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USAF DURABILITY DESIGN HANDBOOK: GUIDELINES FOR THE ANALYSIS AND DESIGN OF DURABLE AIRCRAFT STRUCTURES

S. D. Manning
General Dynamics Corporation
Fort Worth Division
P.O. Box 748
Fort Worth, Texas 76101

and

J. N. Yang
The George Washington University
Washington, D. C. 20052

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James L. Rudd
Project Engineer

Davey L. Smith
Structural Integrity Branch
Structures & Dynamics Division

FOR THE COMMANDER:

Ralph L. Kuster, Jr., Colonel, USAF
Chief, Structures & Dynamics Division

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20. ABSTRACT (Continue on reverse side if necessary and identify by block number):

This is the first edition of the Durability Design Handbook. Objectives of the handbook are: (1) summarize and interpret the essential U.S. Air Force Durability Design requirements for metallic airframes, (2) provide durability analysis criteria for economic life and durability critical parts, (3) provide state-of-the-art durability analysis concepts and methods for quantifying the extent of damage for a population of details (e.g., fastener holes, (Continued)

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fillets, cutouts, lugs, etc.) as a function of time, (4) provide guidelines and design data for implementing the durability methodology and for assisting contractor and USAF personnel in complying with the intent of the durability specifications for metallic airframes. This document, loosely called a "Handbook", provides guidelines, concepts, analytical tools and the framework for incorporating future durability methodology advancements and design data.

The durability analysis methodology, based on a probabilistic fracture mechanics approach, accounts for initial fatigue quality, fatigue crack growth accumulation in a population of structural details, load spectra and structural properties. A statistical distribution of equivalent initial flaw sizes is used to represent the initial fatigue quality of structural details (e.g., fastener holes, fillets, cutouts, lugs, etc.). The equivalent initial flaw size distribution is grown forward, using the applicable design conditions, to a selected service time using a deterministic crack growth relationship. Procedures are described and illustrated. The durability analysis methodology has been demonstrated for quantifying the extent of damage due to relatively small cracks in clearance-fit fastener holes (e.g., $\leq$ 0.10"). Further research is needed to extend the methodology to larger crack sizes and to verify the methodology for other details, such as, fillets, cutouts and lugs. The effects of fretting, faying surface sealant, fastener clamp-up, environment, interference-fit fasteners, etc., on the initial fatigue quality need to be investigated. An evaluation is made of the accuracy of the durability analysis by correlating analytical predictions with test data for a fighter full-scale test article and complex splice specimens subjected to a bomber load spectrum.

FOREWORD

This handbook was prepared by General Dynamics, Fort Worth Division, and by George Washington University under Phase III of the "Durability Methods Development" program (Air Force Contract F33615-77-C-3123) for the Air Force Wright Aeronautical Laboratories (AFWAL/FIBEC). James L. Rudd was the Air Force Project Engineer and Dr. Jack W. Lincoln of ASD/ENFS was a technical advisor for the program. Dr. B. G. W. Yee of the General Dynamics' Materials Research Laboratory was the Program Manager and Dr. Sherrell D. Manning was the Principal Investigator. Dr. J. N. Yang of George Washington University (Washington, D.C.) and Dr. M. Shinozuka of Modern Analysis Incorporated (Ridgewood, New Jersey) were associate investigators.

This program was supported by several General Dynamics' personnel as follows: All tests were performed in General Dynamics' Metallurgy Laboratory by R. O. Nay under the direction of F. C. Nordquist. W. T. Kaarlela was responsible for the fractographic data acquisition. Fractographic readings were made by D. E. Gordon, W. T. Kaarlela, A. Meder, R. O. Nay and S. M. Speaker. S. M. Speaker coordinated the testing and fractographic data acquired and supported the initial fatigue quality model

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calibration/evaluation studies. J. W. Norris developed the computer software for storing and analyzing the fractographic data, supported the initial fatigue quality model calibration/evaluation studies and worked on a preliminary version of the handbook. B. J. Pendley and S. P. Henslee conducted the aircraft structural durability survey. Dr. Y. H. Kim, Dr. W. R. Garver and M. A. Flanders contributed to the durability analysis state-of-the-art assessment. F-16 durability test results and supporting data for the durability analysis demonstration were provided by J. W. Morrow, V. Juarez, D. R. McSwain, and P. D. Hudson. Dr. V. D. Smith supported the modeling and statistical analysis effort. Photoelastic investigations were conducted by T. E. Love. Typing was performed by Peggy Thomas and Ernestine Bruner. Ron Jordan prepared many of the illustrations and Joe Conder provided printing and editorial support.

This handbook is the final product of the "Durability Methods Development" program. The U.S. Air Force durability design requirements are reviewed and methodology (i.e., economic life criteria, analytical tools, guidelines, design data, etc.) for satisfying these requirements are described and discussed.

The following reports (AFFDL-TR-79-3118) were also prepared under the "Durability Methods Development" program:

Phase I Reports

- Vol. I - Phase I Summary
- Vol. II - Durability Analysis: State-of-the-art Assessment
- Vol. III - Structural Durability Survey: State-of-the-art Assessment
- Vol. IV - Initial Fatigue Quality Representation
- Vol. V - Durability Analysis Methodology Development
- Vol. VI - Documentation of Computer Programs for Initial Quality Representation (Vol. IV)

Phase II Reports

- Vol. VII - Phase II Documentation
- Vol. VIII- Test and Fractography Data
- Vol. IX - Documentation of Durability Analysis Computer Program

This handbook covers work accomplished during the period July 1981 through January 1984.

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LIST OF SYMBOLS

a	=	Crack size
a_{DL}	=	Durability limit flaw size
a_e	=	Economic repair size limit
a_0	=	Reference crack size for given TTCI's
$a(0)$	=	Crack size at t=0
a_{RL}	=	Repair limit flaw size
$a(t)$, $a(t_1)$, $a(t_2)$	=	Crack size at time t, t_1 and t_2, respectively
a_U, a_L	=	Upper and lower bound fractographic crack size, respectively, used to define the IFQ model parameters
$a(\tau)$	=	Crack size at service time τ
b, Q	=	Crack growth parameters in the equation $\frac{da(t)}{dt}=Q\,[a(t)]^{b}$. Used in conjunction with the IFQ model.
b_i, Q_i	=	Crack growth constants in $\frac{da(t)}{dt} = Q_i\,[a(t)]^{b_i}$ for the ith stress region when this equation is used in conjunction with the service crack growth master curve (SCGMC).
b_i^*, Q_i^*	=	Crack growth constants in $\frac{da(t)}{dt}=Q_i^*\,[a(t)]^{b_i^*}$ for the ith fractographic data set, where $b_i^* = 1.0$. Notation used in conjunction with fractographic data pooling procedures and the EIFS master curve for the ith fractographic data set.

$b^*_{i,j}$, $Q^*_{i,j}$	=	Crack growth constants in $\frac{da(t)}{dt} = Q^*_{i,j}[a(t)]^{b^*_{i,j}}$ for the ith fractographic data set and the jth fractographic sample from the ith data set, where $b^*_{i,j} \geq 1.0$. Notation distinguishes parameters for each sample in a given fractographic data set.
c	=	b - 1; Used in conjunction with the IFQ model when the crack growth law, $\frac{da(t)}{dt} = Q[a(t)]^b$ is used and $b > 1.0$.
c_i	=	$b_i - 1$; Used in conjunction with the SCGMC when $\frac{da(t)}{dt} = Q_i[a(t)]^{b_i}$ is used. The subscript "i" refers to the ith stress region.
CFA	=	Conventional Fatigue Analysis (Palmgren-Miner rule)
DCGA	=	Deterministic Crack Growth Approach
$\frac{da(t)}{dt}$	=	Crack growth rate as a function of time
EIFS	=	Equivalent initial flaw size
$f_{a(0)}(x)$	=	EIFS probability density function = $\frac{dF_{a(0)}(x)}{dx}$
$f_T(t)$	=	$\frac{dF_T(t)}{dt}$
$F_{a(0)}(x)$	=	EIFS cumulative distribution function
$F_T(t)$	=	TTCI cumulative distribution function
FHQ	=	Fastener hole quality
IFQ	=	Initial fatigue quality

ℓ	=	Factor used in conjunction with the fractographic results for a single TTCI data set and the equation: $\beta = \beta_\ell(\ell)^{1/\alpha}$. ℓ is the number of equally-stressed fastener holes per test specimen in which <u>only</u> the largest fatigue crack in <u>any</u> "one" hole per specimen is included in the fractographic data set.
ℓ_i =	=	Same as ℓ with the subscript "i" referring to the ith TTCI data set and used in the equation: $\beta_i = \beta_{\ell_i}(\ell_i)^{1/\alpha_i}$.
$L(\tau), \bar{L}(\tau)$	=	Total and average number of details, respectively, in the entire component having a crack size $>x_1$ at any service time τ
LT	=	Load transfer through the fastener
N_i	=	Total number of details in the ith stress region
N^*	=	Total number of details in the entire durability critical component
$N(i,\tau)$, $\bar{N}(i,\tau)$	=	Total and average number of details, respectively, having a crack size exceeding x_1 at any service time τ
NLT	=	No load transfer through the fastener
P	=	Probability or exceedance probability
PFMA	=	Probabilistic Fracture Mechanics Approach
$p(i,\tau)$	=	Probability that a detail in the ith stress region will have a crack size $>x_1$ at the service time τ

$\hat{Q}_i$ = $\dfrac{\text{Ave. } Q_i^* \beta_i}{\beta_i} = \dfrac{\frac{1}{n}\sum_{i=1}^{n} Q_i^* \beta_i}{\beta_i}$; Normalized crack growth parameter, Q_i^*, for the ith fractographic data set when the crack growth equation, $\dfrac{da(t)}{dt} = Q_i^* [a(t)]^{b_i^*}$ is used and $b_i^* = 1.0$. Used in conjunction with the EIFS master curve for the ith fractographic data set.

$\hat{Q}_{\ell_i}$ = $\dfrac{\text{Ave. } Q_i^* \beta_{\ell_i}}{\beta_{\ell_i}} = \dfrac{\frac{1}{n}\sum_{i=1}^{n} Q_i^* \beta_{\ell_i}}{\beta_{\ell_i}}$

$Q\beta$ = $\text{Ave. } Q_i^* \beta_i = \frac{1}{n}\sum_{i=1}^{n} Q_i^* \beta_i = \frac{1}{n}\sum_{i=1}^{n} Q_i^* \beta_{\ell_i} (\ell_i)^{1/\alpha_i}$ = constant for "generic" EIFS cumulative distribution.

$Q\beta_\ell$ = $\text{Ave } Q_i^* \beta_{\ell_i} = \frac{1}{n}\sum_{i=1}^{n} Q_i^* \beta_{\ell_i}$. Used when checking IFQ model goodness-of-fit when fractography is available <u>only</u> for the largest fatigue crack in any one ℓ_i fastener hole per test specimen.

SCGMC = Service crack growth master curve

t, t_1, t_2 = Flight hours at t, t_1, t_2, respectively.

T, TTCI = Time-to-crack-initiation

x = Crack size

x_1 = Crack size used for $p(i,\tau)$ predictions

x_u = Upper bound limit for EIFS

$y_{1i}(\tau)$ = An EIFS in the IFQ distribution corresponding to a crack size x_1 at time τ in the ith stress region. Value determined using the SCGMC.

Z	=	No. of standard deviations from the mean
α, β, ε,	=	Weibull distribution parameters for shape, scale, and lower bound TTCI, respectively. Used in conjunction with the IFQ distribution or for a single TTCI data set.
α_i, β_i, ε_i	=	Weibull distribution parameters for the ith TTCI data set for shape, scale, and lower bound TTCI, respectively. Used in conjunction with fractographic data pooling procedures ($i = 1,\ldots,n$ data sets).
β_ℓ	=	Weibull scale parameter for TTCI based on the TTCI's for a given fractographic data set in which only the fractography for the largest fatigue crack in any "one" of ℓ fastener holes per test specimen is used to define β_ℓ. Note: $\beta = \beta_\ell\ (\ell)^{1/\alpha}$
β_{ℓ_i}	=	Same as β_ℓ with the subscript "i" denoting the β_ℓ value for the ith TTCI data set and is used to determine β_i as follows: $\beta_i = \beta_{\ell_i}(\ell_i)^{1/\alpha_i}$.
$\Gamma(\ )$	=	Gamma function
ξ, γ	=	Empirical constants in the equation: $Q_i = \xi\sigma^\gamma$, where σ = stress
σ	=	Stress or standard deviation
$\sigma_N^2(i,\tau)$, $\sigma_L^2(i,\tau)$	=	Variance of $N(i,\tau)$ and $L(\tau)$, respectively
τ	=	A particular service time

TERMINOLOGY

1. Crack Size - is the length of a crack in a structural detail in the direction of crack propagation.

2. Deterministic Crack Growth - Crack growth parameters are treated as deterministic values resulting in a single value prediction for crack length.

3. Durability - is a quantitative measure of the airframe's resistance to fatigue cracking under specified service conditions. Structural durability is normally concerned with relatively small subcritical crack sizes which affect functional impairment, structural maintenance requirements and life-cycle-costs. Such cracks may not pose an immediate safety problem. However, if the structural details containing such cracks are not repaired, economical repairs cannot be made when these cracks exceed a limiting crack size. The entire population of structural details in various components is susceptible to fatigue cracking in service. Therefore, a statistical approach is essential to quantitatively assess the "durability" of a part, a component, or an airframe.

4. Durability Analysis - is concerned with quantifying the extent of structural damage due to fatigue cracking for structural details (e.g.. fastener hole, fillet, cutout, lug, etc.) as a function of service time. Results are used to ensure design compliance with Air Force durability design requrements.

5. Economic Life - is that point in time when an aircraft structure's damage state due to fatigue, accidental damage and/or environmental deterioration reaches a point where operational readiness goals cannot be preserved by economically acceptable maintenance action.

6. Economic Life Criteria - are guidelines and formats for defining quantitative economic life requirements for aircraft structure to satisfy U.S. Air Force durability design requirements. The economic life criterion provides the basis for analytically and experimentally ensuring design compliance of aircraft structure with durability design requirements. Two recommended formats for economic life criteria are:

- o probability of crack exceedance
- o cost ratio: repair cost/replacement cost

7. Economic Repair Limit - is the maximum damage size that can be economically repaired (e.g., repair 0.03" - 0.05" radial crack in fastener holes by reaming hole to next size).

8. Equivalent Initial Flaw Size (EIFS) - is a hypothetical crack assumed to exist in the structure prior to service. It characterizes the equivalent effect of actual initial flaws in a structural detail. It is determined by back-extrapolating fractographic results. An equivalent initial flaw is assumed to have the same flaw shape and origin as the observable crack size at a given time. The EIFS concept is a convenient "mathematical tool" for quantifying the IFQ for structural details and the probability of crack exceedance or extent of damage as a function of time. An EIFS is strictly a mathematical quantity rather than an actual initial flaw size. Within this context, EIFS's can be positive or negative, depending on the fractographic results and the back extrapolation method used. EIFS values depend on several factors, including: the fractographic results used (and the test variables reflected), the fractographic crack size range used, the form of the crack growth equation used for the back-

extrapolation, the goodness of the curve fit to the fractographic data, the manufacturing quality of the structural details, fastener hole type, fastener type and fit, etc. EIFS's for different fractographic data are not comparable unless the applicable IFQ model parameters are determined consistently (e.g., same fractographic crack size range used, same "b" value imposed if the $Q[a(t)]^b$ crack growth model is used, same α values imposed for comparable fractographic data sets, etc.). Fractographic data pooling is essential to quantify the IFQ for different fractographic data sets on a common baseline.

9. EIFS Master Curve - is a curve (e.g., equation, tabulation of a(t) vs. t or curve without prescribed functional form) used to determine the EIFS value at t=0 corresponding to a given TTCI value at a specified crack size. Such a curve is needed to determine the IFQ distribution from the TTCI distribution. The EIFS master curve depends on several factors, such as the fractographic data base, the fractographic crack size range used, the functional form of the crack growth equation used in the curve fit, etc. (Ref. EIFS).

10. Extent of Damage - is a quantitative measure of structural durability at a given service time. For example, the number of structural details (e.g. fastener holes, cutouts, fillets, etc.) or percentage of details exceeding specified crack size limits. Crack length is the fundamental measure for structural damage. The predicted extent of damage is compared with the specified economic life criterion for ensuring design compliance with U.S. Air Force durability requirements.

11. Generic EIFS Distribution - An EIFS distribution is "generic" if it depends only on the material and manufacturing/fabrication processes. Theoretically, the EIFS distribution should be independent of design variables, such as load spectrum, stress level, percent load transfer, environment, etc. For "durability analysis", the EIFS distribution for fastener holes (e.g., given material, drilling procedures, fastener types/fit, etc.) should be justified for different design stress levels and load spectra.

12. Initial Fatigue Quality (IFQ) - characterizes the initial manufactured state of a structural detail or details with respect to initial flaws in a part, component, or airframe prior to service. The IFQ,

represented by an equivalent initial flaw size (EIFS) distribution, must be defined using a consistent fractographic data base. The EIFS distribution depends on the fractographic crack size range used and other factors (Ref. EIFS and EIFS master curve). Whatever EIFS distribution is used, it should be defined specifically for the crack size range of interest for the structural details to be used in the durability analysis. A single EIFS distribution will not necessarily be satisfactory for a wide range of crack sizes (e.g., 0.0005" - 0.10"). Based on current understanding, the EIFS distribution should be defined for a fairly small range of crack sizes (e.g., 0.020" - 0.050" crack size for fastener holes). Further research is required to evaluate the effects and sensitivity of the crack size range on the EIFS distribution and the accuracy of the crack exceedance prediction.

13. Initial Fatigue Quality Model - is a "mathematical tool" for quantifying the IFQ distribution for applicable structural details. Using the IFQ model and fractographic results, an EIFS distribution can be determined which is compatible with the TTCI distribution.

14. Probability of Crack Exceedance ($p(i,\tau)$) - refers to the probability of exceeding a specified crack, x_1, size at a given service time, τ. It can be determined from the statistical distribution of crack sizes and can be used to quantify the extent of damage due to fatigue cracking in fastener holes, cutouts, fillets, lugs, etc.

15. Reference Crack Size (a_0) - This is the specified crack size in a detail used to reference TTCI's. The IFQ distribution is based on a selected reference crack size.

16. Service Crack Growth Master Curve (SCGMC) - This curve is used to determine the EIFS, $y_{1i}(\tau)$, corresponding to an exceedance crack size x_1 at time τ. The probability of crack exceedance, $p(i,\tau)$, can be determined from the EIFS cumulative distribution for a given $y_{1i}(\tau)$. The SCGMC is defined for the applicable design variables (e.g., stress level, spectrum, etc.) and it can be determined using either test data or an analytical crack growth program. All SCGMC's must be consistent with the corresponding EIFS master curve and the fractographic data base. The SCGMC must be consistent with the basis for the IFQ distribution.

17. Structural Detail - is any element in a metallic structure susceptible to fatigue cracking (e.g., fastener hole, fillet, cutout, lug, etc.).

18. Time-To-Crack-Initiation (TTCI) - is the time or service hours required to initiate a specified (observable) fatigue crack size, a_0, in a structural detail (with no initial flaws intentionally introduced).

19. TTCI Lower Bound Limit (ϵ) - ϵ is a cutoff value for TTCI's reflected in the IFQ model. It varies for a given a_0 and it depends on the EIFS upper bound limit, x_u, and the EIFS master curve. TTCI's for a given crack size, a_0, should $\geq \epsilon$. This Weibull distribution parameter provides a basis for quantifying the EIFS distribution for different TTCI crack sizes on a common baseline.

20. Upper Bound EIFS (x_u) - defines the largest EIFS in the initial fatigue quality distribution. The x_u value specified by the user should be consistent with ϵ (TTCI lower bound limit) and the EIFS master curve.

SECTION I

INTRODUCTION

1.1 GENERAL

This is the first edition of the Durability Design Handbook. The purpose of the handbook is to:

- o summarize the essential Air Force durability design requirements for metallic airframes [1-3],
- o describe methodology for satisfying the durability design requirements,
- o provide guidelines and design data for implementing the methodology and for demonstrating design compliance,
- o provide a framework, with a loose-leaf format, for incorporating future durability methodology advancements and design data.

This document is loosely called a "Handbook". Further developments and design data are required to expand and

refine the document for efficient design usage. Therefore, the handbook reflects the current understanding of the Air Force's durability design requirements and provides state-of-the-art concepts, tools and guidelines for satisfying these requirements.

The material presented in this handbook is primarily intended for durability design applications for metallic airframes. However, many of the concepts, analytical tools, data and guidelines can also be used to assess the extent of damage due to cracking for in-service aircraft.

1.2 BACKGROUND

Aircraft structures have thousands of structural details susceptible to fatigue cracking: fastener holes, fillets, cutouts, etc. For example, the wing box assembly shown in Fig. 1.1 has over 3000 fastener holes in the wing skins alone. Fatigue cracking in fastener holes is one of the most prevalent forms of structural damage for in-service aircraft [4-8].

Durability is a measure of the structure's resistance to fatigue cracking. The entire population of structural details in various components is susceptible to fatigue

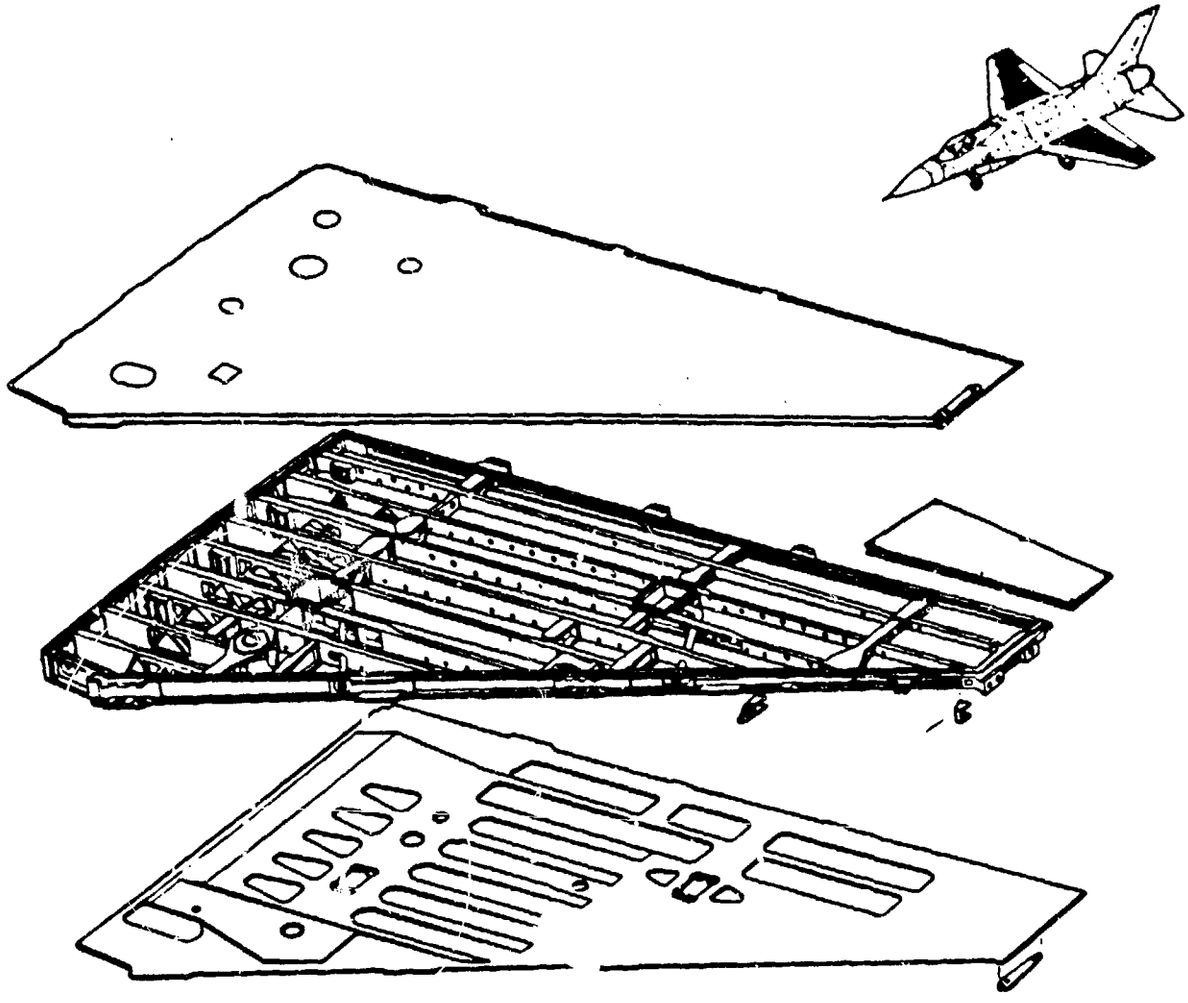


Fig. 1.1 Wing Box Assembly

cracking in service. Therefore, to assess the durability of the structure or extent of damage (i.e., number or percentage of structural details in a part, a structure, a component or airframe exceeding specified crack size limits that cannot be economically repaired) as a function of time, the entire population of structural details must be accounted for. Thus, a statistical approach is essential to quantify the extent of damage as a function of time.

Structural durability is generally concerned with relatively small subcritical crack sizes which affect functional impairment, structural maintenance requirements and life-cycle-costs. Such cracks may not pose an immediate safety problem. However, if the structural details containing such cracks are not repaired, economical repairs cannot be made when these cracks exceed a limiting crack size. For example, a 0.030-0.050" radial crack in a fastener hole can be cleaned up by reaming the hole to the next fastener size. The economical repair limit is the maximum crack size in a detail that can be cleaned-up without further repair or part replacement. If structural details are not repaired or parts replaced at an opportune time, expensive repairs or part replacement may be required. Also, unrepaired cracks may reach sizes which could affect structural safety during the design life of the aircraft.

Aircraft structural safety is governed by damage tolerance conditions whichare concerned with the structure's resistance to failure due to cracking. Damage tolerance is typically concerned with the largest crack size in a single detail. For example, in Fig. 1.1 the damage tolerance of the wing box is limited by a few critical structural details. However, the durability of the wing box is concerned with the entire population of structural details and the size of the largest subcritical crack in each detail.

The conventional fatigue analysis (CFA) approach (i.e., Palmgren-Miner rule, Ref. 9,10) and the deterministic crack growth approach (DCGA) [11] do not provide a quantitative description of the "extent of damage" as a function of service time. The CFA, in its commonly used form, does not quantify crack sizes for a population of details - an essential requirement for any durability analysis method. A DCGA can be used to predict the growth of a single crack in a detail as a function of time. Using the DCGA, details can be grouped and the "worst-case" detail in the group can be used to analytically assure that the largest crack size in the group of details will be ≤ a specified size. However, the DCGA does not quantify the probable crack sizes or ranges of crack sizes for the population of details. CFA

and the DCGA have been evaluated for potential durability analysis applications [12,13].

SECTION II

DURABILITY DESIGN REQUIREMENTS AND ANALYSIS CRITERIA/GUIDELINES

2.1 INTRODUCTION

The purpose of this section is to: (1) briefly review and interpret the important elements of the Air Force's durability design requirements [1-3], (2) discuss durability critical parts criteria and (3) provide guidelines and recommended formats for defining quantitative economic life criteria.

2.2 DURABILITY DESIGN REQUIREMENTS

2.2.1 Objective and Scope

The objective of the Air Force durability design requirements [1-3] is to minimize in-service maintenance costs and maximize operational readiness through proper selection of materials, stress levels, design details, inspections, and protection systems. These design requirements include both analyses and tests.

2.2.2 General Requirements

Essential durability requirements, conceptually described in Fig. 2.1, are as follows:

- The economic life of the airframe must exceed one design service life.

- No functional impairment (e.g., loss of stiffness, loss of control effectivenes, loss of cabin pressure or fuel leaks) shall occur in less than one design service life.

- The economic life of the airframe must be demonstrated analytically and experimentally.

2.2.3 Analytical Requirements

Analyses are required to demonstrate that the economic life of the airframe is greater than the design service life when subjected to the design service loads and design chemical/thermal environments. The economic life analysis must account for initial quality, environment, load sequence, material property variations, etc. The analysis must be verified by tests.

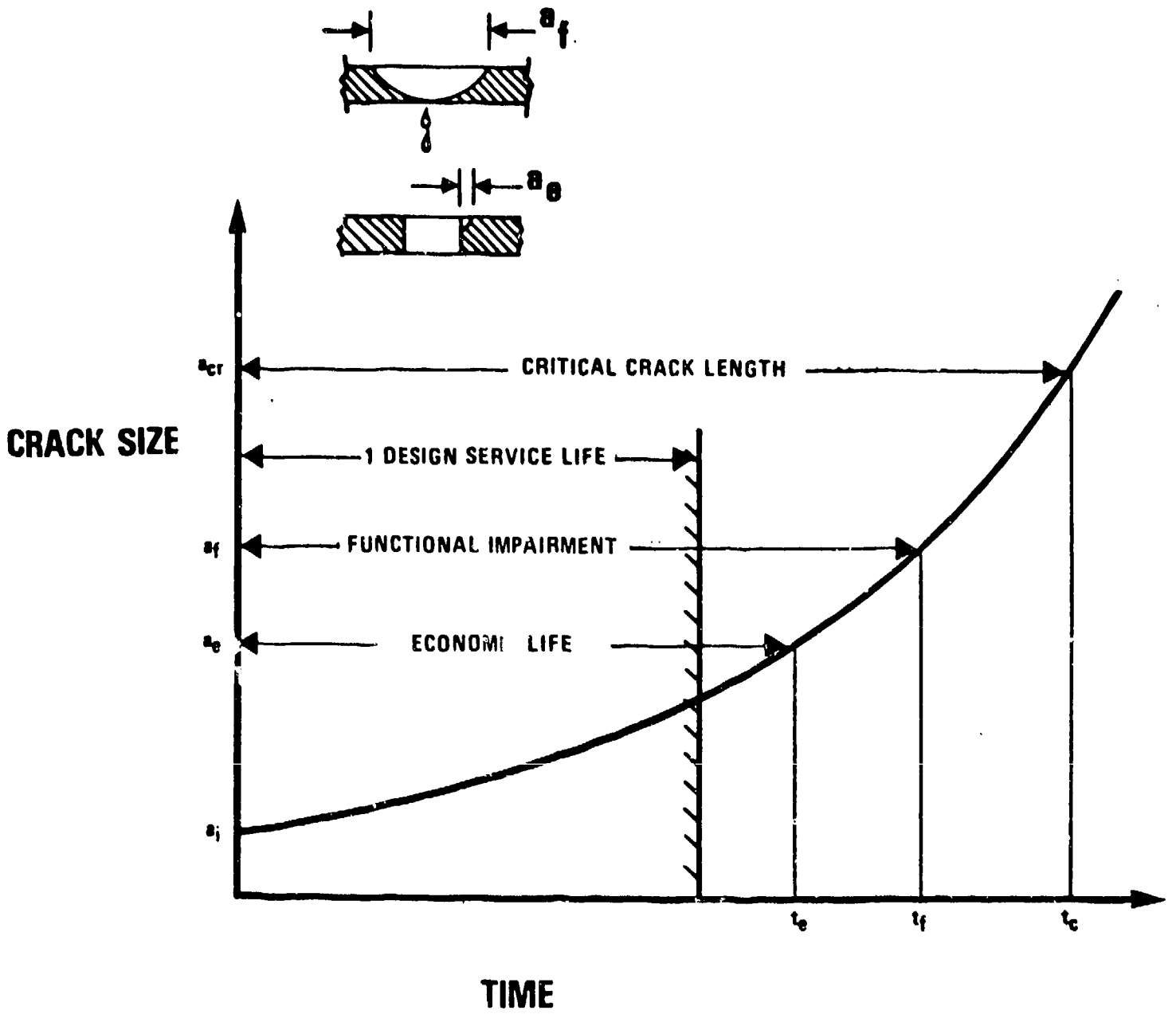


Fig. 2.1 U. S. Air Force Durability Design Requirements

2.2.4 Experimental Requirements

Design development tests are required to provide an early evaluation of the durability of critical components and assemblies as well as the verification of the durability analysis.

A durability test of a full-scale airframe may also be required by the Air Force. The requirements for this test are:

1. The airframe must be durability tested to one lifetime. Critical structural areas must be inspected before the full production go-ahead decision.

2. Two lifetimes of durability testing plus an inspection of critical structural areas must be completed prior to delivery of the first production aircraft.

If the economic life of the airframe is not reached before two lifetimes of durability testing, the following options are available:

1. Terminate the durability testing and perform a nondestructive inspection followed by destructive teardown inspection.

2. Terminate the durability testing and perform damage tolerance testing and nondestructive inspection followed by a destructive teardown inspection.

3. Continue the durability testing for an approved period of time followed by either of the preceding options.

2.3 DURABILITY ANALYSIS CRITERIA

2.3.1 Durability Damage Modes

There are several modes of durability damage, including fatigue cracking, corrosion, wear, etc. Due to its importance and prevalence, fatigue cracking is the form of structural degradation considered in this handbook.

2.3.2 Durability Critical Parts Criteria

Criteria must be developed for determining which parts of an aircraft are durability critical (i.e., which parts

must be designed to meet the durability design requirements). The durability critical parts criteria vary from aircraft to aircraft. They are especially dependent on the definition of economic life for the particular aircraft involved. A typical flow diagram for selecting which parts are durability critical is presented in Fig. 2.2. In Fig. 2.2, durability refers to the ability of an airframe to resist cracking whereas damage tolerance refers to the ability of an airframe to resist failure due to the presence of such cracks.

2.3.3 Economic Life Criteria/Guidelines

Criteria must be developed for determining the economic life of the particular aircraft of interest. Similar to the durability critical parts criteria, economic life criteria vary from aircraft to aircraft. They may be based on fastener hole repair (e.g., reaming the damaged fastener hole to the next nominal hole size), functional impairment (e.g., fuel leakage), residual strength, etc. Two promising analytical formats for quantifying the economical life of an airframe are (1) the probability of crack exceedance, and (2) cost ratio: repair cost/replacement cost. Both formats require a durability analysis methodology capable of quantifying the extent of aircraft structural damage as a

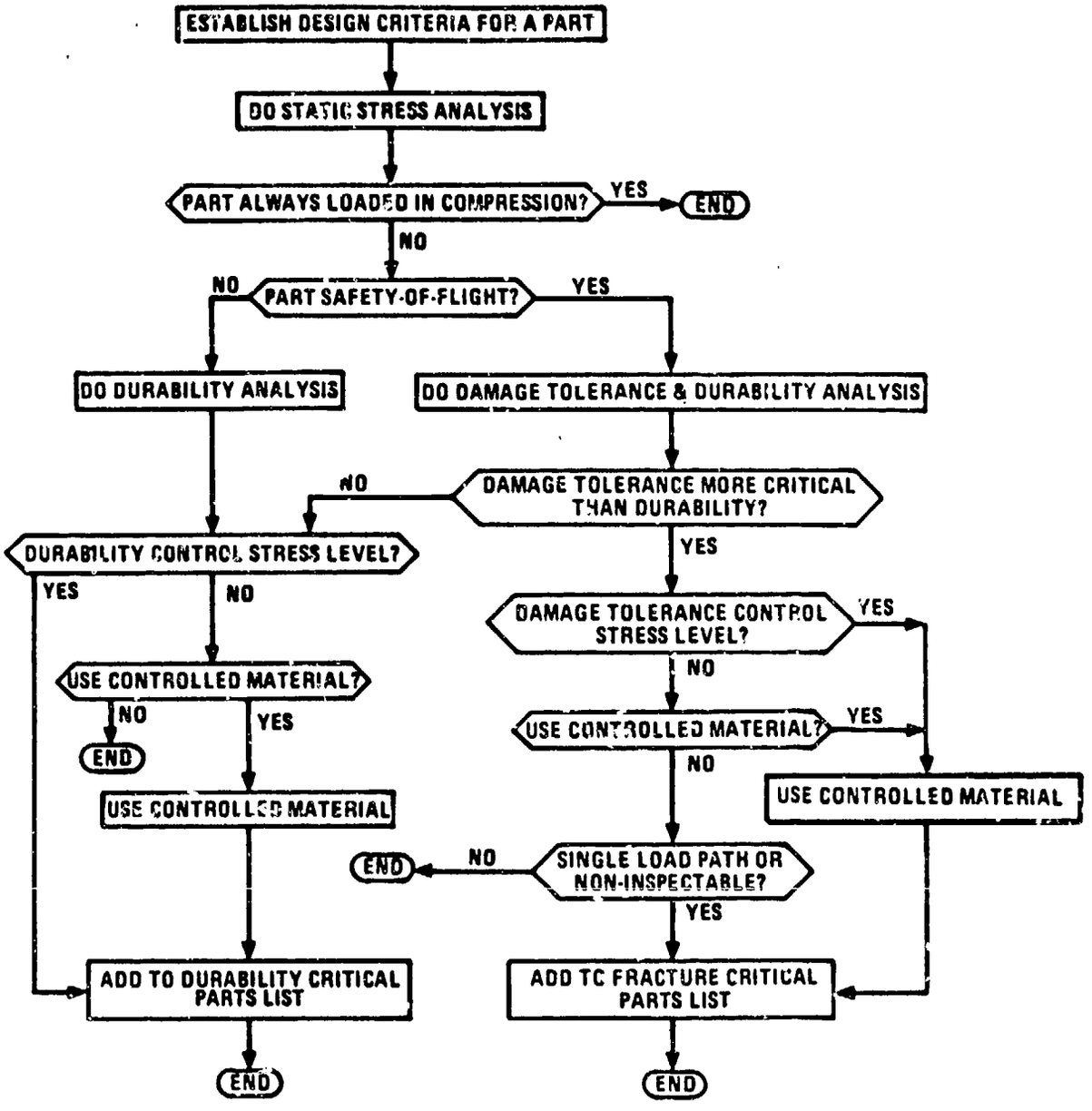


Fig. 2.2 Flow Diagram For Selecting Durability Critical Parts

function of service time. For example, assume the economic life criteria are based on the number of fastener holes which cannot be economically repaired (i.e., number of fastener holes with crack sizes equal to or greater than specified size x_1). Then an analytical format for quantifying economic life is presented in Fig. 2.3. In Fig. 2.3, P is the exceedance probability. Various aspects of economic life are discussed further in the following subsections and elsewhere [11-22].

