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G. R. Inger and A. Deane

West Virginia University
Morgantown, WV 26506

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800 N. Quincy Street
Arlington, VA

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G.R. Inger and A. Deane, West Virginia Univ., Morgantown, WV

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TRANSONIC SHOCK INTERACTION WITH A TANGENTIALLY-INJECTED TURBULENT BOUNDARY LAYER

G. R. Inger* and A. Deane**
West Virginia University, Morgantown, W. Va.

Abstract

A non-asymptotic triple deck theory of transonic shock turbulence boundary layer interaction is described which takes into account the influence of upstream tangential injection on a curved wall, in addition to Prandtl number and the shock strength, the theory is parameterized by arbitrary values of the incoming boundary layer shape factor, wall jet maximum velocity ratio, and the non-dimensional height of the mixing layer. Results of a comprehensive parametric study are then presented. It is shown that the wall jet effects significantly reduce both the upstream and downstream thickness of the interaction zone, while increasing the upstream and downstream friction levels, these effects also reduce the minimum interactive critical wall jet and maintain the onset of incipient separation at the shock foot.

Nomenclature

\( A \) - Von Kármán-Dean wall turbulence length parameter
\( C_f \) - skin friction coefficient, \( \frac{2}{3} \frac{\rho_v}{\rho_0} \nu_0 \frac{\partial U}{\partial y} \frac{1}{U_0} \)
\( \Delta C_f \) - skin friction increment due to wall jet
\( \rho \) - air density
\( \kappa \) - turbulent Prandtl number
\( \sigma \) - turbulent pressure fluctuation, \( P_{\text{j}} \)
\( \beta \) - pressure jump across incident shock
\( \delta \) - boundary layer thickness, respectively
\( u' \) - normal velocity fluctuation, \( \frac{U}{u_\text{inj}} \)
\( u'_{\text{rms}} \) - rms of all turbulent fluctuations
\( U_{\text{inj}} \) - undisturbed incoming boundary layer velocity in x-direction
\( W_{\text{off}} \) - effective wall shift seen by interactive turbulent boundary layers on adiabatic surfaces of small-to-moderate longitudinal curvature.

\( \rho_0 \) - undisturbed inviscid values ahead of incident shock
\( u_{\text{inj}} \) - boundary layer momentum thickness
\( \rho \) - density

\( \delta_\text{SL} \) - inner deck sublayer thickness
\( \delta_\text{t} \) - boundary layer thickness

\( \rho \) - mass density

\( \rho_\text{adi} \) - adiabatic wall

\( \text{inc} \) - incompressible value

\( \iota \) - interaction index

Subscripts

\( \text{off} \) - undisturbed incoming boundary layer properties

1. Introduction

The use of tangential slot-injection to influence and control turbulent boundary layer behavior has been extensively studied in various types of low-speed external and internal aerodynamic flow fields (e.g., on circulation-controlled airfoils, slotted slaps, in film cooling applications and for separation control in inlets and diffusers). In recent years, many applications of such injection have arisen in supercritical transonic flow fields where local shock wave is present; however, little is presently available to provide a basic understanding of how the resulting shock-boundary interaction ("SBLI") alters the influence of tangential injection. Conversely, in such supercritical flows it may be of interest to know how the effects of SBLI may be altered by the use of injection. The present paper addresses these questions for the case of steady non-separating 2-D turbulent boundary layers on adiabatic surfaces of small-to-moderate longitudinal curvature.

The primary objectives of our work are to develop a fundamental theory of a transonic SBLI region occurring downstream of a tangentially-injected turbulent boundary layer on a curved wall (Fig. 1) and then to present the results of a parametric study of this theory showing the
relationship between the dominant physical parameters, the injection and the physics of the SBLI zone. In Section 2, we briefly outline the non-asymptotic triple-deck theory of a SBLI zone on a curved surface without tangential injection. Then by taking the SBLI zone sufficiently far downstream of the injection slot for mixing of the wall jet and overlying turbulent boundary layer to have produced a well-defined "jet-bulled" boundary layer profile, the interactive perturbation field caused by normal shock interaction with this profile is analyzed in Section 3 by an extension of the aforementioned SBLI theory. This is followed in Section 4 by presentation and discussion of the results of a parametric study of this extended solution for the interactive pressure, displacement thickness and skin friction effects.

