formal publications we had to be very technical. While such publications will be easily accessible to research workers in the area, it would certainly be very hard, if not impossible, to be understood by practitioners. To overcome this deficiency we prepared nonformal versions of our results understandable to non-specialists. During the academic year 1982-1983 we prepared a catalog of efficient designs under the title of "A-optimal designs for control-test treatment comparisons".

Remark: Thus far under AFOSR Grants we have prepared 4 catalogs of optimal designs.
C. Production of Ph.D. dissertation

As an institution for higher education and research at University of Illinois at Chicago, we direct our research assistants to prepare and defend their Ph.D. dissertations. During the academic year 1982-1983 Helen L. Hwang prepared and defended her Ph.D. dissertation under the direction of A.S. Hedayat. The title of her thesis was "On (k,t) trades and the construction of BIB designs with repeated blocks".

Remark: Thus far 5 research assistants working on AFOSR projects have completed their Ph.D. dissertations under the direction of A.S. Hedayat. These graduates are now successfully engaged in research and teaching in universities and industries in the United States.

D. Presenting invited talks at international conferences held in the U.S. and abroad, presenting colloquium talks and inviting researchers to our campus

As invited speakers, we presented our newly discovered results at the following international meetings:

1. The First Saudi Symposium on Statistics and Its Applications held in King Saud University, Riyadh, Saudi Arabia, May 2-5, 1983.
2. Jack Kiefer and Jacob Wolfowitz Memorial Statistical Research Conference held at Cornell University, July 6-9, 1983.

As a colloquium speaker A.S. Hedayat was invited to present the newly discovered results by us at the following institutions.

1. Department of Statistics, King Saud University, Riyadh, Saudi Arabia, 9/82.
2. Department of Statistics, North Carolina State University, Raleigh, NC, 12/82.
3. Department of Statistics, University of Chicago, Chicago, IL, 2/83.

During the academic 1982-1983 we sponsored the visit of several internationally known researchers. These researchers presented talks and discussed with us their research activities related to ours.

The participations in international meetings, presenting colloquium talks at various universities and inviting researchers to our campus have been quite stimulating as we were able to discuss in person with other scientists around the world. Through these meetings it has become possible to generate an informal line of communication with other researchers from universities in the United States and other countries—such informal contacts are essential in pursuing our research.
E. **Research work in progress**

Currently several major problems in experimental design are under investigation by Hedayat and his co-researchers in the Department of Mathematics, Statistics, and Computer Science, University of Illinois at Chicago. In addition, Hedayat is collaborating on research via correspondence with his co-researchers at the University of California, Berkeley; University of Indiana, Bloomington; Hiroshima University (Japan); University of Guelph (Canada), Cornell University, Ithaca, Indian Statistical Institute, Calcutta, and several other researchers in other institutions. The area of research includes, but is not limited to, repeated measurements designs, trade off in designs, t-designs, robust designs, and optimal designs for comparing treatments with one or more controls.
PAFT II: RELIABILITY MODELS

I. INTRODUCTION

Since the beginning of the present grant on August 1, 1982 we have continued our study of the following reliability models:

(i) Classes of multistate (degradable) coherent systems.

(ii) Multivariate classes of new better than used (NBU) stochastic processes which are useful in modeling the joint states of degradable components forming a degradable system.

(iii) Multivariate classes of NBU and IFRA (increasing failure rate average) life distributions which are useful in modeling the joint life times of system components which are not necessarily independent.

(iv) The various models of dependence available in the literature such as positive quadrant dependence and association. The relationships between these concepts and necessary and sufficient conditions for them are very useful in understanding more deeply those various forms of statistical dependence.

We have also continued our study of the following topic:

(v) The possible relationships between the theory of multistate systems and the relatively new theory of "fuzzy subsets". Such relationships should enrich both areas with new results and new fields of application.

Our research efforts in the above mentioned topics have been, and are continuing to be, fruitful in yielding useful and interesting results. This is demonstrated by the list of accomplishments given in the following section.

II. RESEARCH ACCOMPLISHMENTS AND RELATED ACTIVITIES

The following paper has been revised and accepted for publication


The following papers have been completed and submitted for publication

2) Characterizations of Multivariate Classes of Life Distributions.


The following report is under preparation

The following invited talk was given
5) Recent Results in Multistate Systems Theory (with F. Proschan), Sixth

The following contributed talk was given
6) A Class of Multivariate New Better Than Used Processes, The First

In order to maintain the flow of the meaningful research ideas which yield
the various useful results demonstrated in the above list, we have maintained an
active program aimed at stimulating such ideas. The following are few examples of
such activities:

i) Participating in a weekly seminar on reliability models.

ii) Serving as a referee for many statistical journals and a reviewer for the
Mathematical Reviews.

iii) Visiting the reliability center at Florida State University for a one month
period.