2.3.3.1 Economic Life Definition

The economic life of an aircraft structure is currently defined in qualitative terms: "...the occurrence of widespread damage which is uneconomical to repair and, if not repaired, could cause functional problems affecting operational readiness" [1-3]. Acceptable limits for "widespread damage" and "uneconomical repairs" must be defined for each aircraft design and such limits must be approved by the Air Force.

A quantitative definition of economic life is not given in this handbook. However, guidelines are presented for specifying economic life criterion (Ref. Section 2.3.3.4). In any case, quantitative criteria for the economic life of

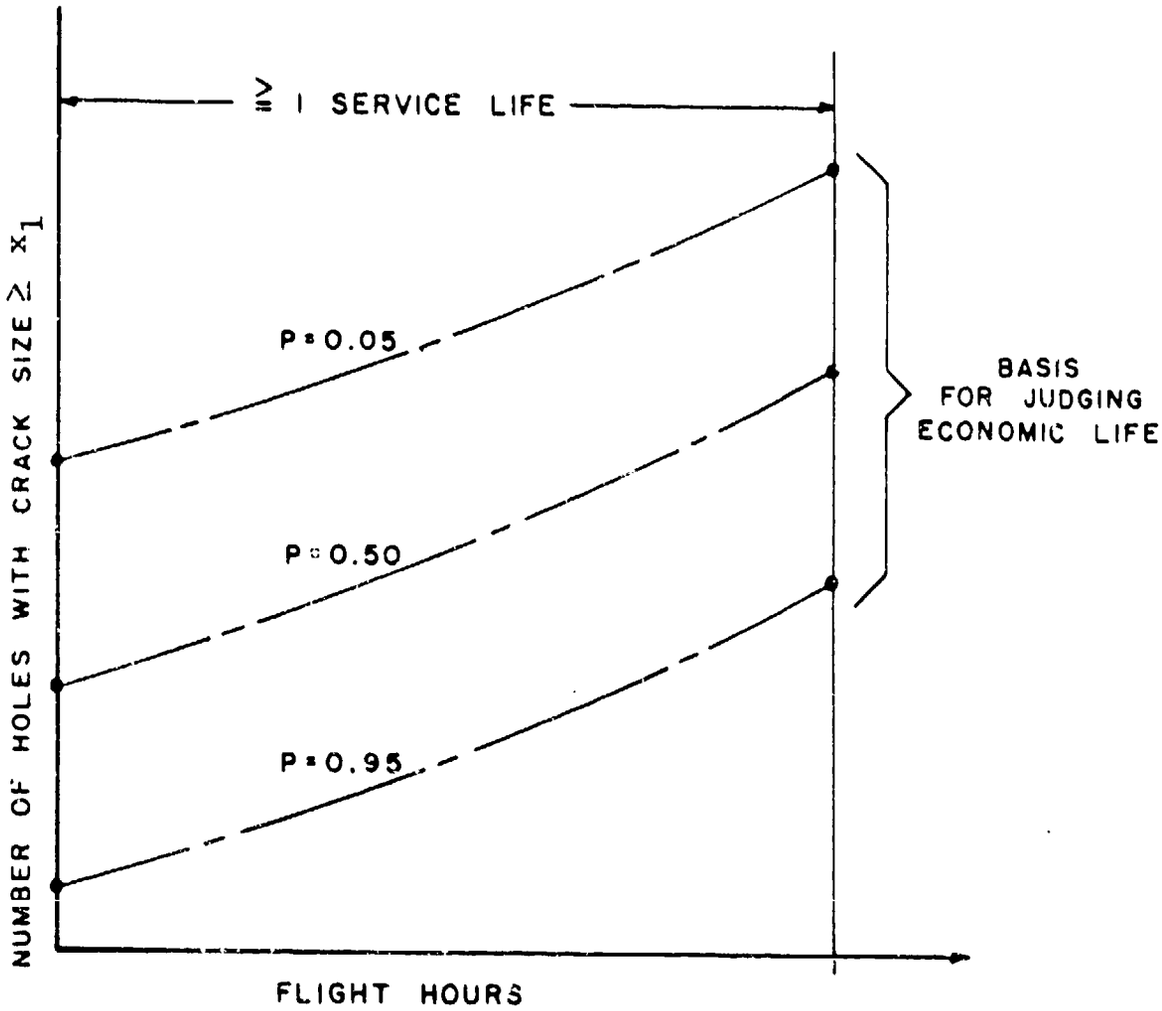


Fig. 2.3 Analytical Format For Economic Life

aircraft structures should be based on specific aircraft requirements and the user's acceptable limits for aircraft performance and maintenance costs.

2.3.3.2 Economic Repair Limit

The "economic repair limit" is the maximum crack size in a structural detail that can be economically repaired. Such limits can easily be defined from geometric considerations for fastener holes but such limits are more difficult to define for structural details such as cutouts, fillets, etc. For example, the economic repair limit for a fastener hole may be governed by the largest radial crack that can be cleaned-up by reaming the hole to the next fastener size (e.g., 0.03" to 0.05" radial crack).

The objective of the durability analysis method presented in this handbook is to analytically predict the number of structural details with a crack size which would cause an uneconomical repair or functional impairment. The user must define the uneconomical repairment or functional impairment crack size for the details to be included in the extent-of-damage assessment. Such crack sizes depend on considerations such as structural detail type, location,

accessability, inspectability, repairability, repair costs, etc.

Structural details may contain one or more cracks. However, structural durability is concerned with the largest crack in each detail which may require repair or part replacement.

2.3.3.3 Extent of Damage

The extent of damage is a quantitative measure of the number of structural details containing cracks that exceed specified crack size limits as a function of service time. Structural maintenance requirements and costs depend on the number of structural details requiring repair. The "durability" of the structure depends on the extent of damage for the population of structural details in a part, a component, or airframe.

The extent of damage can be predicted using the analytical tools provided in this handbook. Extent of damage predictions provide the basis for analytically ensuring design compliance with the governing economic life criterion.

2.3.3.4 Formats For Economic Life Criteria

Two analytical formats for defining quantitative economic life criteria are recommended: (1) probability of crack exceedance and (2) cost ratio: repair cost/replacement cost [14-17]. The analytical tools described in this handbook can be used to predict results in these formats. Various aspects of each format for a quantitative economic life criterion are discussed below, including examples and guidelines (Ref. Fig. 2.3).

2.3.3.4.1 Probability of Crack Exceedance. The probability of a crack occurrence which is larger than a specified crack size is referred to as the "probability of crack exceedance." This quantity is a fundamental output of the durability analysis methodology described in this handbook. For example, in Fig. 2.4 the probability of exceeding crack size x_1 at $t = \tau$ is represented by the cross-hatched area under the crack size density function at $t = \tau$. Crack size rankings in the respective distributions for two different times are preserved; namely, the crack size x_1 at $t = \tau$ has the same rank (or percentile) as the initial crack size at $y_{1i}(\tau)$ at $t = 0$. The probability of crack exceedance can be used to predict the number of expected repairs in a given service interval [15,17]. It

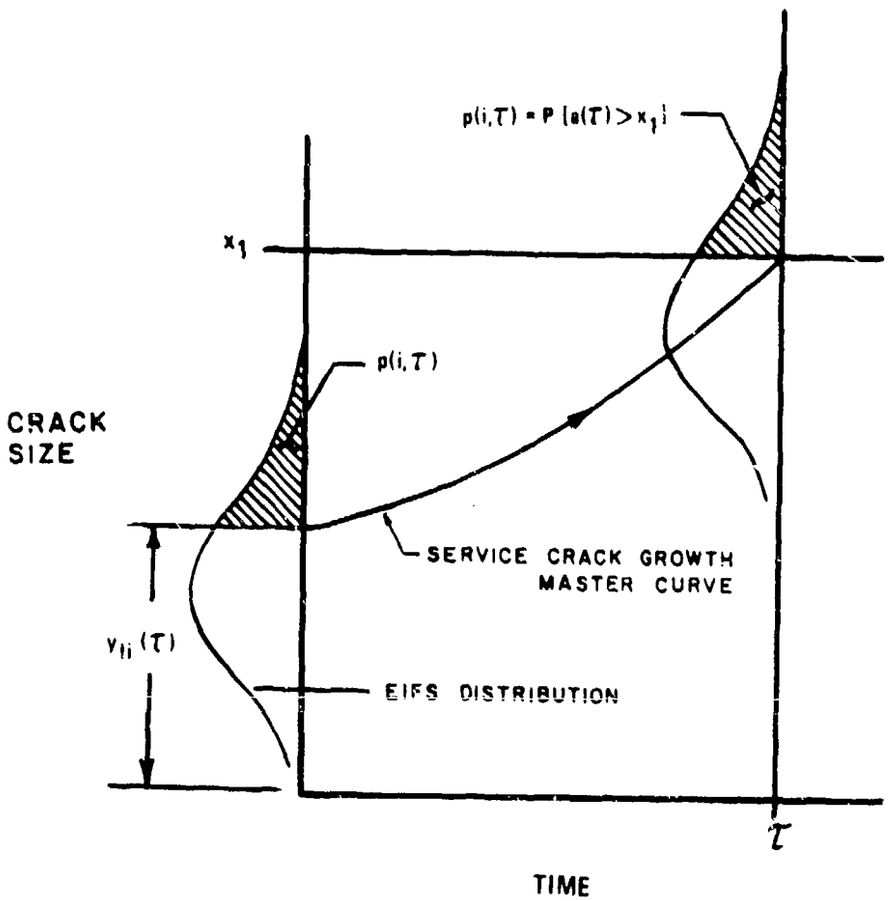


Fig. 2.4 Durability Analysis Approach

also provides a basis for judging airframe durability and for analytically demonstrating design compliance with the governing criterion for economic life.

Another explanation of the probability of crack exceedance concept will now be given. Each common structural detail, in a group of details having a common stress history, has a single dominant crack. Such cracks are a random variable and their "initial" size depends on the manufacturing quality for each structural detail. The population of crack sizes depends on the group of details considered. For example, in Fig. 2.4 assume the initial fatigue quality distribution and the distribution of crack sizes at time τ are for 100 fastener holes (i.e., the population of details).

The probability of exceeding crack size x_1 at time τ is represented by the cross-hatched area under the probability density of crack sizes shown in Fig. 2.4. Suppose the probability of crack exceedance is $p(i,\tau) = 0.05$. This means that on the average 5% of the details (e.g., 5% of the fastener holes) in a part or component would be expected to have a crack size $\geq x_1$ at time τ. $p(i,\tau)$ is a fundamental measure of the extent of damage. Using the binomial distribution, the extent of damage for different groups of

details can be combined to quantify the overall damage for a part, a component or airframe.

The allowable crack exceedance is one criterion recommended for quantifying economic life. Although this handbook provides guidelines for quantifying the allowable crack exceedance, specific values are not presented for demonstrating design compliance with the Air Force's durability design requirements. Such values must be tailored for specific aircraft structure and the user's acceptable limit for structural maintenance requirements/costs, functional impairment, operational readiness, etc. The allowable crack exceedance criterion for economic life design compliance shall be approved by the Air Force.

The allowable crack exceedance for a part or component depends on several factors, including: criticality, accessability, inspectability, repairability, cost, operational readiness, acceptable risk limits, etc. For example, an expensive fracture critical part may be embedded into the wing under-structure. The part is not readily accessible and it is difficult to inspect and repair. Suppose the bolt hole for this part governs its economic life. Then a lower allowable crack exceedance may be desired

than for an equally critical part that is more accessible and inspectable. For example, an average of 2% crack exceedance at 1.2 service lives might be suitable in the first case and an average of 5% might be appropriate for different circumstances.

An example for the probability of crack exceedance criterion is as follows. The economic life of a part or component is reached when 5 percent of the structural details (e.g., fastener holes, cutouts, fillets, etc.) have reached a crack size ≥ a specified limiting crack size at 1.2 service lives. The limiting crack size depends on the type of structural detail, the economic repair limit, and the crack size which would cause functional impairment (limiting case). Structural safety or damage tolerance must not be compromised. Also, the specified limiting crack size for each detail type should account for inspection capabilities and requirements and operational readiness.

The economic life criterion described (i.e., 5% crack exceedance) can be used to demonstrate economic life design compliance analytically and experimentally. The analytical tools presented in this handbook can be used to quantify the extent of damage in terms of crack exceedance. Therefore, given the criterion for economic life, design compliance can be analytically assured. Experimental compliance can be determined based on the results of the durability demonstration test results.

2.3.3.4.2 Repair Cost/Replacement Cost Ratio. The ratio of repair cost/replacement cost is another recommended criterion for quantitative economic life. For example, when the cost to repair a part or component exceeds the cost to replace it, the economic life is reached. In other words, the economic life is reached when the cost ratio = 1 at a specified service life (e.g., 1.2 service lives).

Input from the aircraft user is needed to define acceptable allowable cost ratios for different parts or components. Allowable cost ratios could be specified for particular design situations and user goals.

Repair costs are proportional to the number of structural details (e.g., fastener holes) requiring repair after a specified service time. The analytical tools described in this handbook can be used to quantify the number of details requiring repair as a function of service time. Although specific repair cost data may be difficult to obtain for different circumstances and replacement costs may vary, the cost ratio can be estimated using assumed repair and replacement costs.

The cost ratio criterion for economic life is not recommended for demonstrating design compliance unless

acceptable cost data are available. However, this criterion is recommended for evaluating user design tradeoff options affecting the life-cycle-cost of the airframe. The analytical tools described in this handbook can be used to evaluate the life-cycle-cost design tradeoffs.

SECTION III

SUMMARY OF THE DURABILITY ANALYSIS METHOD

3.1 INTRODUCTION

Essential elements and equations of the durability analysis method are summarized in this section. Details of the approach and implementation procedures are described in Sections IV and V and elsewhere [14-21,23].

3.2 GENERAL DESCRIPTION OF THE METHOD

The basic objective of the durability analysis methodology is to quantify the extent of damage as a function of service time for a given aircraft. The extent of damage is measured by the number of structural details (e.g., fastener holes, cutouts, fillets, lugs, etc.) expected to have a crack whose size is greater than a specified value at a given service time. Hence, the extent of damage is represented by a probability of crack exceedance. The durability analysis results quantitatively describe the extent of damage as a function of service time and serves as a basis for analytically assuring that the

economic life of the structure will exceed the design service life.

The durability analysis includes two essential steps: (1) quantify the initial fatigue quality of the structural details considered, and (2) predict the probability of crack exceedance using the initial fatigue quality and the applicable design conditions (e.g., load spectrum, stress levels, percent load transfer, etc.). Essential elements of the durability analysis method are described in Fig. 3.1.

The durability analysis method has been developed and demonstrated for fatigue cracks in fastener holes [15-21]. However, the basic approach theoretically applies to fatigue cracks in other structural details, such as fillets, cutouts, lugs, etc. Further research is required to evaluate and demonstrate the method for details other than fastener holes.

3.3 ASSUMPTIONS AND LIMITATIONS

1. Fatigue crack length, measured in the direction of crack propagation, is the fundamental measure of durability damage.

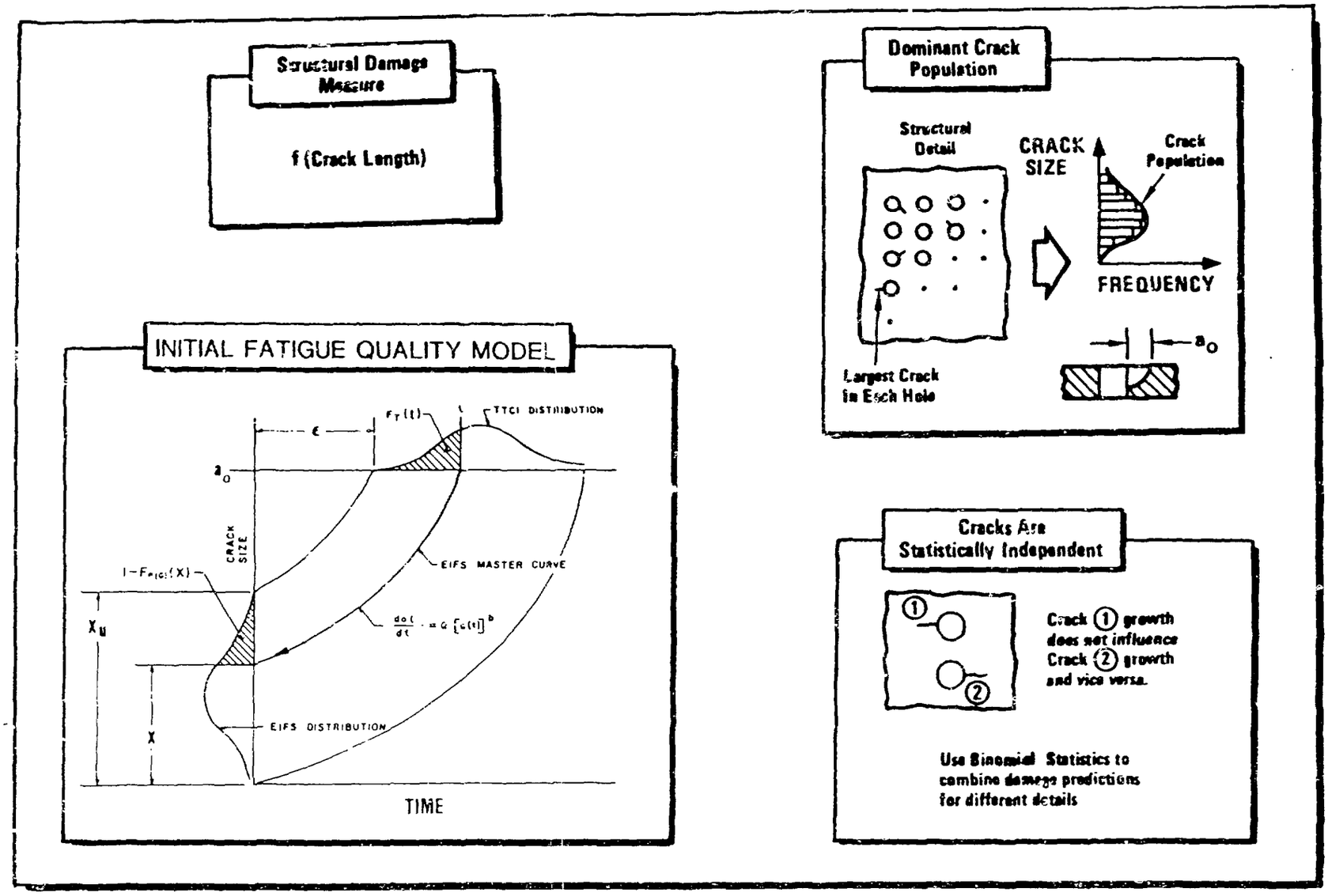


Fig. 3.1 Essential Elements of the Durability Analysis Methodology

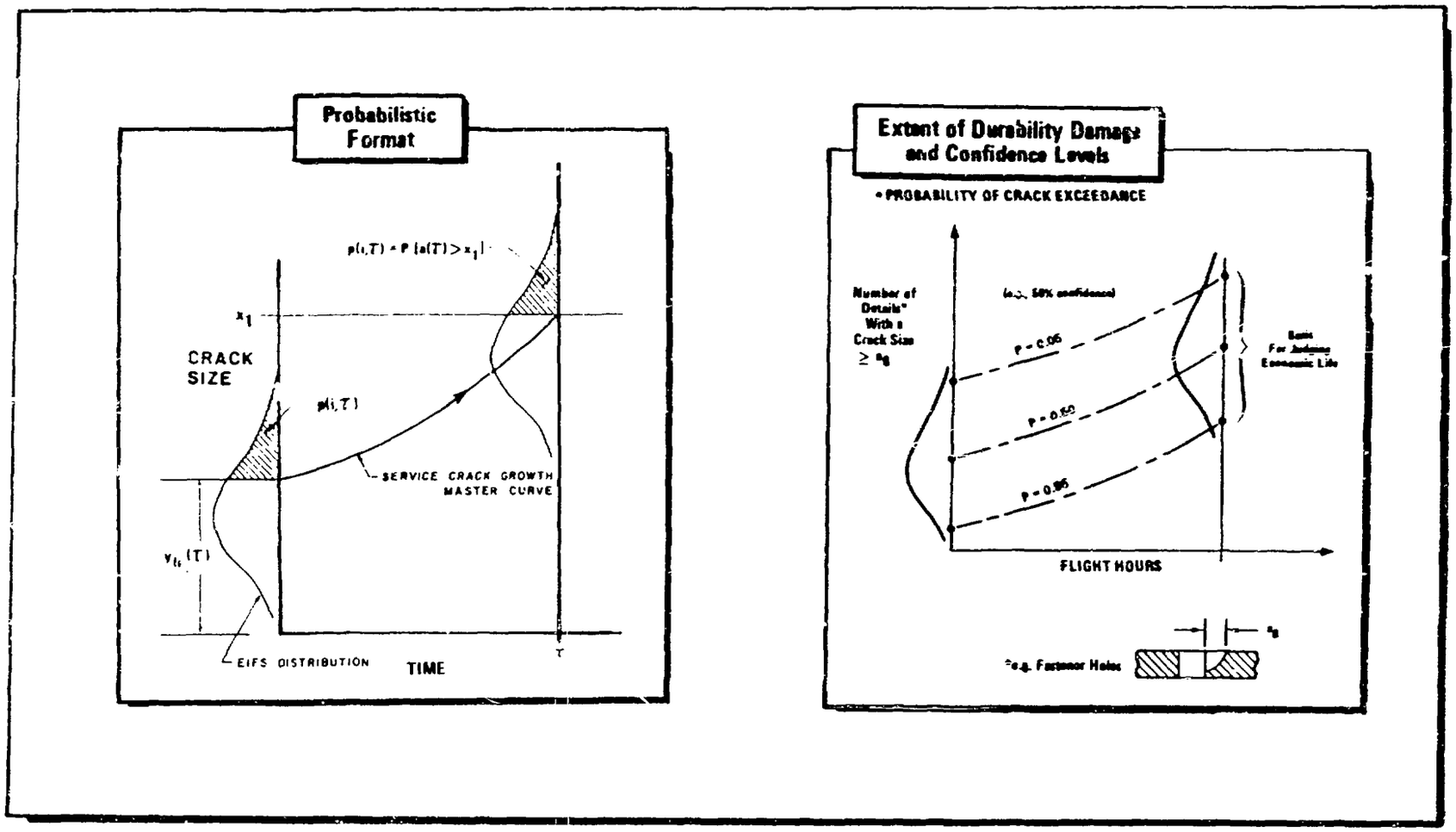


Fig. 3.1 Essential Elements of the Durability Analysis Methodology (Continued)

2. Each detail (e.g., fastener hole , fillet , cutout , lug , etc.) in an aircraft structure has a single dominant fatigue crack which governs the durability of the structure; the size of such a crack is considered to be a random variable.

3. The largest fatigue crack in each detail is relatively small (e.g., ≤ 0.05" corner crack in a fastener hole) and such cracks are statistically independent. Hence, the growth of the largest crack in one detail does not significantly affect the growth of the largest crack in neighboring details and vice versa. Therefore, the binomial distribution can be used to quantify the extent of damage for different details, parts, components or the entire airframe.

4. An equivalent initial flaw size can be determined by back-extrapolating fractographic results using a rational crack growth law. An EIFS is a mathematical quantity describing the IFQ for a given detail. As such, the EIFS is not necessarily an actual initial crack size in the detail.

5. Different EIFS distributions can be developed using the same fractographic data set and fractographic crack size range by using different crack growth master curves.

6. The EIFS distribution is defined for a selected fatigue crack size range (e.g., 0.020" - 0.050" crack in fastener holes).

7. An EIFS distribution can be grown from time zero to a given service time using a single deterministic crack growth curve.

8. A suitable EIFS distribution for durability analysis can be determined using selected fractographic data (i.e., for a given load spectra, % load transfer, stress level, etc.). The derived EIFS distribution can be used to predict the extent of damage for different load spectra, stress levels and % load transfers other than those reflected in the fractographic data base.

9. A suitable service crack growth master curve (SCGMC) can be developed for specific durability analysis conditions.

3.4 INITIAL FATIGUE QUALITY MODEL

The IFQ model provides a means for quantifying the IFQ of structural details (e.g., fastener holes, fillets, cutouts, lugs, etc.) susceptible to fatigue cracking. IFQ can be represented by either a TTCI distribution or by an

EIFS distribution. The IFQ distribution is used to predict the extent of damage or probability of crack exceedance for multiple structural details subjected to fatigue loading and environment. Essential elements of the IFQ model and notations are shown in Fig. 3.2. The model equations are summarized in this section and details are given elsewhere [1,23].

The time-to-crack-initiation (TTCI) cumulative distribution for a reference crack size, a_0, is represented by the three-parameter Weibull distribution as follows:

$$F_T(t) = P[T \leq t] = 1 - \exp\left[-\left(\frac{t-\varepsilon}{\beta}\right)^{\alpha}\right]; \; t \geq \varepsilon \qquad (3\text{-}1)$$

where: T = TTCI

α = Shape Parameter

β = Scale Parameter

ε = Lower Bound of TTCI

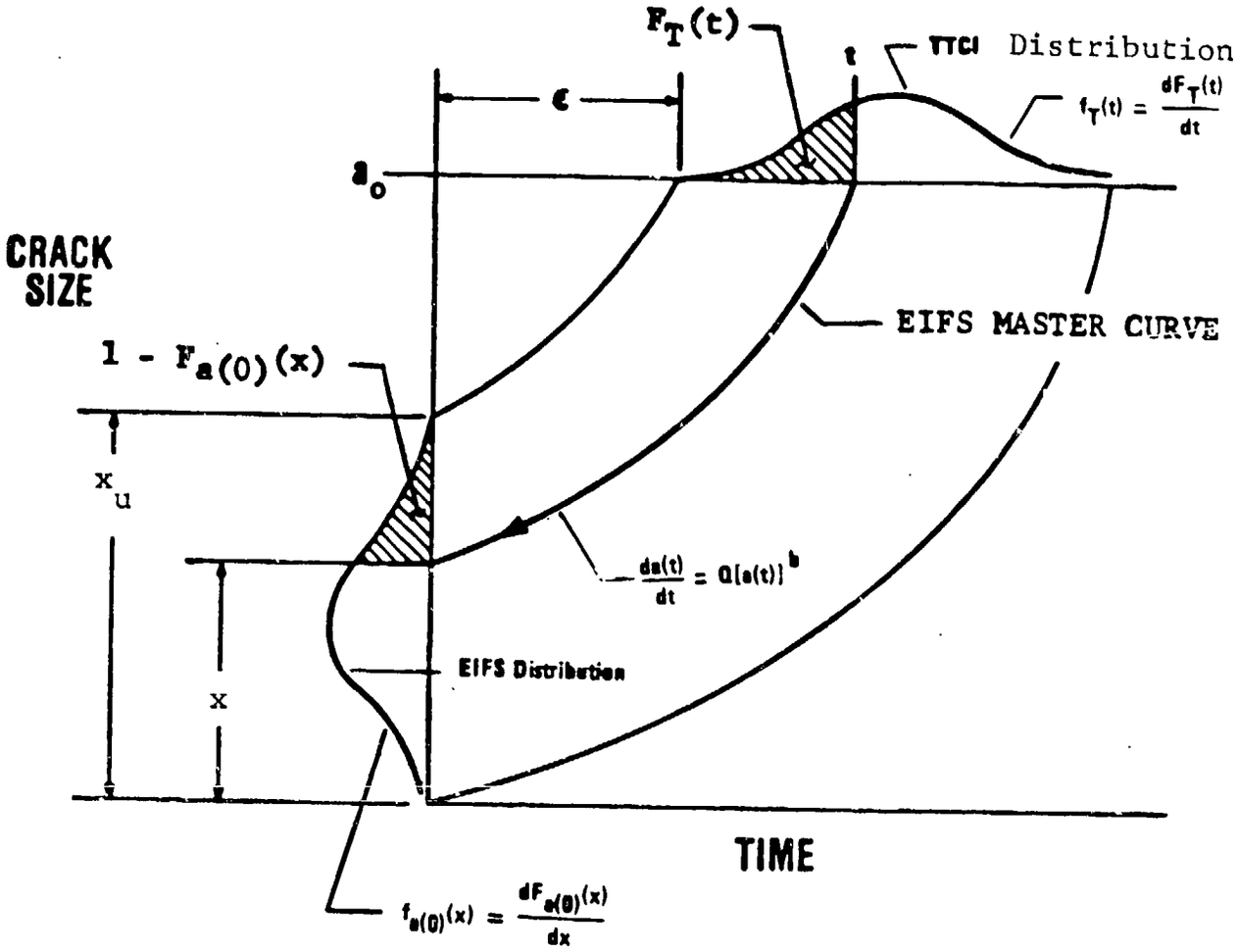


Fig. 3.2 Initial Fatigue Quality Model

The EIFS cumulative distribution, $F_{a(0)}(x)$, is obtained from a transformation of Eq. 3-1 and the following crack growth law in the small crack size region.

$$\frac{da(t)}{dt} = Q[a(t)]^b \tag{3-2}$$

where: Q ,b = Parameters depending on loading spectra, structural and material properties

Eq. 3-2 is used because of its simplicity and general ability to fit crack growth data.

The derived EIFS cumulative distribution, $F_{a(0)}(x)$, is statistically compatible with the TTCI cumulative distribution $F_T(t)$: i.e., $F_T(t)=1 - F_{a(0)}(x)$.

IFQ model equations have been developed for two variations of Eq. 3-2, i.e., $b \neq 1$ and $b=1$ [15,16]. In this section, the IFQ model equations are summarized for both $b>1$ (Case I) and $b=1$ (Case II). Case II is recommended for durability analysis. Hence, the implementation procedures presented in this handbook are tailored for Case II.

Several IFQ model parameter studies were performed for both straight-bore and countersunk fastener holes [16]. These studies were for 7475-T7351 aluminum and clearance-fit fasteners. For the fractographic data sets considered, the computed "b value" in Eq. 3-2 was found to be less than 1. Since negative EIFS values are theoretically possible when $b<1$, b values ≥ 1 are recommended for the present durability analysis. The reader is referred to Section III of Volume VII [16] for further details.

3.4.1 IFQ Model Equations for Case I ($b > 1$)

General Crack Size - Time Relationship

Integrating Eq. 3-2 from t_1 to t_2, one obtains the following,

$$a(t_1) = \left\{ \left[a(t_2)\right]^{-c} + cQ\,(t_2 - t_1) \right\}^{-1/c} \qquad (3\text{-}3)$$

where: $a(t_1)$, $a(t_2)$ = Crack size at time t_1 and t_2, respectively

$c = b-1$

Q = Crack growth model parameter

EIFS MASTER CURVE

Let $t_1=0$, $t_2=T$ and $a(T)=a_0=$ reference crack size at crack initiation. Then Eq. 3-3 becomes

$$EIFS = a(0) = (a_0^{-c} + cQT)^{-1/c} \tag{3-4}$$

EIFS Upper Bound Limit

The upper bound of a(0), denoted by x_u, is obtained from Eq. 3-4 by setting the lower bound ε for T,

$$x_u = (a_0^{-c} + cQ\varepsilon)^{-1/c} \tag{3-5}$$

TTCI Lower Bound Limit

The lower bound ε of T can be expressed in terms of the upper bound x_u of a(0),

$$\varepsilon = \frac{1}{cQ}\left(x_u^{-c} - a_0^{-c}\right) ; \quad a_0 \geq x_u \tag{3-6}$$

EIFS Cumulative Distribution

The distribution of a(0) can be obtained from that of T given by Eq. 3-1 through the transformation of Eq. 3-4 as follows,

$$\left.\begin{aligned} F_{a(0)}(x) &= \exp\left\{ - \left[\frac{x^{-c} - a_0^{-c} - cQ\varepsilon}{cQ\beta} \right]^{\alpha} \right\} ; \quad 0 < x \leq x_u \\ &= 1.0 \qquad\qquad\qquad\qquad\qquad\qquad ; \quad x \geq x_u \end{aligned}\right\} \tag{3-7}$$

or

$$\left.\begin{aligned} F_{a(0)}(x) &= \exp\left\{ - \left[\frac{x^{-c} - x_u^{-c}}{cQ\beta} \right]^{\alpha} \right\} ; \quad 0 < x \leq x_u \\ &= 1.0 \qquad\qquad\qquad\qquad\qquad\qquad ; \quad x \geq x_u \end{aligned}\right\} \tag{3-8}$$

$$\beta = \beta_{\ell} (\ell)^{1/\alpha} \tag{3-9}$$

where, β = Weibull scale parameter for TTCI based on the TTCI's for a given fractographic data set in which only the fractography for the largest fatigue crack in any "one" of ℓ fastener holes per test specimen is used.

ℓ = Number of equally-stressed fastener holes per test specimen in which only the largest fatigue crack in any "one" hole per specimen is included in the fractographic data set.

α = Weibull shape parameter for a given TTCI data set.

Details for Eq. 3-9 are given in Section 4.5 and Ref. 16.

3.4.2 IFQ Model Equation For Case II (b=1)

General Crack Size-Time Relationship

$$a(t_1) = a(t_2) \exp\left[-Q(t_2-t_1)\right] \tag{3-10}$$

EIFS Master Curve

$$a(0) = \text{EIFS} = a_0 \exp(-QT) \tag{3-11}$$

EIFS Upper Bound Limit

$$x_u = a_0 \exp(-Q\varepsilon) \tag{3-12}$$

TTCI Lower Bound Limit

$$\varepsilon = \frac{1}{Q} \ln (a_0/x_u) \tag{3-13}$$

EIFS Cumulative Distribution

$$\left.\begin{aligned} F_{a(0)}(x) &= \exp\left\{ - \left[\frac{\ln (a_0/x) - Q\varepsilon}{Q\beta}\right]^{\alpha}\right\}; \quad 0 < x \leq x_u \\ &= 1.0 \qquad\qquad\qquad\qquad\qquad\qquad ; \quad x \geq x_u \end{aligned}\right\} \tag{3-14}$$

or

$$\left.\begin{aligned} F_{a(0)}(x) &= \exp\left\{ - \left[\frac{\ln (x_u/x)}{Q\beta}\right]^{\alpha}\right\}; \quad 0 < x \leq x_u \\ &= 1.0 \qquad\qquad\qquad\qquad\qquad\qquad ; \quad x \geq x_u \end{aligned}\right\} \tag{3-15}$$

In Eqs. 3-14 and 3-15, $\beta = \beta_\ell (\ell)^{1/\alpha}$ (Ref. Eq. 3-9 and Section 4.5 for further details).

3.5 DURABILITY ANALYSIS PROCEDURES

The general procedure for implementing the durability analysis method developed is described and discussed below:

1. Decide what level the extent of damage will be predicted for (e.g., a single part?, several different parts?, a component?, complete airframe?, fleet of airframes?).

2. Determine which structural details will be included in the durability analysis (e.g., fastener holes, cutouts, fillets, lugs, etc.).

3. Determine the IFQ or EIFS distribution for each type of structural detail to be included in the extent of damage assessment. Use the model shown in Fig. 3.2 and applicable fractographic results for a selected crack size range (e.g., 0.020" - 0.050" crack in fastener holes) to define the EIFS distribution expressed in Eqs. 3-14 and 3-15. Determine α, Q and $Q\beta$ using the procedures described in Sections IV and V and Ref. 16. The x_u selected should be consistent with Eq. 3-12.

Theoretically, the IFQ model developed applies to any structural detail. However, the model has been verified only for fatigue cracks in fastener holes. Further work is

required to develop suitable fatigue test specimens and procedures to acquire appropriate fractographic data for details such as, fillets, cutouts, etc.

4. For each part, component, etc., group the structural details by type into m stress regions where the maximum stress in each region may reasonably be assumed to be equal for every location or detail (e.g., fastener hole).