2. Brief Outline of the Basic SBLI Theory

2.1. The Triple-deck Model

It is well-known experimentally that when separation occurs, the disturbance flow pattern associated with normal shock-boundary layer interaction is a very complicated one involving a bifurcated shock pattern, whereas the unperturbed case pertaining to turbulent boundary layers up to $M = 1.1$ has instead a much simpler type of interaction pattern which is more amenable to analytical treatment (Fig. 2). The flow consists of a known inviscid, isentropic turbulent boundary layer profile $M = 0.1$ subjected to small transverse perturbations due to an impinging wall shock wave. In the practical Reynolds number range of interest here ($Re = 10^5$ to $10^6$) we purposely employ a non-asymptotic triple-deck flow model in the turbulent boundary layer patterned after the Lighthill-Stratford-Hiorns approach that has proven highly successful in treating a variety of other problems involving turbulent boundary layer response to strong rapid adverse pressure gradients and which is supported by a large body of transonic and supersonic interaction data. The resulting flow model, Fig. 2, consists of an inviscid boundary layer profile and a three-layered structure: an upstream influence and skin friction perturbations. An approximate analytic solution is further achieved by assuming small linearized disturbances ahead of and behind the shock interaction plus neglect of the detailed shock structure within the boundary layer, which gives accurate predictions for all the properties of engineering interest when $M > 1.05$. The resulting equations can be solved by operational methods yielding the interactive rise in skin friction, displacement thickness growth and the skin friction behavior upstream and downstream of the shock foot. This solution contains all the essential global features of the actual transonic viscous interaction flow and detailed comparisons with experiment have shown that it gives a very good account of all the important engineering features of non-separating interactions over a wide range of Mach-Reynolds number conditions.

2.2. Wall Curvature and Shock Obliquity Effects

Since SBLI with tangential injection often arises in flows on curved surfaces, it is desirable to account for wall curvature effects in the foregoing interaction theory. For the small to moderate curvatures usually encountered ($K = 0.1$ to $0.4$), details analysis of the transonic small disturbance flow in the outer deck shows that while the explicit new curvature terms in the perturbation equations are negligible, some of the new terms in the inviscid rotational disturbance equations are of the negligible order $K^2$; only the curvature effect on the undisturbed boundary-layer velocity and eddy viscosity profiles are of possible significance. Here again, the explicit $K^2$ terms in the governing equations of this inviscid flow are all negligible; however, curvature can moderately influence (10-20%) the eddy viscosity terms, with a consequent effect on the boundary-layer profile in the form of a skin friction reduction and shape factor increase is described approximately by the relationships:

$$C_{f_1} = 1 \times 10^{-5} K^2 C_{f_1}^{(0)}$$

$$H_{fl} = 1 \times 10^{-5} K^2 H_{fl}^{(0)}$$

where to this order of accuracy the corresponding effect on $C_{f_1}$ is negligibly small. Note, for example, that the typical value $K = 0.01$ yields a reduction and increase in $C_{f_1}$ and $H_{fl}$ of 10% and 5%, respectively. The use of $C_{f_1}$ and $H_{fl}$ with the half velocity profile model and $K^2$, as an additional input parameter, provides a good engineering account of the moderate curvature effects on the middle-deck interaction solution. Within the very thin inner disturbance shear stress deck it is found yet again that the explicit curvature effects on the various inertia, pressure gradient, and laminar viscous terms in the disturbance flow equations are altogether negligible. Moreover, because of the extreme inner-deck thickness, the eddy viscosity curvature effects can also be safely neglected for the high Reynolds number conditions typifying most practical external aerodynamic flows.
Predictive results for the typical influence of $K_i$ on SBLI properties, which agree with experimental observations, may be found in Ref. 4. They show that the curvature effect is most pronounced for adverse pressure gradients along the wall, due primarily to the increasing shape factor. Since the curvature effect slightly reduces the inverse boundary-layer velocity profile fullness and spreads out the interaction, it further acts to advance the downstream boundary layer while slightly increasing the local $f$; around the shock front, owing to the reduced inflectional blending resistance, we conclude the slight shock solubility at the boundary layer in the presence of the inverse curvature. However, it is important to note that the solubility may be equivalent to a normal shock at the effective lower shock Mach number.

\[ \Theta_{\text{eff}} = \sqrt{\frac{\Theta_{\text{in}}}{1 + \Theta_{\text{in}}}} \]