5. For each stress region, ith stress region, determine the corresponding EIFS value, $y_{1i}(\tau)$, that grows to a crack size x_1 at service time τ as illustrated in Fig. 3.3 [16]. If applicable fractographic data are available for different stress levels and fractographic data pooling procedures are used, the crack growth rate expression in Eq. 3-16, where $b_i = 1$, can be integrated from $a(0) = y_{1i}(\tau)$ to $a(\tau) = x_1$ to obtain $y_{1i}(\tau)$ in Eq. 3-17.

$$\frac{da(t)}{dt} = Q_i \left[a(t) \right]^{b_i} \tag{3-16}$$

$$y_{1i}(\tau) = x_1 \exp(-Q_i \tau); \; b_i = 1 \tag{3-17}$$

If suitable fractographic results are available for the design conditions (e.g., load spectra & load transfer,

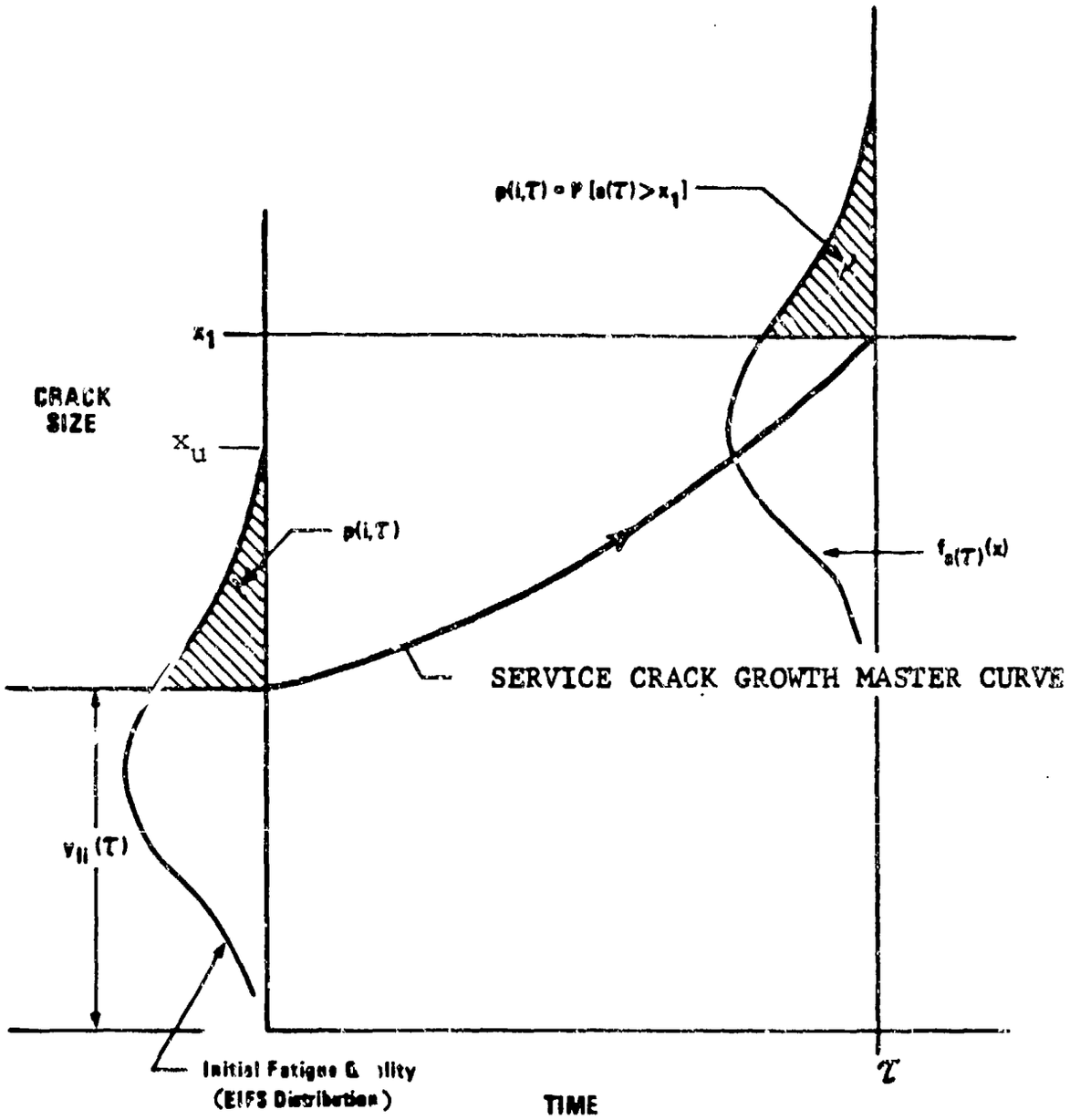


Fig. 3.3 Growth of EIFS Distribution as Function of Time

stress level, etc.), Q_i may be expressed by a power function as follows.

$$Q_i = \xi\sigma^{\gamma} \tag{3-18}$$

In Eq. 3-18, σ is the maximum applied stress in the loading spectrum, and ξ and γ are constants to be determined from the available fractographic data.

If suitable fractographic results are not available, an analytical crack growth program can be used to predict the crack growth over the crack size range of interest. However, the analytical crack growth program should first be "tuned" or curve-fitted to the applicable EIFS master curve before it is used to predict the crack growth damage accumulation a(t). Then, the crack growth parameter b_i and Q_i can be obtained by fitting Eq. 3-16 to predict the crack size a(t) as a function of service life t.

6. Compute the probability of crack exceedance for each stress region, i.e., $p(i,\tau) = P[a(\tau) > x_1] = 1-F_{a(0)}[y_{1i}(\tau)]$, using Eqs. 3-15 and 3-17, with the result

$$\left.\begin{aligned} p(i,\tau) &= 1 - \exp\left\{-\left[\frac{\ln\,(x_u/y_{1i}(\tau)}{Q\beta}\right]^{\alpha}\right\}; \quad 0 < y_{1i}(\tau) \leq x_u \\ p(i,\tau) &= 0 \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad ; \quad y_{1i}(\tau) \geq x_u \end{aligned}\right\} \tag{3-19}$$

in which β is defined by Eq. 3-9 (ref. Section 4.5).

7. The average number of details $\bar{N}(i,\tau)$, and the standard deviation $\sigma(i,\tau)$ in the ith stress region with a crack size greater than x_1 at service-time τ are determined using the binomial distribution and are expressed as follows:

$$\bar{N}(i,\tau) = N_i \; p(i,\tau) \tag{3-20}$$

$$\sigma_N(i,\tau) = \left\{N_i \; p(i,\tau) \left[1-p(i,\tau)\right]\right\}^{\frac{1}{2}} \tag{3-21}$$

in which N_i denotes the total number of details in the ith stress region. The average number of details with a crack size exceeding x_1 at service time τ for m stress regions, $\bar{L}(\tau)$, and its standard deviation, $\sigma_L(\tau)$, can be computed using Eqs. 3-22 and 3-23.

$$\bar{L}(\tau) = \sum_{i=1}^{m} \bar{N}(i,\tau) \tag{3-22}$$

$$\sigma_L(\tau) = \left[\sum_{i=1}^{m} \sigma_N^2 \; (i,\tau)\right]^{\frac{1}{2}} \tag{3-23}$$

Equations 3-22 and 3-23 can be used to quantify the extent of damage for a single detail, a group of details, a part, a component, or an airframe.

The reference crack size for crack exceedance, x_1, can be defined for different detail types according to the limiting crack size that can be economically repaired. Upper and lower bounds for the prediction can be estimated using $\bar{L}(\tau) \pm Z\sigma_L(\tau)$, where Z is the number of standard deviations, $\sigma_L(\tau)$, from the mean, $\bar{L}(\tau)$. Eqs. 3-20 through 3-23 are valid if cracks in each detail are relatively small and the growth of the largest crack in each detail is not affected by cracks in neighboring details. Hence, the crack growth accumulation for each detail is statistically independent [15].

SECTION IV

INITIAL FATIGUE QUALITY DETAILS

4.1 INTRODUCTION

Initial fatigue quality (IFQ), or EIFS distribution, is the "cornerstone" for the durability analysis method developed. Much has been learned about the characteristics and traits of an EIFS distribution during the course of this program [14-23]. The purpose of this section is to (1) discuss the current understanding of the EIFS distribution based on fastener hole experience, (2) present guidelines for acquiring the data needed to quantify the EIFS distribution and (3) describe and illustrate the procedures for calibrating the IFQ model parameters from available fractographic data.

The IFQ model described in Section III should be evaluated further using existing fractographic data for fatigue cracks in fastener holes [e.g., 24-27]. This experience is needed to further advance the understanding of the EIFS distribution for different materials and design conditions (e.g., load spectra, stress level, % load transfer, fastener hole type/fit, etc.).

4.2 EIFS DISTRIBUTION

Initial fatigue quality (IFQ) defines the initial manufactured state of a structural detail or details with respect to initial flaws in a part, component or airframe prior to service. The IFQ for a group of replicate details is represented by an equivalent initial flaw size (EIFS) distribution. An equivalent initial flaw is a hypothetical crack assumed to exist in a detail prior to service. An EIFS is the initial size of a hypothetical crack which would result in an actual crack size at a later point in time. As such, the EIFS is strictly a "mathematical quantity" rather than an actual initial flaw in a detail. Observed cracks from fatigue tests (fractography) are extrapolated backwards using a crack growth analysis to estimate their EIFS.

The time required for an initial defect, of whatever type, to become a fatigue crack of size a_0 is defined as the time-to-crack-initiation (TTCI). a_0 is an arbitrary crack size which can be reliably observed fractographically following a fatigue test. An EIFS distribution quantitatively describes the EIFS crack population for a group of replicate details. Using the IFQ model described in Section III, the EIFS distribution is determined by

coupling the TTCI distribution with a deterministic crack growth law. The IFQ model is a convenient "mathematical tool" for quantifying the EIFS distribution, which is statistically compatible with the TTCI distribution.

An EIFS distribution, $F_{a(0)}(x)$, can be established by fitting the IFQ model parameters to observed fractogaphic data for a given material, load spectra, stress level, % bolt load transfer, fastener type/fit, etc. The basic premise of the durability analysis approach is this: Once the EIFS distribution has been established, the cumulative distribution of fatigue cracks at a given time and the cumulative distribution of TTCI, $F_T(t)$, for a given a_0 can be analytically predicted for different service conditions (e.g.. load spectrum, stress level, etc.).

Intuitively, EIFS is an inherent property of such factors as the material, manufacturing/assembly techniques, and workmanship. As such, EIFS should be independent of load spectrum, stress level, and % bolt load transfer.

Necessary traits of the EIFS distribution are as follows:

- The EIFS distribution, when grown forward during service, must accurately predict the observed cumulative distribution of crack sizes at any time.

- Alternatively, it must predict observed TTCI values for any crack size a_0.

- An EIFS distribution should not depend on subsequent service, i.e., spectrum and load level. This implies that a set of identical test specimens, if divided into two or more groups and tested using different stress levels or spectra, should produce the same EIFS distribution. This is called a "generic" EIFS distribution.

The EIFS distribution is not:

- necessarily the distribution of actual physical defects or cracks in the material initially.

- unique. In fact many different EIFS distributions can predict the same observed flaw distributions over a fairly wide range reasonably well. Each EIFS distribution is obtained using a different

crack growth model. An example is shown in Ref. 16.

Equations for the EIFS distribution (Eqs. 3-8 and 3-15), presented in this handbook, are based on a given crack growth law (Eq. 3-2). Other equations for $F_{a(0)}(x)$ could also be developed for different crack growth laws. However, the durability analysis approach proposed herein is quite general and the user can adapt the method to his crack growth model, analytical crack growth program, fractographic data base, etc.

4.3 TEST/FRACTOGRAPHY GUIDELINES

Fractographic data (i.e., crack size versus time) are needed to define the IFQ or EIFS distribution for those details (e.g., fastener holes, fillets, lugs, cutouts, etc.) used in the durability analysis. The test and fractography guidelines recommended in this section are based on the current understanding for fatigue cracking in fastener holes. As such, these guidelines should be considered preliminary.

Further work is required to develop test/fractography guidelines for details other than fastener holes (e.g.,

fillets, cutouts, lugs, etc.). Also, suitable test specimens should be standardized for generating the fatigue cracking data needed for each detail type to be included in the extent of damage assessment.

4.3.1 Test Guidelines

The following guidelines are for fastener holes:

1. Whatever test specimen is used to generate the fatigue cracking data, it should account for the applicable design variables (e.g., material, hole preparation technique/tools, fastener type/fit, percent load transfer, stress level, load spectrum, environment, etc.).

2. The specimen design used should provide a maximum amount of information for a single test. For example, fatigue cracking data can be obtained for multiple details in a single specimen. To justify using a specimen with multiple details, each detail should be exposed to a comparable stress level. And, the largest crack in any one detail should not significantly affect the crack growth in a neighboring detail and vice versa. Structural details must be spaced far enough apart so that they will crack independently.

3. Single dog-bone and reverse dog-bone type specimens were successfully used for this program. The single dog-bone type specimen shown in Fig. 4.1 was used with one or two holes. This type of specimen can be used to generate fatigue cracking data for no load transfer cases. Studies performed during Phase II suggested that IFQ is independent of the percent load transfer [16]. Further work is required to justify using no load transfer specimens to define the IFQ for fastener holes with different percentages of load transfer. If no load transfer specimens can be justified, then the specimen types shown in Figs. 4.1 and 4.2 could be promising for economically generating the fatigue cracking data needed.

4. The no load transfer specimen shown in Fig. 4.2 contains two fastener holes. This specimen design provides a positive way to assure the independence of fatigue cracking in the two fastener holes. This specimen can be fatigue tested to a specified service time or until failure occurs in a single hole. If there are fatigue cracks in both holes when the test is stopped, fractography can be performed for both holes or for the largest crack in either hole and TTCI results can be scaled using the procedure described in Section 4.5. If an observable fatigue crack occurs in only one hole when the test is stopped, the test

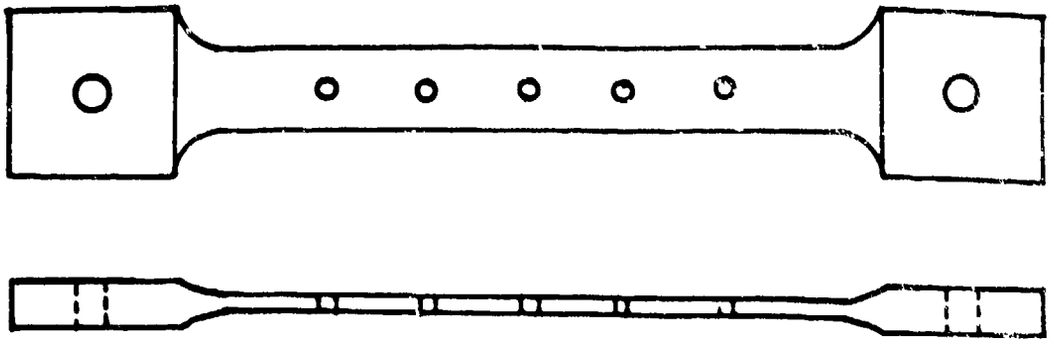


Fig. 4.1 No-Load Transfer Specimen With Multiple Details

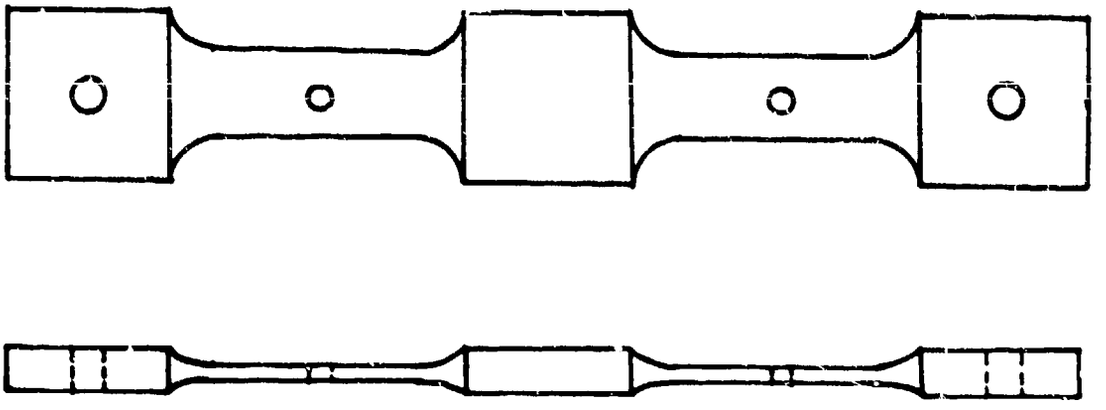


Fig. 4.2 Two-For-One No-Load Transfer Specimen Design

can be continued for the uncracked hole by reworking the specimens as follows: cutoff the test sections containing the hole with a crack from the thick part at the middle of the specimen. The thick part then becomes the lug end that can be used to continue the testing until a visible crack occurs in the remaining fastener hole.

5. The no load transfer specimens should be tested with the desired type of fastener and fit in the test holes.

6. The reverse double dog-bone specimen design concept shown in Fig. 4.3 seemed to work fairly well for this program, particularly the "15% load transfer design". The transition between the lug end and test section should be smooth and gradual. Also, the specimen length and geometry should allow an adequate range of axial deformations to obtain the desired shear load transfer through the fasteners and mating holes.

7. The number of specimens required for testing depends on factors, such as: (1) type of specimen used (single hole or multiple hole), (2) design variables to be accounted for (e.g., no. of materials, fastener type/fit, fastener diameter, bolt torque, manufacturing variations, fretting, environments, stress level, load spectra, etc.). (3)

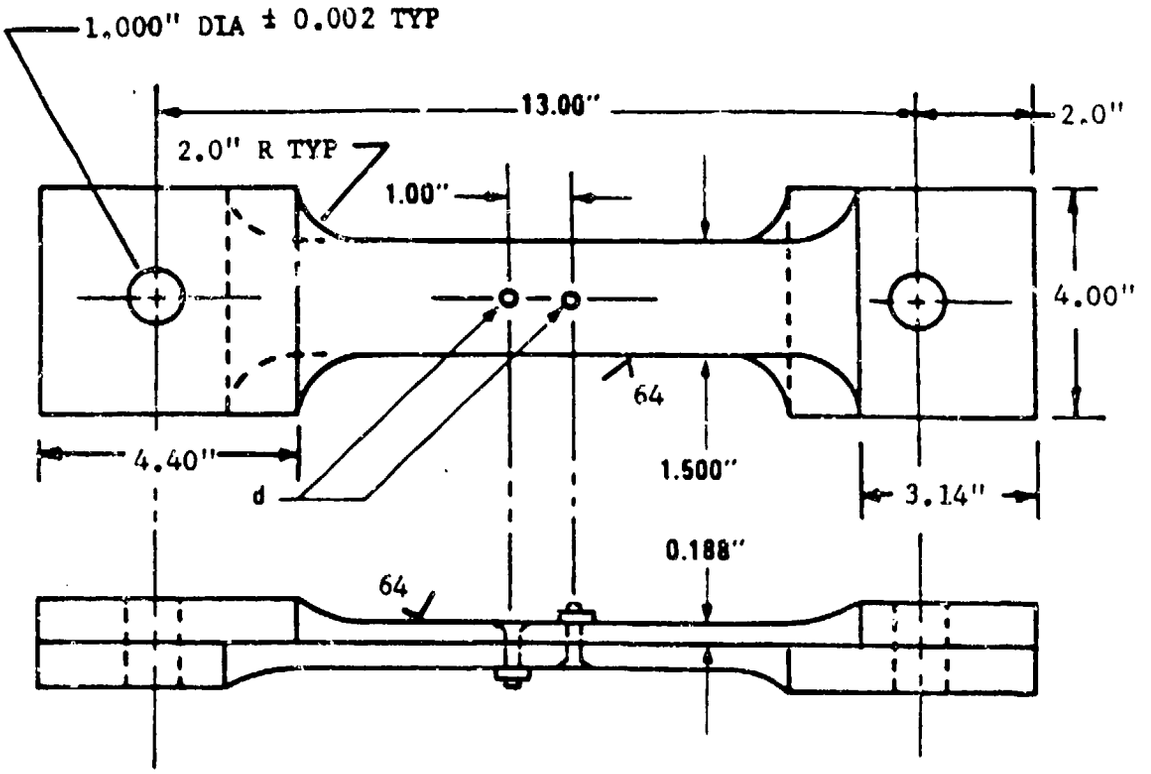


Fig. 4.3 Reverse Double Dog-Bone Specimen (15% Bolt Load Transfer)

confidence level desired for calibrated IFQ model parameters, (4) expected scatter in fatigue cracking results, etc. The schedule and budget also influence this decision.

8. Room temperature tests are recommended for quantifying the IFQ. Effects of environment can be accounted for in the service crack growth master curve.

9. For a given load spectrum, test replicate specimens using at least two different stress levels. Three stress levels are preferred if affordable. If only two stress levels are used, tests should be conducted at a high and a low stress level. Select stress levels that will cover the range of expected design stress levels. This information can be used to define the service crack growth master curve for different stress levels without using an analytical crack growth program.

4.3.2 Guidelines for Fractographic Data

1. Select a minimum crack size to be read, such that it is consistent with the capabilities of the fractographic reading equipment and technicians. Budget and schedule

should also be considered. The minimum fractographic crack size to be read depends on the smallest crack size, x_1, for which crack exceedance predictions will be made. For example, a crack size range of 0.020" - 0.050" would be reasonable for assessing the durability of fastener holes.

2. Take advantage of the β scaling technique described in Section 4.5 to minimize fractographic reading requirements. For example, use test specimens with multiple fastener holes and then fractographically evaluate the largest crack per specimen in a given fastener hole.

3. Use automated crack monitoring techniques as much as possible to minimize fractographic acquisition costs. Also, automatic storing of the fractographic results directly into the computer can minimize the time and costs for plotting results and for calibrating the IFQ model parameters.

4.4 PROCEDURES FOR CALIBRATING THE IFQ MODEL PARAMETERS

Suggested procedures for determining the IFQ model parameters are described and discussed in this section, including guidelines. However, it is felt that further work and experience is needed in the evaluation and optimization of the IFQ model parameters using pooled fractographic data

before values can be tabulated for different materials for design purposes.

During the course of this program various techniques and their variations for determining consistent IFQ model parameters have been investigated. Model calibration studies were performed in Phase II for both straight bore and countersunk fastener holes [Ref. 16, Appendices A and B]. The lessons learned from these studies are reflected in this section.

4.4.1 Generic Nature of IFQ and Data Pooling Concepts

The purpose of this section is to: (1) discuss fractographic data pooling concepts for determining the EIFS distribution, (2) explain the "generic" nature of the IFQ distribution and (3) describe how the IFQ model parameters can be determined using pooled fractographic results for different data sets.

The IFQ model parameters in Eq. 3-15 can be determined for fastener holes using applicable fractographic results for one or more data sets. A fractographic data set refers to the fractographic results (i.e., a(t) versus t values) for replicate fatigue tests (e.g., same: material, specimen

geometry, hole drilling procedure, load spectrum, stress level, % bolt hole transfer, fastener type/fit, etc.). Fractographic results can be used to determine the TTCI and crack growth behavior for fastener holes. This information is used to determine an appropriate IFQ distribution for durability analysis.

Fractographic results for different data sets can be "pooled" to quantify the IFQ model parameters for durability analysis. Pooling the fractographic results for different data sets (e.g., same material, fastener type/fit, and hole drilling procedure but different load spectra and stress levels) is recommended because this increases the sample size available for fitting the IFQ model parameters (x_u, α and $Q\beta$) in Eq. 3-15. Therefore, the resulting EIFS distribution can be applied to different design stress levels, load spectra, etc.

An EIFS distribution is "generic" if it depends only on the material and manufacturing/fabrication processes. Theoretically, the EIFS distribution should be independent of design variables, such as load spectrum, stress level, percent bolt load transfer, environment, etc.

Fractographic results are used to determine the IFQ for fastener holes. Since the fractography reflects the influence of specific fatigue test variables (e.g., load spectrum, stress level, percent bolt load transfer, etc.), the derived IFQ distribution may not be strictly "generic". However, the fractographic data pooling procedures described herein can be used to determine an appropriate IFQ distribution for practical durability analyses.

TTCI distributions for different fractographic data sets (i.e., same material, drilling procedure, fastener type and fit but different load spectra, stress level, etc.) should transform into the same EIFS distribution to obtain compatible results (Ref. Fig. 4.4). For example, each TTCI value in a given TTCI data set is grown backward from a_0 to an EIFS value, a(0), at time t=0 using the applicable crack growth relationship for each TTCI data set (Ref. Fig. 4.5). The resulting a(0) value for each TTCI value in each fractographic data set maps into the same EIFS distribution (Ref. Fig. 4.4).

To distinguish the crack growth parameters Q and b in Eq. 3-2 for each fractographic data set to be pooled, Eq. 4-1 is used. In Eq. 4-1

$$\frac{da(t)}{dt} = Q_i^* \, [a(t)]^{b_i^*} \qquad (4\text{-}1)$$

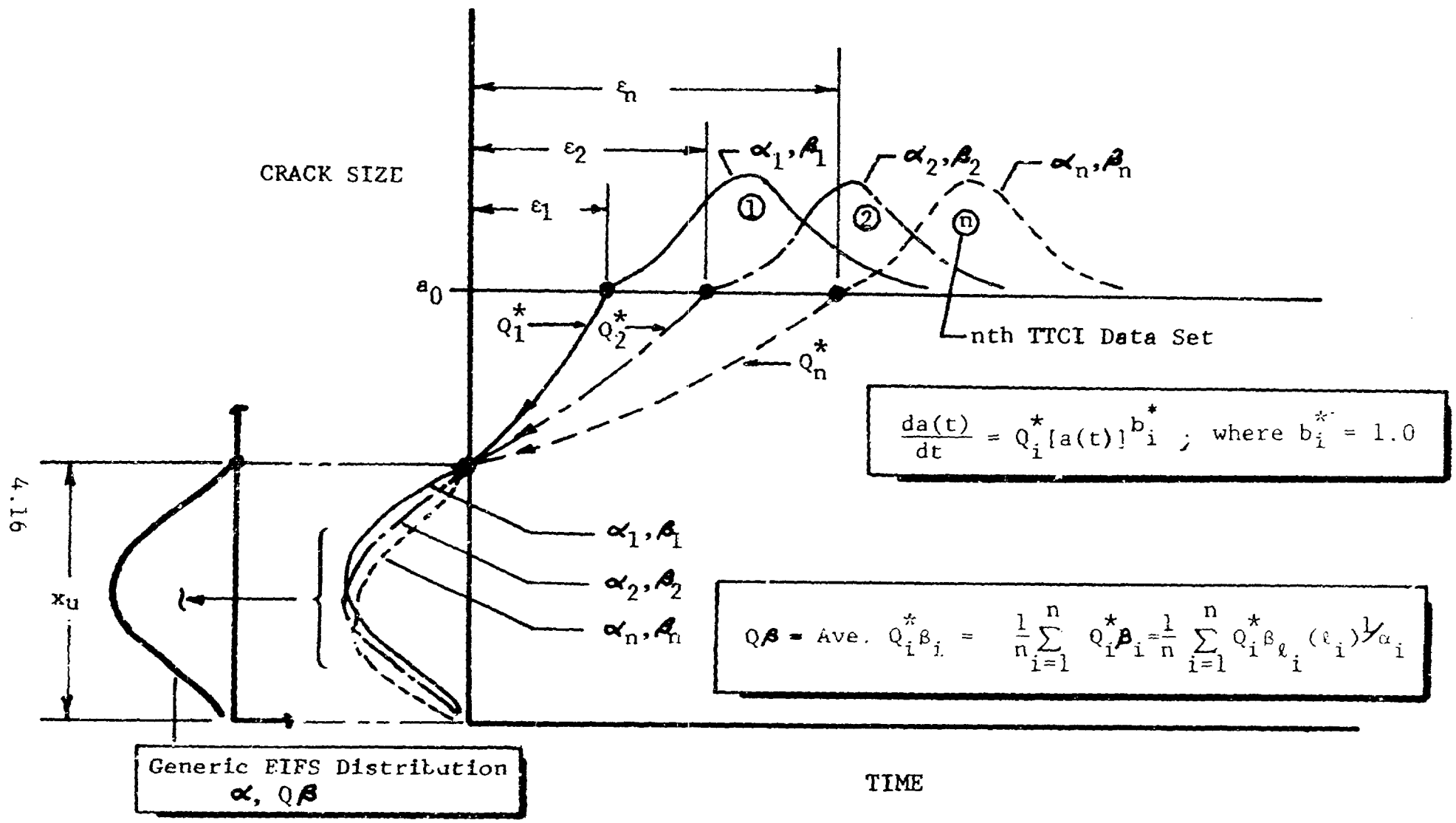


Fig. 4.4 Illustration of Concept of Transforming Individual TTCI Distributions for Different Data Sets Into a "Generic" EIFS Distribution

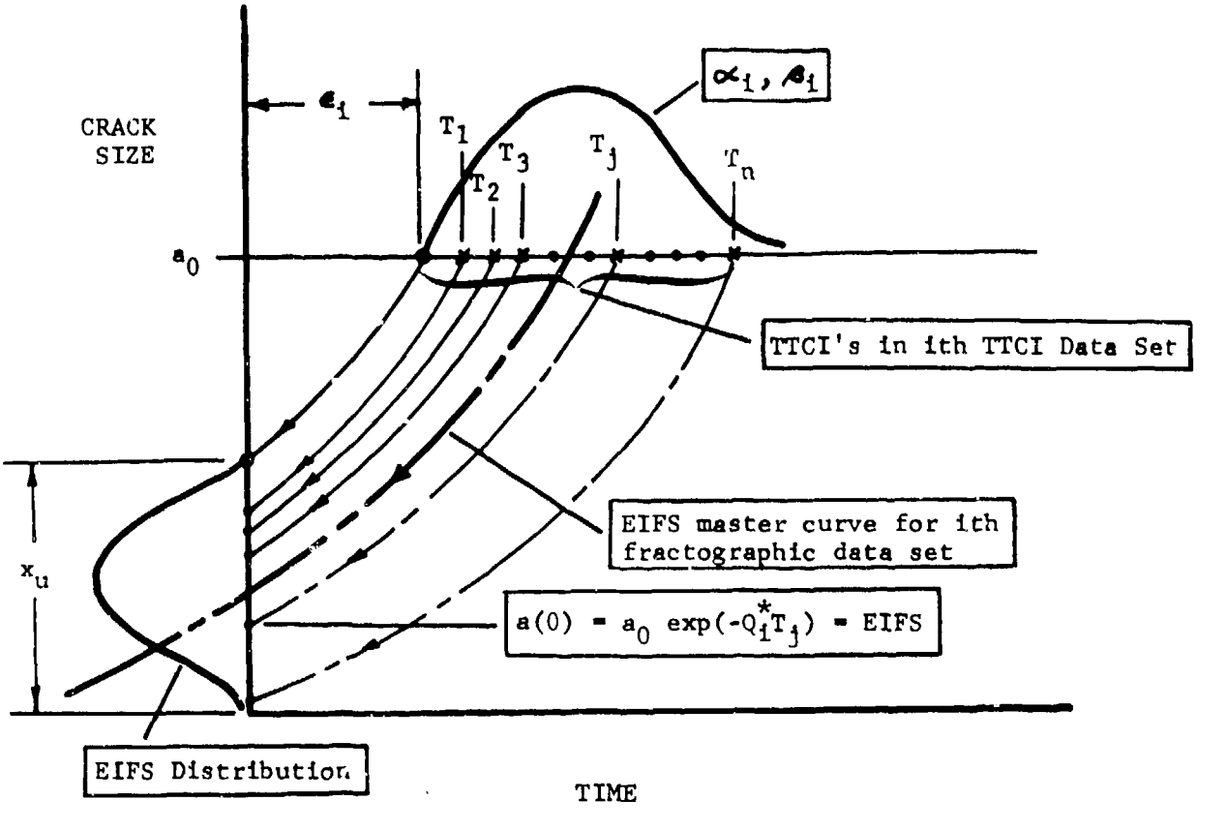


Fig. 4.5 Illustration Showing TTCI's and EIFS Master Curve for the ith Fractographic Data Set

Q_i^* and b_i^* are crack growth constants for the ith fractographic data set. To make the fractographic results for different data sets "compatible" for a single IFQ distribution, <u>each</u> fractographic data set pooled should have the same b_i^* value. For IFQ model Case I (Ref. Section 3.4.1), b > 1; for Case II, b = 1 (Ref. Section 3.4.2).

The subscript "i" is added to the parameters α, β, and ϵ in the TTCI distribution of Eq. 3-1 to distinguish values for the different fractographic data sets to be pooled. For example, α_i, β_i, and ϵ_i denote the shape, scale and lower bound of TTCI for the ith fractographic data set, respectively (Ref. Fig. 4.4).

Theoretically, a "generic" EIFS distribution can be obtained for pooled fractographic data sets by imposing the following conditions (Ref. Fig. 4.4):

$$\left.\begin{array}{c} \alpha_1 = \alpha_2 = \alpha_3 = \ldots\ldots\ldots = \alpha_n \\ b_1^* = b_2^* = b_3^* = \ldots\ldots\ldots = b_n^* \\ Q_1^*\beta_1 = Q_2^*\beta_2 = Q_3^*\beta_3 = \ldots\ldots\ldots = Q_n^*\beta_n \\ x_{u_1} = x_{u_2} = x_{u_3} = \ldots\ldots\ldots = x_{u_n} \end{array}\right\} \qquad (4\text{-}2)$$

The conditions of Eq. 4-2 are necessary to obtain the same EIFS cumulative distribution, $F_{a(0)}(x)$, for each of the fractographic data sets to be pooled.

Because of sampling fluctuations due to limited amounts of test data, Eq. 4-2 cannot be satisfied exactly. However, based on the experiences of this program, it appears that the conditions of Eq. 4-2 can be reasonably satisfied - at least to an acceptable degree for practical durability analysis. Also, it appears that the parameters α_i, b_i^* and $Q_i^*\beta_i$ may be material constants which are independent of the load spectra and stress level [16]. Further investigation using existing fractographic results [e.g., 24,25] is needed. In the present investigation, Eq. 4-2 is forced to be satisfied using data poolin procedures and the parameter optimization method (Ref. subsection 4.4.2.4).

4.4.2 Calibration and Data Pooling Procedures

4.4.2.1 Determination of Q_i^*

The crack growth parameter Q_i^* in Eq. 4-1 for each data set can be determined in various ways. Two different methods are described in this section. Both methods are based on the least squares fit criterion.

Method 1

Equation 4-1 can be transformed in a linear least squares fit form as shown in Eq. 4-3.

$$\ln \frac{da(t)}{dt} = \ln Q_i^* + \ln a(t) \tag{4-3}$$

Using the least squares criterion, an expression for Q_i^* can be determined as follows:

$$E = \sum \left[\ln \frac{da(t)}{dt} - \ln Q_i^* - \ln a(t) \right]^2 \tag{4-4}$$

$$\frac{\partial E}{\partial Q_i^*} = 0 = 2 \sum \left[\ln \frac{da(t)}{dt} - \ln Q_i^* - \ln a(t) \right] \left(- {}^1/_{Q_i^*} \right) \tag{4-5}$$

Solving Eq. 4-5 for Q_i^*, Eq. 4-6 is obtained.

$$Q_i^* = \exp \left[\frac{\Sigma \ln \frac{da(t)}{dt} - \Sigma \ln a(t)}{N} \right] \tag{4-6}$$

Values of da(t)/dt can be estimated for a given a(t) using the fractographic data in the selected crack size range, i.e., a_L to a_U. There are various numerical techniques for estimating da(t)/dt values. This includes, for example, the Direct Secant Method [28], the Modified

Secant Method [29,30] and the Incremental Polynomial Method [28].

Method 2

The crack growth parameter Q_i^* can also be determined from the generalized crack size-time relationship of Eq. 4-7.

$$a(t_j) = a(t_i) \exp\left[-Q_i^* (t_i - t_j)\right] \tag{4-7}$$

The notations for Eq. 4-7 are described in Fig. 4.6.

Equation 4-7 can be transformed into the least squares fit form as follows:

$$E = \sum \left\{ \ln \left[\frac{a(t_i)}{a(t_j)} \right] - Q_i^* (t_i - t_j) \right\}^2 \tag{4-8}$$

The following expression for Q_i^* is obtained from Eq. 4-8 by invoking the condition $\frac{\partial E}{\partial Q_i^*} = 0$,

$$Q_i^* = \frac{\sum (t_i - t_j) \ln \left[\frac{a(t_i)}{a(t_j)} \right]}{\sum (t_i - t_j)^2} \tag{4-9}$$

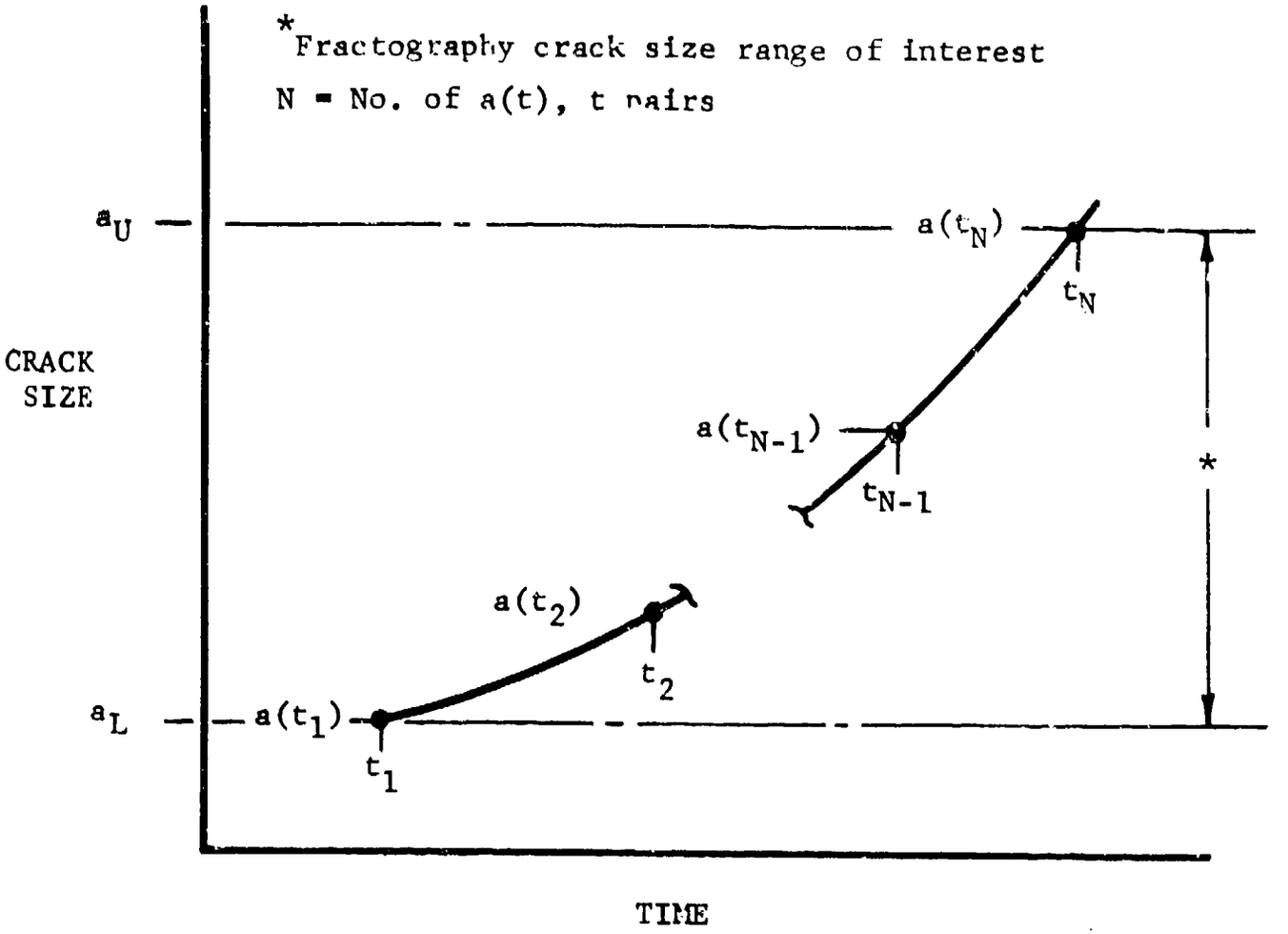


Fig. 4.6 Notational Scheme for Determination of Q_i^*

Equation 4-9 is rewritten in Eq. 4-10 in the notational form shown in Fig. 4.6.