Thereby allowing the solubility effect to be accurately accounted for in the present theory. (4)

### Extension to Include Essential Function

We recall that the aforementioned interaction theory leads to a shock strength and unsaturated inverse turbulent velocity profile model characterized by the overall parameters $\Theta_{\text{in}}$, $\Theta_{\text{out}}$, and $K_i$. In the problem at hand we have a new unique shape of velocity profile that exists due to the wall jet effect. In the present interaction, we will be concerned with modeling such a profile and its associated wall-region eddy viscosity behavior by a convenient set of parameters that characterize the essential aerodynamic features and yet are flexible in the sense that they allow parametric sensitivity analysis.

When air is tangentially injected through a slot or entrance into an overlying boundary layer it is entrained by the boundary layer flow (Fig. 5a). Immediately downstream of the slot, entrainment mixing causes a complex flow field involving a complicated inner which may not be validly treated by laminar flow theory, in any event the resulting complex velocity profile assumes a unique character with a maximum and a minimum (Fig. 5a). As the flow proceeds further downstream, experimental studies have shown that the minimum is rapidly eliminated by further mixing so that when $x$ with the profile attains a fully-developed "metastable" shape (Fig. 5b), composed of an unblown or turbulent boundary layer profile plus a wall jet component containing a velocity maximum near the wall. As this fully-developed shape continues downstream, further mixing gradually screen and spreads out the jet maximum (Fig. 5a) until the boundary layer ultimately tends toward an infinity profile. Of particular interest in the present study of weak transonic normal shock interaction with the boundary layer downstream of a normal injection slot, we will deal with the case where the shock interacts with a jet-affected type of velocity profile (Fig. 5b); this the most interesting encountered in practice.

While providing a boundary layer profile that can be analytically modeled in a manner appropriate to the SBLI solution (see below), this case also permits a simplified treatment of the eddy viscosity aspects of the interactive decks in the boundary layer, as follows. Experimental studies have shown that the usual Law of the Wall behavior and its associated mixing length eddy viscosity model applies to the lower portion below the jet maximum when the injection effect is small-to-moderate (e.g., $u_0 = 0.01 u_\infty$). Since the thin disturbance or jet inner layer of the SBLI region lies well within this layer of the Wall region, while there are no eddy viscosity-associated perturbation terms in the overlying plume, we can use the inviscid frozen-turbulence nature of its disturbance flow, height, it can be shown that the form of all the basic triple-deck equations in the aforementioned SBLI theory can be carried over to the present problem provided that one fully accounts for the wall jet effects in the unsteady flow system $C_{\text{in}}$, displacement thickness Reynolds number $R_{\text{in}}$, and especially, the characteristics of $K_i$, $B_{\text{in}}$, and $B_{\text{out}}$ as well as the profile distribution itself.

An appropriate analytical model of the important boundary layer profile was developed which accounts for the essential new wall-jet features of the flow while also being well-suited to the laminar pressure disturbance equation that is involved in the triple-deck solution. It is constructed as the sum of a wall-jet component and an "unknown" component, where to be consistent with SBLI theory of the SBLI composite profile characterized by the three parameters $B_{\text{in}}$, $B_{\text{out}}$, and $K_i$, see Appendix A. Thus if $u_{\text{max}}$ is the height of the maximum velocity $u_{\text{max}}$, with $u_{\text{max}}$ denoting the corresponding difference between $u_{\text{max}}$ and the unilaminar velocity due to the wall jet effect (see Fig. 6), the total profile is expressed as

\[ u = u_0 + \left( \frac{y}{y_{\text{max}}} \right)^{1.2} \]

where the wall jet component varies from 0 at $y = 0$ (no slip) to its maximum value $u_{\text{max}}$ at $y = y_{\text{max}}$ and then decays outwardly toward zero, becoming negligible beyond some characteristic jet-spread height $s$ above $y_{\text{max}}$ (we presume $y_{\text{max}} < s$). Above $y_{\text{max}}$, we have followed the experimentally-based work of Carriere et al. and represented $u_{\text{max}}$ by a modified "smooth" function whose slope is $y_{\text{max}}$ equals $u_0$ in the wall. Such that the total composite profile correctly has a maximum at $y_{\text{max}}$:

\[ u_{\text{max}} = \frac{\left( \frac{y}{y_{\text{max}}} \right)^{1.2}}{\text{SHEAR}^2} \left( \frac{y}{y_{\text{max}}} \right)^{1.2} \]

where

\[ \text{SHEAR} = \left( \frac{y_{\text{max}}}{y} \right) \left( \frac{u_{\text{max}}}{u_0} \right) \]

The reasons upstream of the slot and very far downstream where the profile maximum has disappeared can of course be handled by the existing "unknown" version of the present SBLI theory.
where
\[ v = \ln(1 + \frac{C}{2}) - \ln(1 - \frac{C}{2}) \] (5B)

is a phasing factor insuring the maximum in total velocity at \( y_{\text{max}} \)
and
\[ C = \frac{\delta_{\text{max}}}{\delta_{\text{max}}} - \frac{\delta_{\text{max}}}{\delta_{\text{max}}} \] (5B)

is a lateral spreading constant typically \( \approx 0.15 \) to avoid secondary profile maxima above \( y_{\text{max}} \).

Below \( y_{\text{max}} \) on the other hand, we require a functional representation that gives a reasonable monotonic shape and matched smoothly to Eq. (5) at \( y_{\text{max}} \). Furthermore, we desire some control over the wall slope in order to represent injection effects on the local skin friction \( \Delta C_f \). The specific constraints on this functional choice are (a) only one maximum in the total composite profile at \( y = y_{\text{max}} \), (b) a match with the value and slope of the upper \( f(y) \) function at \( y_{\text{max}} \), and (c) positive values of the non-dimensional slope
\[ S_{\text{w}} = \frac{(\delta_{\text{w}} \text{vel} - \delta_{\text{w}} \text{vel} \text{vel})}{\delta_{\text{w}} \text{vel} \text{vel}} \] (6)

leading to physically reasonable skin friction increments
\[ C_f = S_{\text{w}} \frac{(\delta_{\text{w}} \text{vel} - \delta_{\text{w}} \text{vel} \text{vel})}{\delta_{\text{w}} \text{vel} \text{vel}} \] (6)

Now condition (a) so severely restricts the class of monotone functions it admits that no general solution can be generated to accommodate a completely arbitrary combination of conditions (b) and (c); what can be found, however, are functions which allow either an arbitrary choice of all three parameters \( S_{\text{w}}, \delta_{\text{max}}, \delta_{\text{max}} \) within a restrictive range or the choice of a wide range of values for the two key parameters \( \delta_{\text{max}}, \delta_{\text{max}} \) with \( S_{\text{w}} \) then subsequently determined (but still within an interesting range of resulting values). One such function which has proven quite satisfactory for the purposes of this investigation is
\[ \frac{\delta_{\text{w}} \text{vel} - \delta_{\text{w}} \text{vel} \text{vel}}{\delta_{\text{w}} \text{vel} \text{vel}} = C_1 \frac{\delta_{\text{max}}}{\delta_{\text{max}}} - C_3 \frac{(\exp(C_2 \delta_{\text{max}}) - 1)}{\delta_{\text{max}}} \] (7A)

where the aforementioned matching conditions are fulfilled if the constants \( C_1, 2, 3 \) satisfy the three simultaneous relations
\[ C_1 - C_3 (\exp(C_2) - 1) = \delta_{\text{max}} \] (7B)
\[ C_1 - C_3 \exp(C_2) s - \delta_{\text{max}} \] (7C)
\[ C_1 - C_3 = S_{\text{w}} \] (7D)

This trio is readily solved numerically during the implementation of the velocity profile model by using a standard non-linear simultaneous root-finder subroutine.

The aforementioned provides a smooth, piecewise-continuous and physically realistic analytical model of a fully-turbulent boundary layer downstream of a tangential injection slot; it captured the velocity, overshoot and negative vorticity, region features unique to this kind of flow (Ref. 4) while retaining sufficient basic parameterization to permit sensitivity studies of how the jet-injection effect influences the SBLI zone. Moreover, it has the advantage of allowing current and later experimental data on turbulent wall-jet boundary layer behavior to be incorporated into the interaction study without tying the present research down to the much more difficult and lengthy effort of such experimental studies. The weak boundary layer compressibility effects on this profile for adiabatic transonic flow are quite satisfactorily accounted for by the reference temperature method.