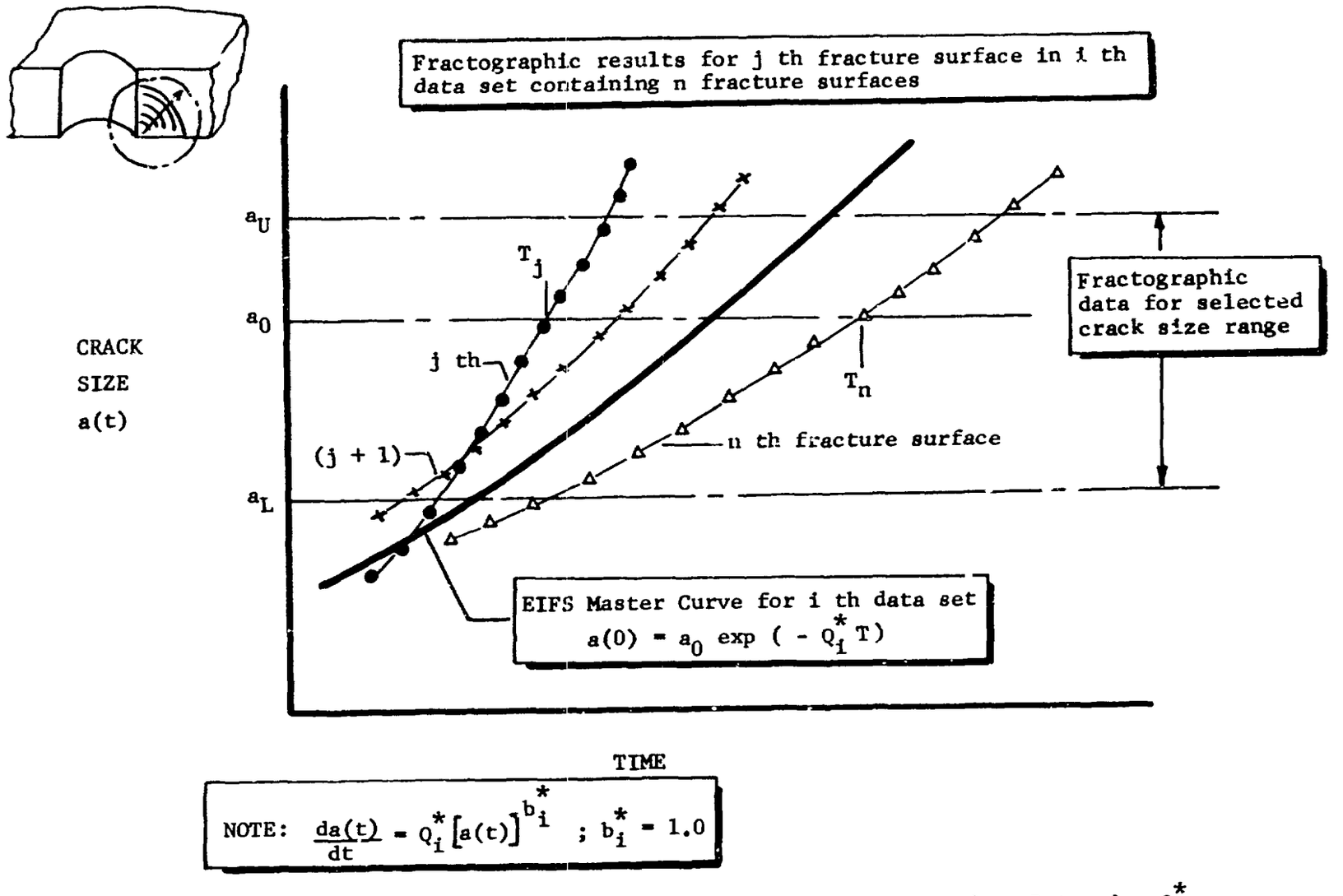
$$Q_i^* = \frac{\sum_{i=N}^{2} \sum_{j=1}^{i-1} (t_i - t_{i-j}) \ln \left[\frac{a(t_i)}{a(t_{i-j})} \right]}{\sum_{i=N}^{2} \sum_{j=1}^{i-1} (t_i - t_{i-j})^2} \tag{4-10}$$

In Eq. 4-10, N = number of [a(t),t] pairs in the selected fractographic crack size range, a_L to a_U.

Discussion

The crack growth parameter Q_i^* in Eq. 4-1 can be determined using the fractographic data for a single fatigue crack or for all fatigue cracks in a given data set. Q_i^* can be determined using either Eq. 4-6 or 4-10. Analytically, Eq. 4-10 is more appealing than Eq. 4-6 because Q_i^* can be determined directly from the a(t) and t values without having to approximate da(t)/dt values.

The two methods for determining Q_i^* are illustrated and compared in Section 4.6.1. A conceptual description of the fractographic data needed to determine Q_i^* for a given data set is given in Fig. 4.7.



NOTE: $\frac{da(t)}{dt} = Q_i^* [a(t)]^{b_i^*}$; $b_i^* = 1.0$

Fig. 4.7 Conceptual Description of Fractographic Data used to Determine Q_i^* for the i th Fractographic Data Set

4.4.2.2 Determination of α_i, β_i and ε_i

For a given fractographic data set, the Weibull distribution parameters for shape, scale and lower bound of TTCI are denoted by α_i, β_i and ϵ_i, respectively. These parameters can be determined as follows:

1. Define TTCI values for a selected reference crack size, a_0, in the range: $a_L \leq a_0 \leq a_U$ (Ref. Fig. 4.7).

2. Select an upper bound EIFS value, x_u, in the range: $a_L \leq x_u \leq a_0 \leq a_U$. The x_u value selected should result in an ϵ_i value (Ref. Eq. 3-13) $\leq$ the smallest TTCI for the given a_0. For a given x_u there is a corresponding ϵ_i value and vice versa (Eq. 3-12) (Ref. Fig. 4.5).

3. Estimate the cumulative distribution of TTCI by ranking the $(t - \epsilon_i)$ values for the reference crack size, a_0, in ascending order using Eq. 4-11.

$$F_T(t) = \frac{r}{n+1} \tag{4-11}$$

where r = rank of $(t - \epsilon_i)$ (1, 2, ...n)

n = No. of $(t - \epsilon_i)$ values in the fractographic data set for reference crack size, a_0.

4. Determine α_i and β_i. The three parameter Weibull distribution, Eq. 3-1, can be transformed into the following least squares fit form.

$$Z = \alpha_i Y + U \tag{4-12}$$

where $Z = \ln \{ -\ln [1-F_T(t)]\}$

$Y = \ln (t-\varepsilon_i)$

$U = -\alpha_i \ln\beta_i$

α_i and β_i can be determined using Eq. 4-13 and 4-14, respectively.

$$\alpha_i = \frac{N\Sigma YZ-(\Sigma Y)(\Sigma Z)}{N\Sigma Y^2-(\Sigma Y)^2} \tag{4-13}$$

$$\beta_i = \exp\left[\frac{\alpha_i \Sigma Y-\Sigma Z}{\alpha_i N}\right] \tag{4-14}$$

where N = No. of TTCI values for a_0; and Y and Z are defined previously (Eq. 4-12).

4.4.2.3 Determination of α and $Q\beta$

α and $Q\beta$ in Eq. 3-15 can be determined as follows:

1. Compute the product $Q_i^* \beta_i$ for each fractographic data set to be pooled. Then determine $Q\beta$ = Ave. $Q_i^* \beta_i$.

2. Compute α using the normalized TTCI results for all the fractographic data sets pooled. Two suggested methods are described below.

Method 1

Compute $(t-\epsilon_i)/\beta_i$ for each fatigue crack in each fractographic data set for the specified a_0. Pool the resulting values for all the data sets and estimate the cumulative distribution, $F_T(t)$, using Eq. 4-11. In Eq. 4-11, r = rank of $(t-\epsilon_i)/\beta_i$ and n = number of $(t-\epsilon_i)/\beta_i$ values for all the fractographic data sets combined. Note that β_i is the Weibull scale parameter for each TTCI data set separately.

Equation 3-1 with the subscript "i" notation can be transformed into Eq. 4-15 as follows,

$$Z = \alpha X \tag{4-15}$$

where $Z = \ln \{ -\ln [1-F_T(t)]\}$

$$X = \ln (t-\epsilon_i)/\beta_i$$

Using Eq. 4-15 and the least squares criterion, the following equation for α is obtained.

$$\alpha = \frac{\Sigma XZ}{\Sigma X^2} \tag{4-16}$$

Therefore, α can be obtained using Eq. 4-16 and the results for the pooled fractography.

Method 2

A non-dimensional form for $F_T(t)$ is given by Eq. 4-17.

$$F_T(t) = 1 - \exp\left\{ -\left[\frac{\hat{Q}_i t - \ln(a_0/x_u)}{Q\beta}\right]^{\alpha} \right\} \tag{4-17}$$

where $\hat{Q}_i$ =(Ave. $Q_i^* \beta_i)/\beta_i$ (normalized Q_i^* value for data set i)

and $Q\beta$ = Ave. $Q_i^* \beta_i$ = constant (for generic EIFS distribution).

Equation 4-17 is obtained by substituting Eq. 3-13 into Eq. 3-1 and rearranging terms.

Equation 4-17 can be transformed into the following least squares fit form,

$$Z = \alpha X + B \tag{4-18}$$

where, $Z = \ln \{ - \ln [1 - F_T(t)] \}$

$$X = \ln [\hat{Q}_i t - \ln (a_0/x_u)]$$

$$B = - \alpha \ln Q\beta$$

$$Q\beta = \text{Ave. } \overset{*}{Q}_i \beta_i = \text{constant.}$$

With the least square criterion, Eq. 4-19 for α is obtained.

$$\alpha = \frac{\Sigma Z}{\Sigma X - N \ln Q\beta} \tag{4-19}$$

α can then be obtained using the pooled fractographic results and Eq. 4-19.

4.4.2.4 Optimization of Parameters

The parameters x_u, α and $Q\beta$ in Eq. 3-15 should be optimized to meet the user's criterion for acceptable fit. There are different ways this can be accomplished. For example, the sum squared error (SSE) can be minimized for a given combination of parameters (i.e., a_0, x_u, α, $Q\beta$). a_0 and x_u values are assumed and the corresponding α, $Q\beta$ and SEE are determined. This procedure is continued until the SSE is minimized.

4.4.2.5 General Steps

The essential steps for determining the IFQ model parameters are as follows:

1. Select suitable fractographic data for fitting the model parameters.

2. Select a fractographic crack size range of most interest for durability analysis. For example, a crack size range of 0.020" to 0.050" might be used for fastener holes. Therefore, a_L = 0.020" and a_U = 0.050" (Ref. Fig. 4.7). The fractographic readings for each fatigue crack may not cover the selected crack size range of interest, i.e., a_L to a_U. In such cases, the fractography should be extrapolated,

forward or backwards, to cover the range from a_L to a_U (Ref. Fig. 4.7).

3. Select a reference crack size, a_0, for TTCI's (Ref. Fig. 4.7). The following "rule-of-thumb" for a_0 is based on the IFQ modeling experience of this program. More experience with pooled fractographic results and IFQ model parameter optimization is needed to provide additional guidelines.

$$\left(\frac{a_L + a_U}{2}\right) \leq a_0 \leq a_U$$

4. Determine the crack growth parameter, Q_i^*, in Eq. 4-1 for each fractographic data set. Use fractographic results in the range from a_L to a_U.

5. Determine the TTCI for a selected a_0 for each fatigue crack. Use interpolation or extrapolation procedures as required (Ref. Fig. 4.7).

6. For a given x_u determine the corresponding ε_i and β_i for each fractographic data set separately. Then, compute the product $Q_i^* \beta_i$ for each data set (Ref. Fig. 4.4).

7. Compute the average $Q_i^* \beta_i$ for the pooled fractographic data sets. With the average $Q_i^* \beta_i$ value, determine the normalized crack growth parameter , $\hat{Q}_i$, for each data set as follows:

$$\hat{Q}_i = \frac{\text{ave. } Q_i^* \beta_i}{\beta_i} = \frac{\frac{1}{n} \sum_{i=1}^{n} Q_i^* \beta_i}{\beta_i} \tag{4-20}$$

where n = No. of TTCI data sets pooled.

8. Normalize the TTCI results for each data set as described in Section 4.4.2.3; pool the results and determine the corresponding α for the pooled data sets.

9. Using the resulting values for x_u, α and ave. $Q_i^* \beta_i$ determine how well the EIFS distribution fits the fractographic data used. The fit may not be satisfactory because the x_u chosen is not optimum. To improve the fit for the fractographic data used, one can choose another value of x_u and repeat the same procedures described in steps 6 through 8. By iteration, one can determine the combination of x_u, a_0, α, and ave. $Q_i^* \beta_i$ giving the "best-fit".

4.5 STATISTICAL SCALING OF β FOR MULTIPLE DETAILS

It's not always practical or cost-effective to initiate a fatigue crack in each fastener hole in a test specimen that will be large enough (e.g., $a_U \geq 0.050''$) to provide useful fractographic data for quantifying IFQ. A statistical method has been developed for determining the Weibull scale parameter, β, for the cumulative distribution of TTCI using only the largest fatigue crack in any one of ℓ fastener holes per specimen [16]. This method accounts for the TTCI in each fastener hole per specimen and minimizes the fractographic data needed to determine the IFQ. For example, fractography is required for only the largest fatigue crack in any one of ℓ fastener holes per specimen in the data set. Further details are discussed below and elsewhere [16].

A fatigue test specimen may contain one or more replicate fastener holes (e.g., same: drilling technique, diameter, fastener type, fastener fit, etc.). In practice, the test specimens are fatigue tested until a fatigue crack $\geq a_U$ is initiated in at least one of the ℓ fastener holes per specimen. When the fatigue test is stopped, only one fastener hole per specimen may have a fatigue crack $\geq a_U$ and the remaining fastener holes may or may not contain an observable fatigue crack.

The resistance of each fastener hole per specimen to fatigue cracking should be accounted for when defining the TTCI and EIFS distributions. Fastener holes with the higher resistance to fatigue cracking have a longer TTCI. Although each fastener hole in a specimen may be drilled and fatigue tested the same way, the TTCI will typically vary. Therefore, the IFQ for the fastener holes should account for the TTCI for each fastener hole.

If each replicate fastener hole per test specimen is subjected to a common stress history, the Weibull scale parameter for the TTCI distribution can be determined using Eq. 4-21 [16],

$$\beta = \beta_{\ell} (\ell)^{1/\alpha} \tag{4-21}$$

where: β = Weibull scale parameter for the TTCI distribution.

β_{ℓ} = Weibull scale parameter for the TTCI distribution based on the largest fatigue crack in one of ℓ fastener holes per specimen in the complete fractographic data set.

α = Weibull shape parameter

ℓ = Number of fastener holes per specimen

Eq. 4-21 can be used to "scale" β_ℓ to account for the TTCI for each fastener hole in the data set. A conceptual description of β_ℓ scaling is illustrated in Fig. 4.8 and the effects of scaling on the TTCI distribution and the EIFS distribution are noted. For illustrative purposes, the fatigue test specimen is assumed to contain four equally stressed fastener holes (i.e., $\ell_i = 4$).

If the TTCI data is available for each fastener hole in the data set, there's no need to use Eq. 4-21 because β can be determined directly using the TTCI data. Eq. 4-21 has been successfully used for both straight bore and countersunk fastener hole applications [16,19-21].

Actual TTCI observations are needed to check the goodness-of-fit of the theoretical TTCI distribution.

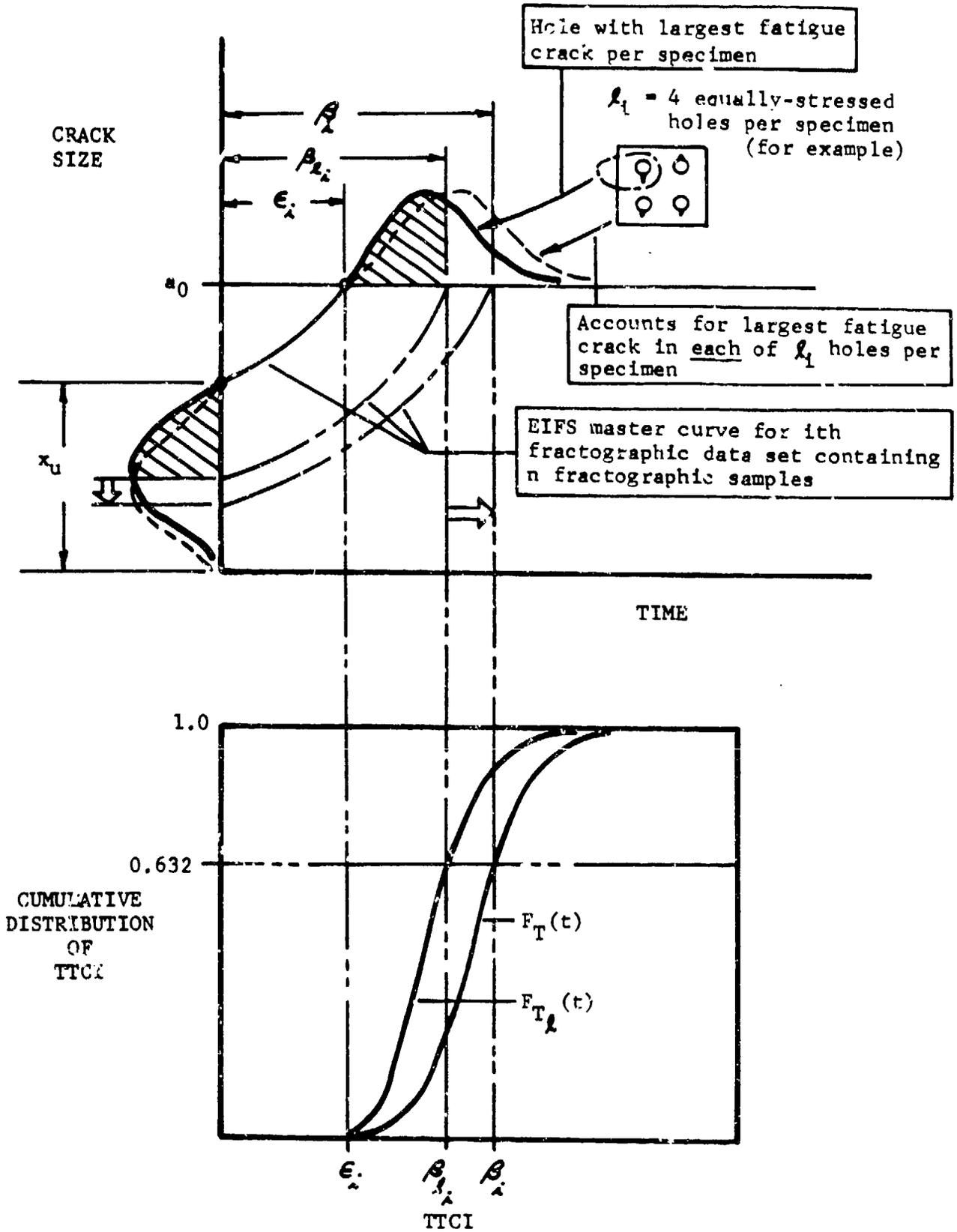


Fig. 4.8 Illustration of β Scaling Concept for the i th Fractographic Data Set

Therefore, if fractography is available for only the largest fatigue crack in any one of ℓ fastener holes in a test specimen, these observations must be used to check the goodness-of-fit. For example, this means that β_ℓ must be used instead of β to check the TTCI distribution goodness-of-fit because fractographic results are not available for each fastener hole in the data set. In this case, the goodness-of-fit of the theoretical cumulative distribution of crack size must also be checked using actual fractography available. As a result, β is used to define the IFQ of the durability analysis, but β_ℓ is employed to verify the goodness-of-fit. This aspect is further discussed in Section 4.6.3.

4.6 ILLUSTRATION OF PROCEDURES FOR DETERMINING IFQ

The purpose of this section is to illustrate how the IFQ model parameters can be determined using the procedures described in Section 4.4. To illustrate the procedures, including data pooling methods, the IFQ model parameters will be determined for the three fractographic data sets described in Table 4-1. Essential features of the calibration procedures will be illustrated and the goodness-of-fit of the derived EIFS distribution will be demonstrated for pooled data sets as well as for individual data sets.

Table 4-1 Description of Fractographic Data Sets

Data Set	Material	% Load Transfer	Gross Stress & (ksi)	Fastener		Load Spectrum	No. Specimens Used
				Dia	I.D.		
AFXLR4	7475-T/351 Aluminum	15	32	¼"	MS90353	F-16 400h	10
AFXMR4	↓	↓	34	↓	↓	↓	9
AFXHR4	↓	↓	38	↓	↓	↓	9

Notes:

(a) Countersunk rivet (Blind, Pull-Through)

(b) Fastener holes drilled using Winslow Spacematic drill

(c) Fastener - hole: clearance fit

(d) Ref. Fig. 4.3 for specimen details

(e) Fractography basis: Largest crack for 1 of 4 holes per specimen (ℓ = 4)

The resulting EIFS distribution will be discussed, including β-scaling considerations.

4.6.1 Determination of TTCI and Q_i^* Values

TTCI and Q_i^* values can be determined from the fatigue crack growth data (fractography). The fractographic data (a(t) versus t) for data sets AFXLR4, AFXMR4 and AFXHR4 is obtained from Volume VIII [24]. The procedures for determining the TTCI values for a selected reference crack size, a_0, and for determining Q_i^* values are illustrated in Tables 4-2 and 4-3, respectively, for data set AFXMR4.

A fractographic size range of a_L = 0.020" to a_U = 0.050" will be used to define the EIFS distribution. Using the fractographic data for specimen AFXMR4 (563HB) from Volume VIII, the TTCI values can be determined for selected a(t) values by interpolation. TTCI's are desired for five reference crack sizes: 0.020", 0.0275", 0.035", 0.0425" and 0.050". A three-point Lagrangian interpolation is used to determine the TTCI for the selected reference crack size. Results are shown in Table 4-2, including a comparison of interpolated TTCI's versus observed values from the fractographic results.

Table 4-2 Illustration of Procedure for Determining TTCI Values for Selected a(t)'s

Specimen	Fractographic Data **		Interpolation Results	
	a(t) (In.)	t (Flt. Hrs.)	a(t) (In.)	t (Flt. Hrs.)
AFXMR4 (563HB)	0.0196*	4800*	0.0196	4800
			0.020	4847
	0.0237	5200	0.0237	5257
			0.0275	5643
	0.0262	5600	0.0262	5515
	0.0314*	6000*	0.0314	6000
			0.035	6295
	0.0367	6400	0.0367	6423
			0.0425	6803
	0.0433	6800	0.0433	6848
			0.0500	7166
	0.0509*	7200*	0.0509	7200

Notes

* Fractographic results used for three-point Lagrangian interpolation

** Ref. Vol VIII [24]

The following procedures are illustrated in Table 4-3 for specimen AFXMR4 (567HB):

- o extrapolation of fractography to a_L
- o Modified Secant Method for defining da(t)/dt's [30]
- o determination of Q_i^* for a single fatigue crack

Since the fractography for this fatigue crack starts at a crack size > a_I (0.020"), the TTCI at a_L = 0.020" must be determined by extrapolation. Various extrapolation techniques could be used [e.g., 31-34]. In this case, the generalized crack growth relationship of Eq. 3-10 is used to determine the TTCI value for a_L = 0.020". The crack growth parameter Q in Eq. 3-10 (Note: $Q = Q_i^*$ to be consistent with notation used for fractographic data pooling) was determined using the a(t) versus t values shown in Table 4-3. Q_i^* values for Eq. 3-10 were determined using Eqs. 4-6 and 4-10. However, the extrapolation for a(t) = 0.020" was made using $Q_i^* = 5.202 \times 10^{-4}$ as shown in Table 4-3.

The procedures illustrated in Tables 4-2 and 4-3 were used to determine the TTCI's for five reference crack sizes for each specimen in three fractographic data sets. Results

Table 4-3 Illustration of Procedures for Determining Q_i^* for Data Set AFXMR4 (Specimen No. 567 HB)

Flight Hours t	a (t) (In.)	$\frac{da(t)}{dt} \times 10^6$ (In./Hr.) ①	$Q_i^* \times 10^4$ (1/Hr.) Eq. 4-6	$Q_i^* \times 10^4$ (1/Hr.) Eq. 4-10
(4243)②	0.0200	(10.83)③		
4400	0.0217	12.00	4.947	5.202
4800	0.0265	14.00	↓	↓
5200	0.0329	13.63		
5600	0.0374	13.88		
6000	0.0440	16.00		
(6387)④	0.0500	(15.44)⑤		
6400	0.0502	13.13		
6800	0.0545	13.38		

Notes

① Based on Modified Secant Method [29, 30]

② Extrapolation from $a(t_2) = 0.0217''$ to $a(t_1) = 0.0200''$
Using Eq. 3-10 and: $Q_i^* = 5.202 \times 10^{-4}$, $t_2 = 4400$

③ $$\frac{da(t)}{dt} = \frac{0.0217 - 0.020}{4400 - 4243} = 10.83 \times 10^{-6}$$

④ Three - point Lagrangian interpolation

$$t = \frac{(0.050 - 0.044)\ (0.050 - 0.0374)\ (6400)}{(0.0502 - 0.044)\ (0.0502 - 0.0374)} + \frac{(0.050 - 0.0502)\ (0.050 - 0.0374)\ (6000)}{(0.044 - 0.0502)\ (0.044 - 0.0374)} + \frac{(0.050 - 0.0502)\ (0.050 - 0.044)\ (5600)}{(0.0374 - 0.0502)\ (0.0374 - 0.044)} = 6387$$

⑤ $$\frac{da(t)}{dt} = \frac{\frac{0.0502 - 0.050}{(6400 - 6387)} + \frac{0.0500 - 0.044}{(6387 - 6000)}}{2} = 15.44 \times 10^{-6}$$

are summarized in Tables 4-4, 4-5 and 4-6 for data sets AFXLR4, AFXMR4 and AFXHR4, respectively. In Tables 4-4 through 4-6, average TTCI values are shown for each of the five reference crack sizes. Also, the crack growth parameter Q_i^* for two different equations (Eqs. 4-6 and 4-10) is shown for individual fractographic data sets. Q_i^* values, based on Eq. 4-6, for data sets AFXLR4, AFXMR4 and AFXHR4, are shown in Table 4-7, along with the input data used.

4.6.2 Determination of α and $Q\beta$

The procedures for determining α and $Q\beta$, described in Section 4.4, are illustrated in this section. Input/output for individual fractographic data sets is summarized in Table 4-8. For illustration purposes, results are shown for a_0 = 0.035" and for x_u = 0.025". The parameters ϵ_i, α_i, β_{ℓ_i} and product $Q_i^*\beta_{\ell_i}$ were first determined for individual data sets.

Using the average $Q_i^*\beta_{\ell_i}$ value of 2.155 for the three data sets, the normalized Q_i^* value was determined for each data set. For example, $\hat{Q}_{\ell_i}$ values of 1.739×10^{-4}, 2.818×10^{-4}, and 4.040×10^{-4} were determined for data sets AFXLR4, AFXMR4 and AFXHR4, respectively. $\hat{\epsilon}_i$ values were then determined

Table 4-4 Summary of TTCI Values for Data Set AFXLR4
(7475 - T7351 Alumimum)

Data Set	Specimen No.	σ (ksi)	TTCI (Flt. Hours)				
			a_0 =0.020"	a_0 =0.0275"	a_0 =0.035"	a_0 =0.0425"	a_0 =0.050"
AFXLR4	33TA	32	9835	11433	12818	13990	14949
	35TA		5100	6267	7474	8734	10034
	36HA		11070	12717	14066	15117	15869
	120HB		9851	13650	16676	18934	20419
	121HB		14608	16791	18653	20193	21411
	559HB		3649	4089	4511	4917	5305
	123TA		20844	22334	23536	24451	25078
	124TA		5707	7964	9905	11532	12844
	557HB		5973	6435	6875	7293	7689
	558TA		8566	9469	10262	10945	11517
		Ave.	9520	11115	12478	13611	14512
			$Q_i^* = 1.822 \times 10^{-4}$/Hr(Eq. 4-10)				
			$Q_i^* = 1.857 \times 10^{-4}$/Hr(Eq. 4-6)				

Table 4-5 Summary of TTCI Values for Data Set AFXMR4
(7475 - T7351 Aluminum)

Data Set	Specimen No.	σ (ksi)	TTCI (Flt. Hours)				
			a_0 =0.020"	a_0 =0.0275"	a_0 =0.035"	a_0 =0.0425"	a_0 =0.050"
AFXMR4	562TA	34	2629	5379	7821	9955	11780
	563HB		4847	5643	6295	6803	7166
	564TB		11272	12491	13455	14163	14617
	565HA		6476	8412	10077	11471	12595
	566HA		2643	5277	7308	8738	9566
	567HB		4243	4793	5354	5885	6387
	568HB		11078	12669	14014	1'111	15962
	569HB		3076	4541	5765	6750	7495
	570HB		2509	2832	3099	3313	3471
		Ave.	5415	6893	8132	9132	9893
			$Q_i^* = 2.027 \times 10^{-4}$/Hr (Eq. 4-10)				
			$Q_i^* = 2.091 \times 10^{-4}$/Hr (Eq. 4-6)				

Table 4-6 Summary of TTCI Values for Data Set AFXHR4
(7475 - T7351 Aluminum)

Data Set	Specimen No.	σ (ksi)	TTCI (Flt. Hours)				
			a_0 =0.020"	a_0 =0.0275"	a_0 =0.035"	a_0 0.0425"	a_0 = 0.050"
AFXHR4	571HB	38	2651	3539	4192	4611	4794
	572HA		5090	5600	6086	6548	6985
	573HA		5726	6559	7321	8012	8633
	574TB		2973	3312	3607	3859	4067
	575TB		8119	8945	9678	10315	10859
	576HA		3194	3665	4051	4353	4569
	577HB		1784	2085	2321	2492	2597
	579HA		6159	6776	7306	7747	8101
	580HA		1894	2394	2814	3154	3415
		Ave.	4177	4764	5264	5677	6002
			$Q_i^* = 4.981 \times 10^{-4}$/Hr(Eq. 4-10)				
			$Q_i^* = 5.092 \times 10^{-4}$/Hr(Eq. 4-6)				

Table 4-7 Summary of Q_i^* Results Based On Eq. 4-6.

a(t) (In.)	AFXLR4			AFXMR4			AFXHR4		
	t Flt. Hrs.	$\frac{da(t)}{dt} \times 10^6$ (In./Hr.)	$Q_i^* \times 10^4$ (1/Hr.)	t Flt Hrs.	$\frac{da(t)}{dt} \times 10^6$ (In./Hr.)	$Q_i^* \times 10^4$ (1/Hr.)	t Flt. Hrs.	$\frac{da(t)}{dt} \times 10^6$ (In./Hr.)	$Q_i^* \times 10^4$ (1/Hr.)
0.020	9520	4.70	1.857	5415	5.07	2.091	4177	12.78	5.092
0.0275	11115	5.10	↓	6893	5.56	↓	4764	13.89	↓
0.035	12478	6.06	↓	8132	6.78	↓	5264	16.58	↓
0.0425	13611	7.47	↓	9132	8.68	↓	5577	20.62	↓
0.050	14512	8.32	↓	9893	9.86	↓	6002	23.08	↓

Table 4-8 Summary of IFQ Model Parameters and Results for Individual Data Sets (a_0 = 0.035", x_u = 0.025")

Fractographic Data Set I.D.	i	t(TTCI) (FLT. HRS)	ℓ_i	$Q_i^* x10^4$ (1/Hr.)	Rank r	r/n+1	ε_i (Hrs.)	α_i	β_{ℓ_i} (Hrs.)	$Q_i^* \beta_{\ell_i}$	$\hat{Q}_{\ell_i} x10^4$ (1/Hr.)	$\hat{\varepsilon}_i$ (Hrs.)
AFXLR4	1	4511	4	1.822	1	.091	1847	1.575	12388	2.257	1.739	1935
		6875			2	.182						
		7474			3	.273						
		9905			4	.364						
		10262			5	.455						
		12818			6	.546						
		14066			7	.637						
		16676			8	.728						
		18653			9	.819						
		23536			10	.909						
AFXMR4	2	3099	4	2.027	1	.1	1659	1.456	7[illegible]	1.550	2.818	1194
		5354			2	.2						
		5765			3	.3						
		6295			4	.4						
		7308			5	.5						
		7821			6	.6						
		10077			7	.7						
		13455			8	.8						
		14014			9	.9						
AFXHR4	3	2321	4	4.981	1	.1	676	1.704	5334	2.657	4.040	833
		2814			2	.2						
		3697			3	.3						
		4051			4	.4						
		4192			5	.5						
		6086			6	.6						
		7306			7	.7						
		7321			8	.8						
		9678			9	.9						

$Q\beta_\ell$ = Ave $Q_i^* \beta_{\ell_i}$ = 2.155

using the applicable $\hat{Q}_{f_k}$ values, a_0 = 0.035", x_u = 0.025", and Eq. 3-13.

The α value for the three fractographic data sets was determined using the data shown in Table 4-9 and the procedures described in Section 4.4. An α = 1.805 was determined using Eq. 4-19 and the data given in Table 4-9.

IFQ model parameters based on the three data sets are summarized in Table 4-10. Pertinent parameters and concepts for the EIFS distribution are also summarized and illustrated in Fig. 4.9.

4.6.3 Goodness-of-Fit of IFQ Model

How well does the EIFS distribution established for the pooled data sets fit the observed fractographic data? This question must be resolved before the EIFS distribution is accepted for durability analysis applications.