3.2 Implementation of the Extended Theory

The foregoing approach may be implemented by several straightforward modifications to the existing computer program for the zero-blowing SBLI theory, as follows. To include small-to-moderate wall curvature effects (Ref. 5), we add \( M_0 \) as an independent input variable and accordingly modify the input values of \( \delta_{\text{max}} \) and \( C_f \) according to Eqs. (1) and (2); furthermore, we eliminate the inviscid curvature singularity; altering the normal shock to a slightly oblique one at the boundary layer edge, by modifying the input effective normal shock Mach number according to Eq. (3). The influence of tangential injection is accommodated by introducing the two new input parameters \( M_{\text{max}}, u_{\text{max}} \), and \( y_{\text{max}}/\text{y} \), characterizing the magnitude and height, respectively, of the wall jet component effect; in addition, values of the auxiliary parameters \( C_1 \) and \( S_{\text{w}} \) can be set within certain restricted ranges. The program subroutine which evaluates the wall turbulent boundary layer velocity profile model is modified to add the matched upper and lower wall jet-component increments pertaining to these inputs (Eqs. 4-7), using a Reference Temperature-Method compressibility correction of the appropriate parameters. Figure 7 illustrates some typical boundary layer velocity profiles containing these tangential injection effects. Using the adiabatic temperature-velocity relationship
\[ T = T_{\text{w}, \text{AD}} - (T_e - T_{\text{w}, \text{AD}}) \frac{U_e}{U_e} \] (8)

the associated Mach number profile \( M(y) \) and its derivative \( M_y/y \) which are both needed in the subsequent SBLI solution routine are calculated, the corresponding mass flow and momentum defect distributions \( \delta_{\text{w}} \text{vel} - \delta_{\text{w}} \text{vel} \text{vel} \) and \( (\delta_{\text{w}} \text{vel} - \delta_{\text{w}} \text{vel} \text{vel}) \) are also integrated across the boundary layer to obtain the values of \( \delta_{\text{w}}/\delta_{\text{w}} \) and \( \delta_{\text{w}}/\delta_{\text{w}} \), respectively, associated with the wall jet effect. The resulting values of the displacement thickness and shape factor are shown in Figs. 8A and 8B, to illustrate how the mass and momentum addition to the boundary layer from the wall jet substantially decreases \( \delta_{\text{w}} \) and produces a greater profile "fullness", reflected in a significantly reduced shape factor, increasing...
the height of the jet maximum is seen to have a similar effect, because this enhances the effective strewness of the injection effect on the boundary layer profile. Awareness of these overall integral property effects proves helpful in interpreting the predicted interaction properties given below.

Implementation of these wall jet-modifications is quite straightforward, except to note that feedback of the aforementioned modified integral properties into the solution sequence must be properly phased: since the wall jet effect on the incoming boundary layer profile shape is already included in the $M_{ij}$ distribution used in solving the Lighthill interactive pressure equation, the feedback must be done after this pressure disturbance solution is carried out. Subsequent use of the jet-affect values of $\delta$ and $C_f$ then further influences the local interactive displacement thickening and skin friction solution results. To illustrate the importance of this proper feedback of the jet-influenced profile integral properties and to compare the linearized interactive layer profile fullness and shape factor reduction due to injection causes a significant streamwise contraction of the interactive pressure rise; this is in agreement with experimental observations [see, e.g., Figs. 11a, 11b at Vol. 17]. Accompanying this contraction of the interaction zone, the two thin effects of injection on the ratio $\delta^* / \delta$ are seen to act with opposite and nearly equal influence: while the profile shape-factor effect of injection reduces $\delta^*$, the corresponding reduction of $C_f$ is approximately of the same magnitude so that the overall change in $\delta^* / \delta$ is small. This implies that the net interaction effect on $\delta^*$ scales appropriately with the corresponding effect on $C_f$. Turning to the interactive skin friction behavior typified in Fig. 9e, it can be seen that the increased $C_f$ level due to the wall jet effect dominates most of the interaction zone both fore and aft of the shock except in the vicinities of the shock foot and the sonic point within the boundary layer. In these two isolated cases, there is a local breakdown of the linearization underlying the Lighthill equation and the resulting transonic singularity which causes fundamental difficulties with the numerical solution of this equation that can only be cured by restoring (at least locally) the appropriate non-linear transonic correction term. For all other maximum wall jet velocities (including, interestingly enough, the so-called "overblown" cases where $U_{\text{max}} > M_{\text{ac}}$, the boundary layer contains only one local sonic point that is well-removed from $dM_{\text{ac}}/dy = 0$ (for subsonic $U_{\text{max}}$ it lies above $U_{\text{max}}$ while for supersonic $U_{\text{max}}$ it lies below). Such local cases, no fundamental difficulties were discerned.

4. Discussion of Parametric Study Results

The present theory has been used to carry out a systematic study of how the key tangential injection parameters influence the essential properties of a subsequent SBLI zone. We now present and discuss the results.
We note here that the absolute values of the effects of thinning out and delaying value with Reynolds number is in agreement with seen that the theoretical prediction of a gradual pressure gradient effect. A careful examination of a large number of corresponding increase in approximate experimental boundary determined by a as a phenomenon studied. The results showing the influence of the incoming uniform shape factor, shock strength and Reynolds number on the wall jet effects are presented in Figures 12 and 13 as a function of both the magnitude and location of the jet velocity maximum for a typical supercritical flow of $M_0 = 1.20$. Additional plots showing the influence of the streamwise slope of $\Delta\theta$ at the shock foot, which relate to the effective "viscous wedge" angle sensed by the outer inviscid flow; this effect is illustrated in Fig. 19, where the strong increase of this slope with wall jet strength may be clearly seen.

4.1. Interactive Pressure and Displacement Thickness

Typical pressure distributions, showing the strong systematic contraction of the streamwise interactive thickness distribution $\Delta(x)$, is illustrated in Figure 11. A comprehensive summary of such results showing the upstream and downstream influence distances (the distance ahead and behind the shock interaction, respectively, of the overall shock jump value). The results are presented in Figures 12 and 13 as a function of both the magnitude and location of the jet velocity maximum for a typical supercritical flow of $M_0 = 1.20$. Additional plots showing the influence of the incoming uniform shape factor, shock strength and Reynolds number on the wall jet effects are presented in Figures 14-17. Taken overall, these results show that tangential injection has a parametrically significant effect on the downstream streamwise scale of the interaction to a degree comparable to, or greater than, the uniform shape factor and/or Mach number effects. When nondimensionalized in terms of $M_0$, the results are not very sensitive to Reynolds number.

The corresponding systematic influence of injection on the relative interactive displacement thickness distribution $\Delta^+(x)/\Delta^+(x)$ is illustrated in Figure 16, where we see that the effect on $\Delta^+(x)$ and $\Delta^+(x)$ is largely canceled over a wide range of wall jet strengths when presented in this ratioed manner. However, there is a significant injection effect on the streamwise slope of $\Delta^+(x)$ at the shock foot, which relates to the effective "viscous wedge" angle sensed by the outer inviscid flow; this effect is illustrated in Fig. 19, where the strong increase of this slope with wall jet strength may be clearly seen.

4.2. Incipient Separation

The present theory, although it breaks down at separation, does yield a useful indication of incipient separation. For a given value of $C_f$ at $N = 0$, owing to the particular attention paid to the treatment of the local interactive skin friction behavior. Since this indicates is of great practical interest, a parametric study of incipient separation conditions inherent in the present theory was carried out.

As a basis for comparison, the results for flow without any tangential injection are shown in Fig. 20a where the shock Mach number above which incipient separation occurs is plotted as a function of the Reynolds number with the shape factor as a parameter. Similarly, the approximate experimental boundary determined by a careful examination of a large number of transonic interaction tests, besides Nusselt's $N < 1.30$ criterion for turbulent flow. It is seen that the theoretical prediction of a gradual increase in the incipient separation Mach number value with Reynolds number is in agreement with the trend of this data. The theoretical prediction of only a small influence of shape factor on the incipient separation conditions is also borne out by more recent data as indicated in Fig. 20b. We note here that the absolute values of the incipient separation Mach number predicted on the basis of a normal shock are consistently under-

4.3. Downstream Effects

The SBLI effect, has been shown in several comprehensive studies of supercritical airfoil flow fields to have an appreciable influence on both shock location and downstream boundary layer behavior, and hence on the global aerodynamic characteristics. Therefore, the predicted influence of tangential injection on the post-interactive boundary layer properties was of interest, as would the extent to which SBLI alters the boundary layer behavior that otherwise exists downstream.