The IFQ model parameters α and $Q\beta$ in Table 4-10 were determined for given a_0 and x_u values. This combination of a_0, x_u, α and $Q\beta$ may not necessarily be the one that minimizes the sum squared error (SSE). In any case, the

Table 4-9 Illustration of Procedures and Results for Determination of α for Given $Q\beta_\ell$

(a_0 = 0.035"; x_u = 0.025")

Fractographic Data Set I.D.	i	t(TTCI) (FLT HRS)	$\hat{Q}_{\ell_i}$ x10^4 (1/Hrs.)	$\hat{Q}_{\ell_i}$ t	Rank r	r/n+1	α	$Q\beta_\ell$
AFXLR4	1	4511	1.739	.784	1	.034	1.805	2.155
		6875		1.196	5	.172		
		7474		1.299	6	.207		
		9905		1.722	12	.414		
		10262		1.784	14	.483		
		12818		2.229	17	.586		
		14066		2.445	18	.621		
		16676		2.899	21	.724		
		18653		3.244	24	.828		
		23536		4.093	28	.966		
AFXMR4	2	3099	2.818	.873	2	.069		
		5354		1.509	8	.276		
		5765		1.624	9	.310		
		6295		1.774	13	.448		
		7308		2.059	15	.517		
		7821		2.204	16	.552		
		10077		2.839	20	.689		
		13455		3.792	25	.862		
		14014		3.949	27	.931		
AFXHR4	3	2321	4.040	.937	3	.103		
		2814		1.137	4	.138		
		3607		1.457	7	.241		
		4051		1.636	10	.345		
		4192		1.693	11	.379		
		6086		2.458	19	.655		
		7306		2.951	22	.759		
		7321		2.957	23	.793		
		9678		3.909	26	.897		

Note: $Q\beta_\ell$ = Ave. $Q_i^* \beta_{\ell_i}$

Table 4-10 Summary of IFQ Model Parameter Results

Fractographic Data Set I.D.	i	σ (ksi)	ℓ_i	$a_0 = 0.035"$; $x_u = 0.025"$								
				α_i	β_{ℓ_i} (Hrs.)	$Q_i^* \times 10^4$ (1/Hrs.)	$Q_i^* \beta_{\ell_i}$	$\hat{Q}_{\ell_i} \times 10^4$ (1/Hrs.)	β_i (Hrs.)	$Q_i^* \beta_i$	$\hat{Q}_i \times 10^4$ (1/Hrs.)	α
AFXLR4	1	32	4	1.575	12388	1.822	2.257	1.739	29872	5.443	1.724	1.805
AFXMR4	2	34	4	1.456	7648	2.027	1.550	2.818	19818	4.017	2.599	↓
AFXHR4	3	38	4	1.704	5334	4.981	2.657	4.040	12033	5.994	4.281	↓
						Ave $Q_i^* \beta_{\ell_i}$ =	2.155		Ave $Q_i^* \beta_i$ =	5.151		

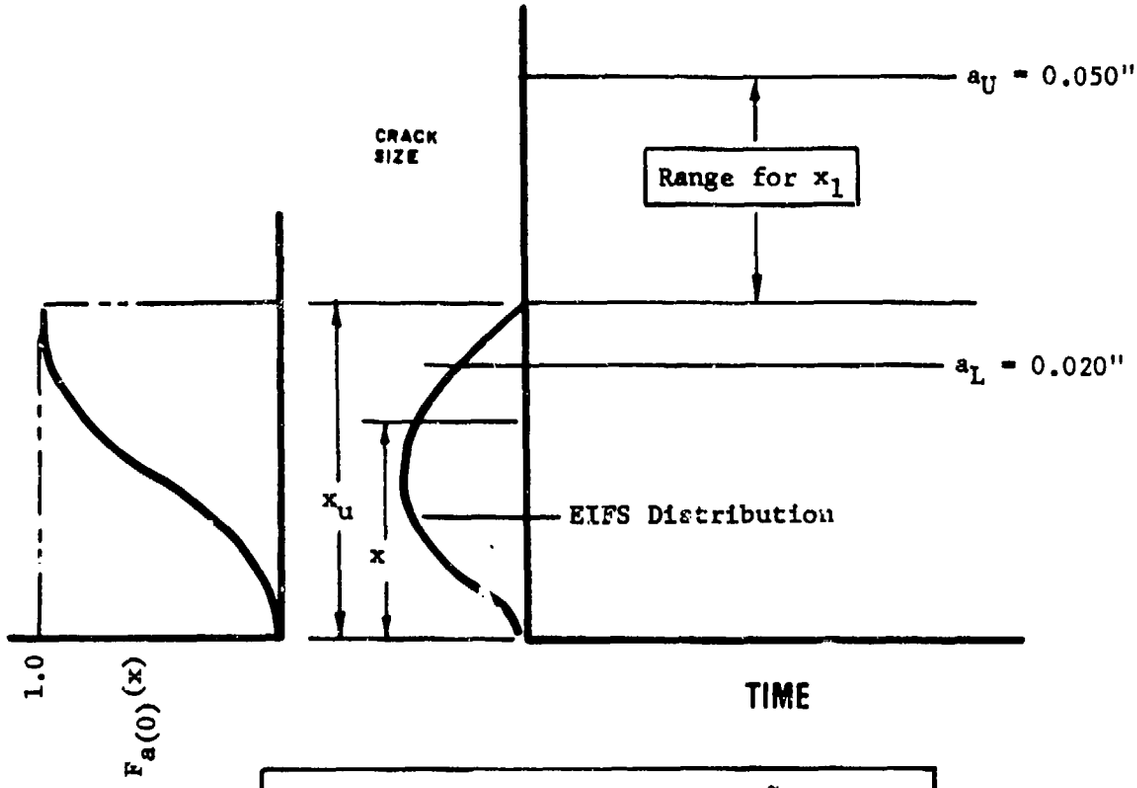
Notes

$$\text{Ave. } Q_i^* \beta_{\ell_i} = \frac{1}{n} \sum_{i=1}^{n} Q_i^* \beta_{\ell_i} = Q\beta_\ell \quad (\ell_i = 1.0;\ \text{no scaling})$$

$$\text{Ave. } Q_i^* \beta_i = \frac{1}{n} \sum_{i=1}^{n} Q_i^* \beta_{\ell_i} (\ell_i)^{1/\alpha_i} = \frac{1}{n} \sum_{i=1}^{n} Q_i^* \beta_i = Q\beta \quad (\ell_i = 4;\ \text{with scaling})$$

$$\hat{Q}_{\ell_i} = \frac{\text{Ave. } Q_i^* \beta_{\ell_i}}{\beta_{\ell_i}}$$

$$\hat{Q}_i = \frac{\text{Ave. } Q_i^* \beta_i}{\beta_i}$$



$$F_{a(0)}(x) = \exp\left\{ - \left[\frac{\ln (x_u/x)}{Q\beta} \right]^{\alpha} \right\}; \quad 0 < x \leq x_u$$

$$= 1.0 \qquad ; \quad x \geq x_u$$

$\alpha = 1.805$

$Q\beta_\ell = 2.155$

$x_u = 0.025"$

$\ell = 4$

$a_0 = 0.035"$

$a_L \leq x_u \leq a(t) \leq a_U$

$x_u \leq x_1 \leq a_U$

Note: $\beta = \beta_\ell (\ell)^{1/\alpha}$

Fig. 4.9 EIFS Cumulative Distribution Parameters for Pooled Fractographic Data Sets (AFXLR4, AFXMR4, AFXHR4)

user must decide which combination of IFQ parameters will give an acceptable fit.

A goodness-of-fit plot for the distribution of TTCI's for a_0 = 0.035" is shown in Fig. 4.10 for the three data sets pooled. This plot is based on the data given in Table 4-9 and Eq. 4-17.

The EIFS distribution ($F_{a(0)}(x)$), Eq. 3-15, and the EIFS master curve, Eq. 3-11, can be used to predict the TTCI cumulative distribution for a given a_0. By comparing the predicted results with the observed TTCI's, the goodness-of-fit plots for TTCI (a_0 = 0.035") are shown in Figs. 4.11, 4.12, and 4.13 for data sets AFXLR4, AFXMR4, AFXHR4, respectively. These plots show that the IFQ model parameters, based on the pooled fractographic results for three data sets, fit the observed fractographic data very well for the individual data sets.

TTCI goodness-of-fit plots for a crack size of 0.050" are shown in Figs. 4.14, 4.15 and 4.16 for data sets AFXLR4, AFXMR4 and AFXHR4, respectively. These plots show that the EIFS distribution, based on a_0 = 0.035", can be used to predict the TTCI cumulative distribution for a_0 = 0.050".

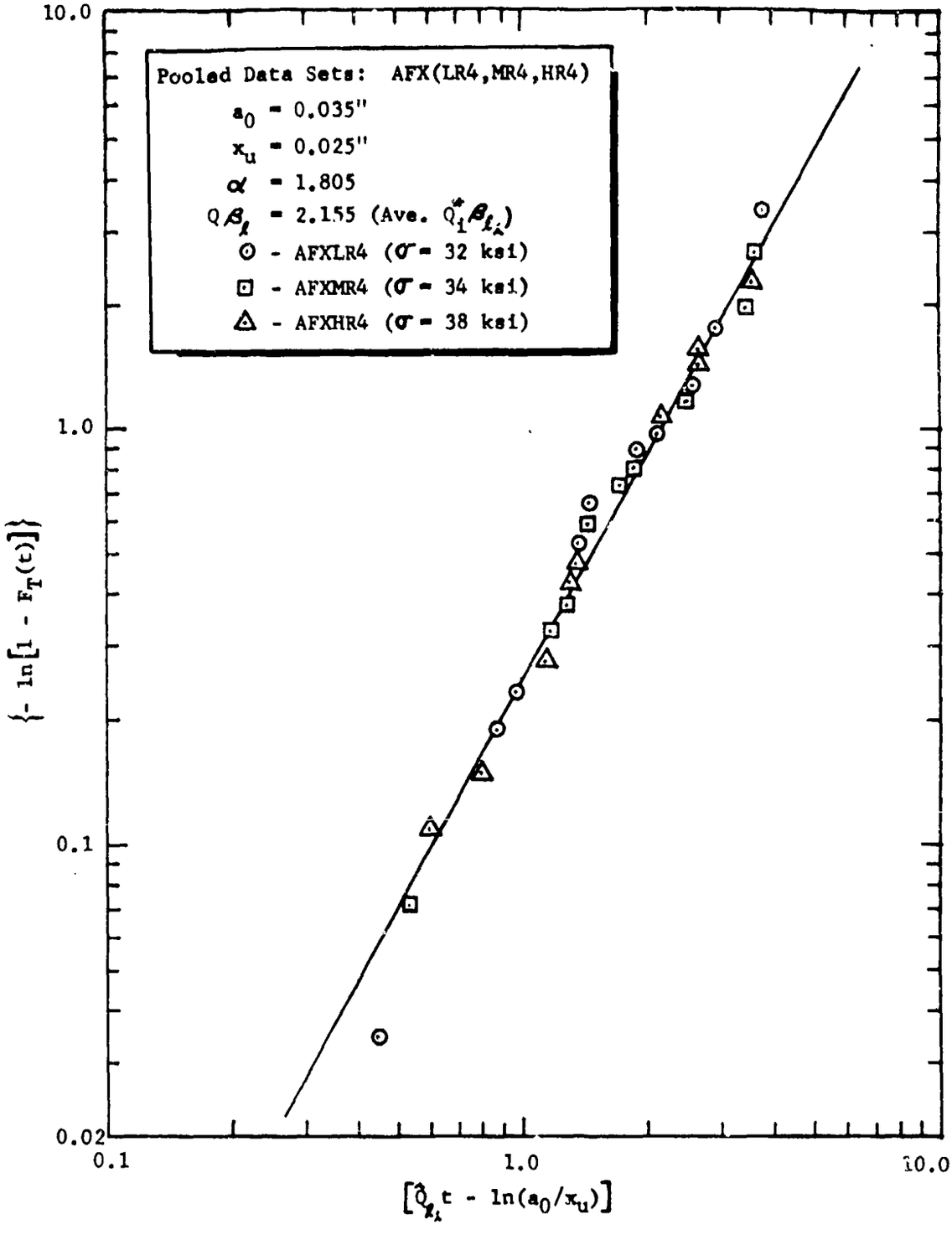


Fig. 4.10 TTCI Goodness-of-Fit Plot for Pooled Fractographic Data Sets (AFXLR4, AFXMR4, AFXHR4) ; a_0 = 0.035"

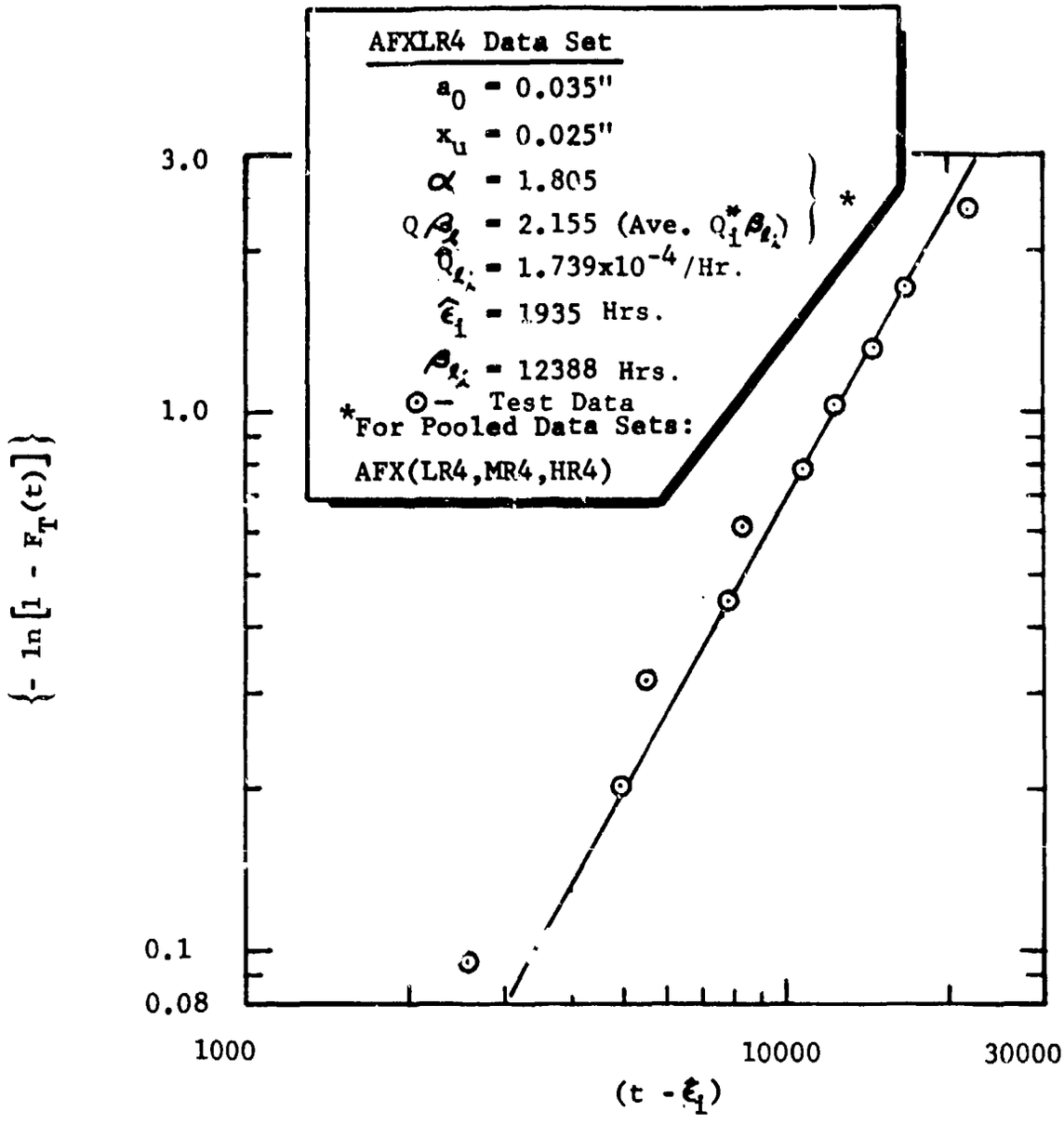


Fig. 4.11 TTCI Goodness-of-Fit Plot for AFXLR4 Data Set Based on IFQ Model Parameters for Pooled Data Sets; a_0 = 0.035"

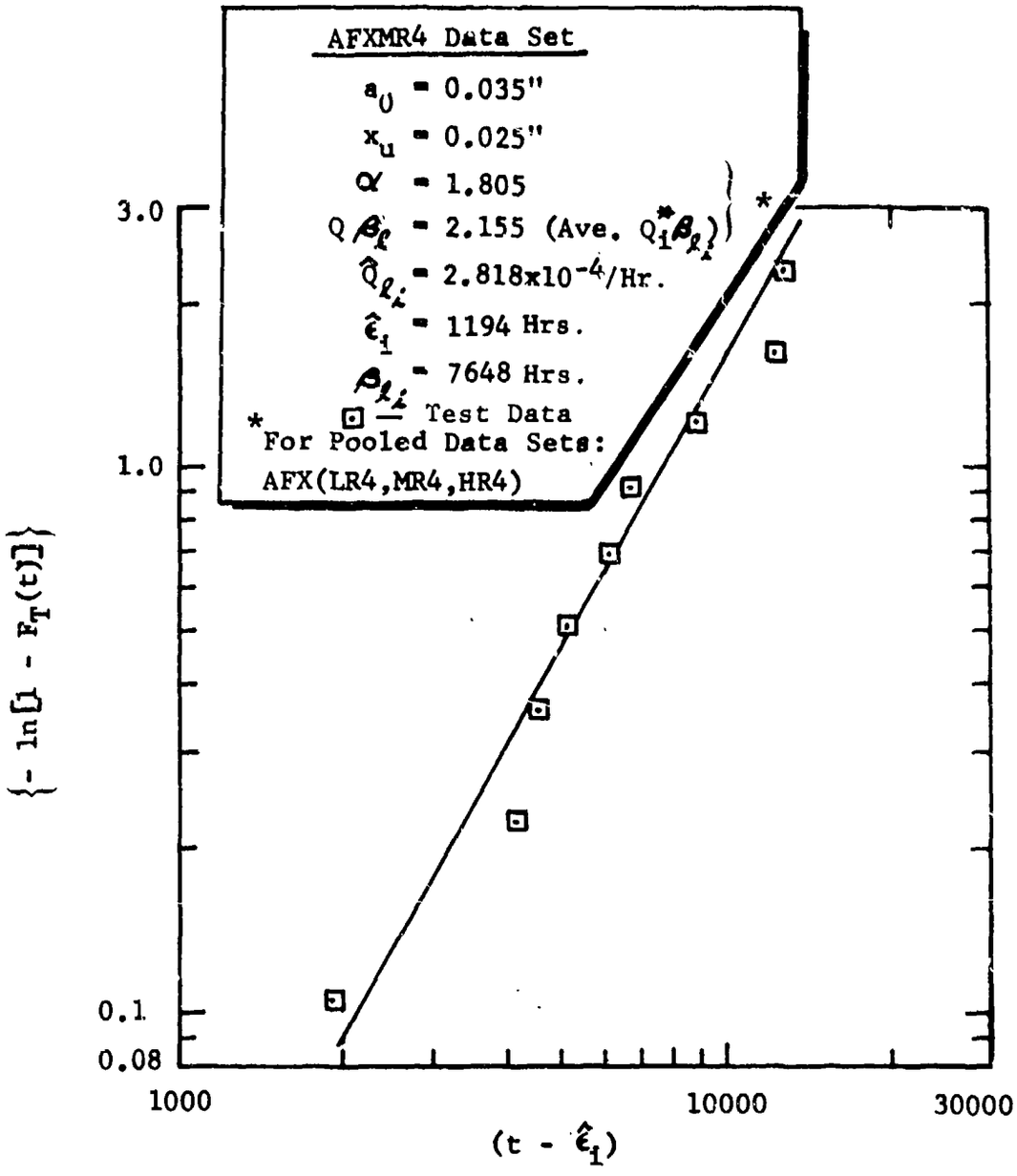


Fig. 4.12 TTCI Goodness-of-Fit Plot for AFXMR4 Data Set Based on IFQ Model Parameters for Pooled Data Sets; a_0 = 0.035"

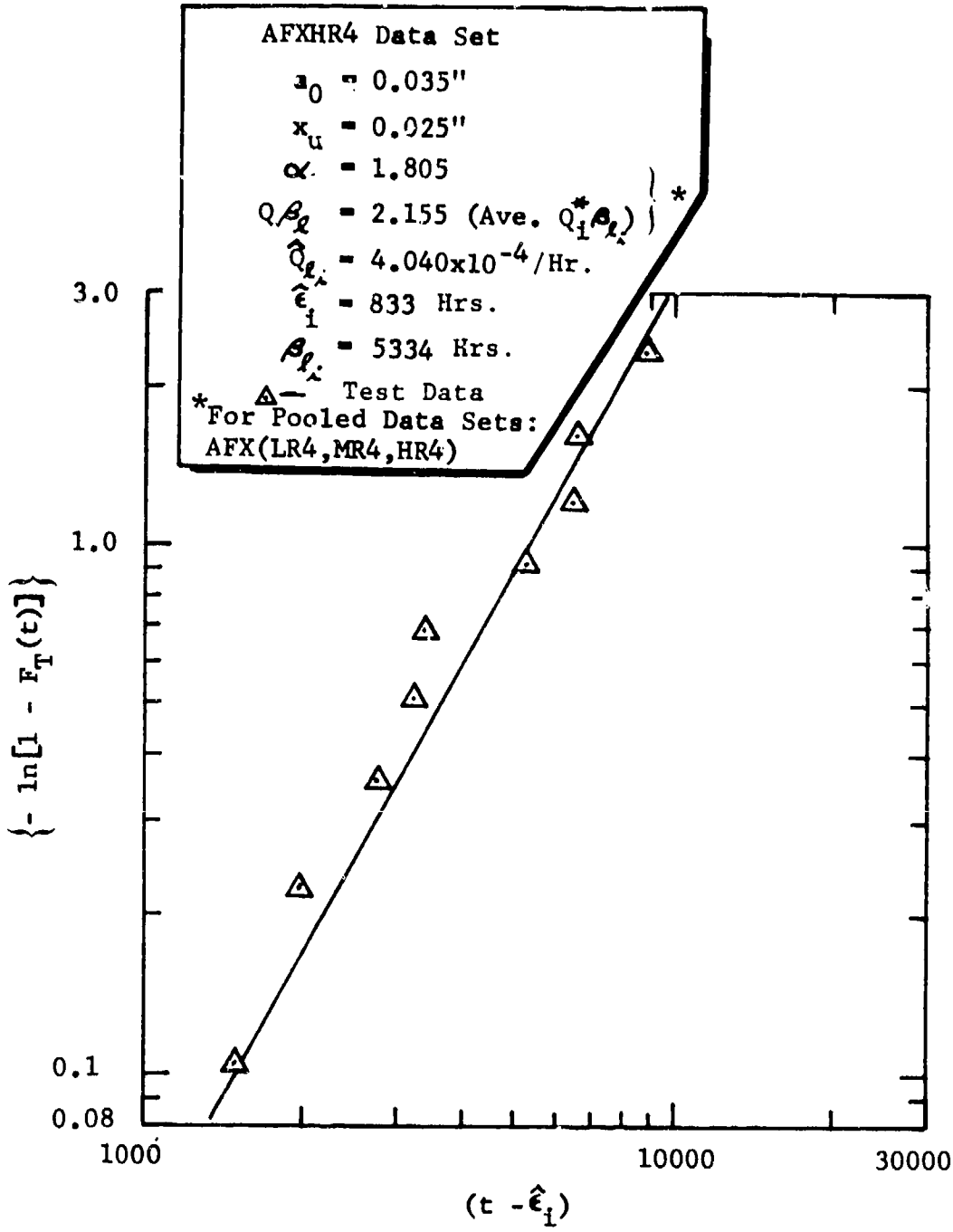


Fig. 4.13 TTCI Goodness-of-Fit Plot for AFXHR4 Data Set Based on IFQ Model Parameters for Pooled Data Sets; a_0 = 0.035"

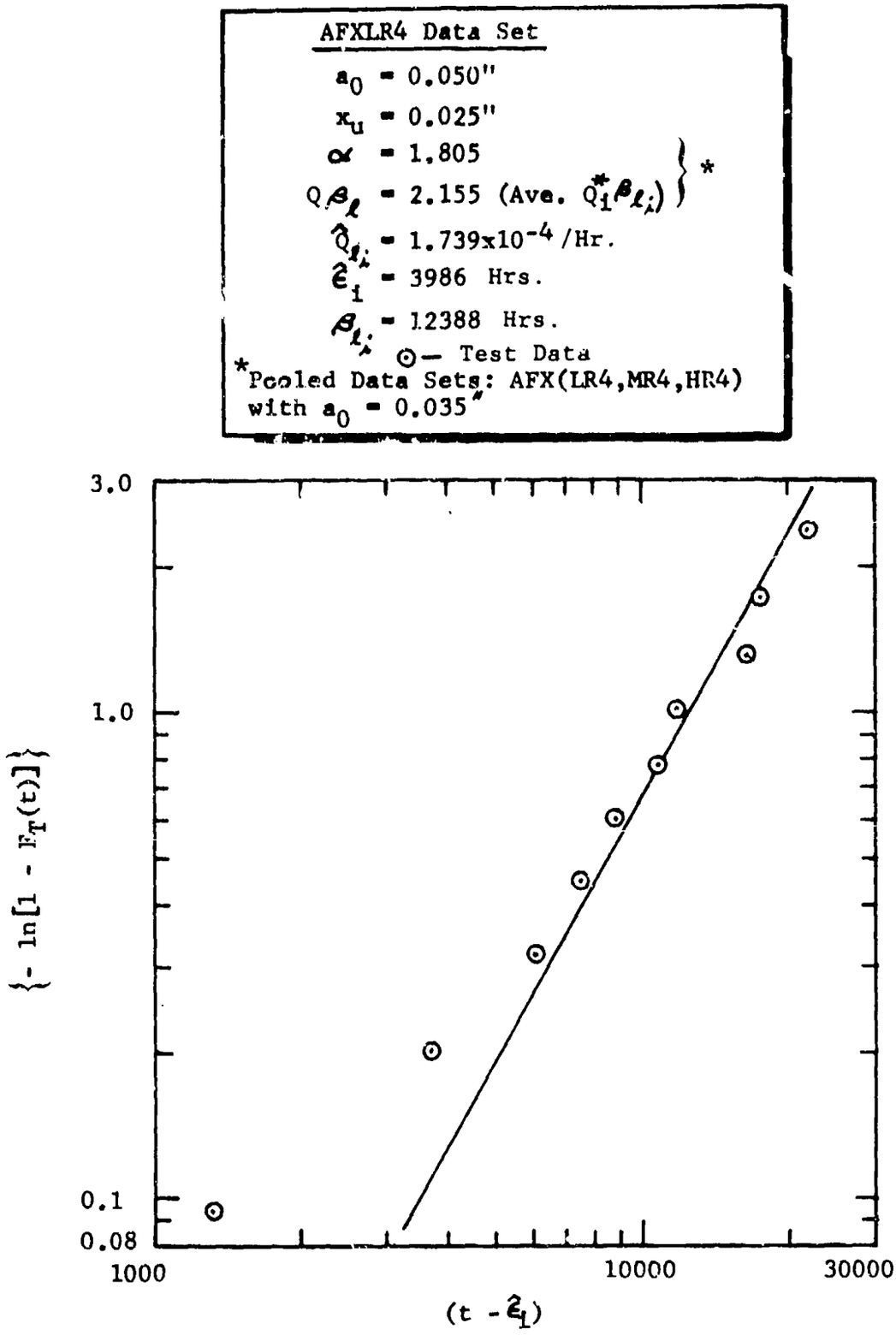


Fig. 4.14 TTCI Goodness-of-Fit Plot for AFXLR4 Data Set Based on IFQ Model Parameters for Pooled Data Sets; a_0 = 0.050"

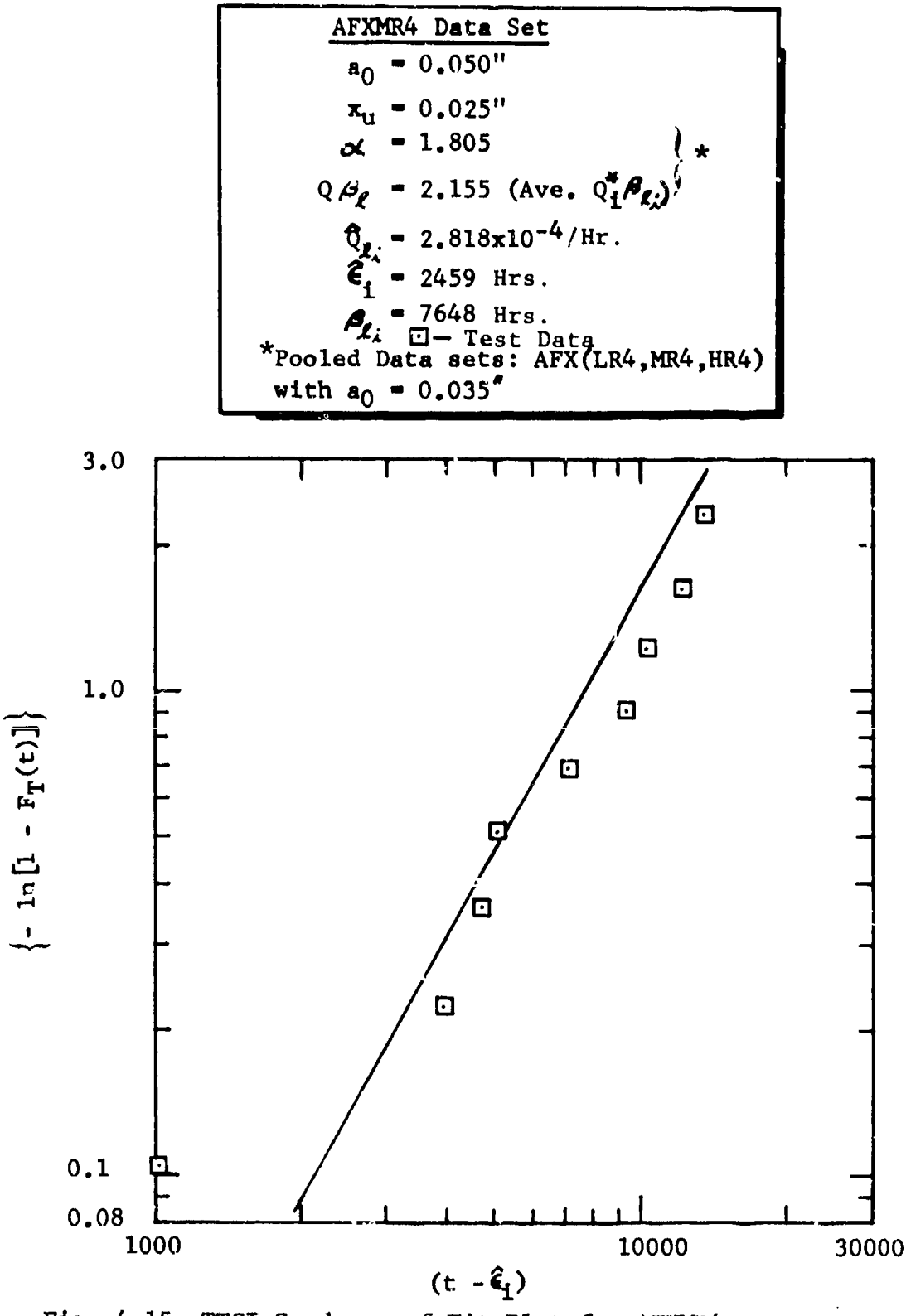


Fig. 4.15 TTCI Goodness-of-Fit Plot for AFXMR4 Data Set Based on IFQ Model Parameters for Pooled Data Sets; a_0 = 0.050"

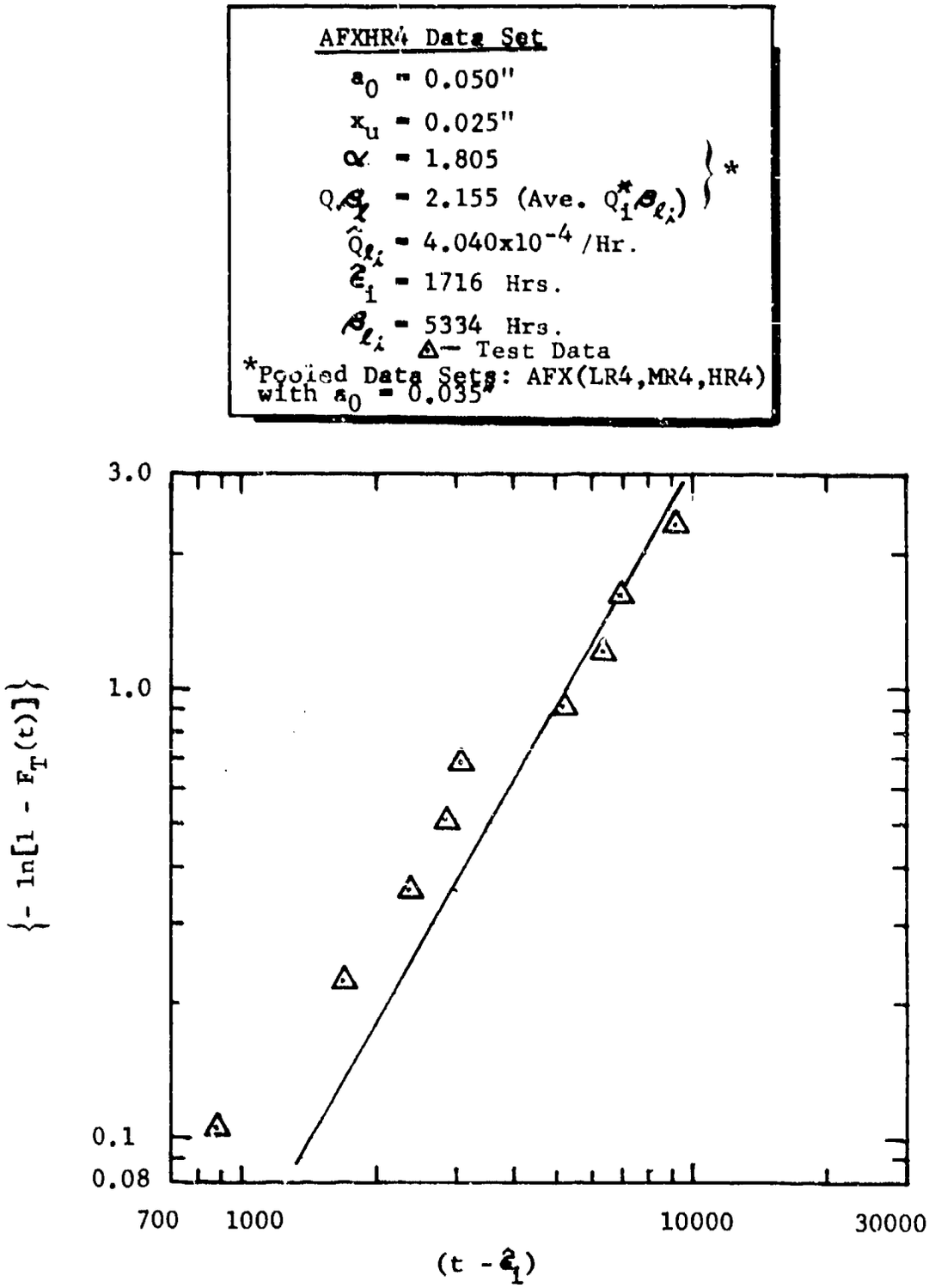


Fig. 4.16 TTCI Goodness-of-Fit Plot for AFXHR4 Data Set Based on IFQ Model Parameters for Pooled Data Sets; a_0 = 0.050"

A plot of log $\hat{Q}_{\ell_i}$ versus log σ is shown in Fig. 4.17 for the three data sets. A best fit line for Eq. 3-18 is also shown. In this case, Eq. 3-18 fits the data very well. Moreover, it would be reasonable to use Eq. 3-18 and the EIFS distribution defined by parameters in Table 4-10 to predict the cumulative distribution of TTCI's for different stress levels.

4.6.4 Discussion of EIFS Distribution

Parameters for the EIFS cumulative distribution are shown in Table 4-10. These parameters were based on the pooled results for three fractographic data sets (AFXLR4, AFXMR4, AFXHR4).

The goodness-of-fit plots in Figs. 4.10 - 4.16 reflected $\ell = 1$ rather than $\ell = 4$ (actual). The IFQ model parameters in Table 4-10 were based on the fractographic data for the largest fatigue crack in 1 of 4 fastener holes per test specimen. Since the fractographic data is not available for the fatigue crack in each fastener hole in each specimen, the goodness-of-fit plots were made for $\ell = 1$. However, for durability analysis, the EIFS cumulative

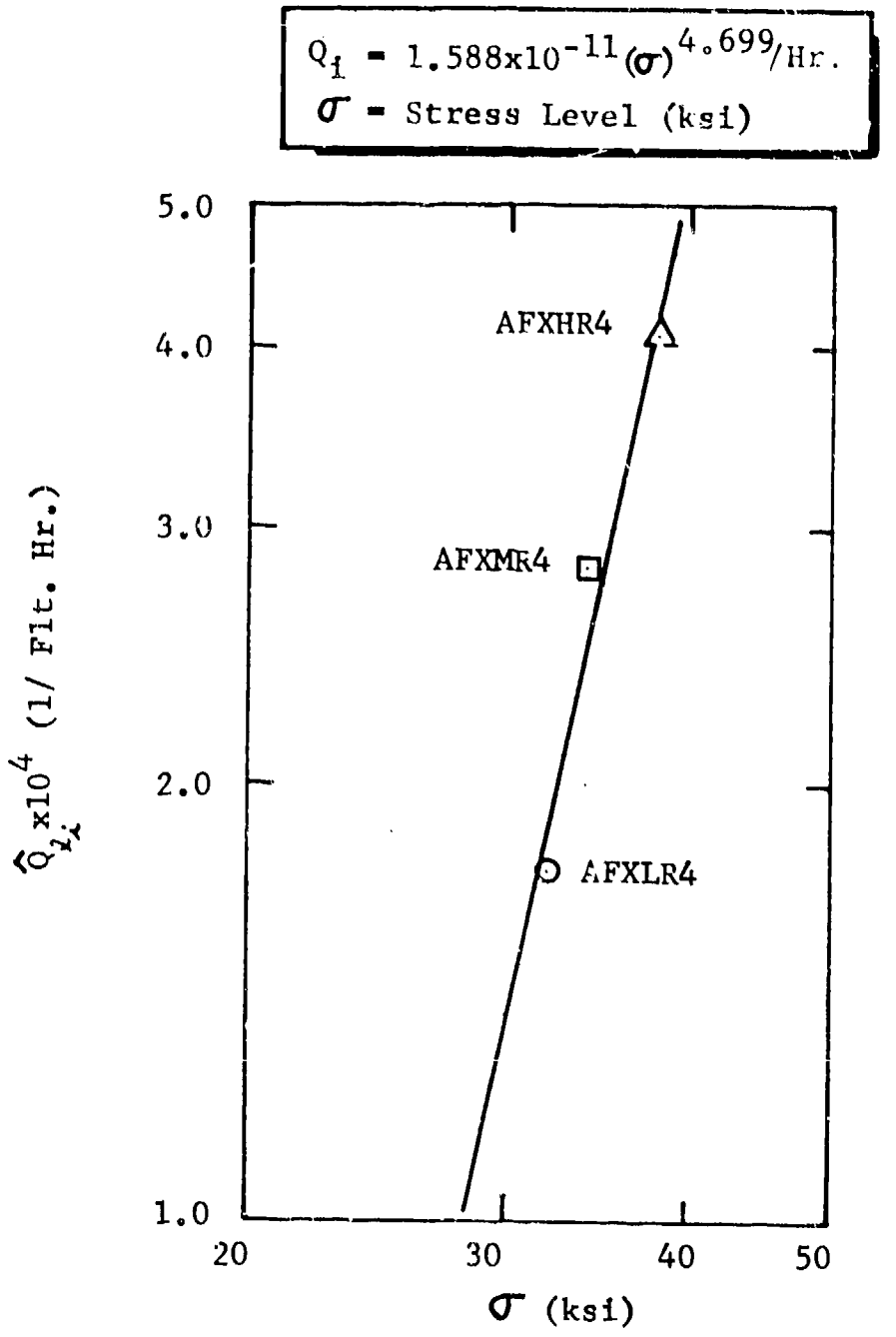


Fig. 4.17 Plot of $\hat{Q}_{\ell_i}$ versus σ to Determine Applicable SCGMC

distribution, Eq. 3-15, should be used with $\ell = 4$ (Ref. Section 4.5).

Suggested procedures have been described and illustrated in this handbook for establishing the EIFS distribution for pooled fractographic data sets. Using these procedures and suitable fatigue crack fractographic data [e.g., 24, 25], appropriate EIFS distributions for durability analysis can be determined.

4.6.5 Practical Aspects

The following analytical tools or data are needed to efficiently determine an acceptable EIFS distribution for fastener holes:

- Suitable fractographic data (i.e., crack size versus time information) for fatigue cracks in fastener holes.

- an analytical crack growth program [e.g., 35-38].

- computer program for manipulating large amounts of fractographic data and for determining optimized IFQ model parameters,

- a plotting program to evaluate the goodness-of-fit of the derived EIFS distribution to the observed crack sizes and times-to-crack initiation.

IFQ can be quantified for fastener holes using suitable fractogaphy results, if available, or using assumed IFQ model parameters and an analytical crack growth program [e.g., 36]. User judgment and experience are also required to quantify IFQ for different durability analysis applications.

In general, fractographic results will not be available for the desired set of design variables: material, fastener type/fit, stress level, load spectra, etc. In this case, the user has three basic options: (1) Use available fractographic results and interpret for the particular design conditions, (2) acquire suitable fractographic data, or (3) use assumed IFQ model parameters, based on similar design variables, and an analytical crack growth program. Whatever option is used, the user should satisfy his requirements, schedule and budget.

A wealth of fractographic data is available for fatigue cracks in fastener holes [e.g., 24-27]. The IFQ model parameters should be quantified using such data to provide

a broad data base and experience for selecting IFQ parameter values for practical durability analyses.

Recommended parametric values for the EIFS distribution of fastener holes are not tabulated in this handbook for durability analysis applications. However, the procedures and guidelines herein can be used to develop appropriate parameter values for a given condition. Tabulated values of IFQ model parameters for different materials and variables will be incorporated into the handbook later.

SECTION V

DETAILS FOR PERFORMING DURABILITY ANALYSIS PREDICTIONS

5.1 INTRODUCTION

The purpose of this section is to: (1) describe and illustrate procedures for determining the service crack growth master curve (SCGMC), (2) discuss crack exceedance predictions, (3) present different formats for extent of damage and (4) illustrate and discuss related durability analysis considerations.

5.2 SERVICE CRACK GROWTH MASTER CURVE

A service crack growth master curve (SCGMC) is needed for each stress region where a probability of crack exceedance, $p(i,\tau)$, prediction is desired. For reliable predictions of $p(i,\tau)$, the SCGMC <u>must</u> be compatible with the EIFS distribution used. Guidelines are presented in this section, and recommended procedures for determining the SCGMC are illustrated. Also, refer to references 15, 16, 20 and 21 for further discussions and applications.

The SCGMC is used to determine an EIFS, $y_{1i}(\tau)$, that will grow to a selected crack size x_i at service time τ. Mathematically, the SCGMC can be expressed by Eq. 3-17. For a given stress region, the same SCGMC is used to grow the EIFS distribution from t = 0 to t = τ (Ref. Fig. 3.3). Once $y_{1i}(\tau)$ has been determined, the corresponding $p(i,\tau)$ can be determined from the EIFS distribution (Ref. Section 3.3 and 5.3).

5.2.1 Guidelines

The SCGMC must be compatible with the applicable EIFS distribution. If this principle is not strictly followed, the $p(i,\tau)$ predictions will not be consistent and the accuracy of such predictions may be questionable.

To obtain a compatible SCGMC, use the same: (1) crack size range used to define the EIFS distribution (i.e., a_L to a_U), (2) crack growth law as the EIFS master curve (i.e., Eq. 3-2), and (3) procedures and goodness-of-fit criteria to

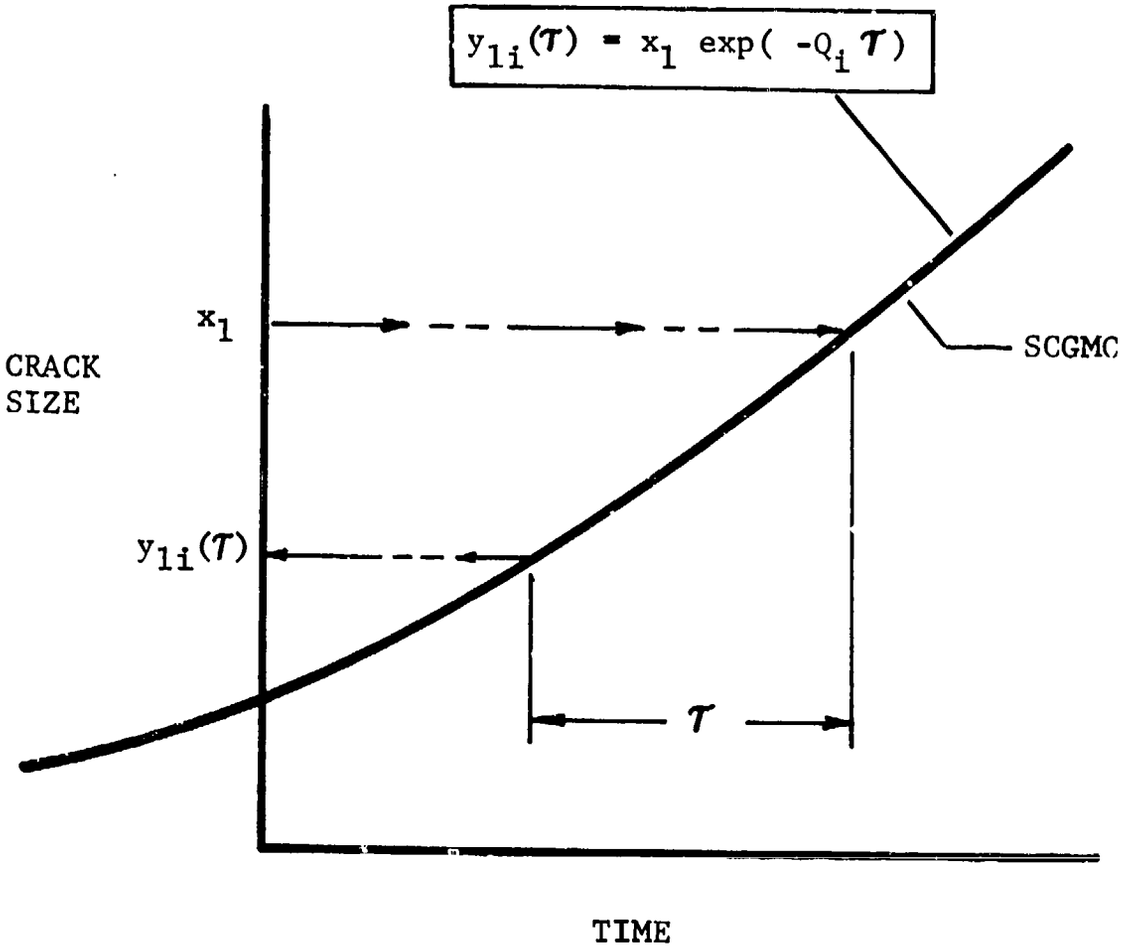


Fig. 5.1 Service Crack Growth Master Curve

determine the SCGMC as used to determine the EIFS master curve(s). The following guidelines for determining a compatible SCGMC are based on the understanding developed under this program.

Two basic situations should be considered in determining the SCGMC: (1) applicable fractographic results are available to define the SCGMC and (2) an analytical crack growth program must be used to determine the SCGMC because applicable fractography is not available for the selected design conditions. In the first case, principles 1 and 2 below apply; whereas, in the second case principle 3 applies.

1. Use the same crack size range used to establish the EIFS distribution to determine the SCGMC for given design conditions (e.g., load spectra, stress level, and bolt load transfer, etc.). For example, suppose the fractographic crack size range used to determine the IFQ model parameters was: a_L = 0.020" to a_U = 0.050". Then, this same range should also be used to determine the SCGMC.

2. The same crack growth law used to determine the EIFS distribution, $F_{a(0)}(x)$, should be used to determine a compatible SCGMC. For example, Eq. 3-16 can be used to

define the SCGMC if Eq. 3-2 has been used to define $F_{a(0)}(x)$ (Ref. Eqs. 3-8 and 3-15). In Eq. 3-16, the subscript "i" refers to the i th stress region. The SCGMC should have the same "b" value as the EIFS master curve(s). Furthermore, the Q_i and b_i parameters for the SCGMC should be determined for the same fractographic crack size range used to determine the IFQ model parameters.

3. If an analytical crack growth program [e.g., 35-38] is used to determine the SCGMC, then the crack growth program should first be "tuned" to the "normalized" EIFS master curve(s), Eq. 3-11. The EIFS distribution established may be based on several fractographic data sets. Therefore, the analytical crack growth program should be tuned to selected EIFS master curves. After tuning, the crack growth program can be used to predict a(t) versus t values for the desired service conditions. The Q_i and b_i parameters in Eq. 3-16 should then be fitted to the a(t) versus t results in the designated crack size range: a_L to a_U.

5.2.2 Illustrations

Two examples are presented in this section for determining the SCGMC for the following cases:

- Case 1 - $y_{1i}(\tau)$ can be defined using applicable fractographic results.

- Case 2 - an analytical crack growth program is required to determine the SCGMC for the desired load spectrum and stress levels.

5.2.2.1 Case 1

In this case, applicable fractographic data are available for the desired material and load spectrum but not for the desired stress levels. It is assumed that applicable fractographic data are available for two or more stress levels.

The crack growth parameter, Q_i, in Eq. 3-18 can be determined using the $Q^*_{\ell_i}$ values for the three fractographic data sets shown in Table 4-10. A plot of log $\hat{Q}_{\ell_i}$ versus log σ is shown in Fig. 4.17 and the empirical equation for $\hat{Q}_{\ell_i}$ is also shown. In this case, the SCGMC, given by Eq. 3-17, can be used to determine the $y_{1i}(\tau)$ value for selected stress levels.

The empirical relationship, given by Eq. 3-18, worked very well in this case. Equation 3-18 also worked very well

for selected straight-bore and countersunk fractographic data sets investigated under Phase II of this program [16]. However, there's no guarantee that Eq. 3-18 will be adequate for all fractographic data sets and further evaluation of existing fractographic data is needed.

5.2.2.2 Case 2

In this case, an analytical crack growth program [e.g., 35] is used to determine the SCGMC because fractographic data is not available for the desired load spectrum, stress level, and % bolt load transfer. It is assumed that the EIFS distribution has already been established using applicable fractographic results. Also, the analytical crack growth program has been "tuned" to the EIFS master curves - represented by Eq. 3-11 and the $\hat{Q}_{\ell_i}$ parameters (note: $Q = \hat{Q}_{\ell_i}$ in this case) in Table 4-10 (for example). The IFQ model parameters in Table 4-10 are based on fractographic data sets AFXLR4, AFXMR4 and AFXHR4 and the procedures described in subsection 4.4.2.

A SCGMC is needed for load spectrum "A" and for a maximum stress level of 42 ksi. A suitable analytical crack growth program can be used to predict the a(t) versus t values for the specified conditions.

Analytical crack growth results for a(t) versus t are shown in Table 5-1 for 0.020" $\leq$ a(t) $\leq$ 0.050". These results apply to load spectrum "A" and σ = 42 ksi. Note that the predicted a(t) versus t values cover the same crack size range used to determine the IFQ model parameters in Table 4-10.

The SCGMC, Eq. 3-17, can be determined using the data in Table 5-1 and the procedures described in subsection 4.4.2.1. For example, Q_i in Eq. 3-17 can be determined using Eq. 4-6 and the data from Table 5-1. In this case, a "best-fit" $Q_i = 1.697 \times 10^{-4}$ was obtained. For a given crack size x_1, in the range from a_L to a_U, and service time, t, the corresponding $y_{1i}(\tau)$ can be determined using Eq. 3-17. Two examples for $y_{1i}(\tau)$, based on Eq. 3-17 and $Q_i = 1.697 \times 10^{-4}$, are given below for different x_1 and τ values.

- o x_1 = 0.025", τ = 4000 flight hours : $y_{1i}(\tau)$ = 0.0127"

- o x_1 = 0.035", τ = 8000 flight hours : $y_{1i}(\tau)$ = 0.0090"

Table 5-1 Analytical Crack Growth Results for Spectrum "A" (σ = 42 ksi)

t (Flt. Hrs.)	a(t) (In.)
0	0.0200
500	0.0209
1000	0.0220
1500	0.0232
2000	0.0246
2500	0.0267
3000	0.0297
3500	0.0336
4000	0.0383
4500	0.0427
5000	0.0486
5114	0.050

5.3 CRACK EXCEEDANCE PREDICTIONS

The probability of crack exceedance, $p(i,\tau)$, can be analytically predicted for a given crack size, x_i, and service time, τ. For a given EIFS distribution and applicable SCGMC, $p(i,\tau)$ can be determined using Eqs. 3-9, 3-17 and 3-19.

Suppose the EIFS distribution has been established and the following parameters from Table 4-10 apply: a_0 = 0.035", x_u = 0.025", α = 1.805, $Q\beta_\ell$ = 2.155 and ℓ = 4. The SCGMC for load spectrum "A" and σ = 42 ksi is defined by Eq. 3-17 where Q_i = 1.697 x 10^{-4} (Ref. Subsection 5.2.2.2). Using these parameters and applicable equations, $p(i,\tau)$ can be predicted, for example, at x_i = 0.030" and τ = 8000 flight hours. In this case, $y_{1i}(\tau)$ = 0.00772" and the $p(i,\tau)$ = 0.080. Therefore, approximately 8% of the fastener holes in the specified stress region (σ = 42 ksi) would be expected to have a crack size ≥ 0.030" at τ = 8000 flight hours.

The overall extent of damage and the corresponding variance for different stress regions can be determined using Eqs. 3-20 through 3-23. Extent of damage formats are described and illustrated in Section 5.4.

5.4 EXTENT OF DAMAGE FORMATS/ILLUSTRATIONS

The objective of the durability analysis is to quantify the "extent of damage" as a function of time for selected details. Various formats are described and illustrated in this section for presenting the durability analysis results.

5.4.1 Extent of Damage Formats

Several example formats for "extent of damage" are illustrated in Fig. 5.2. The basic objective of the durability analysis results is to analytically assure design compliance with the specified economic life criterion and to evaluate design tradeoffs affecting structural maintenance requirements and user life-cycle-costs.

5.4.2 Extent of Damage Illustrations

The overall extent of damage provides a quantitative measure of structural durability. As long as the largest crack in each detail included in the damage assessment is relatively small and such cracks are statistically independent, the extent of damage can be estimated using the binomial distribution. Eqs. 3-20 and 3-21, derived from the binomial distribution, can be used to predict the extent of damage and its standard deviation, respectively. The extent

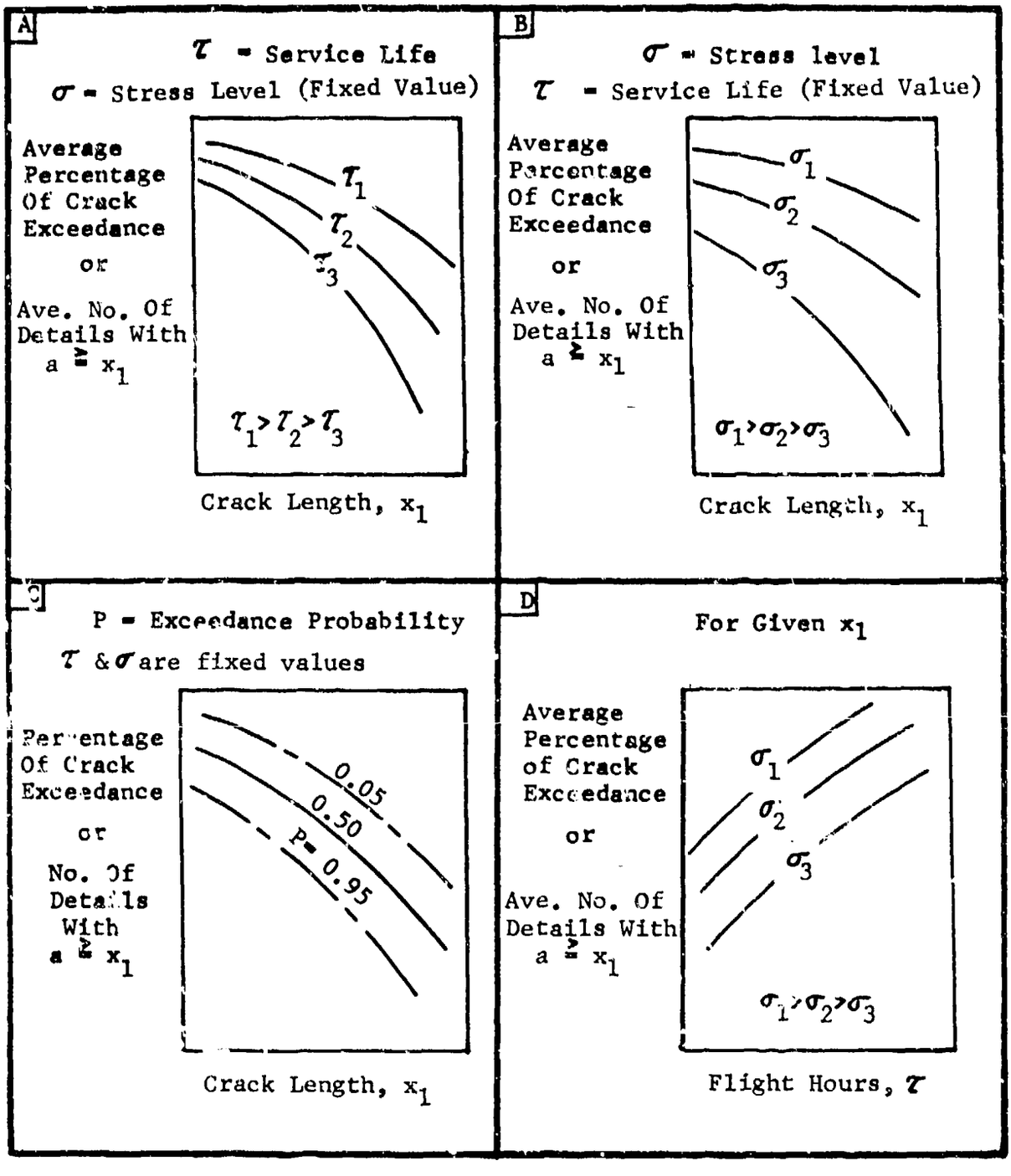


Fig. 5.2 Formats for Presenting Extent of Damage Results

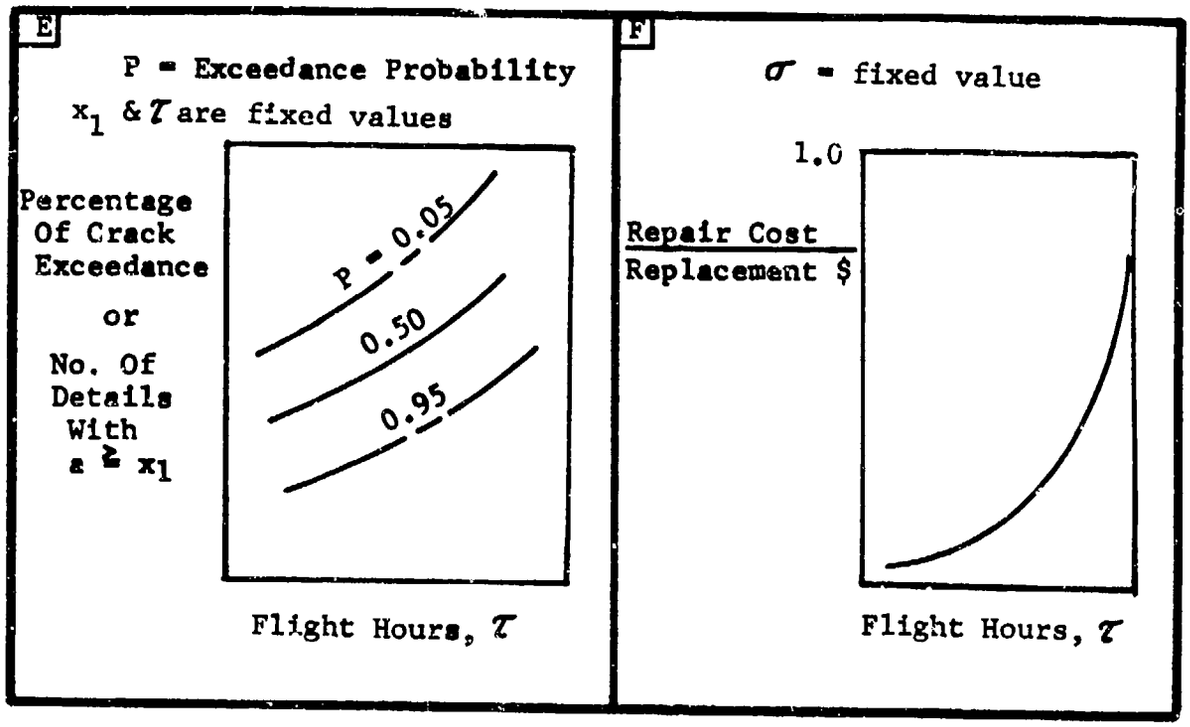


Fig. 5.2 Formats for Presenting Extent of Damage Results (Continued)

of damage for the desired combination of details, parts, components, etc., can be determined using Eqs. 3-22 and 3-23.

Three examples are presented to illustrate how Eqs. 3-20 through 3-23 can be used to quantitatively define the extent of damage for different levels. The following situations are considered:

- Example 1 - One detail type: two stress regions; extent of damage for one control area.
- Example 2 - One detail per airplane; extent of damage for fleet.
- Example 3 - Three different detail types; different stress regions; different crack exceedance crack sizes (x_1); extent of damage for a component.

Example 1

An extent of damage estimate is desired for a wing lower skin at a critical control area containing 90 fastener holes for x_1 = 0.05" at = τ= 8000 flight hours. The skin is

divided into two stress regions. For illustration purposes, $p(i,\tau)$ values are assumed for each stress region: other durability analysis details are shown in Table 5-2.

Results from Table 5-2 may be interpreted as follows: The average number of fastener holes with a crack size $\geq x_1$ at $\tau = 8000$ hours is $N_i p(i,\tau) = 7.6$ holes. The corresponding standard deviation is $\sigma_N(i,\tau) = 2.60$. The average percentage of fastener holes with a crack size $\geq x_1$ at $\tau = 8000$ flight hours is 8.4%. Using these average values for $N_i p(i,\tau)$ and $\sigma_N(i,\tau)$, upper and lower bound estimates can be made for the extent of damage at desired exceedance probabilities.

Example 2

Suppose an extent of damage assessment (including average plus upper and lower bound estimates) is desired for one detail per airplane in a fleet of 1000 airplanes. Assume the following probability of crack exceedance has been computed for the applicable initial fatigue quality, stress level and load spectra for $x_1 = 0.03"$ and $\tau = 10,000$ flight hours: $p(i,\tau) = 0.05$. Using Eq. 3-20, the average number of details in the fleet with a crack size $\geq 0.03"$ is $\overline{N}(i,\tau) = 1000 \times 0.05 = 50$ details per fleet.

Table 5-2 Extent of Damage Assessment for Wing Skin Containing Fastener Holes

STRESS REGION	NO. FASTENER HOLES (N_i)	$p(i,\tau)$	$N_i p(i,\tau)$	$\sigma_N(i,\tau)$
1	80	0.07	5.6	2.28
2	10	0.20	2.0	1.26
	90		7.6	2.60

Ave. percentage of details with a crack size $\geq x_1$ = $\frac{7.6 \times 100\%}{90}$

= 8.4%

Upper and lower bound predictions for the extent of damage can be estimated for selected probabilities using $\bar{N}(i,\tau) + Z\sigma_N(i,\tau)$, where Z is the number of standard deviations, $\sigma_N(i,\tau)$, from the mean, $\bar{N}(i,\tau)$. For example, assume $\bar{N}(i,\tau)$ is normally distributed and $Z = \pm 3$. From statistical tables for areas under the normal distribution, Z = 3 and Z = -3 correspond to a probability of 0.0013 and 0.9987, respectively. The standard deviation for $\bar{N}(i,\tau)$, based on Eq. 3-21, is: $\sigma_N(i,\tau) = [50(0.95)]^{\frac{1}{2}} = 6.89$. Using the information above, the upper and lower bound prediction for $\bar{N}(i,\tau)$ is 70.67 and 29.33, respectively.

These results may be interpreted as follows: The probability of exceeding 70.67, 50 and 29.33 details with $x_1 \geq 0.03''$ at τ = 10000 flight hours is P = 0.0013, 0.50 and 0.9987, respectively (with 50% confidence). This information provides average as well as upper and lower bound estimates for the extent of damage for the fleet.

Example 3

An aircraft component contains countersunk fastener holes, fillets and cutouts in selected control areas governing the structures durability. The objective of this example is to show how the extent of damage for different details with different crack exceedance crack sizes (x_1) can be combined to quantitatively define the overall damage for the component. Details of the analysis and results are shown in Table 5-3. The total number of details in the component (two control areas) with a crack size $\geq x_1$ is estimated to be 15 with a standard deviation of 3.72.

Table 5-3 Extent of Damage Assessment for a Component Containing Different Detail Types

CONTROL AREA	TYPE DETAIL	NO. DETAILS (N_i)	STRESS REGION NO.	x_1* (in.)	τ (Flt. Hrs.)	$p(i,\tau)$	$N_i p(i,\tau)$	$\sigma_N(i,\tau)$
I	CSK FASTENER HOLE	100	1	0.03	10000	0.05	5	2.18
		10	2			0.10	1	0.95
		20	3			0.08	1.6	1.21
	FILLET	40	1	0.06		0.06	2.4	1.50
II	CUTOUT	50	1	0.10		0.10	5.0	2.12
		220					15.0	3.72

*Limiting crack size for economical repair

Average percentage of details with a crack size $\geq x_1 = \frac{15 \times 100\%}{220} = 6.8\%$

5.5 ADDITIONAL CONSIDERATIONS

5.5.1 Details Other Than Fastener Holes

The durability analysis method described in Section III theoretically applies to different types of structural details susceptible to fatigue cracking (e.g., fastener holes, lugs, cutouts, fillets, etc.). Although the method has been demonstrated for clearance-fit fastener holes, further research is required to verify the method for other detail types and for different combinations of details.

Theoretically, the IFQ for a given detail type can be quantified using applicable fractographic results for the desired detail. Suitable specimens need to be designed for acquiring fatigue cracking data for lugs, fillets, cutouts, etc. Crack initiation data should be generated for different detail types for different: materials, stress levels, load spectra, manufacturing techniques, etc. These data should be used to verify and refine, if required, the durability analysis method for those details which may have a significant effect on the structural maintenance requirements and economic life.

5.5.2 Large Crack Sizes

The durability analysis method was developed for predicting crack exceedance for relatively small crack sizes (e.g., ≤ 0.10") in structural details. The largest crack in each detail was assumed to be statistically independent to justify using the binomial distribution for details. If the largest crack in a given detail doesn't significantly affect the growth of cracks in neighboring details, perhaps the proposed durability analysis method can be extended to crack sizes > 0.10". The crack growth power law (Eq. 3-2) used in the IFQ model may not be suitable for defining the EIFS cumulative distribution for crack sizes > 0.10". Other functional forms for crack growth rate may be required to justify the same EIFS distribution for both small and large crack sizes. Eq. 3-2 may be acceptable for making $p(i,\tau)$ predictions for larger crack sizes if the "EIFS master curve" is curve fit to the larger crack sizes. This would result in a different EIFS cumulative distribution for the small and the large crack size range.

5.5.3 Effects of Scale-Up and Hole Interactions

In general, experimental crack growth results for coupon specimens, full-scale structural components and prototype structures exhibit scale-up and interaction effects. The

possible effects of scale-up on the IFQ or EIFS distribution, based on coupon fatigue test results, are not accounted for in the durability analysis demonstrations of Section VII. A preliminary investigation of the effects of scale-up on the durability analysis has been made [16] but further research is required. Major sources of scale-up effects are:

- Increase in the number of fastener holes
- Change in stress field
- Increased variability in workmanship
- Increased variability in material properties.

The interaction effects of the dominant crack in neighboring fastener holes on crack exceedance predictions have not been evaluated under the present program. Such effects are not considered to be significant when the largest crack in a fastener hole is relatively small (e.g., $\leq$ 0.10") and the spacing between fastener holes is considerably larger than 0.10".

5.5.4 Functional Impairment

The durability analysis method can also be used to investigate functional impairments such as fuel leakage and ligament breakage. For example, a leak may occur when a through-the-thickness crack develops in a fuel tank. Cracks frequently originate at fastener holes. Therefore, the resistance of the structure to functional impairment due to fuel leaks can be estimated from the predicted number of fastener holes with a through-the-thickness crack.

The durability analysis method has been verified only for relatively small crack sizes (e.g., $\leq$ 0.10" in a fastener hole). Since through-the-thickness cracks may exceed 0.10", further work is required to verify the method for larger crack sizes. Also, through-the-thickness type cracks need to be further investigated for fillets and other details to assess structural durability.

SECTION VI

COMPARISON OF DETERMINISTIC AND PROBABILISTIC APPROACHES FOR DURABILITY ANALYSIS

6.1 INTRODUCTION

A probabilistic fracture mechanics approach (PFMA) for durability analysis has been developed. The deterministic crack growth approach (DCGA) was used to analytically assure the Air Force's durability design requirements for the F-16 airframe [39,40]. For several years now, variations of the well-known DCGA have been used extensively for damage tolerance analyses.

The objectives of this section are to: (1) conceptually describe the DCGA used for the F-16 airframe durability analysis, (2) compare the essential features of the DCGA with the PFMA developed under this program and (3) discuss and compare the type of output that can be obtained using the two approaches and their significance for quantifying "structural durability."

A hypothetical durability problem is used to explain the essential features and differences in the DCGA and the PFMA for fastener hole applications.

A durability analysis state-of-the-art assessment has been documented [12,13]. Details of the DCGA used for the F-16 durability analysis are given in Refs. 39 and 40. Details of the PFMA are documented herein and elsewhere [14-21].

6.2 F-16 DURABILITY ANALYSIS APPROACH

Two different approaches were used for the durability analysis of the F-16 airframe: (1) deterministic crack growth approach (DCGA) and (2) conventional fatigue analysis [9,10]. Details of the F-16 airframe durability analysis methods are given in References 39 and 40. In this section, only the DCGA will be considered. Also, the approach will be discussed for the durability analysis of fastener holes.

The essential features of the DCGA approach, used for the durability analysis of the F-16 airframe, are conceptually described in Fig. 6.1. For fastener holes, the basic objective of the F-16 durability analysis was to show that no fastener hole in a part or component would have a

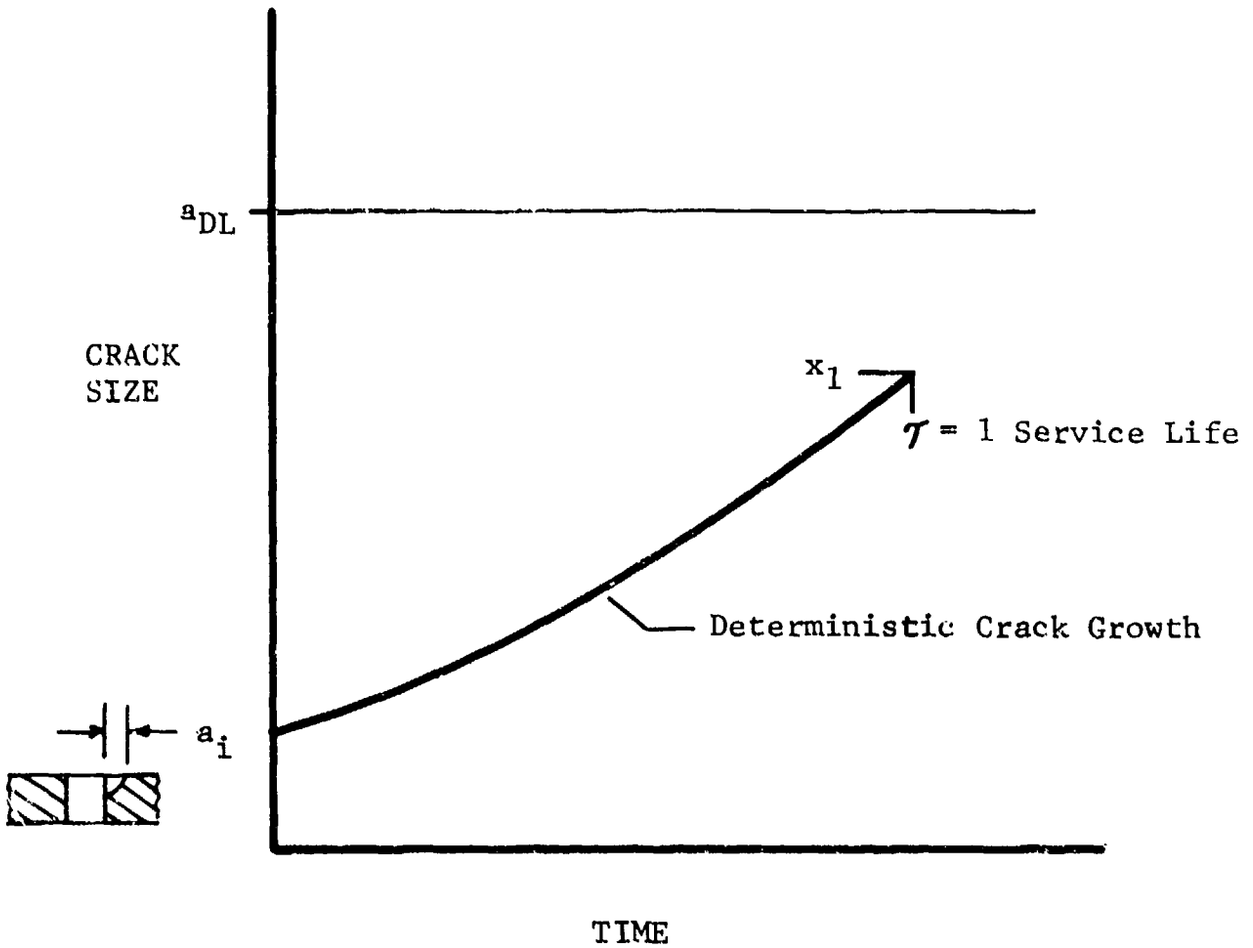


Fig. 6.1 F-16 Durability Analysis Approach

crack size greater than the repair, a_{RL}, after one service life. a_{RL} is the maximum crack size in a fastener hole that can be cleaned up by reaming the hole to the next fastener size. Typical a_{RL} values range from 0.03" to 0.040".

Each fastener hole was assumed to have an initial flaw. F-16 durability analyses were performed using an initial flaw size of either a_i = 0.005" or 0.010". These initial flaw sizes are based on the results of the F-4 tear-down inspection [7].

The following general procedures were used to evaluate the durability of fastener holes in the F-16 airframe.

1. Select part for durability analysis.

2. Divide a part into control areas or stress regions.

3. Group the structural details (e.g., fastener holes) according to applicable stress region.

4. Select the most critical detail in each stress region for durability analyses.

5. Assume an initial flaw size (a_i) for the most critical detail in each region. The initial flaw size is "representative" of the initial quality of the detail.

6. Use a suitable deterministic crack growth computer program, [e.g., 36] to grow a_i to a crack size x_1 at a specified service life for the applicable maximum stress for each stress region.

7. Show for the most critical detail in each stress region that x_1 is $\leq a_{DL}$ (durability limit flaw size for functional impairment and/or economic repair) at $\tau = 1$ service life. a_{DL} is the maximum crack size in a fastener hole, for example, that can be economically repaired by reaming the fastener hole to the next size. $a_{DL} = 0.03"$ was commonly used.

6.3 PROBABILISTIC FRACTURE MECHANICS APPROACH

The PFMA is conceptually described in Fig. 6.2 and details are given in Section III of this handbook. This approach will be further described later for the example problem.