Now, we have seen above that tangential injection reduces the SBLI displacement thickness growth while increasing the downstream post-interaction $C_f$ (Fig. 9c). Conversely, we may view SBLI as increasing the downstream $\Delta^+$, and hence countering the thinning effect otherwise obtained by the wall jet, while reducing the injection-produced $C_f$ enhancement; both these SBLI effects make the boundary layer less resistant to separation in any subsequent adverse pressure gradient region it may encounter, and hence diminish the effectiveness of injection in otherwise delaying downstream separation. Regarding the skin friction, these conclusions are summarized in Fig. 23, where there is shown the typical influence of increasing wall jet strength on the post-interactive $C_f$. It is seen that while separation at first increases it slightly due to the corresponding increase in $C_f$, stronger injection rates have the opposite effect of lowering it (as well as $C_{min}$) because of the intensified adverse pressure gradient effect.

5. Concluding Remarks

Viewed overall, the present study has shown that the usual favorable tangential injection effects of thinning out and delaying the separation of turbulent boundary layers in subsonic flow can be significantly compromised by transonic
shock boundary layer interaction. Conversely, such an
intersection was seen to appreciably reduce the
streamwise extent of an SBLI zone albeit with the
allied consequence of intensifying the local inter-
active adverse pressure gradient and onset of
short foot separation. It has further been estab-
lished that a fundamentally-based triple-deck
theory of SBLI with injection is now available to
treat these effects in both external or internal
supercritical flow fields, moreover, this theory
has been contrived to serve as a locally-invis-
cible interactive module inside the inviscid shock
location driven by the attendant local boundary
layer properties including an arbitrary non-
equilibrium shape factor. Consequently it would
be possible to investigate in the future interest-
ing problems of allowing in supercritical flow
fields, including the use of tangential injection to
mitigate the influence of SBLI upon the viscous
streamlines edge effect of supercritical airfoils [12],
and the inclusion of SBLI effects in viscous-
supercritical flow field analysis programs for circula-
tion-controlled airfoils and wings flying at
supercritical flight speeds.

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Appendix

Because of its convenient analytical form,
accurate similarity representation of the so-called
law of the wall of the wake behavior and
separation, we have adopted Valz's model for the
inviscid turbulent boundary layer upstream of the
intersection. For the low Mach number small heat
transfer conditions appropriate to transonic inter-
sections, it may be satisfactorily corrected for
compressibility effects by the present Reference
Toward Ref. 1 and 2, under these conditions
accurate, comparable in accuracy to, but for
which to implement the "Constant" assumptions the
compressible transonic approach.

Let - be known, (incompressible) wake function
\[ \frac{U}{U_0} = 1 + \frac{1}{2} \left( \frac{U}{U_0} \right) \left( \frac{T}{T_0} \right) \left( \frac{R}{R_0} \right) \frac{1}{1 + \frac{1}{2} \left( \frac{T}{T_0} \right) \left( \frac{R}{R_0} \right) \left( 1 - \frac{1}{2} \frac{T}{T_0} \right)^2 \right) \]
and \( \frac{T}{T_0} = 1 + \frac{1}{2} \left( \frac{T}{T_0} \right) \left( \frac{R}{R_0} \right) \frac{1}{1 + \frac{1}{2} \left( \frac{T}{T_0} \right) \left( \frac{R}{R_0} \right) \left( 1 - \frac{1}{2} \frac{T}{T_0} \right)^2 \right) \)
subject to the following condition limiting \( T \) to
\( T_0 \) and \( R \) to:

\[ 2 \left. \frac{U}{U_0} \right|_{T = T_0} - \frac{1}{2} \left( \frac{T}{T_0} \right) \left( \frac{R}{R_0} \right) \left( 1 - \frac{1}{2} \frac{T}{T_0} \right)^2 \left( 1 - \frac{1}{2} \frac{T}{T_0} \right) \left( \frac{R}{R_0} \right) \left( 1 - \frac{1}{2} \frac{T}{T_0} \right)^2 \] \]

Eqs. (A-1) and (A-2) have the following desirable properties: (a) for \( \frac{U}{U_0} \geq 1 \) or \( \frac{U}{U_0} \leq 1 \) the wake is domin-

ated by a law of the wake behavior which correctly
satisfies both the outer limit conditions \( U_0 \), \( U_0 = 1 \)
and \( U_0 \) for \( U_0 = 1 \) and (b) on the other hand, for
very small \( R \) values, \( U_0 \) assumes a law of the wall
behavior consisting of a logarithmic term that \n
is exponentially damped out extremely close to the
wall into a linear laminar profile \( \frac{U}{U_0} = R \) for \( R = 0 \); (c) Eqs. (A-1) may be differen-
tialized wrt \( y \) to yield an analytical expression
for \( \frac{U}{U_0} \) also, which proves advantageous in
solving the middle and inner deck interaction problem
these texts where \( \frac{U}{U_0} \) must be known and essential to
the boundary layer edge.