6.4 EXAMPLE DURABILITY PROBLEM

A durability analysis is required for a structure containing 200 fastener holes. For analysis purposes, assume that: (1) the fastener holes are grouped into three stress regions, (2) all fastener holes in a given stress region are equally-stressed, and (3) the number of fastener holes in each region is known. If the economic repair limit for each fastener hole is 0.03", how "durable" is the structure at the end of one service life?

Conceptually describe the durability analysis of this structure using the DCGA and the PFMA. Then, compare and discuss the type of information that can be obtained from each approach.

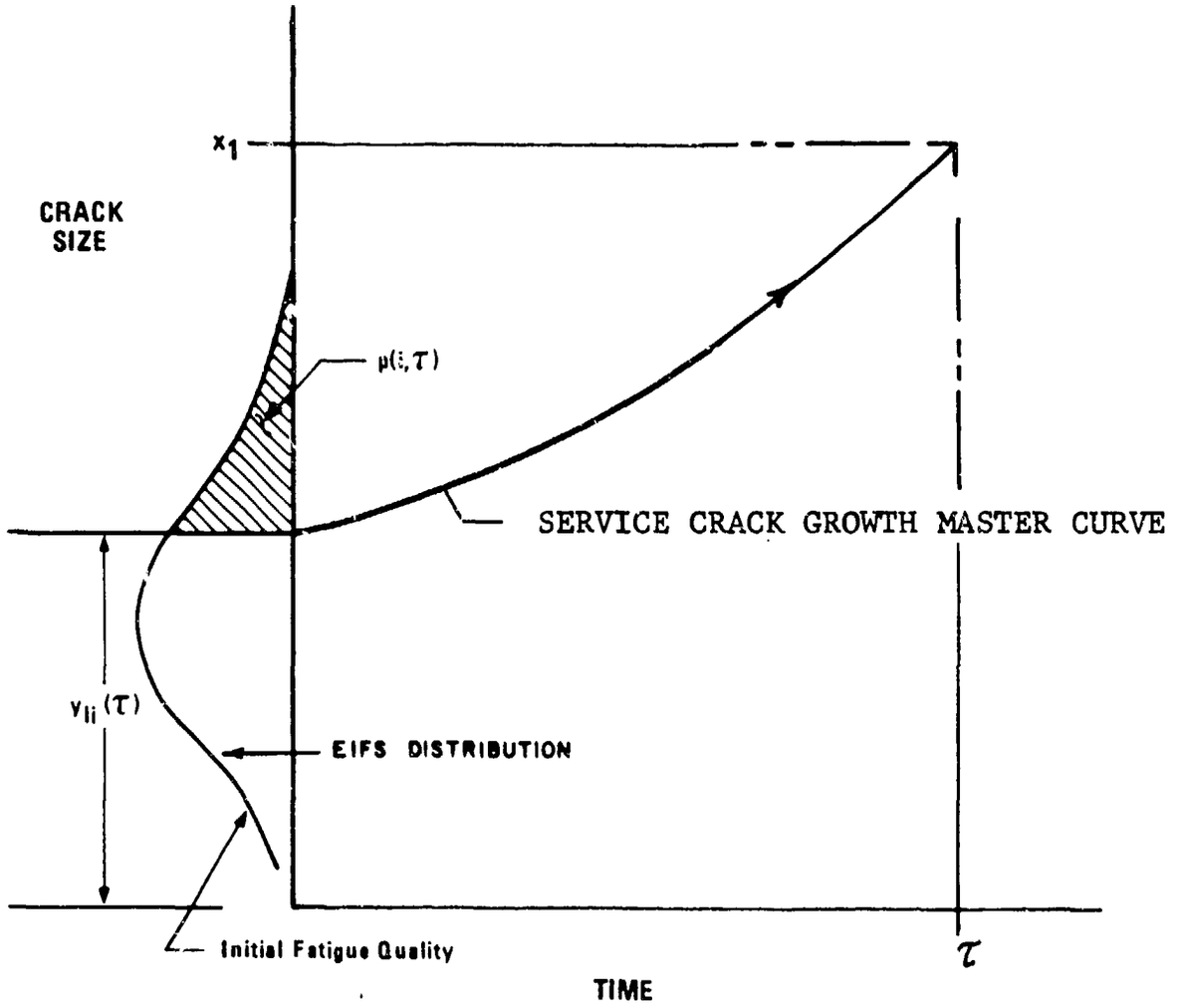


Fig. 6.2 Probabilistic Fracture Mechanics Approach

6.4.1 Durability Analysis Based on the DCGA

The "worst-case" fastener hole from each stress region is used for the durability analysis. Using the DCGA, the analysis proceeds as follows for each stress region. First, an initial flaw size, a_i, is assumed to exist in the fastener hole in the most adverse position. The size of the flaw is considered to be representative of typical initial flaws in fastener holes. The size of the initial flaw, a_i, at the end of one service life, is predicted for each stress region using a deterministic crack growth computer program [e.g., 36,37], applicable material properties, the applicable stress level, and load spectra. Assumed results for the analysis, for illustration purposes, are summarized in Table 6-1.

The assumed results shown in Table 6-1 can be interpreted as follows. All fastener holes in each stress region will have a crack size less than the economic repair limit [e.g., a_{RL} = 0.03"] at the end of one service life. The following can also be stated for stress region I: at least 1 out of 100 fastener holes will have a crack size = 0.020" at the end of one service life.

Similarly, at least 1 out of 50 fastener holes in stress regionsII and III will have a crack size = 0.027" and 0.015", respectively, after one service life.

Table 6-1 Illustration of the Deterministic Crack Growth Approach and the Type of Information Obtained from the Analysis

Stress Region	No. Holes/Region	a_i	Max Crack Size x_1 @ τ = 1 SL
I	100	.010"	0.020"
II	50		0.027"
III	50		0.015"
	200		

SL = Service Life

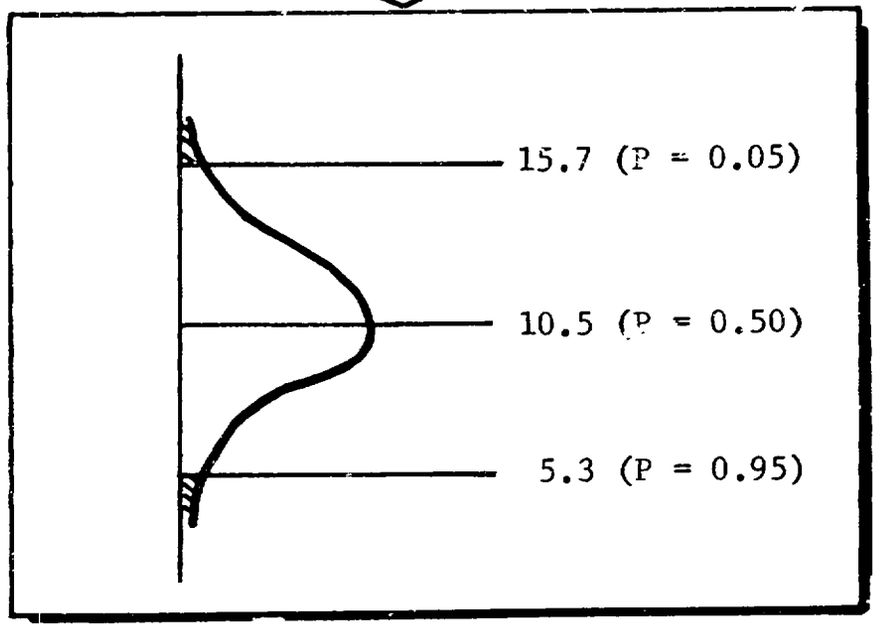
6.4.2 Durability Analysis Based on the PFMA

Using the PFMA, the durability analysis is performed as follows. First, the initial fatigue quality (IFQ) or cumulative EIFS distribution is defined for the fastener holes in each stress region. IFQ model parameters are determined using available fractographic results [e.g., 24,25] and the procedures described and illustrated in Section IV of this handbook. A service crack growth master curve (SCGMC), compatible with the EIFS master curve, is determined for each stress region using the procedure described in Section 5.2 herein. Then, the probability of exceeding a crack size, x_1 = 0.03" at one service life is predicted for each stress region using the cumulative distribution, $F_{a(0)}(x)$, and the applicable SCGMC.

For illustration purposes, probability of crack exceedance values, $p(i,\tau)$, are assumed for each stress region as shown in Table 6-2. Refer to Eq. 3-19 and the durability analysis procedures described in Section 3.5 herein for details on computing $p(i,\tau)$ for each stress region. Next, the average number of fastener holes $\overline{N}(i,\tau)$ and the standard deviation $\sigma_N(i,\tau)$ for each stress region with a crack size greater than x_1 = 0.03" at τ = 1 service life is determined using Eqs. 3-20 and 3-21. The extent of

Table 6-2 Illustration of the Probabilistic Fracture Mechanics Approach and the Type of Information Obtained from the Analysis

STRESS REGION	NO. OF HOLES PER REGION (N_i)	$p(i,\tau)$	AVE. NO. OF HOLES WITH A CRACK SIZE $\geq x_1$ $\bar{N}(i,\tau)$	$\sigma_N(i,\tau)$
I	100	0.05	5	2.179
II	50	0.08	4	1.918
III	50	0.03	1.5	1.206
	200		$\bar{L}(\tau)$ = 10.5	$\sigma_L(\tau)$ = 3.143



damage $\overline{L}(\tau)$ and its standard deviation $\sigma_L(\tau)$ for the three stress regions combined are determined using Eqs. 3-22 and 3-23, respectively. Results for the analysis described above are summarized in Table 6-2.

Upper and lower bounds for the predicted extent of damage can be estimated using $\overline{L}(\tau) \pm Z\sigma_L(\tau)$, where Z is the number of standard deviations, $\sigma_L(\tau)$, from the mean, $\overline{L}(\tau)$. "Extent of damage" predictions are illustrated in Table 6-2 for three probabilities. For example, the probability of exceeding $\overline{L}(\tau)$ = 15.7, 10.5 and 5.3 is P = 0.05, 0.50 and 0.95, respectively.

6.4.3 Conclusions

The DCGA does not account for the initial fatigue quality variation for the population of fastener holes. A single initial flaw size is used to characterize "initial quality" and the results of the analysis do not provide a quantitative description of the extent of damage for all the fastener holes.

The "damage" is determined for a single fastener hole in a given stress region and it is assumed that all of the other fastener holes in the region are no worse than the

hole analyzed. For the simple problem considered, only the "worst-case" hole out of 200 holes would have to be analyzed to show that the size of the crack in any one hole would be $\leq a_{RL}$.

The PFMA provides a lot more information than the DCGA. For example, the PFMA provides the following information:

- Average number of fastener holes and its standard deviation in each stress region with a crack size $\geq a_{RL}$.

- The extent of damage and its standard deviation for the population of fastener holes.

- Upper and lower bounds for the extent of damage for selected probabilities.

The information above gives a quantitative description of the "extent of damage" as a function of time. This information can be used to judge the durability of the structure, to assess structural maintenance requirements and costs, and to evaluate durability design tradeoffs.

SECTION VII

DURABILITY ANALYSIS DEMONSTRATION

7.1 INTRODUCTION

A demonstration of the durability analysis methodology, described in Sections III-V, is presented in this section for: (1) the F-16 lower wing skins and (2) a complex splice subjected to a B-1 bomber load spectrum. Both analyses are correlated with test data. Also, the durability analysis results and practical aspects are discussed.

7.2 F-16 LOWER WING SKIN

A durability analysis of the F-16 lower wing skins (durability test article) is presented to illustrate the methodology described. Analytical predictions of the extent of damage in each wing skin are presented in various formats, and results are compared with observations from the tear-down inspection of the F-16 durability test article.

The F-16 durability test article was tested to 16,000 flight hours (equivalent to 2 service lives) using a 500-hour block spectrum. Each wing received the same loading.

Following the test, all fastener holes in the lower wing skins were inspected using eddy current techniques. Fastener holes with crack indications were confirmed by fractographic evaluation. The right hand and left hand lower wing skins were found to have twenty six and seven fastener holes, respectively, with a crack size ≥ 0.03" at τ = 16,000 flight hours.

A preliminary durability analysis for the F-16 lower wing skins was presented in Ref. 19. This analysis reflected: (1) fastener hole IFQ based on fractographic results for protruding head fasteners, (2) crack growth rates for the IFQ model based on Eq. 3-2 ($b \neq 1$), (3) three-parameter Weibull distribution used in the IFQ model, (4) model parameters based on a single data set (one stress level, F-16 400-hour block spectrum, Ref. 25), (5) three stress regions considered for the lower wing skin, and (6) an analytical crack growth program [35] and 500-hour block spectrum used to define the "service crack growth master curve" for each stress region.

Essential features of the present analysis are: (1) fractographic results for countersunk fasteners used to quantify IFQ (countersunk fasteners were used on the F-16 durability test article), (2) crack growth rates for the IFQ

model based on Eq. 3-2 (b=1), (3) three-parameter Weibull distribution used in the IFQ model, (4) model parameters based on pooled fractographic results for three different data sets (three stress levels, 400-hour block spectrum), (5) lower wing skin divided into 10 stress regions, and (6) service crack growth master curve for each stress region based on Eqs. 3-17 and 3-18 and applicable fractographic results. There were no significant differences in the 400-hour and 500-hour spectra.

The F-16 lower wing skin was divided into ten stress regions as shown in Fig. 7.1. Applicable stress levels and the corresponding number of fastener holes in each stress region are shown in Table 7-1. The stress levels for Zones I-IV were based on strain gage data and finite element analysis results. The stress levels for Zones V, VII-IX were determined using a coarse grid finite element analysis and a theoretical stress distribution for a circular hole in an infinite plate under uniaxial tension. The stress levels for Zones VI and X were based on a fine grid finite element analysis.

Fractographic results for three data sets (i.e., AFXLR4, AFXMR4, and AFXHR4) [24] for maximum gross stress levels of 32 ksi, 34 ksi, and 38 ksi were used to calibrate the IFQ

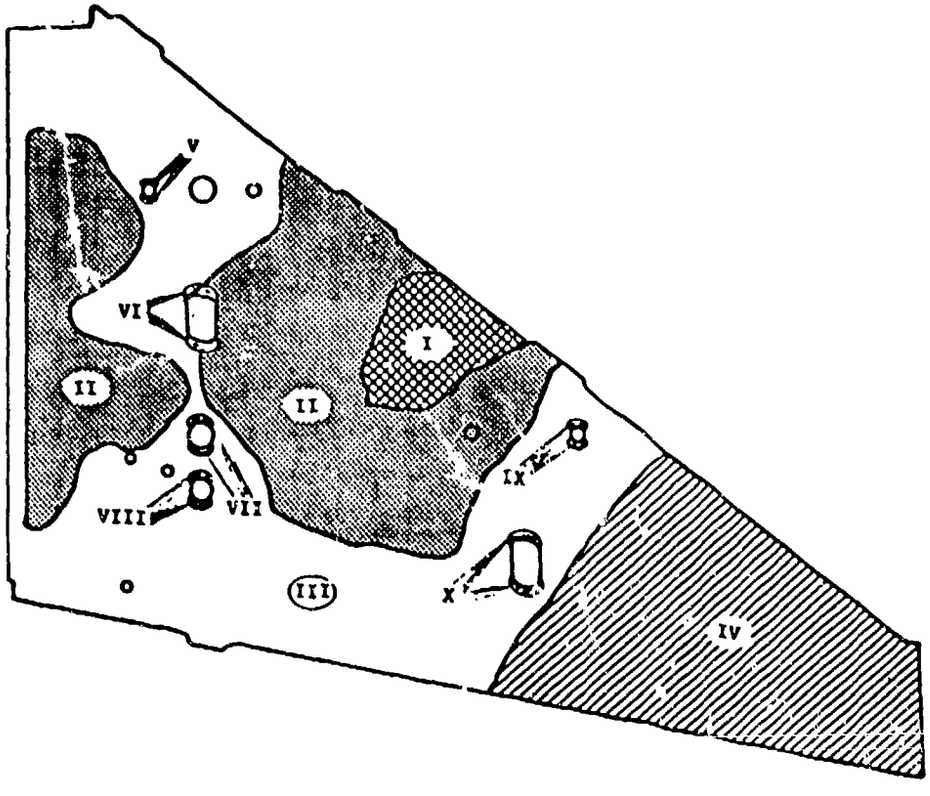


Fig. 7.1 Stress Zones for F-16 Lower Wing Skin

Table 7-1 Stress Levels and Number of Fastener Holes for F-16 Lower Wing Skin

STRESS ZONE	LIMIT STRESS LEVEL (ksi)	NUMBER OF FASTENER HOLES
I	28.3	59
II	27.0	320
III	24.3	680
IV	16.7	469
V	28.4	8
VI	29.2	30
VII	32.4	8
VIII	26.2	8
IX	26.2	12
X	25.7	20
		1614

model parameters. The F-16 400-hour block load spectrum was used. The AFX series specimens were designed for 15% load transfer. The specimens were made of 7475-T7351 aluminum and contained two MS90353-08 (1/4 dia.) blind, countersunk rivets as shown in Fig. 4.3. All specimens reflect typical aircraft production quality, tolerances and fastener fits. Nine specimens were tested per data set.

The crack growth rate parameter Q_i^* in Eq. 4-1 ($b_i^*=1$) was determined for each of the three AFX data sets. Q_i^* was determined from the fractographic results using a least-square fit of Eq. 4-1 [16]. A fractographic crack size range 0.005" - 0.10" was used. An upper bound EIFS of x_{u1} = 0.03" was assumed for the IFQ distribution. Using Eq. 3-13 and $Q = Q_i^*$, the corresponding lower bound of TTCI value, ϵ, for each reference crack size, a_0, was determined for each data set. The results of Q_i^* and ϵ are shown in Table 7-2.

Fractographic results for the three AFX series data sets were combined to determine the corresponding "pooled" α value using the following procedures. Time-to-crack-initiation (TTCI) results for three different reference crack sizes (a_0 = 0.03", 0.05", and 0.10") were used for each of the three data sets. The TTCI-ϵ results for each reference crack size were normalized for each data set using

Table 7-2 Summary of IFQ Model Parameters for F-16 400-Hour Spectrum

DATA SET	σ_{MAX} (KSI)	a_0 (INCH)	$Q_i^* \times 10^4$ (HRS^{-1})	τ (HRS)	α	β_{ℓ_i} (HRS)	$Q_i^* \beta_{\ell_i}$
AFXLR4	32	0.03	1.201	0	1.823	15,033	1.805
		0.05		4,253		12,916	1.551
		0.10		10,025		13,421	1.612
AFXMR4	34	0.03	2.037	0		8,721	1.777
		0.05		2,508		7,759	1.581
		0.10		5,910		9,093	1.852
AFXHR4	38	0.03	4.731	0		5,469	2.587
		0.05		1,079		5,098	2.412
		0.10		2,545		4,598	2.175
POOLED α = 1.823, AVERAGE $Q_i^* \beta_{\ell_i}$ = 1.928							

Notes

1. $x_u = 0.03''$
2. $0.005'' \leq a(t) \leq 0.10''$ (fractographic crack size range used)
3. $\ell = 4$

the average TTCI values ($\bar{x}$). Results for the three data sets were pooled together and the $(TTCI-\epsilon)/\bar{X}$ data were ranked in ascending order. Equation 3-1 was transformed into a least-squares fit form to determine the pooled α value [16]. The pooled value was found to be 1.823 (Table 7-2).

After determining α for the pooled data sets, the adjusted TTCI's for each reference crack size for each data set were considered separately to determine the corresponding β_{ℓ_i} values [16]. These values are presented in Table 7-2. Also summarized in Table 7-2 are the $Q_i^* \beta_{\ell_i}$ values for the nine cases considered. For generic EIFS, the α and $Q\ \beta_{\ell_i}$ values should be constants. An average $Q_i^* \beta_{\ell_i}$ = 1.928 and α = 1.823 are used for the present durability analysis. A plot of Q_i^* versus β_{ℓ_i} is shown in Fig. 7.2 for the three data sets considered (9 cases).

A SCGMC is needed for each of the ten stress regions shown in Fig. 7.1 to compute the corresponding $p(i,\tau)$. In this case, suitable fractographic results are available to determine the SCGMC for each stress region. For example, the Q_i^* results from Table 7-2 can be used to define a generalized SCGMC based on Eqs. 3-17 and 3-18.

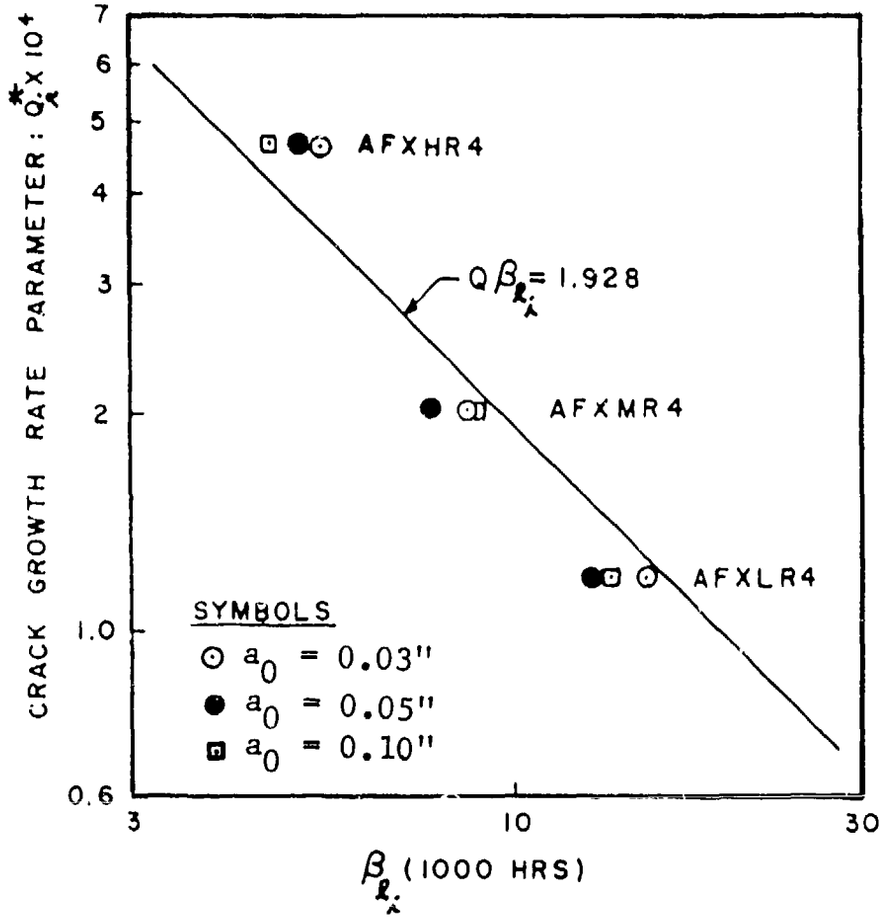


Fig. 7.2 Q_i^* Versus β_{ℓ_i} for the F-16 400-Hour Spectrum

In Fig. 7.3, ln Q_i^* is plotted against ln σ. The parameters ξ and γ in Eq. 3-18 can be determined using a least-squares fit and the results of Fig. 7.3. The resulting crack growth equation for Q_i (note: no superscript "*" is used for a SCGMC), expressed in ksi units, is given in Eq. 7-1.

$$Q_i = 1.427 \times 10^{-16} \sigma^{7.928} \tag{7-1}$$

A generalized SCGMC, as a function of stress level, can be obtained by substituting Eq. 7-1 into Eq. 3-17.

Crack exceedance predictions for the F-16 lower wing skin were determined using Eqs. 3-9, 3-17, 3-19 and 7-1 as well as the following parameters: x_u = 0.03", α = 1.823, $Q_i^* \beta_{\ell_i}$ = 1.928 (average), ℓ = 4 and various τ values. The results are presented in various formats as described below.

The extent of damage predictions for the F-16 lower wing skin are summarized in Table 7-3 at τ = 16000 flight hours for each of the ten stress regions shown in Fig. 7.1. The number of fastener holes with a crack size ≥ 0.03", $\bar{L}(\tau)$, and the standard deviation, $\sigma_L(\tau)$, were estimated to be 17.6 and 4.077, respectively. Based on the test results for the right hand and left hand lower wing skins, an average of

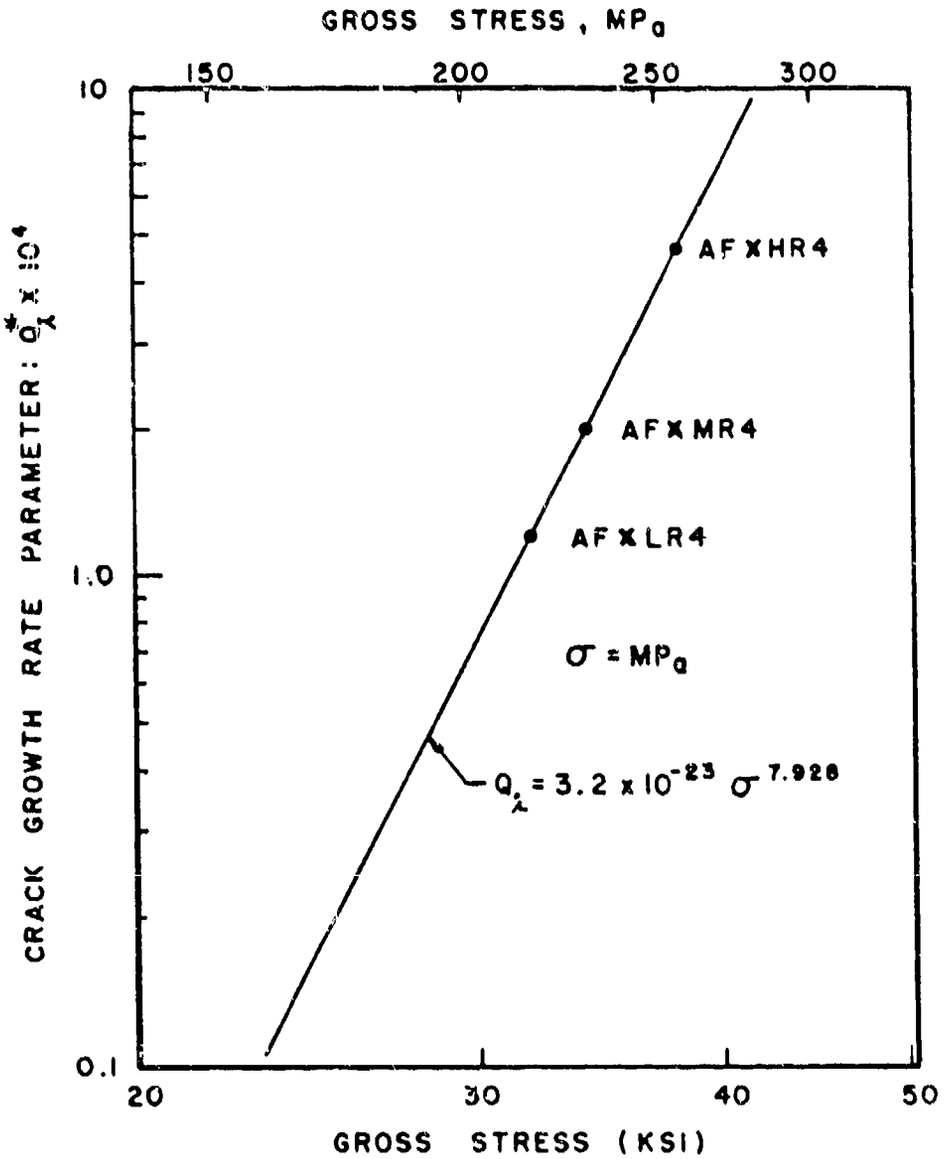


Fig. 7.3 Q_i^* Versus Gross Stress for the F-16 400-Hour Spectrum

Table 7-3 Durability Analysis Results for F-16 Lower Wing Skin

STRESS REGION	$Q_i \times 10^4$ (HRS^{-1})	$p(i,\tau)$	NO. HOLES WITH $x_1 \geq 0.03''$ @ τ = 16,000 HRS				
			PREDICTED		TEST		
			$\bar{N}(i,\tau)$	$\sigma_N(i,\tau)$	R.H. WING	L.H. WING	AVERAGE
I	0.4620	0.0426	2.5	1.547	7	0	3.5
II	0.3182	0.02182	6.9	2.598	7	2	4.5
III	0.1380	0.00480	3.3	1.812	4	1	2.5
IV	0.0071	0.00002	0.0	0.	0	0	0
V	0.4751	0.0448	0.4	0.618	1	0	0.5
VI	0.5921	0.0662	1.9	1.332	5	1	3.0
VII	1.3504	0.2649	2.1	1.242	0	2	1.0
VIII	0.2507	0.0142	0.1	0.314	1	1	1.0
IX	0.2507	0.0142	0.2	0.444	1	0	0.5
X	0.2152	0.0108	0.2	0.445	0	0	0
$\bar{L}(\tau)$ = 17.6, $\sigma_L(\tau)$ = 4.077, TOTAL TEST AVERAGE = 16.5							

16.5 fastener holes had a crack size ≥ 0.03" at τ = 16000 hours. In Table 7-3, the predicted extent of damage results track the average test results for the individual stress regions very well.

In Fig. 7.4, the predicted percentages of crack exceedance versus fastener hole crack size are plotted for the F-16 lower wing skin at τ = 16000 flight hours. Curves 1, 2 and 3 are based on $L(\tau)$ x 100%/N*, $[\bar{L}(\tau) + \sigma_L(\tau)]$ x 100%/N* and $[\bar{L}(\tau) - \sigma_L(\tau)]$ x 100%/N*, respectively. $\bar{L}(\tau)$ and $\sigma_L(\tau)$ are defined by Eqs. 3-22 and 3-23, respectively. N* is the total number of fastener holes in the F-16 lower wing skin (i.e., 1614 holes). Since the number of fastener holes in each stress region is large, it is reasonable to approximate the binomial distribution by the normal distribution. The corresponding exceedance probabilities for curves 1, 2, 3 are shown in Fig. 7.4 in parentheses.

Test results for the right and left hand lower wing skin (at x_1 = 0.03" and τ = 16000 hours) are plotted as a circle and a square, respectively, in Fig. 7.4. Approximatley 1.1% of the fastener holes in the F-16 lower wing skin are predicted to have a crack size ≥ 0.03" at τ = 16000 hours. This compares with an average of 1.02% based

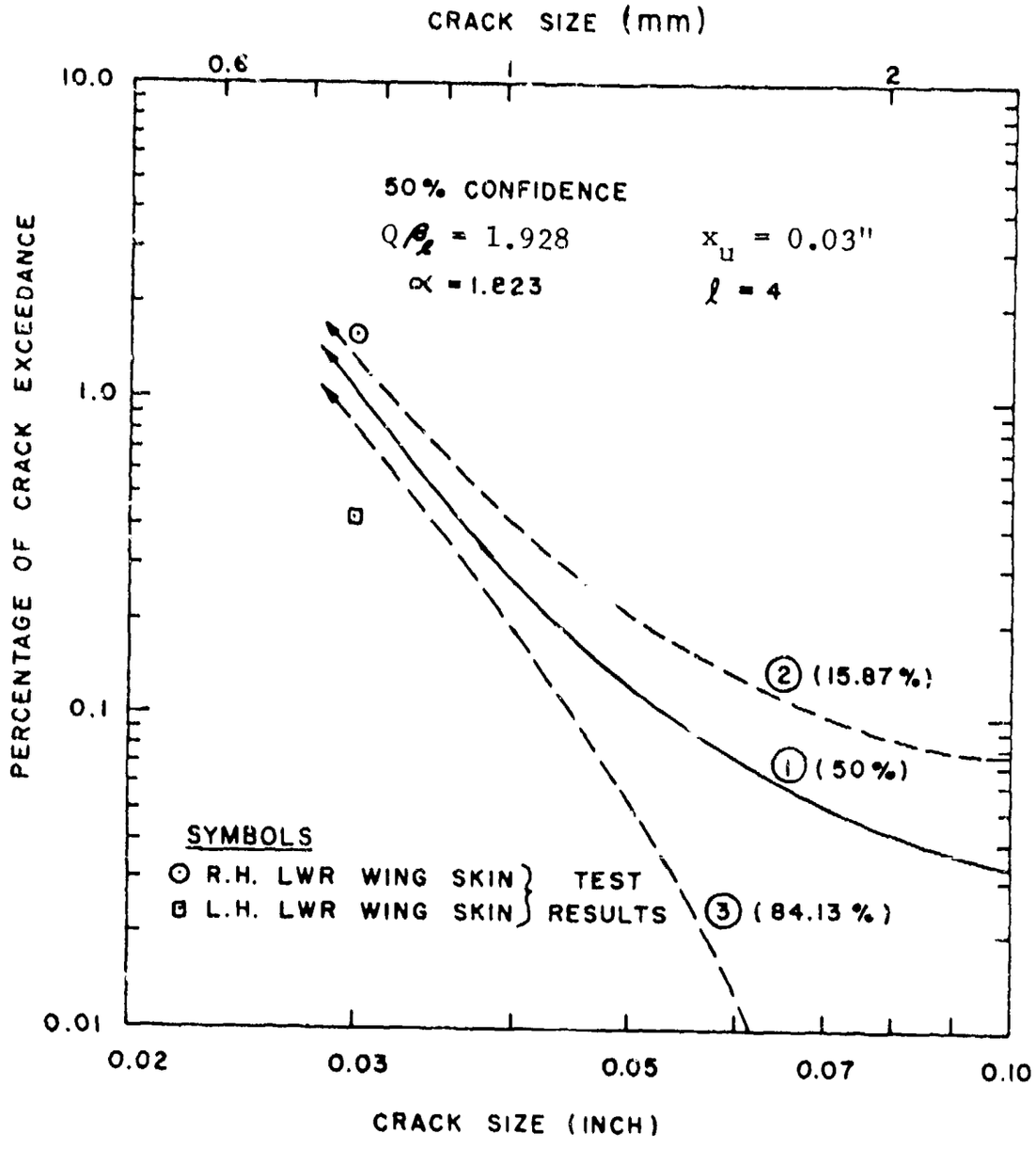


Fig. 7.4 Percentage of Crack Exceedance Versus Crack Size at 16000 Hours for 3 Probability Levels (F-16 Fighter)

on test results for the right hand and left hand lower wing skins.

In Fig. 7.4, the predicted average percentage of crack exceedance decreases rapidly for larger crack sizes. For example, the average percentage of crack exceedance for the fighter lower wing skin decreases from approximately 1.1% at x_1 = 0.03" to approximately 0.14% at x_1 = 0.05". Crack exceedance predictions are based on the service crack growth master curve defined by Eqs. 3-17 and 3-18. A single service crack growth master curve may not adequately fit the full range of desired crack sizes for all crack exceedance predictions. For example, different service crack growth master curves are required to fit two different crack size ranges as illustrated in Fig. 7.5. Curve 1 and Curve 2 shown in Fig. 7.5 apply to crack size ranges A_1 and A_2, respectively. Crack exceedance predictions based on curves 1 and 2 of Fig. 7.5 will be different for the same crack exceedance size, x_1. For example, $p(i,\tau)$ predictions based on Curve 2 for x_1, τ_1 and x_2, τ_2 will be larger than those based on Curve 1.

The extrapolation of crack exceedance predictions to larger crack sizes should be consistent with the applicable crack growth process for given design conditions and the

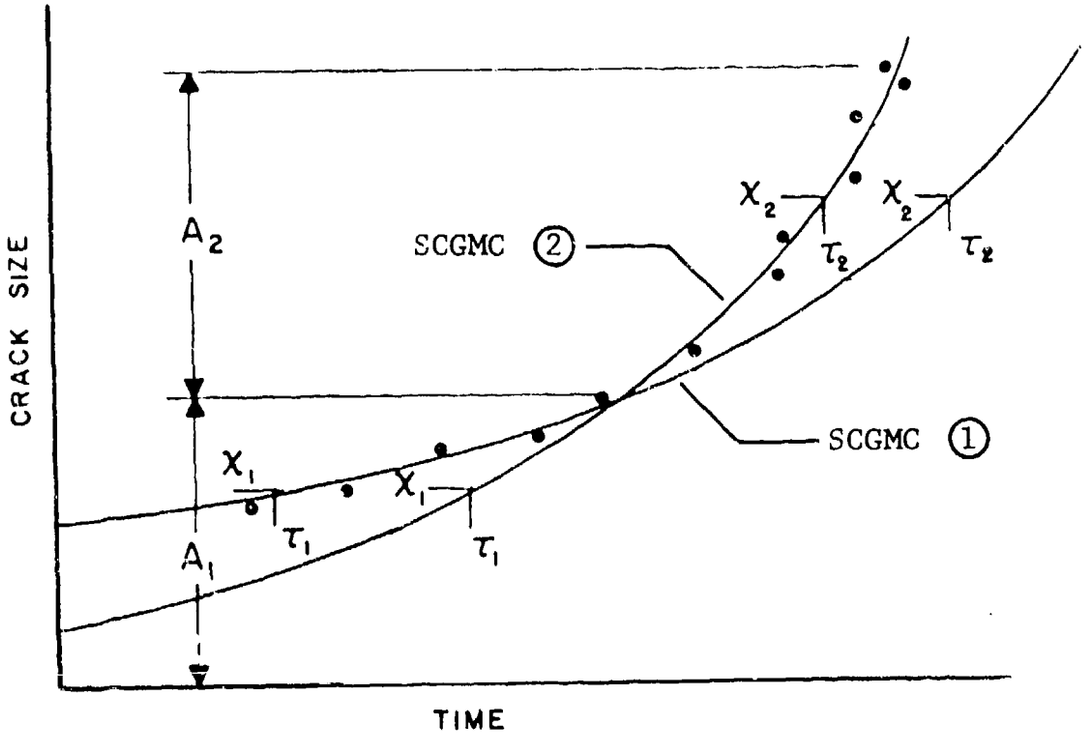


Fig. 7.5 Service Crack Growth Master Curves for Different Crack Size Ranges

crack exceedance crack size, x_1. Further research is needed to develop a better understanding and confidence in crack exceedance predictions for different crack sizes, materials, and design conditions.

Analytical predictions of the extent of damage are presented in Fig. 7.6 in an exceedance probability format. In this case, the predicted number of fastener holes in the F-16 lower wing skin with a crack size ≥ 0.03" are plotted as a function of flight hours for different exceedance probability values (i.e., P = 0.05", 0.50, 0.95). The plots are based on Eq. 3-19, x_1 = 0.03", α = 1.823, $Q_i^* \beta_{\ell_i}$ = 1.928 (average), ℓ = 4, N_i = 1614 fastener holes, Z = ±1.65 and $\bar{L}(\tau) \pm Z\sigma_L(\tau)$. For example, at τ = 16,000 hours, $\bar{L}(\tau)$ = 17.6 fastener holes and $\sigma_L(\tau)$ = 4.077. The upper bound prediction, $\bar{L}(\tau) + Z\sigma_L(\tau)$, is approximately 24.3 fastener holes. In other words, there is a probability of 0.05 that more than 24.3 fastener holes in the F-16 lower wing skin will have a crack size ≥ 0.03" at 16000 flight hours. There is a probability of 0.50 and 0.95, respectively, that more than 17.6 and 10.9 fastener holes will have a crack size ≥ 0.03" at τ = 16000 flight hours. The average and upper/lower bound predictions for the F-16 lower wing skin compare very well with test results for the right hand and

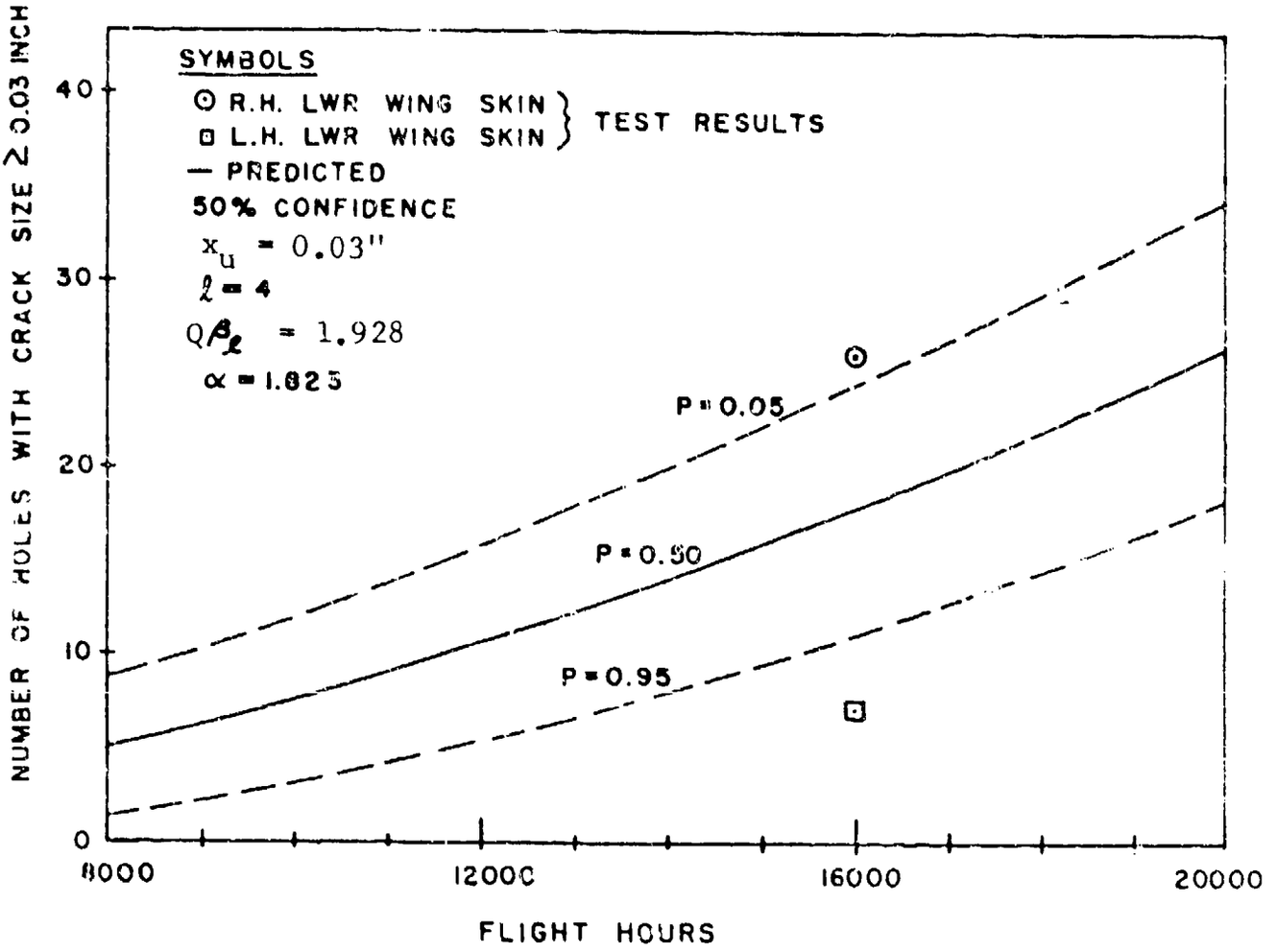


Fig. 7.6 Number of Holes with Crack Size $\geq$ 0.03" Versus Flight Hours--Exceedance Probability Format (F-16 Fighter)

left hand lower wing skins at τ = 16000 flight hours (Fig. 7.6).

The extent of damage predictions are presented in a stress level format in Fig. 7.7. Curves are shown for the baseline stress (σ), 1.1σ, and 1.2σ. Results are based on Eqs. 3-9, 3-17, 3-19 and 7-1. The "baseline stress" refers to the maximum stress level for each of the ten stress zones. For prediction purposes, the baseline stresses for each stress zone were all increased by the same percentage. The results shown in Fig. 7.7 can be used to assess the extent of damage as a function of stress level and flight hours. This format is particularly useful for evaluating durability design tradeoffs in terms of the extent of damage. For example, at τ = 16000 flight hours, approximately 1.1% of the fastener holes in the F-16 lower wing skin would be predicted to exceed a crack size of 0.03" for the baseline stress levels. If the baseline stresses were increased to 1.1σ and 1.2σ, the predicted average percentage of holes with a crack size ≥ 0.03" would be approximately 4% and 12%, respectively. This provides a quantitative measure of the structural durability as a function of stress level and flight hours.

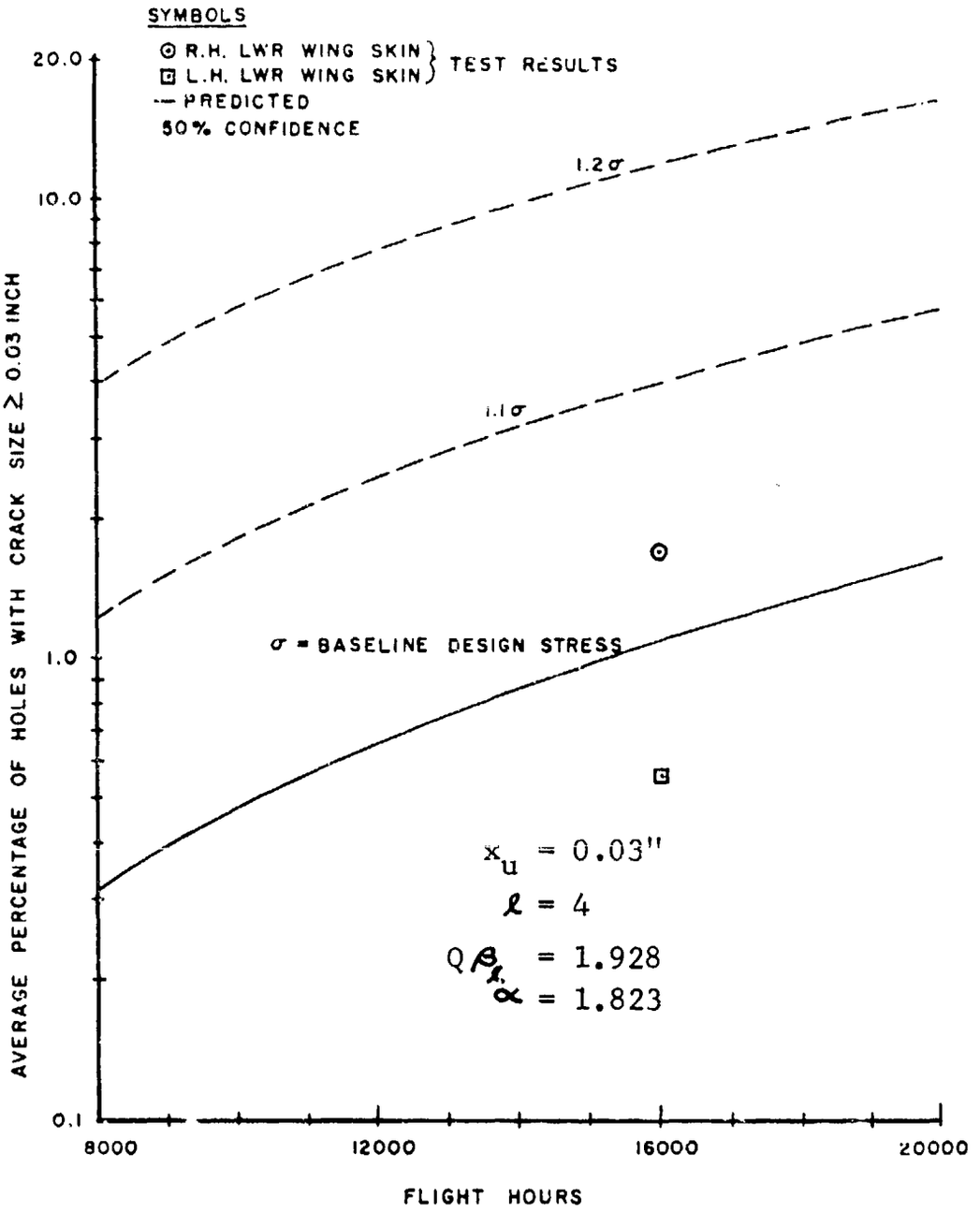


Fig. 7.7 Average Percentage of Holes with Crack Size ≥ 0.03" Versus Flight Hours--Stress Level Format (F-16 Fighter)

7.3 COMPLEX SPLICE SPECIMENS SUBJECTED TO A B-1 BOMBER SPECTRUM

A durability analysis of complex-splice specimens subjected to a B-1 bomber load spectrum is presented. Analytical predictions of the extent of damage in the specimens are presented in various formats and compared with fractographic results. The analytical/experimental results are summarized here and described in more detail in Ref. 20.

The complex-splice specimen geometry is presented in Fig. 7.8. Specimens were made of 7475-T7351 aluminum plate; and countersunk steel rivets were used. A B-1 bomber load spectrum [16,24] was applied. Based on a simplified stress analysis and strain gage results, the maximum gross stress in the outer row of fastener holes at the faying surface was estimated to be 35.8 ksi. The eleven specimens were tested to two service lifetimes (27,000 flight hours) or failure, whichever came first.

After testing, all fastener holes in the outer rows were inspected. Fractography was performed for the largest crack in each fastener hole in the outer rows. Twenty-five out of 110 fastener holes in the outer rows had a crack size ≥ 0.05" at 13,500 hours. Hence, 22.7% of the fastener holes in the outer rows had a crack size ≥ 0.05" at 13,500 hours.

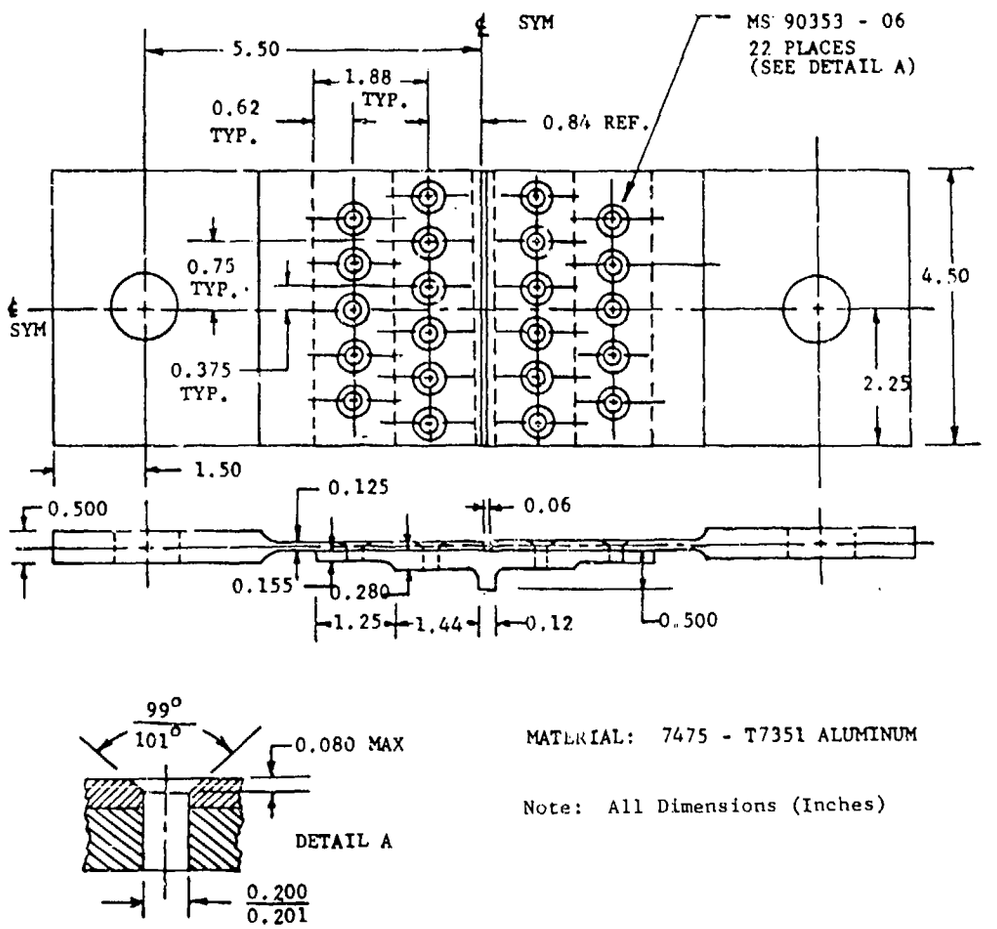


Fig. 7.8 Complex Splice Specimen

The IFQ of the fastener holes was based on the fractographic results for nine data sets and three different reference crack sizes. The specimens were made of 7475-T7351 aluminum and contained 2 countersunk rivets. Load transfer levels of 15%, 30% and 40% were considered. All specimens had the same configuration (Fig. 4.3) with the same overall length and basic test section dimensions. However, the lug end dimensions varied depending on the amount of load transfer. Three maximum stress levels were considered for each load transfer level. Specimens were designed for a given % load transfer assuming a perfect fit between the mating fasteners and holes. Therefore, the actual % load transfer for specimens varied depending on factors such as fastener-hole fit, axial stiffness of test specimen, stress level, etc.

A fractographic crack size range of 0.005" - 0.1" was considered. An upper bound EIFS of x_u = 0.05" was used for the IFQ distribution. Since the largest fatigue crack in one of four fastener holes per specimen was used, ℓ = 4 (Ref. subsection 4.5).The same data pooling procedures were used to determine the IFQ model parameters which were previously described for the fighter demonstration. The average α and $Q_i^{*}\beta_{\ell_i}$ values were found to be 2.702 and 2.823, respectively.

A generalized SCGMC, based on Eqs. 3-17 and 3-18, was obtained using Q_i^* and gross stress (σ) values for 9 fractographic data sets. The Ln Q_i^* versus lnσ is plotted in Fig. 7.9. The solid line represents the least-square best fit through the plot points. The dashed lines have the same slope as the solid line and they encompass all the plot points. The corresponding best-fit equation for Q_i (Eq. 3-18) as a function of gross stress level when stress is expressed in ksi units is as follows:

$$Q_i = 6.151 \times 10^{-13} \sigma^{5.381} \tag{7-2}$$

Crack exceedance predictions for the complex-splice specimens were determined using Eqs. 3-9, 3-17, 3-19, and 7-2. At τ = 13,500 hours, an average of 9 fastener holes (8.3%) were predicted to exceed a crack size of 0.05". The test results showed an average of 25 fastener holes (22.7%) exceeding a crack size of 0.05". The difference in the predicted and test crack exceedances is attributed mainly to the stress level used in the predictions. The actual stress level and distribution in the outer row of fastener holes is far more complex, due to lateral bending effects, than those considered for the damage assessment. The crack exceedance predictions are very sensitive to the gross applied stress level used. This is illustrated in Fig. 7.10. The solid

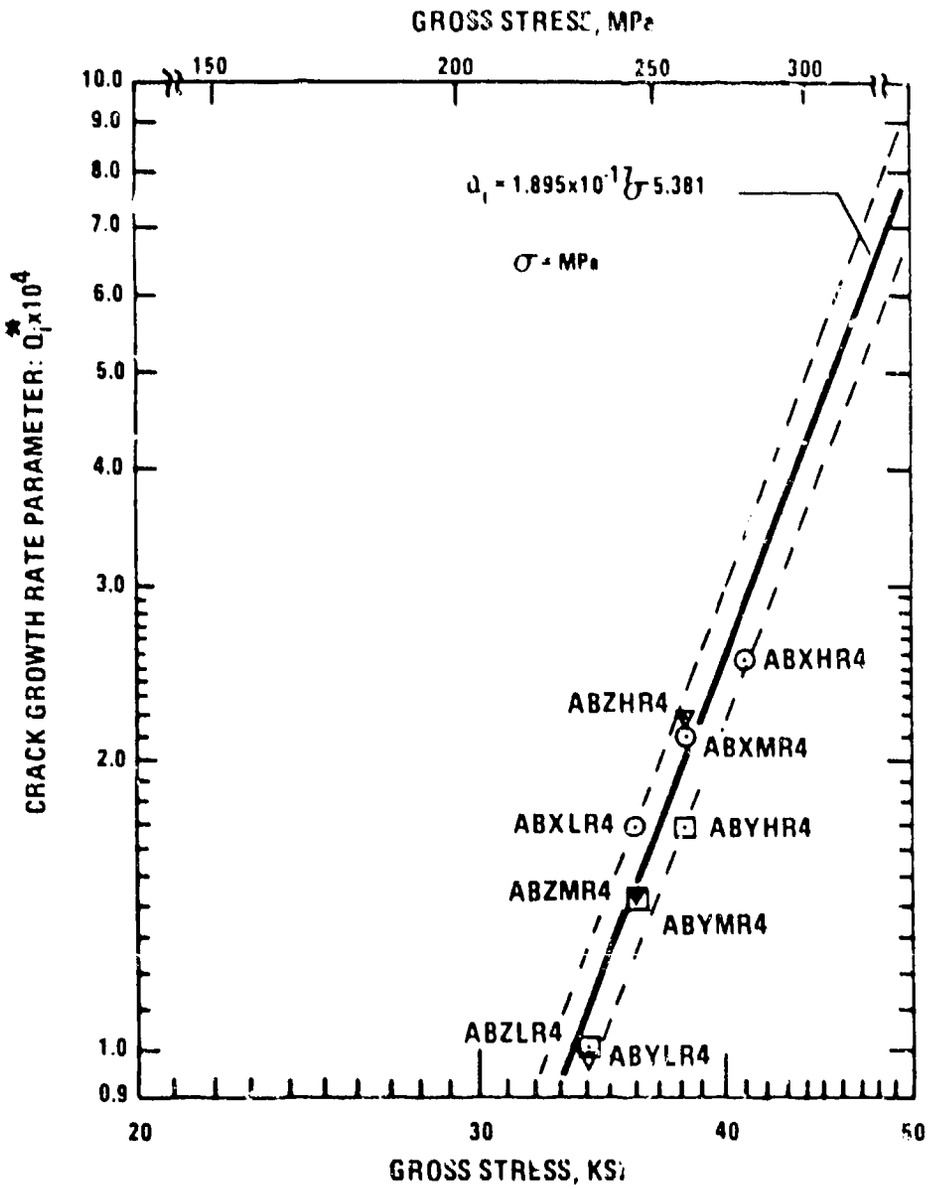


Fig. 7.9 Q_i^* Versus Gross Stress for B-1 Bomber Load Spectrum

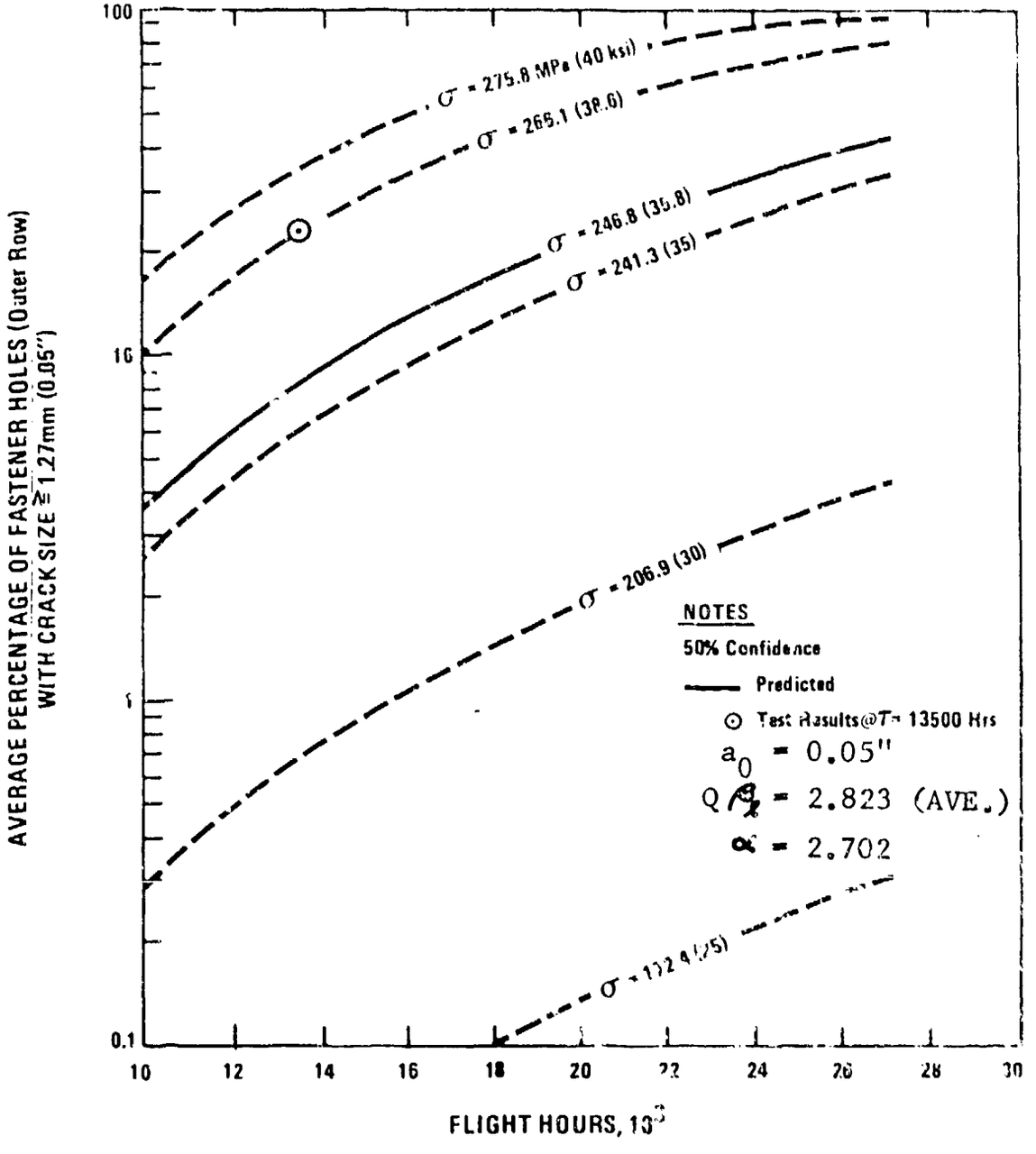


Fig. 7.10 Average Percentage of Holes with Crack Size ≧ 0.05" Versus Flight Hours--Stress Level Format (B-1 Bomber)

line represents average crack exceedance predictions for the gross stress level of 35.8 ksi obtained using the simplified stress analysis approach. The dashed line represents average crack exceedance for other gross stress levels. Also plotted as a single point is the average test crack exceedance at τ = 13,500 hours. It can be seen that if the gross applied stress level used in the predictions were 38.6ksi rather than 35.8 ksi, the predicted crack exceedance at τ = 13,500 hours would match the test results. Hence, a more accurate stress analysis could result in improved predictions.

Other useful crack exceedance formats, previously discussed for the fighter demonstration, are presented in Figs. 7.11 and 7.12 for the complex-splice specimens.

7.4 CONCLUSIONS AND RECOMMENDATIONS

Probabilistic fracture mechanics methods for durability analysis have been described and demonstrated for both a full-scale fighter aircraft structure and for a complex splice subjected to a bomber spectrum. These methods can be used to analytically assure compliance with the Air Force's durability design requirements. The analytical tools

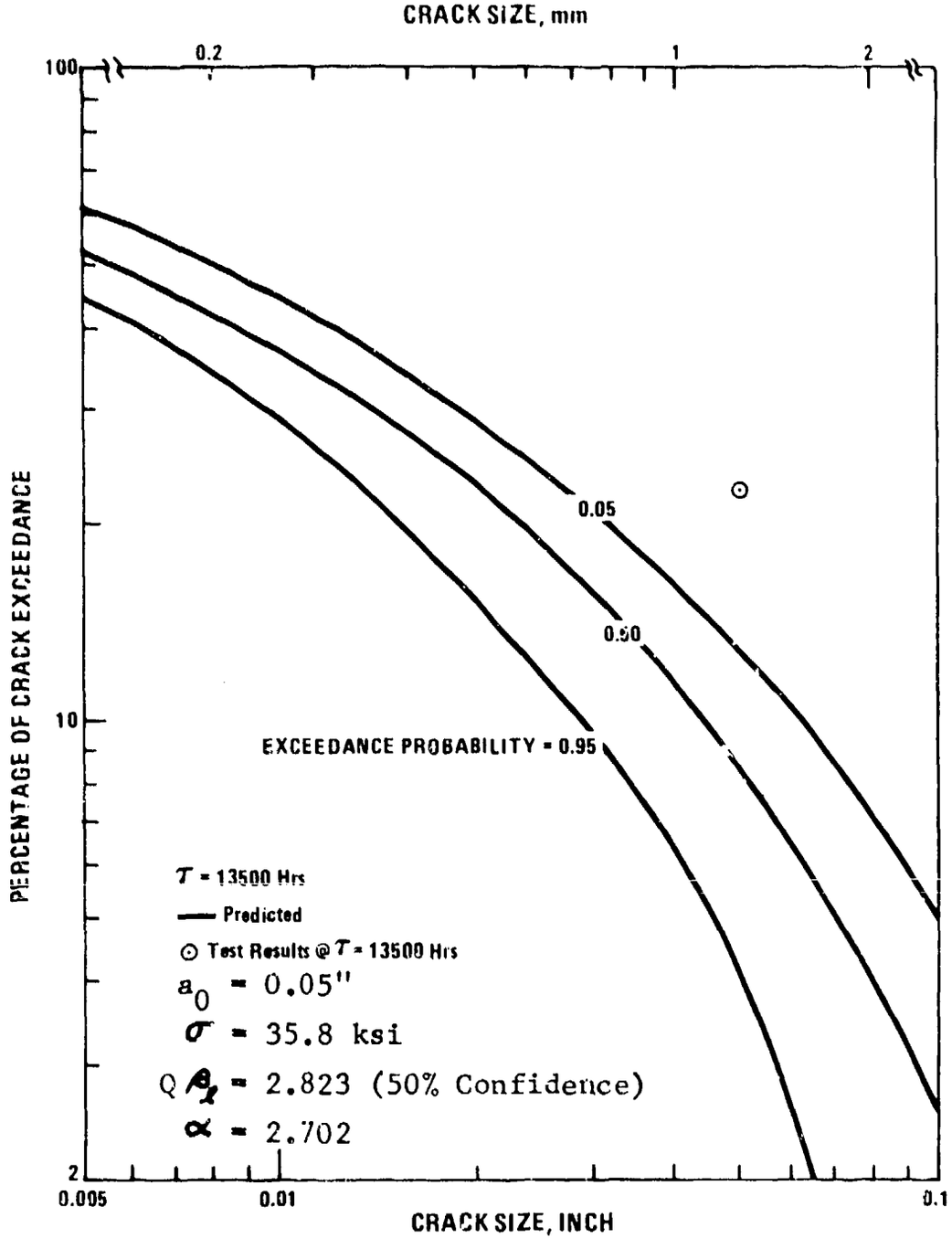


Fig. 7.11 Percentage of Crack Exceedance Versus Crack Size and Exceedance Probability at 13500 Hours (B-1 Bomber)

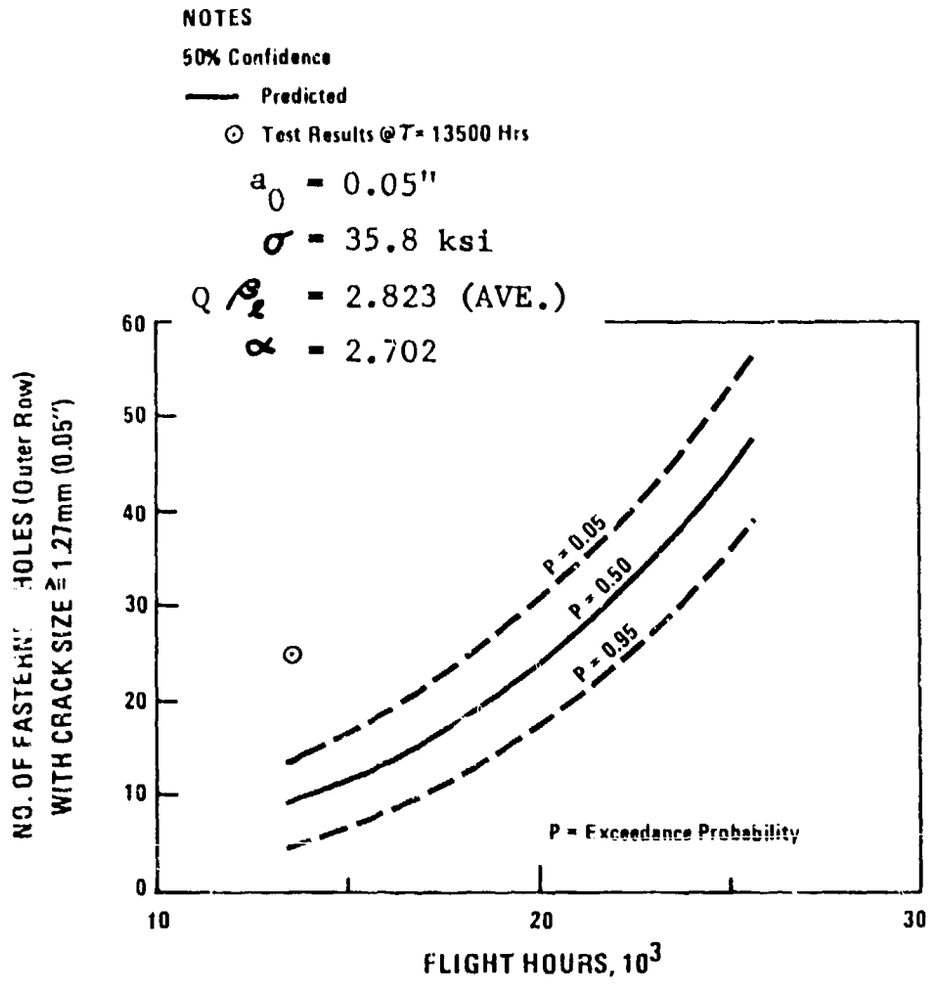


Fig. 7.12 Number of Holes (Outer Row) with Crack Size ≥ 0.05" Versus Flight Hours--Exceedance Probability Format (B-1 Bomber)

described can be used to quantify the extent of damage as a function of the durability design variables for structural details in a part, a component or airframe. Once the economic life and durability critical parts criteria are established, the extent of damage predictions can be used to assure design compliance with Air Force durability requirements.

An initial fatigue quality model can be used to define the EIFS cumulative distribution using suitable fractographic results. Procedures and guidelines have been developed for determining the IFQ model parameters for pooled fractographic data sets and for scaling TTCI results. The parameters α and $Q_i^* \beta_i$ provide the basis for putting fractographic results on a common baseline for quantifying the initial fatigue quality. For generic EIFS, α and $Q_i^* \beta_i$ should be constants for different fractographic data sets (same material, fastener type/fit, and drilling technique per data set), loading spectra, stress levels and percent load transfer. Encouraging results have been obtained to justify the use of the same EIFS cumulative distribution for crack exceedance predictions for different design conditions. Further research is required to confirm the IFQ distributions for different materials, load spectra, stress levels, fastener types/diameters/fit, % load transfer, etc. A considerable amount of fractographic results exist

which need to be evaluated using the IFQ model [e.g., 16, 24].

The effects of fretting, clamp-up, corrosion, size effect (scale-up from coupon to component), faying surface sealant, interference-fit fasteners, etc., on IFQ need to be investigated. Also the feasibility of using no-load transfer specimens with multiple holes to quantify the IFQ should be evaluated using spectrum and constant amplitude loading. This could provide an economical way to generate the fractographic results needed to quantify the IFQ.

Theoretically, the IFQ model can be used to quantify the EIFS cumulative distribution for various structural details as long as fractographic results are available for the details to be included in the durability analysis. The IFQ model has been evaluated using fractographic results for fastener holes. Suitable specimens and guidelines need to be developed for generating crack initiation and crack growth results for other details such as, cutouts, fillets, lugs, etc. Fractographic results should be developed and evaluated for such details so that the durability analysis methods described can be efficiently applied to different types of structural details in typical aircraft structures.

The accuracy of crack exceedance predictions, based on the same EIFS cumulative distribution, needs to be evaluated for different design conditions. Also, IFQ model parameter sensitivity studies need to be performed to better understand the average parameter values and variances and the impact of these parameters on the IFQ for different fractographic data sets.

The durability analysis methodology was developed for crack exceedance predictions for relatively small crack sizes (e.g., ≤ 0.10") in structural details. The largest crack in each detail was assumed to be statistically independent to justify using the binomial distribution for combining crack exceedance predictions for structural details. If the largest crack in a given detail doesn't significantly affect the growth of cracks in neighboring details, perhaps the proposed durability analysis methodology can be extended to crack sizes > 0.10". The crack growth law of Eq. 3-2 may not be suitable to use for crack exceedance predictions for crack sizes > 0.10". However, a general service crack growth master curve can be generated under given design conditions which is valid for crack sizes > 0.10" [14-16]. Nevertheless, this approach has not been demonstrated in the present study and further

research is required to extend the probabilistic fracture mechanics approach developed to larger crack sizes.

Two different $F_{a(0)}(x)$ equations (i.e., Eqs. 3-8 and 3-15) were presented for representating the IFQ. Either equation works but Eq. 3-15 is recommended for two reasons: (1) It assures all EIFS's in the IFQ distribution will be > 0, and (2) the crack growth rate parameter Q_i^* can be easily determined from the fractographic results and the resulting Q_i^* values for different data sets will be directly comparable. If Eq. 3-8 is used, the same b value (Eq. 3-2) must be imposed for different fractographic data sets to put the $Q_i^*\beta_i$ values on a comparable baseline. As long as b > 1, all EIFS's in Eq. 3-8 will be > 0. Further studies are needed to evaluate the accuracy of Eqs. 3-8 and 3-15.

The EIFS cumulative distribution, $F_{a(0)}(x)$, is independent of the reference crack size, a_0. This is illustrated in Eqs. 3-8 and 3-15. Therefore, the TTCI distribution for different reference crack sizes will transform into a common $F_{a(0)}(x)$.

The IFQ model is simply a "mathematical tool" for quantifying the IFQ of structural details. Therefore, the resulting EIFS's must be considered in the context of the

IFQ model and the fractographic results used to calibrate the model parameters. EIFS's should be considered as hypothetical cracks used for crack exceedance predictions rather than actual initial flaws per se.

Back extrapolations of fractographic data must be done consistently to put the EIFS's on a common baseline for different data sets. Inconsistent EIFS results will be obtained if the EIFS distribution is determined by back extrapolating the fractography results for individual specimens then fitting a statistical distribution to the EIFS results for different data sets. Two problems result if this approach is used: (1) the EIFS's are not on a common baseline for different data sets, and (2) the resulting EIFS distribution is not statistically compatible with the TTCI distribution and the fatigue wear out process. The resulting EIFS distribution should be statistically compatible with the TTCI distribution. The IFQ model presented in this section satisfies this requirement.

Several useful applications of the durability analysis methodology developed are: (1) the evaluation of durability design tradeoffs in terms of structural design variables, (2) the evaluation of structural maintenance requirements before or after aircraft are committed to service, and (3)

the evaluation of aircraft user options affecting life-cycle-costs, structural maintenance requirements, and operational readiness.

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