The use of the incompressible form of Eq. (A-1)
in the defining integral relations for \( \beta \) and \( \mu \)
yields the following relationship that links the
wake parameter to the resulting incompressible shape
factor \( \frac{H_1}{H_1} = \left( \frac{1}{H_1} \right) \left( \frac{1}{H_1} \right) \)

\[ \frac{H_1}{H_1} = \frac{1}{3} \left( \frac{T}{T_0} \right) \left( \frac{R}{R_0} \right) \left( 1 - \frac{1}{2} \frac{T}{T_0} \right)^2 \left( 1 - \frac{1}{2} \frac{T}{T_0} \right) \left( \frac{R}{R_0} \right) \left( 1 - \frac{1}{2} \frac{T}{T_0} \right)^2 \] \]

Eqs. (A-1) and (A-2) together with the defining
relation for \( R \) enable a rather general and convenient
parameterization of the profile and hence the
interaction that depends on it in terms of three
important physical quantities: the shock strength
(Sw), the displacement thickness Reynolds number
Re, the wall temperature ratio \( T_0/T \), and the
shape factor \( H_1 \). That reflects the prior upstream
history of the incoming boundary layer including
possible pressure gradient and surface mass trans-
fer effects. With these parameters prescribed, the
aforementioned three equations may be solved simu-
larly for the attendant skin friction \( C_f \), the
value of \( R \) and, if desired, the value appropriate
to these flow conditions.

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**Fig. 1 Interaction Problem Configuration**
Figure 1: Shape and Factor Effect on Interaction-Theoretical
- Wall Pressure
- Displacement Thickness
- Skin Friction

Figure 2: Density Effect due to Wall and Fluid.
Fig. 5
Schematic of Turbulent Boundary Layer Development Downstream of a Wall Jet

Fig. 6
Model for the Wall Jet Effect on the Turbulent Boundary Layer Velocity Profile

Fig. 7
Typical Turbulent Boundary Layer Profiles with Injection

\[ M_i = 1.20, \text{Re}_x = 3.5 \times 10^6 \]
\[ H_i = 1.10 \text{ (unblown)} \]

\[ \Delta U_{\text{max}} / U_e = 0.50 \]

\[ y/D_x = 0.05 \]
\[ y/D_x = 0.10 \]
Fig. 8: Blowing Effect on Integral Properties of the Boundary Layer

![Diagram showing displacement thickness and shape factor with various parameters.]

Fig. 9: Typical Blowing Effects on Interactive Property Distributions

![Diagram showing interaction between blowing effect and profile shape.]

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\[ M_1 = 1.20 \]
\[ Re_{\delta^*} = 3.5 \times 10^4 \]
\[ H_i_{H,I} = 1.40 \]

\[ \Delta U_{max}/U_e = 0.5 \]
\[ y_{max}/d_0 = 0.05 \]
\[ \Delta \rho_{\text{blow}} = 1.5 \times 10^{-6} \]
Sonic and Supersonic Regions within a Blown Boundary Layer (Schematic)

Parametric Study of Wall Jet-Effect on Interaction Pressure Distribution
Fig. 12: Blowing Effect on Downstream Influence Distance

Fig. 13: Blowing Effect on Downstream Influence Distance

Fig. 14: Reynolds and Mach Number Effects on Blown Downstream Influence Distance

Fig. 15: Reynolds and Mach Number Effects on Blown Downstream Influence Distance
Fig. 15 Shape Factor Effect on Blown Pocket in Times

Fig. 16 Parametric Study of Wall Jet Effect on Interactive Displacement Thickness Distribution

Fig. 17 Shape Factor Effect on Blown Downstream Influence

Fig. 19 Injection Effect on Viscous Wedge Angle
Fig. 22 Schematic of Global Viscous-Inviscid Interaction Problem on Supercritical Airfoils

Fig. 23 Blowing Effects on Downstream Skin Friction Level
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